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# A METHOD OF MODAL PROOF IN ARISTOTLE

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## 1. Introduction

If it is possible to walk from Athens to Sparta, and if walking from Athens to Sparta entails passing through the isthmus of Corinth, then it is possible to pass through the isthmus of Corinth. This is an illustration of a general principle of modal logic: if B follows from A, then the possibility of B follows from the possibility of A. We may call this the possibility principle. The first philosopher known to have formulated and employed the possibility principle is Aristotle. Along with the principle, Aristotle also developed a rule of inference, which we may call the possibility rule: given the premiss that A is possible, and given a derivation of B from the assumption that A is the case, it may be inferred that B is possible.

The aim of the present paper is to offer a detailed account of Aristotle's understanding and use of the possibility rule. This topic is worthy of study for at least two reasons. First, the possibility rule is of interest in its own right as a theoretical achievement on Aristotle's part. It is well known that Aristotle was a pioneering figure both in modal logic and in philosophical thought about modality more generally, and the possibility rule stands among his significant contributions in this field.

The second reason derives from the varied uses to which the possibility rule is put by Aristotle. He applies the rule not only in his modal logic, in *Prior Analytics* 1. 15, but also in physical and metaphysical contexts, in works such as the *Physics*, *De caelo*, *De*



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*generatione et corruptione*, and the *Metaphysics*. Many of these latter applications figure in the justification of claims that are central to Aristotle's philosophical teaching. For example, he uses the possibility rule in *De caelo* 1. 12 in order to establish that the cosmos is imperishable, and in *Physics* 8. 5 in order to establish the existence of a first, unmoved mover. A study of the possibility rule can help to clarify Aristotle's justification of these claims.

Many of Aristotle's applications of the possibility rule have received considerable attention from commentators. However, they have proved difficult to understand, partly because each individual application leaves the nature of the possibility rule indeterminate in various respects. There is often no agreement as to how exactly the rule is applied in a given passage; in some cases commentators even disagree as to whether the rule is applied at all. This lack of consensus can best be addressed through a comprehensive approach, gathering and comparing the use of the possibility rule throughout Aristotle's works. Such a synoptic treatment has not been attempted before, and so we undertake to provide one in the present paper. The undertaking requires some length of discussion, but we hope that this will be rewarded by a much better understanding both of the nature of Aristotle's possibility rule and of the single arguments in which the rule appears.

We begin, in Section 2, by discussing how Aristotle introduces the possibility principle and the possibility rule in *Prior Analytics* 1. 15. We offer a formal framework for representing the possibility rule, drawing on resources from modern natural deduction systems. Using this framework, we describe a pattern of proof in which the possibility rule is embedded within a *reductio ad absurdum*. We will then go on, in Sections 3–8, to discuss Aristotle's various applications of the possibility rule, and to show that they conform to that pattern of proof. We provide semi-formal reconstructions of the arguments in which the rule is applied, analysing these arguments within a unified framework.

In what follows we will treat all passages of which we are aware in which the possibility rule is applied by Aristotle, except for one. The exception is a passage from *Prior Analytics* 1. 15 containing proofs of several modal syllogisms ($34^a34$–$35^b2$). These proofs involve a number of extraneous technical details which it would be tedious to discuss here, and so we leave them aside for a separate paper.[1] The passages to be discussed in the present paper, then, are the following:

2. *Prior Analytics* 1. 15
   - The possibility principle ($34^a5$–24)
   - The possibility rule ($34^a25$–33)
3. *De caelo* 1. 12
   - Falsehood vs. impossibility ($281^b9$–15)
   - Whatever is eternal is imperishable ($281^b20$–5)
   - Nothing is one-directionally eternal ($283^a11$–17)
   - Nothing is one-directionally eternal, second argument ($283^b6$–17)
4. *Physics* 7. 1 and 8. 5
   - There is no infinite chain of moved movers (7. 1, $242^a49$–$243^a31$)
   - Not everything moved is moved by something else that is moved (8. 5, $256^b3$–13)
5. *De generatione et corruptione* 1. 2
   - No magnitude is divisible everywhere, first argument ($316^a14$–$^b16$)
   - No magnitude is divisible everywhere, second argument ($316^b19$–27)
6. *Metaphysics Θ* 4
   - It is not possible to measure the diagonal ($1047^b3$–14)
   - A proof of the possibility principle ($1047^b14$–26)
7. *Posterior Analytics* 1. 6
   - Premisses of demonstrations are true of necessity ($74^b32$–9)
8. Three borderline cases: *Metaphysics Λ* 6, *Physics* 6. 3, *De motu animalium* 4
   - The essence of the first mover is not a capacity (*Metaph. Λ* 6, $1071^b12$–20)
   - Nothing moves in an instant of time (*Phys.* 6. 3, $234^a24$–31)
   - The indestructibility of the cosmos (*MA* 4, $699^b23$–9)

In Section 2 we lay out a theoretical framework which will be presupposed in the subsequent sections. Sections 3–8 are not strictly presupposed by one another, and can be read selectively, according to the reader's interest.

[1] 'Proof by Assumption of the Possible in *Prior Analytics* 1. 15', manuscript under review.

## 2. Justifying the possibility rule: *Prior Analytics* 1. 15, $34^a5$–33

Aristotle's most detailed treatment of the possibility rule is found in *Prior Analytics* 1. 15. We consider how Aristotle explains the rule there, and how he justifies it by means of the possibility principle. We will also introduce formal tools which will help us to represent and analyse applications of the possibility rule throughout Aristotle's works.

### (*a*) *The possibility principle* ($34^a5$–24)

In order to justify the possibility rule, Aristotle begins by stating the following principle:

> First, it must be said that if it is necessary for B to be when A is, then it will also be necessary for B to be possible when A is possible. (*Pr. An.* 1. 15, $34^a5$–7)

The principle takes the form of a conditional. Its antecedent is: 'it is necessary for B to be when A is', i.e. that B is a necessary consequence of A. We will often express this by saying that B follows from A, and represent it by the formula '$A \Rightarrow B$'.[2] The consequent of the principle is that the possibility of B follows from the possibility of A. Thus, the principle is that if B follows from A, then the possibility of B follows from the possibility of A. It can be represented by the following schema:

Possibility principle: If $A \Rightarrow B$ then $\text{Poss}(A) \Rightarrow \text{Poss}(B)$

In this schema, it has been left deliberately open what kinds of item the letters 'A' and 'B' may stand for. Aristotle suggests that the possibility principle is applicable to a wide variety of items such as events, properties, and statements ($34^a12$–15). However, he seems especially interested in its application to statements; thus, he describes an instance of the possibility principle in which 'A' stands for the two premisses of a valid syllogism and 'B' for its conclusion:

> [i] If C is predicated of D and D of F, then necessarily C is also predicated

[2] Aristotle does not give an explanation of the relation of necessary consequence, but seems to treat it as a primitive notion; see J. Lear, *Aristotle and Logical Theory* (Cambridge, 1980), 2–14. We too will not attempt an analysis of it, but will simply use the symbol '⇒' to express whatever exactly Aristotle has in mind when he employs phrases such as 'it is necessary for . . . to be when . . . is'.

of F; [ii] and if each of the two premisses is possible, then the conclusion is also possible—[iii] as, if someone should put A as the premisses and B as the conclusion, [iv] it would result not only that when A is necessary then B is simultaneously also necessary, [v] but also that when A is possible B is possible. (*Pr. An.* 1. 15, 34ᵃ19–24)

To aid the discussion of this passage, we have divided it into five points. In point [i] Aristotle sketches a syllogism, in which, as he emphasizes, the conclusion is a necessary consequence of the two premisses. Skipping for the moment to point [iii], we find Aristotle there stipulating that 'A' should stand for the two premisses of the syllogism and 'B' for its conclusion. Thus B follows from A. In point [iv] Aristotle infers from this, as an aside, that if A is necessary then B is also necessary. Finally, in point [v] Aristotle states that if A is possible then B is also possible. This can be inferred from the fact that B follows from A together with the possibility principle. The statement in point [v] is presumably meant to be equivalent to the statement in point [ii], that if each of the two premisses is possible then the conclusion is also possible.

Now, when 'A' stands not for a single item but for the two premisses of a syllogism, the claim that A is possible may be interpreted in two different ways. It may be taken to mean that the premisses are jointly possible, or that they are separately possible. Two statements are separately possible if it is possible for the one to be true and it is possible for the other to be true. They are jointly possible if it is possible for both statements to be true together. Joint possibility implies separate possibility, but not vice versa. For example, the two statements 'some horses are sick' and 'no animals are sick' are separately possible, but they are evidently not jointly possible. As we will see, the distinction between joint and separate possibility is important for the correct understanding of the possibility principle.

In point [ii] the phrase 'if each of the two [ἑκάτερον] is possible' suggests separate possibility; for an explicit indication of joint possibility, we would expect a word such as 'both' (ἄμφω) or 'together' (ἅμα). However, if Aristotle means separate possibility then his claim is false. The separate possibility of the premisses of a valid syllogism does not entail the possibility of its conclusion. For example, the conclusion 'some horses are not animals' follows from the premisses 'some horses are sick' and 'no animals are sick'. The premisses are separately possible, yet the conclusion is not possible.

It has been suggested by Peter Geach that Aristotle simply made a mistake in point [ii].[3] On the other hand, one may argue that Aristotle's Greek does not completely rule out a joint-possibility reading, despite the pronoun 'each' (*ἑκάτερον*). If Aristotle means joint possibility, then his claim is true: the joint possibility of the premisses of a valid syllogism entails the possibility of the conclusion. Therefore several commentators have argued that this is what Aristotle means to say in points [ii] and [v].[4] In other writings, Aristotle shows awareness of the difference between separate and joint possibility, and of the fact that the former does not entail the latter.[5] Given this, it is preferable on grounds of charity not to attribute to him the mistake attributed to him by Geach, but to assume that Aristotle meant the correct joint-possibility reading.

Aristotle's possibility principle as applied to an arbitrary finite number of premisses can then be formulated as follows: if B follows from one or more premisses, then the possibility of B follows from the joint possibility of those premisses. More formally, this may be represented as follows, where the letters '$A_1$', …, '$A_n$', and 'B' each stand for a single statement:

$$\text{If } A_1, \ldots, A_n \Rightarrow B \text{ then } \mathrm{Poss}(A_1, \ldots, A_n) \Rightarrow \mathrm{Poss}(B)$$

As this schema indicates, we are assuming that the abbreviation 'Poss' can be applied to any finite number of statements $A_1, \ldots, A_n$.[6]

[3] P. Geach, 'Aristotle on Conjunctive Propositions', *Ratio*, 5 (1963), 33–45 at 44–5; similarly R. Patterson, *Aristotle's Modal Logic: Essence and Entailment in the* Organon (Cambridge, 1995), 158, 161, and 171.

[4] P. Thom, *The Logic of Essentialism: An Interpretation of Aristotle's Modal Syllogistic* (Dordrecht, 1996), 27; U. Nortmann, *Modale Syllogismen, mögliche Welten, Essentialismus: Eine Analyse der aristotelischen Modallogik* [*Modale Syllogismen*] (Berlin, 1996), 202–3; T. Ebert and U. Nortmann (trans. and comm.), *Aristoteles: Analytica Priora*, *Buch I* (Berlin, 2007), 549; G. Striker (trans. and comm.), *Aristotle:* Prior Analytics, *Book I* [*Prior Analytics*] (Oxford, 2009), 144–5.

[5] For example, at *De caelo* 1. 12, 281$^b$15–18, Aristotle states that a man is separately capable of sitting and of standing but not jointly capable of both; see also *SE* 4, 166$^a$23–30.

[6] Consequently, it is difficult to treat 'Poss' either as a modal sentential operator or as a modal predicate as used in modern logic; for operators and predicates typically have a fixed number of arguments. In modern logic, the joint possibility of a number of statements can be expressed as the possibility of their conjunction. However, the notion of conjunction as a sentential operator does not seem to be available in Aristotle's logic. Nor is it clear whether Aristotle would envisage alternative ways of unifying a number of statements into a single item (for example, into a set of statements). For our purposes it is not necessary to give a precise account of the logical syntax of 'Poss' and its arguments.

When applied to a single statement it means that the statement is possible, and when applied to more than one statement it means that the statements are jointly possible.

(*b*) *The possibility rule* ($34^a25$–33)

After his discussion of the possibility principle in points [i]–[v], Aristotle proceeds to use the principle in order to establish the possibility rule, as follows:

> [vi] Now that this has been shown, [vii] it is clear that if something false but not impossible is hypothesized, what follows because of the hypothesis will be false but not impossible. [viii] For example, if A is false but not impossible, and if when A is B is, then B will also be false but not impossible. (*Pr. An.* 1. 15, $34^a25$–9)

In point [vi] Aristotle refers back to his discussion of the possibility principle. In [vii] and [viii] he derives a consequence from this principle. The consequence concerns the hypothesis of something 'false but not impossible'. Aristotle's phrasing in [vii] and [viii] seems to imply that whatever follows from such a hypothesis is itself false but not impossible. However, it is unlikely that Aristotle really means to assert this; for as he explains elsewhere, something *true* can follow from something false (*Pr. An.* 2. 2–4), and given this, he must realize that something true can follow from something false but not impossible. For example, from the premisses 'every man is walking' and 'no horse is walking' there follows the conclusion 'no horse is a man'; the premisses are false but not impossible, and the conclusion is true. Therefore, when Aristotle says 'false but not impossible' it is best to understand him to mean 'at worst false, perhaps true, but not impossible'.[7] This is equivalent to 'possible'.[8] Thus, Aristotle can be understood in point [viii] to state that if A is possible and B follows from A, then B is possible. But this seems to be a mere restatement of the possibility principle. What, then, is Aristotle adding in points [vii]–[viii] that is different from what he already said in his discussion of the principle?

[7] Authors who hold this view include Alex. Aphr. *In Pr. An.* 185. 17–20 Wallies; Nortmann, *Modale Syllogismen*, 196–7; I. Mueller (trans. and comm.), *Alexander of Aphrodisias:* On Aristotle's *Prior Analytics* 1. 14–22 (Ithaca, NY, 1999), 39. A similar view is endorsed by Striker, *Prior Analytics*, 145.

[8] Throughout this paper, we take 'possible' to be equivalent to 'not impossible', thus understanding it in what is known as the one-sided sense (as opposed to the two-sided sense, where 'possible' is equivalent to 'neither impossible nor necessary').

Gisela Striker suggests that the difference is this: instead of being concerned with the notion of possibility as employed in the possibility principle, Aristotle's claim in points [vii]–[viii] concerns the notion of compatibility with some given premisses. She takes this latter claim to be:

If a proposition A is compatible with given premisses $S_1 \ldots S_n$, then any proposition logically implied by A is also compatible with $S_1 \ldots S_n$.[9]

Thus Striker holds that Aristotle's use of the word 'possible' (*δυνατόν*) shifts from indicating possibility in points [i]–[v] to indicating compatibility in points [vii]–[viii]. However, Aristotle gives no indication of such a shift in usage, and Striker does not justify her interpretation persuasively. It is preferable to take 'possible' in points [vii]–[viii] to have the same meaning as in points [i]–[v], indicating possibility rather than compatibility.

A better clue to the difference between the possibility principle and what Aristotle says in points [vii]–[viii] is his use of the verb 'hypothesize' (*ὑποτίθεσθαι*) in [vii]. This verb did not occur in Aristotle's formulation of the possibility principle. It is typically used in the description of proof procedures. For example, in proofs by *reductio*, 'hypothesize' is used to describe the step in which we assume the contradictory of the statement we want to establish.[10] Based on what we derive from this assumption, we may be able to conclude the *reductio* proof. The appearance of the term 'hypothesize' in point [vii] suggests that in our present passage, too, Aristotle wishes to describe a procedure of proof in which an assumption is made and certain consequences are derived from this assumption.

If this is correct, then in point [vii] Aristotle is not simply repeating the possibility principle, but is offering a procedure of proof based upon that principle. The procedure in question can be taken to be the following: having stated that a given statement is possible, and having derived some consequence from the assumption that this statement is true, we infer that the consequence too is possible. This procedure can be cast into the following rule:

> Possibility rule: Given the premiss that A is possible, and given a deduction of B from A, you may infer that B is possible.

[9] Striker, *Prior Analytics*, 141; see also 145.
[10] *Pr. An.* 1. 16, $36^a23$; 1. 29, $45^b2$; 2. 11–13 *passim*.

In order to supply the required deduction of B from A, it is not necessary first to establish that A is actually the case. It suffices to introduce A as an assumption, or hypothesis, which serves as the starting-point of the deduction, and on the basis of which we derive the consequence B. We will call such a deduction of B from A a subordinate deduction, and will refer to B as the conclusion of the subordinate deduction.

In order to present arguments that employ the possibility rule, we will write them as sequences of numbered statements, where statements that belong to a subordinate deduction are indented and prefixed by a vertical line. Proof [**P1**] gives a simple example, establishing the possibility of BeA ('B belongs to no A') given the possibility of AeB ('A belongs to no B'):

1. Poss(AeB) [premiss] [**P1**]
2. | AeB [assumption]
3. | BeA [from 2, conversion]
4. Poss(BeA) [possibility rule: 1, 2–3]

The proof begins by stating the premiss that it is possible that A belongs to no B. In the second line, corresponding to Aristotle's step of 'hypothesizing', we assume that A does in fact belong to no B. This marks the beginning of the subordinate deduction. Our subordinate deduction is very short: we simply apply one of Aristotle's conversion rules to infer that B belongs to no A.[11] This statement, found in the third line, is the conclusion of the subordinate deduction. From the existence of this subordinate deduction, and the premiss in the first line, the possibility rule allows us to infer that it is possible that B belongs to no A.

The above way of presenting subordinate deductions is borrowed from certain modern natural deduction systems. In particular, we have found it useful to employ a Fitch-style notation, such as is used by Kit Fine in his discussion of Aristotle's use of the possibility rule in *Metaphysics Θ* 4.[12]

[11] Aristotle sometimes states that at least two premisses are required for a deduction (*Pr. An.* 1. 15, $34^{\mathrm{a}}17$–18; 1. 23, $40^{\mathrm{b}}35$–7; 2. 2, $53^{\mathrm{b}}17$–20; *Post. An.* 1. 3, $73^{\mathrm{a}}7$–11). For present purposes, however, we may ignore this problematic restriction.

[12] K. Fine, 'Aristotle's Megarian Maneuvers' ['Megarian Maneuvers'] (forthcoming), 1–22 at 17–20 (page references are to the preprint available from http://philosophy.fas.nyu.edu/object/kitfine). Our possibility rule is essentially the same as what Fine calls the rule of $\Diamond$-Introduction. Although Fine's paper is not primarily concerned with this rule, he introduces it as a tool to interpret Aristotle's proof

(*c*) *The problem of iteration*

Aristotle thinks that the possibility principle justifies the possibility rule, as is clear from point [vi] above. He also purports to justify the rule by means of the principle in the lines immediately after point [viii], at $34^a29$–33. Although his justification in these lines is brief and not very informative, it is at least in some cases easy to see how the principle can justify an application of the rule.

For example, let us consider the simple argument given in [**P1**] above. The subordinate deduction in lines 2–3 establishes that BeA follows from AeB, i.e. it establishes AeB ⇒ BeA. This is the antecedent of an instance of the possibility principle, and so we can infer Poss(AeB) ⇒ Poss(BeA) by *modus ponens*. Now, line 1 of the argument states Poss(AeB). So again by *modus ponens*, we can infer Poss(BeA). This is line 4, the conclusion of the argument. Thus the possibility principle justifies the application of the possibility rule in this argument.

However, this kind of justification will not suffice for most of Aristotle's actual applications of the possibility rule. The difficulty lies in the means by which Aristotle reaches the conclusion of the subordinate deduction. As we will see, in deriving this conclusion he often relies not only on the assumption with which the subordinate deduction begins, but also on other statements. In particular, he often relies on statements that occurred before the subordinate deduction began. This is problematic. In order to facilitate discussion of Aristotle's procedure in such cases, it is helpful to analyse it into two steps: the statement is first copied into the subordinate deduction, and only then used to draw an inference within it. The step of copying a statement from outside a subordinate deduction into it will be referred to as 'iteration', borrowing terminology commonly used in modern natural deduction systems.[13]

In order both to illustrate the move of iteration and to obtain a clearer view of the problems associated with it, we have devised the

of the possibility principle in *Metaphysics Θ* 4, which we examine at pp. 234–42 below. The present paper has benefited from Fine's discussion of the rule, and shows that the framework introduced by him is applicable to a wide range of arguments in Aristotle.

[13] However, it must be noted that modern natural deduction systems do not allow iteration into the subordinate deduction in the context of the possibility rule. Instead, they may allow for an analogous step such as Fine's □-Elimination; see n. 17 below.

argument given in proof [**P2**], in which the premiss in line 1 is iterated into the subordinate deduction:[14]

| | | | |
|---|---|---|---|
| 1. | AiB | [premiss] | [**P2**] |
| 2. | Poss(AeC) | [premiss] | |
| 3. | │ AeC | [assumption] | |
| 4. | │ AiB | [iterated from 1] | |
| 5. | │ CoB | [from 3, 4, Festino] | |
| 6. | Poss(CoB) | [possibility rule: 2, 3–5] | |

This argument purports to establish that it is possible that C does not belong to some B, given the premisses that A belongs to some B and that it is possible that A belongs to no C. Is the argument valid, and in particular, is the step of iteration in it permissible? The answer is: no. This is clear if we consider an instance of the argument in which the term B is identified with the term C:

| | | | |
|---|---|---|---|
| 1. | AiB | [premiss] | [**P3**] |
| 2. | Poss(AeB) | [premiss] | |
| 3. | │ AeB | [assumption] | |
| 4. | │ AiB | [iterated from 1] | |
| 5. | │ BoB | [from 3, 4, Festino] | |
| 6. | Poss(BoB) | [possibility rule: 2, 3–5] | |

We now have the conclusion that it is possible that B does not belong to some B, given the premisses that A belongs to some B and that it is possible that A belongs to no B.[15] But there are terms A and B such that AiB is true and AeB is possible, without BoB being possible.[16] For example, some horses are sick and it is possible that no horse is sick, but it is not possible that some horses are not horses. Hence the arguments in [**P2**] and [**P3**] are invalid. This shows that it is not generally permissible to iterate statements into subordinate deductions in the context of the possibility rule.

Correspondingly, the possibility principle cannot be used to justify the application of the possibility rule in the above two argu-

[14] As usual, 'AiB' means 'A belongs to some B', and 'CoB' means 'C does not belong to some B'.

[15] For the inference in the subordinate deduction see *Pr. An.* 2. 15, 63$^b$40–64$^a$15, where Aristotle explains that BoB can be inferred from the contradictory pair AeB and AiB by means of the syllogism Festino.

[16] By Aristotle's lights it seems that it is never possible that BoB, for any term B; see *Pr. An.* 2. 15, 64$^b$7–13 (in conjunction with 64$^a$4–7, 23–30). See also J. Łukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, 2nd edn. (Oxford, 1957), 9; P. Thom, *The Syllogism* (Munich, 1981), 92.

ments. In [**P3**], for example, the subordinate deduction does not establish that BoB follows from AeB alone. Rather, it establishes that BoB follows from the pair of statements AeB and AiB, i.e. it establishes (AeB, AiB) ⇒ BoB. Given this, the possibility principle allows us to infer Poss(AeB, AiB) ⇒ Poss(BoB), i.e. that the possibility of BoB follows from the joint possibility of AeB and AiB. Now, line 2 of the argument states that AeB is possible; line 1 states that AiB is actually the case, from which it presumably follows that AiB is possible. Consequently, AeB and AiB are each separately possible. Still, nothing in the argument implies that they are *jointly* possible; in fact, being contradictories, they clearly are not. As a result, we have no grounds to infer by means of the possibility principle that BoB is possible.

Thus a rule allowing for unrestricted iteration in the context of the possibility rule would yield arguments which cannot be justified by means of the possibility principle, and which are invalid. A valid rule of iteration would have to be restricted so as to incorporate some sort of guarantee of joint possibility. Specifically, there must be a guarantee that the assumption which initiates the subordinate deduction, together with any statements iterated into it, are all, taken together, jointly possible. The possibility rule already requires a statement to the effect that the assumption initiating the subordinate deduction is possible; but this does not suffice to guarantee that the assumption is jointly possible with any statements one might wish to iterate into it. On the other hand, such a guarantee can be effected in one special case: namely, when it is known that the statements to be iterated are true of necessity. For if something is necessary, then anything possible is jointly possible with it. Accordingly, iteration into subordinate deductions is permissible here if there is a guarantee that the statements iterated are true of necessity.

Now, if there is a guarantee that some statement A is true of necessity, this would license the introduction of the statement 'A is true of necessity' as a separate line in the proof. Given this, one may propose the following restriction on iteration in the context of the possibility rule: a statement A may be iterated into a subordinate deduction if, and only if, the statement 'A is true of necessity' is present earlier in the proof.[17] Such a formulation would be desir-

[17] This leads in effect to the rule which Kit Fine calls □-Elimination (Fine, 'Megarian Maneuvers', 17): given that □A is present outside the subordinate deduction, A may be introduced into it.

able from the standpoint of formal rigour. Unfortunately, however, it does not match Aristotle's overall practice in arguments employing the possibility rule. As we will see, Aristotle never asserts the necessity of statements which he iterates into the subordinate deduction. What is more, he sometimes iterates statements for which there is no guarantee that they are true of necessity, and which in fact have the status of contingent truths. In these cases, Aristotle's arguments appear to be invalid. But regardless of their invalidity, they show that when he iterates a statement A, he does not think of himself as performing a step which requires the presence of a statement of A's necessity. It is unclear what general restrictions, if any, Aristotle imposed on iterations in the context of the possibility rule, and although an answer to this question would be most valuable, we will have to leave it open in this paper. Thus, when reconstructing Aristotle's arguments, we will not impose any formal restrictions on iteration; instead we will discuss in each case separately whether or not a given iteration can be justified.

### (*d*) Reductio *arguments*

It is a curious fact that Aristotle uses the possibility rule only within the context of arguments by *reductio*. Aristotle never gives a systematic description of *reductio* arguments, but the general strategy is to assume the contradictory of the intended conclusion, and to show that an unacceptable consequence follows from this. Such arguments can be understood as applications of the following rule:

> *Reductio* rule: Given the premiss A, and given a subordinate deduction from the contradictory of B to the contradictory of A, you may infer B.[18]

A simple example of an application of this rule can be represented as follows:

1. AiB [premiss] **[P4]**
2. | BeA [assumption]
3. | AeB [from 2, conversion]
4. BiA [*reductio*: 1, 2–3]

This argument establishes that B belongs to some A given the

[18] By the contradictory of a statement we mean its negation. Aristotle sometimes employs a more generous version of the *reductio* rule, allowing for a subordinate deduction from the contradictory of B to a *contrary* of A.

premiss that A belongs to some B (*Pr. An.* 1. 2, $25^{\mathrm{a}}20$–2). It begins with the premiss that A belongs to some B. In the second line we assume that B belongs to no A, which is the contradictory of the intended conclusion. This assumption marks the beginning of the subordinate deduction. In the subordinate deduction we apply one of Aristotle's conversion rules to infer that A belongs to no B. This statement, the conclusion of the subordinate deduction, is the contradictory of the premiss in line 1. Thus, the *reductio* rule allows us to conclude in line 4 that B belongs to some A.

Most of Aristotle's applications of the *reductio* rule are more complex than this, in that the subordinate deduction makes use not only of the assumption for *reductio*, but also of other statements which occurred earlier in the argument. In other words, Aristotle iterates statements from outside the subordinate deduction into it. For example, his proof of the syllogism Bocardo can be reconstructed as follows (*Pr. An.* 1. 6, $28^{\mathrm{b}}17$–20):

1. PoS [major premiss] [**P5**]
2. RaS [minor premiss]
3. │ PaR [assumption]
4. │ RaS [iterated from 2]
5. │ PaS [from 3, 4, Barbara]
6. PoR [*reductio*: 1, 3–5]

In this reconstruction the conclusion of the subordinate deduction, PaS, is the contradictory of the premiss PoS in line 1. On the basis of this, the *reductio* rule allows us to conclude PoR in line 6, which is the contradictory of the assumption for *reductio* in line 3.[19]

As we saw earlier, iteration into a subordinate deduction leads to problems when the subordinate deduction is used in an application of the possibility rule. However, if it is being used only for an application of the *reductio* rule, these problems do not arise. As is well known, *reductio* arguments allow for free, unrestricted iteration. Because of this difference in the permissibility of iteration, some modern natural deduction systems make a distinction between two types of subordinate deduction: one which figures in *reductio* and other non-modal rules, and another which figures in modal rules such as the possibility rule. Kit Fine distinguishes them by distinguishing two kinds of assumption with which a subordinate deduc-

[19] Thus the premiss PoS in line 1 corresponds to A in the above formulation of the *reductio* rule, and the statement PoR in line 6 corresponds to B.

tion may begin.[20] One kind of assumption, which he calls straight supposition, is used in non-modal rules and allows for free iteration. The other kind of assumption, which he calls modal supposition, is used in modal rules and does not allow for iteration (although a related move called $\Box$-Elimination is allowed: see nn. 13 and 17 above).

In principle, this is a good way to proceed. On the other hand, Aristotle himself does not seem to distinguish between different kinds of subordinate deductions or assumptions. At least, he does not make a terminological distinction between the assumptions that initiate subordinate deductions for the *reductio* rule and those that initiate subordinate deductions for the possibility rule. He uses the same three verbs to indicate both kinds of assumption, namely 'posit' (*τιθέναι*), 'take' (*λαμβάνειν*), and 'hypothesize' (*ὑποτίθεσθαι*, also rendered 'assume').[21] He also uses the noun 'hypothesis' (*ὑπόθεσις*) for assumptions in both contexts.[22]

Following Aristotle, we will not formally distinguish different kinds of subordinate deduction. However, we will indicate whether a given subordinate deduction is going to be used for an application of the *reductio* rule or rather of the possibility rule.[23] Thus the assumption at the beginning of a subordinate deduction will be labelled either 'assumption for *reductio*' or 'assumption for possibility rule'. Subordinate deductions that are exploited in an application of the *reductio* rule will be referred to as '*reductio* subordinate deductions', and those that are exploited in an application of the possibility rule as 'modal subordinate deductions'.

[20] Fine, 'Megarian Maneuvers', 17

[21] Assumption in *reductio* contexts: *τιθέναι*, *Pr. An.* 1. 15, 34b29; 1. 23, 41a26–7; 2. 11, 61a20; 2. 14, 62b30; *λαμβάνειν*, *Pr. An.* 2. 14, 63b16; *Post. An.* 1. 26, 87a7; *ὑποτίθεσθαι*, *Pr. An.* 1. 16, 36a23; 1. 29, 45b2; 2. 11–13 *passim*. Assumption in the context of the possibility rule: *τιθέναι*, *Pr. An.* 1. 13, 32a19; 1. 15, 34b1, 34b3, 34b26; *Metaph.* *Θ* 4, 1047b19; *Phys.* 7. 1, 243a31; 8. 5, 256b11; *De caelo* 1. 12, 281b23, 283a17, 283b15–16; *λαμβάνειν*, *Phys.* 7. 1, 242b49, b67; *ὑποτίθεσθαι*, *Pr. An.* 1. 15, 34a25–6; *Metaph.* *Θ* 4, 1047b10; *De caelo* 1. 12, 281b14.

[22] Assumption in *reductio* contexts: *Pr. An.* 1. 15, 34b29; 2. 11, 61b32, 62a4; 2. 13, 62b12, b20; 2. 14 *passim*; 2. 17 *passim*. Assumption in the context of the possibility rule: *Pr. An.* 1. 15, 34a26; *Phys.* 7. 1, 243a30; on the last of these passages, see p. 210 below.

[23] We will not encounter any cases in which a single subordinate deduction is used both for an application of the *reductio* rule and for an application of the possibility rule.

(*e*) *Combining the* reductio *rule and the possibility rule*

We now have in place all the tools we need to understand Aristotle's applications of the possibility rule. As mentioned above, these applications always occur within the context of a proof by *reductio*. To illustrate how the *reductio* rule can be combined with the possibility rule, let us begin with a simple example that does not employ iteration. The following argument establishes the impossibility of BeA, given the impossibility of AeB:

| | | | |
|---|---|---|---|
| 1. | Not Poss(AeB) | [premiss] | [**P6**] |
| 2. | │ Poss(BeA) | [assumption for *reductio*] | |
| 3. | │ │ BeA | [assumption for possibility rule] | |
| 4. | │ │ AeB | [from 3, conversion] | |
| 5. | │ Poss(AeB) | [possibility rule: 2, 3–4] | |
| 6. | Not Poss(BeA) | [*reductio*: 1, 2–5] | |

Aristotle's own arguments are more complicated than this example. As we will see later, the main complication derives from his use of iteration. In addition, there are two small points to take note of. First, the order of certain elements in the argument can vary; in particular, the premiss in line 1 of [**P6**] could also have occurred only after both subordinate deductions had been completed. Second, the assumption for *reductio* need not be identical with the statement of possibility that serves as a premiss for the possibility rule (line 2 of [**P6**]); the statement of possibility may also be a consequence derived from the assumption for *reductio*, or it may be introduced on independent grounds. Thus, a more general pattern combining the two rules can be outlined as follows:

| | | |
|---|---|---|
| │ Not C | [assumption for *reductio*] | [**P7**] |
| │ . . . | | |
| │ Poss(A) | [. . .] | |
| │ │ A | [assumption for possibility rule] | |
| │ │ . . . | | |
| │ │ B | [deduced from A and, perhaps, iterated statements] | |
| │ Poss(B) | [by possibility rule] | |
| . . . | | |
| Not Poss(B) | [. . .] | |
| C | [by *reductio*] | |

The modal subordinate deduction in this pattern is the inner deduction extending from A to B; the *reductio* subordinate deduction is the outer deduction extending from 'Not C' to Poss(B). The first line of the modal subordinate deduction contains the assumption for the possibility rule, and our pattern requires a statement within the *reductio* subordinate deduction to the effect that this assumption is possible (Poss(A)). The pattern also requires a statement outside the *reductio* subordinate deduction to the effect that the conclusion of the modal subordinate deduction is impossible ('Not Poss(B)'). Several details are deliberately left unspecified by the pattern. In particular, it is not specified how the assumption for *reductio* 'Not C' is used in deriving the conclusion of the *reductio* subordinate deduction, Poss(B).[24]

The pattern given in [**P7**] is general enough to capture the variety of uses to which the possibility rule is put in Aristotle's works: all of his applications of the rule can be reconstructed as instances of the pattern. The purpose of Sections 3–8 below is to establish this claim, for at least the great majority of cases (the few remaining cases are dealt with elsewhere: see n. 1 above).

## 3. The eternity of the cosmos: *De caelo* 1. 12

*De caelo* 1. 12 contains a series of arguments concerning the eternity and necessary existence of the cosmos. One of Aristotle's main claims in this chapter is that the cosmos is not only eternal but also imperishable, i.e. that it is impossible for it to perish. Among Aristotle's targets is the view put forward in Plato's *Timaeus* that the cosmos was created and is perishable, but that it will never cease to exist, being maintained by god.

Three of Aristotle's arguments in *De caelo* 1. 12 employ the possibility rule ($281^{b}20$–5, $283^{a}11$–17, $283^{b}6$–17). The first has been the subject of considerable attention and controversy among scholars, whereas the other two have been less discussed. Aristotle's reasoning in all three arguments is complex and in some ways problematic, especially in his use of iteration into the modal subordinate deduction. At the same time, he takes special care in *De caelo* 1. 12 to explain the working of the possibility rule, offering among other

[24] As we will see, in Aristotle's individual applications of the possibility rule the assumption for *reductio* either plays a role in arriving at the statement Poss(A), or else by way of iteration it plays a role inside the modal subordinate deduction; sometimes it does both.

things a useful preparatory discussion of falsehood and impossibility at the beginning of the chapter. We will first briefly consider this preparatory discussion, and then discuss the three arguments in order.

### (*a*) *Falsehood vs. impossibility* ($281^b9$–15)

As a preliminary to his arguments, Aristotle emphasizes the difference between being merely false and being impossible:

> When you are not standing, to say that you are standing is false, but not impossible. Likewise, if someone is playing the cithara but not singing, to say that he is singing is false, but not impossible. But to stand and sit simultaneously, and for the diagonal to be commensurate, is not only false but also impossible. It is not the same to hypothesize something false and something impossible. It is from something impossible that something impossible follows [*συμβαίνει δ' ἀδύνατον ἐξ ἀδυνάτου*]. (*De caelo* 1. 12, $281^b9$–15)

Given that you are standing, the statement 'you are sitting' is false but not impossible (under ordinary circumstances). By contrast, the statement 'you are simultaneously standing and sitting' is not only false but impossible. And so is the statement 'the diagonal of a square is commensurate with its side'. The distinction between falsehood and impossibility is relevant to the upcoming applications of the possibility rule because, as we saw at the end of the previous section, the general pattern of argument requires a statement to the effect that the conclusion of the modal subordinate deduction is (not only false but) impossible.

The last two sentences of the passage, in which 'hypothesizing' and 'following' are connected with the notion of impossibility, express Aristotle's commitment to the possibility principle and the possibility rule. The second of these sentences raises some questions of translation and interpretation. Its simplest translation would be 'something impossible follows from something impossible'. Where B follows from A, this is ambiguous between saying that if A is impossible then B is impossible, and saying that if B is impossible then A is impossible. While the first of these two claims is false, the second one is true and moreover is equivalent to Aristotle's possibility principle. Therefore we have chosen a translation that clearly favours the second reading.[25]

[25] In contrast, Barnes's revised Oxford translation seems to favour the first read-

(*b*) *Whatever is eternal is imperishable* ($281^{b}20$–5)

The above preparatory remarks lead into a well-known argument in which Aristotle applies the possibility rule in order to prove that whatever is eternal is imperishable: in other words, if something always exists then it is impossible for it to cease to exist. Aristotle's proof appears as follows:

> [i] Consequently, if something that exists for an infinite time is perishable, it would have a capacity for not existing. [ii] If, then, it exists for an infinite time, let that of which it is capable obtain. [iii] Then it will actually exist and not exist simultaneously. [iv] Now something false would follow because something false was posited. But if the hypothesis were not impossible, then what follows would not be impossible as well. [v] Therefore, everything that always exists is imperishable without qualification. (*De caelo* I. 12, $281^{b}20$–5)

The proof proceeds by *reductio*. The claim which is to be refuted is that something which is eternal is perishable. So Aristotle begins, in point [i], by assuming for *reductio* that some item X is both eternal and perishable. The perishability of X implies that it is possible that X does not exist for some time.[26] In [ii] we are asked to assume that X does not exist for some time, as indicated by the phrase 'let it obtain' (*ἔστω ὑπάρχον*). This is an assumption for the possibility rule, marking the beginning of a modal subordinate deduction.

In [iii] Aristotle immediately states the conclusion of the modal subordinate deduction, namely that X exists and does not exist simultaneously for some time. He does not explain how the conclusion is reached, but evidently what he is doing is to employ within the modal subordinate deduction the earlier statement that X exists for ever, which was part of the assumption for *reductio*. This statement combined with the assumption that X does not exist for some time leads to the conclusion in question. Thus, speaking in terms of the framework introduced above, the statement that X exists for ever is iterated into the modal subordinate deduction. This step is problematic, and we will return to it soon.

ing: 'from the impossible hypothesis impossible results follow' (J. Barnes (ed.), *The Complete Works of Aristotle: The Revised Oxford Translation* [*Complete Works*], 2 vols. (Princeton, 1984), i. 466). Leggatt's translation, like ours, favours the second reading: S. Leggatt (trans. and comm.), *Aristotle:* On the Heavens *I and II* [*Heavens*] (Warminster, 1995), 101.

[26] Cf. *De caelo* I. 11, $280^{b}20$–4, and I. 12, $281^{b}27$–9.

At this stage of the argument, the possibility rule allows us to infer that the conclusion of the modal subordinate deduction is possible, i.e. that it is possible that X exists and does not exist simultaneously for some time. Aristotle does not explicitly state this inference, but for the purposes of our interpretation we take it to be part of the logical structure of his argument.

Aristotle thinks that the conclusion of the modal subordinate deduction is impossible, as is clear from point [iv]. Thus he takes it as a premiss that it is impossible that X exists and does not exist simultaneously for some time. This premiss is guaranteed by the principle of non-contradiction (PNC). By *reductio*, it then follows that X is not both eternal and perishable. Since X was arbitrary, Aristotle can infer in [v] that nothing is both eternal and perishable. The argument as a whole can thus be reconstructed as follows:

| | | | |
|---|---|---|---|
| 1. | X is eternal and perishable | [assumption for *reductio*] | [**P8**] |
| 2. | X exists for all time | [from 1] | |
| 3. | It is possible that X does not exist for some time | [from 1] | |
| 4. | X does not exist for some time | [assumption for possibility rule] | |
| 5. | X exists for all time | [iterated from 2] | |
| 6. | X exists and does not exist simultaneously for some time | [from 4, 5] | |
| 7. | It is possible that X exists and does not exist simultaneously for some time | [possibility rule: 3, 4–6] | |
| 8. | It is not possible that X exists and does not exist simultaneously for some time | [premiss: PNC] | |
| 9. | It is not the case that X is eternal and perishable | [*reductio*: 1–7, 8] | |
| 10. | Nothing is both eternal and perishable | [generalization: 9] | |

The main problem with this proof is the iteration of 'X exists for all time' from line 2 into the modal subordinate deduction in line 5. As explained above, the iteration of statements into a modal subordinate deduction is not in general permissible. Nevertheless, it seems unavoidable to attribute to Aristotle this step of iteration, given that in point [iii] of his proof he draws an inference which relies both on the claim that X exists for all time and on the assumption for the possibility rule that X does not exist for some time. By attributing a step of iteration to Aristotle, we do not mean that he understood himself as making precisely this step. Rather, our claim is that he has performed a move which, in the framework introduced above, corresponds to a step of iteration.

Perhaps Aristotle does not recognize that there are restrictions on (the move which in our framework corresponds to) iteration into the modal subordinate deduction. Or perhaps he has reasons which he takes to justify the iteration in this case. As we saw above, one such reason would consist in a guarantee that it is necessary that X exists for all time. But where could such a guarantee come from? One might assume that Aristotle relies on a principle to the effect that whatever is eternal is necessarily eternal; but this is in essence what the whole argument aims to establish, so that there would be a *petitio principii*. This, then, is not a promising approach to justify the iteration in question. Nor is there any other obvious justification available. In the absence of such a justification, the natural conclusion to draw is that Aristotle's reasoning in the argument under consideration is not valid.

That Aristotle's reasoning in this argument is highly problematic is generally agreed by commentators. Some diagnose the problem in a similar way to our analysis, although they describe it in different terms.[27] Others understand the source of the problems with Aristotle's argument in a very different way.[28] Some commentators think that the problems can be solved, so that Aristotle has a valid argument after all.[29] It is beyond the scope of this paper to dis-

[27] For example, Judson speaks of an illicit 'insulated realization manœuvre', in which 'a candidate for possibility [is supposed to be realized] without regard to whether the supposition of its holding requires changes in what else can be taken to be true' (L. Judson, 'Eternity and Necessity in *De caelo* I. 12', *Oxford Studies in Ancient Philosophy*, 1 (1983), 217–55 at 230). For a similar diagnosis, see R. Sorabji, *Necessity, Cause and Blame: Perspectives on Aristotle's Theory* [*Necessity, Cause and Blame*] (London, 1980), 129–30; M. Mignucci, 'Aristotle's *De caelo* I 12 and his Notion of Possibility', in D. Devereux and P. Pellegrin (eds.), *Biologie, logique et métaphysique chez Aristote* (Paris, 1990), 321–34 at 328–30; Leggatt, *Heavens*, 216–17.

[28] C. J. F. Williams, 'Aristotle and Corruptibility: A Discussion of Aristotle, *De caelo* I, xii', *Religious Studies*, 1 (1965–6), 95–107 and 203–15 at 98–102; J. Hintikka, *Time & Necessity: Studies in Aristotle's Theory of Modality* [*Time & Necessity*] (Oxford, 1973), 212–13.

[29] S. Waterlow, *Passage and Possibility: A Study of Aristotle's Modal Concepts* (Oxford, 1982), 75; J. van Rijen, *Aspects of Aristotle's Logic of Modalities* [*Logic of Modalities*] (Dordrecht, 1989), 93–4; N. Denyer, 'Aristotle and Modality: Never Will and Cannot', *Proceedings of the Aristotelian Society*, suppl. 74 (2000), 163–78 at 163–4 and 177–8; S. Broadie, 'The Possibilities of Being and Not-Being in *De caelo* 1.11–

cuss the enormous variety of interpretative approaches that have been taken to the argument advanced by Aristotle in the passage under consideration. Our primary aim here has been to show how his argument can be represented within the formal framework developed above. The adequacy of this framework will receive further confirmation from the fact that it is applicable to all other arguments in which Aristotle applies the possibility rule. According to this framework, the problem with Aristotle's argument lies in the step of iteration into the modal subordinate deduction. If the iteration is not justified, the argument is invalid.

### (*c*) *Nothing is one-directionally eternal* (283ᵃ11–17)

There are two further arguments in *De caelo* 1. 12 in which Aristotle applies the possibility rule. These two arguments aim to establish a claim which is different from, but related to, the conclusion of the argument just discussed. They both aim to establish that nothing is one-directionally eternal, i.e. that nothing is generated and thenceforth always exists, or always exists up to some time and then perishes. These two arguments are presented by Aristotle in compressed form, but they can be reconstructed in some detail by looking to the earlier argument.

The first of the two arguments builds upon some preliminary considerations about the capacities that would be had by any supposed one-directionally eternal object:

> [i] Moreover, why is it at this point rather than another that the thing which always existed earlier perished, or the thing which did not exist for an infinite time came into being? [ii] If there is no reason why at this point rather than another, and the points are infinite, then it is clear that it was in a way generable or perishable for an infinite time. [iii] Therefore, it is capable for an infinite time of not existing, since it will simultaneously have a capacity for not existing and for existing: in the one case earlier, namely if the thing is perishable, in the other case later, namely if it is generable. (*De caelo* 1. 12, 283ᵃ11–16)

Suppose that there is an item X which always exists up to some time, and then perishes. The passage establishes that throughout the time until it perishes, the item X is capable of not existing. Or suppose that there is an item Y which is generated at some time, and thence-

12', in A. C. Bowen and C. Wildberg (eds.), *New Perspectives on Aristotle's* De caelo (Leiden, 2009), 29–50 at 30.

forth always exists. Then the passage establishes that throughout the time after it is generated, the item Y is capable of not existing. Thus, throughout the time of its existence, any one-directionally eternal item has a capacity for not existing. Aristotle's argument for this result appears to turn on the arbitrariness of the time at which the item is generated or perishes: it could have been generated arbitrarily later, or perished arbitrarily earlier, than it actually did.

Aristotle seems to regard this result as significant in its own right, but he also goes on to make use of it in a passage which contains an application of the possibility rule:

> [iv] Consequently, if we posit that that of which it is capable obtains, [v] opposites will obtain simultaneously. (*De caelo* 1. 12, 283ᵃ16–17)

The passage outlines an argument in highly compressed form. In point [iv] Aristotle introduces an assumption for the possibility rule, marking the beginning of a modal subordinate deduction. The conclusion of this subordinate deduction is indicated in point [v], and seems to be similar to what we saw in the earlier argument. Aristotle does not explicitly state what the assumption for the possibility rule is in the present argument; but in view of the capacities discussed in point [iii] above, the assumption can be taken to be that the item in question does not exist for a certain time. More specifically, in the case of a generated item which comes into existence at *t* and thenceforth always exists, the assumption seems to be that the item does not exist for some time after *t*. Aristotle's reasoning in points [i]–[iii] can be taken to show that this assumption is possible.[30] Aristotle does not explain how the conclusion of the modal subordinate deduction can be derived from the assumption for the possibility rule. A natural approach is to model the derivation on the previous argument, as relying on a step of iteration in the same way as above. Aristotle's argument for the case of a generated item can then be reconstructed as follows (the argument for the case of a perishing item is strictly parallel):

[30] For an item Y generated at *t*, points [i]–[iii] can be taken to show that it is possible that Y does not exist for some time after *t*. The argument would proceed as follows: the item Y was generated at *t*, but could have been generated arbitrarily later than it actually was; consequently, for every time *u* after *t*, the item Y could have been generated after *u*, and therefore could have failed to exist at *u*. (This is subtly different from what Aristotle actually asserts in [iii], namely that for every time *u* after *t*, the item Y is, at *u*, capable of not existing.) Likewise for the case of a perishing item.

| | | | |
|---|---|---|---|
| 1. | Y is generated at *t* and exists always after *t* | [assumption for *reductio*] | [**P9**] |
| 2. | Y exists always after *t* | [from 1] | |
| 3. | It is possible that Y does not exist for some time after *t* | [from 1 via [i]–[iii]] | |
| 4. | Y does not exist for some time after *t* | [assumption for possibility rule] | |
| 5. | Y exists always after *t* | [iterated from 2] | |
| 6. | Y exists and does not exist simultaneously for some time after *t* | [from 4, 5] | |
| 7. | It is possible that Y exists and does not exist simultaneously for some time after *t* | [possibility rule: 3, 4–6] | |
| 8. | It is not possible that Y exists and does not exist simultaneously for some time after *t* | [premiss: PNC] | |
| 9. | It is not the case that Y is generated at *t* and exists always after *t* | [*reductio*: 1–7, 8] | |
| 10. | Nothing is generated and then exists always | [generalization: 9] | |

This reconstruction has exactly the same structure as the above reconstruction of Aristotle's earlier argument ([**P8**]). It also attributes the same weakness to Aristotle's argument, namely the iteration of line 2 into the modal subordinate deduction in line 5. As before, unless some special justification for this iteration can be found, Aristotle's argument is invalid.

(*d*) *Nothing is one-directionally eternal, second argument* (283$^{b}$6–17)

The third passage in which the possibility rule is applied in *De caelo* 1. 12 aims to establish the same result as the passage just discussed. However, this time the argument proceeds along rather different lines, apparently turning on linguistic considerations about time and tense. As before, Aristotle considers two cases of a one-directionally eternal item, namely that of a generated and that of a perishing item. Aristotle begins with the case of a generated item:

[i] But certainly it is not true to say of anything now that it is last year, nor last year that it is now. [ii] Therefore it is impossible for something not to exist at some time and later be eternal: [iii] for afterwards it will also have the capacity for not existing, only not for not existing then when it exists—since it is actually existing—but for not existing last year and in past time. [iv] Then let that for which it has the capacity obtain actually. [v] It will follow that it is true to say now that it does not exist last year. [vi] But this

is impossible: no capacity is a capacity for having been, but for being or going to be. (*De caelo* 1. 12, 283^b6–14)

In point [ii] Aristotle states the thesis he wishes to establish, namely that there is no one-directionally eternal generated item. At the same time, he implicitly introduces the assumption for *reductio* that there is such an item. In [iii] he derives a consequence from this assumption, namely that the item in question has a special kind of capacity during the time for which it exists. This capacity is described as a capacity for not existing at some time in the past. It is not clear why Aristotle thinks the item should have this capacity, and for our purposes the question can be set aside. The item's having this capacity can be taken to entail a statement of possibility, which Aristotle will use in his application of the possibility rule. In [iv] Aristotle introduces the assumption for the possibility rule. In [v] he gives the conclusion of the modal subordinate deduction, namely that it is true to say now that the item does not exist last year. In order to complete the argument, a statement is required to the effect that this conclusion is impossible; this statement is given in [vi], and was also already indicated in [i]. The impossibility appears to result from a linguistic incompatibility between the present tense of the verb 'exist' and the temporal adverb 'last year'.

It is not clear how the conclusion of the modal subordinate deduction is derived from the assumption for the possibility rule. In fact, it is not clear how this assumption itself should be spelt out. We can therefore offer only a schematic reconstruction of the argument in proof [**P10**] below, which displays the role of the possibility rule but leaves the assumption for the possibility rule unspecified.

Although we are not in a position to explain the inference within the modal subordinate deduction in [**P10**], Aristotle gives the impression that its conclusion follows directly from the assumption for the possibility rule without relying on any substantive statements from outside the modal subordinate deduction. If this is so, then his argument does not suffer from the difficulties with iteration from which the earlier two arguments suffered.

After having treated the case of a one-directionally eternal generated item, Aristotle goes on to treat the case of a perishing item in an analogous way:

Similarly also if it is eternal earlier and will not exist later. For it will have a capacity for that which it is not in actuality. Consequently, if we posit what

| | | | |
|---|---|---|---|
| 1. | Y is generated at *t* and exists always after *t* | [assumption for *reductio*] | [**P10**] |
| 2. | Y has, after *t*, a capacity for not having existed at some time in the past | [from 1] | |
| 3. | It is possible that P | [from 2] | |
| 4. | P | [assumption for possibility rule] | |
| 5. | It is true to say now that Y does not exist last year | [from 4] | |
| 6. | It is possible that it is true to say now that Y does not exist last year | [possibility rule: 3, 4–5] | |
| 7. | It is not possible that it is true to say now that Y does not exist last year | [premiss] | |
| 8. | It is not the case that Y is generated at *t* and exists always after *t* | [*reductio*: 1–6, 7] | |
| 9. | Nothing is generated and then exists always | [generalization: 8] | |

is possible, it will follow that it is true to say now that this item exists last year, and more generally in past time. (*De caelo* 1. 12, 283$^{b}$14–17)

This argument can be reconstructed in a strictly parallel way. The new reconstruction can be obtained by replacing 'not exist' with 'exist' in lines 2–7 of proof [**P10**], and making appropriate changes in the remaining lines.

We have now considered all the applications of the possibility rule in *De caelo* 1. 12, and shown that they can be reconstructed as following the same general pattern. As we have seen, two of these applications suffer from problems with iteration into the modal subordinate deduction. The last two applications do not seem to suffer from these problems, although they are perplexing in other ways. A number of questions must remain open concerning Aristotle's reasoning in this difficult chapter, but we hope to have shed some light on the structure of his arguments.

## 4. The existence of a first mover: *Physics* 7. 1 and 8. 5

We next want to discuss two passages from the *Physics* in which Aristotle applies the possibility rule. Both concern the existence of first movers. The first passage, in *Physics* 7. 1, aims to establish that there is no infinite chain of moved movers, but that every chain terminates in a first mover. The second passage forms part of a complex argument in *Physics* 8. 5 to the effect that everything moved is ultimately moved by a first, unmoved mover. Within this argument, the possibility rule is used to prove that not everything moved is moved by something else that is also moved.

(*a*) *There is no infinite chain of moved movers* (7. 1, $242^{a}49$–$243^{a}31$)

Aristotle's argument in *Physics* 7. 1 concerns chains of moved movers, in which a first item is moved by a second, the second is moved by a third, and so on. Aristotle claims that any such chain must ultimately terminate by reaching what he calls a first mover: an item which imparts motion but is not moved by anything else. His proof of this claim begins with an assumption for *reductio*, as follows:

> It is necessary that something is the first mover, and that the chain does not proceed to infinity. For let there not be a first mover, but let the chain become infinite. Thus let A be moved by B, B by C, C by D, and always the next by the next. (*Phys*. 7. 1, $242^{a}53$–7)

The assumption for *reductio* is that some chain of moved movers does not terminate, i.e. that there are infinitely many items A, B, C, … such that A is moved by B, B is moved by C, and so on to infinity. Aristotle sets out to derive from this assumption the consequence that it is possible for an infinite motion to occur in a finite time—a consequence which he has established elsewhere to be false.

The derivation of this consequence is long, and relies on several theorems from Aristotle's physics. Aristotle begins by establishing three preliminary claims. First, he argues that the motions undergone by A, B, C, … are all simultaneous, meaning that they all occupy exactly the same interval of time ($242^{a}57$–62). Second, he asserts that every one of these motions traverses a finite distance ($242^{a}65$–6).[31] Third, Aristotle infers from the foregoing claim that the time occupied by the motion of A is finite ($242^{b}44$–5).[32] From the first and third claims it follows that the motions undergone by A, B, C, … all occur in the same finite time. Aristotle's arguments for these preliminary results seem to apply generally to any arbitrary chain of moved movers. Thus, he in effect endorses the thesis

[31] This assertion is justified by a theorem proved at *Phys*. 6. 10, $241^{a}26$–$^{b}12$.

[32] This inference is justified by a theorem, proved in *Phys*. 6, to the effect that no motion takes an infinite time to traverse a finite distance; see *Phys*. 6. 2, $233^{a}31$–4; 6. 7, $237^{b}24$–5.

that for any chain of moved movers, the individual motions in this chain all occur in the same finite time.

With this result in hand, Aristotle begins to set up an application of the possibility rule.

> [i] Since the movers and moved items are infinite, the motion EFGH composed out of all their motions will be infinite: [ii] for it is possible that the motions of A, B, and the others are equal, and it is possible that the motions of the others are greater than the motion of A. [iii] Consequently, whether they are equal or greater, in both cases the whole is infinite. [iv] For we suppose what is possible. (*Phys.* 7. 1, 242$^{b}$45–50)

Aristotle's presentation does not strictly adhere to the logical order of the argument. In point [ii] he gives the possibility statement that serves as a premiss for the possibility rule: it is possible that the motions of B, C, D, … are each at least as great as the motion of A.[33] Here the size or 'greatness' of a motion seems to be identified with the distance traversed by the motion.[34] In [iv] Aristotle makes the assumption that the motions of B, C, D, … are in fact each as great, i.e. each traverse at least as great a distance, as the motion of A.[35] This is the assumption for the possibility rule, marking the beginning of the modal subordinate deduction.

In points [i] and [iii] Aristotle has already stated a consequence of this assumption. The consequence is that the sum of the motions of A, B, C, …, i.e. the whole motion composed out of all these motions, is infinite. Again, this means that the whole motion traverses an infinite distance. Aristotle seems to assume that the distance traversed by the whole motion is the sum of the distances traversed by the individual motions. The role of the assumption for the possibility rule is to exclude cases in which the distances traversed by A, B, C, … form a decreasing, converging series. For in such a case the sum of the distances traversed would not be infinite. For example, if A moves 1 metre, B moves ½ metre, C moves ¼ metre, and so on, the sum of the distances traversed will not be infinite but only 2 metres.[36] Thus, if he did not make an assumption to exclude cases

[33] Aristotle repeats this statement at 7. 1, 242$^{b}$66.

[34] Thus, for example, the statement that the whole motion 'is infinite' (242$^{b}$46–7, 53) is later spelt out in terms of the statement that the motion 'traverses an infinite distance' (242$^{b}$70).

[35] Aristotle repeats this assumption at 7. 1, 242$^{b}$66–7: *ὃ γὰρ ἐνδέχεται, ληψόμεθα ὡς ὑπάρχον*.

[36] See *Phys.* 3. 6, 206$^{b}$7–9.

of this kind, Aristotle would not be in a position to draw the consequence he draws in points [i] and [iii].

The consequence drawn there is not yet the conclusion of the modal subordinate deduction. In order to reach this conclusion, Aristotle proceeds to argue that the whole motion composed of the motions of A, B, C, … will occur in a finite time:

> [v] Since A and each of the others move simultaneously, the whole motion will be contained in the same time as the motion of A. [vi] But the motion of A is contained in a finite time. [vii] Consequently, there would be an infinite motion in a finite time. [viii] But this is impossible. (*Phys*. 7. 1, 242$^{b}$50–3)

Points [v] and [vi] repeat two of the preliminary claims established earlier: that the motions of B, C, D, … are all simultaneous with the motion of A, and that the motion of A occurs within a finite time. From these two claims it follows that the motions of A, B, C, … all occur within the same finite time, and therefore that the whole motion composed of them occurs within this finite time. So, given that the whole motion traverses an infinite distance ([i] and [iii]), Aristotle can infer in [vii] that a motion traverses an infinite distance in a finite time. This is now the conclusion of the modal subordinate deduction.

In point [viii] Aristotle concludes his presentation of the proof by asserting that it is impossible for a motion to traverse an infinite distance in a finite time.[37] He is presumably relying on the theorem stated in *Physics* 6. 2, and proved in detail in *Physics* 6. 7, that no motion traverses an infinite distance in a finite time.[38]

We are now in a position to reconstruct Aristotle's argument as a whole, as displayed in proof [**P11**] below. In this reconstruction, the assumption for the possibility rule, that the distances traversed by B, C, D, … are each at least as great as the distance traversed by A, is abbreviated to the statement that A, B, C, … is a 'non-decreasing chain of moved movers'.

Many points in Aristotle's argument and our reconstruction of it call for further discussion; we will focus here on three issues which especially concern the use of the possibility rule. First, let us

[37] Later, Aristotle qualifies this claim, saying instead that it is impossible for the motion of a single subject (as opposed to a plurality of subjects) to traverse an infinite distance in a finite time (*Phys*. 7. 1, 242$^{b}$55–7). He therefore adds an argument that A, B, C, … constitute a single subject of motion (242$^{b}$53–72).

[38] *Phys*. 6. 2, 233$^{a}$31–4; 6. 7, 238$^{a}$20–31, 238$^{b}$17–20.

| | | | |
|---|---|---|---|
| 1. | S is an infinite chain of moved movers A, B, C,… | [assumption for *reductio*] | [**P11**] |
| 2. | Every chain of moved movers is possibly a non-decreasing chain of moved movers | [premiss] | |
| 3. | It is possible that S is a non-decreasing chain of moved movers A, B, C,… | [from 1, 2] | |
| 4. | S is a non-decreasing chain of moved movers A, B, C,… | [assumption for possibility rule] | |
| 5. | The individual motions in any chain of moved movers all occur in the same finite time | [premiss] | |
| 6. | The motions of A, B, C,… all occur in the same finite time | [from 4, 5] | |
| 7. | The sum of the motions of A, B, C,… occurs in a finite time | [from 6] | |
| 8. | S is an infinite chain of moved movers A, B, C,… | [iterated from 1] | |
| 9. | The sum of the motions of A, B, C,… is a motion that traverses an infinite distance | [from 4, 8] | |
| 10. | A motion traverses an infinite distance in a finite time | [from 7, 9] | |
| 11. | It is possible that a motion traverses an infinite distance in a finite time | [possibility rule: 3, 4–10] | |
| 12. | It is not possible that a motion traverses an infinite distance in a finite time | [premiss] | |
| 13. | S is not an infinite chain of moved movers | [*reductio*: 1–11, 12] | |
| 14. | There is no infinite chain of moved movers | [generalization: 13] | |

consider Aristotle's justification for the premiss in line 12 of [**P11**]. As mentioned above, this premiss seems to be based on arguments in *Physics* 6 to the effect that no motion traverses an infinite distance in a finite time. In describing what these arguments establish, Aristotle varies in *Physics* 6 between a modal and a non-modal claim, namely between the claim that it is not possible for a motion to traverse an infinite distance in a finite time, and the claim that no motion actually does so.[39] If Aristotle took himself in book 6 to establish the modal claim, then this claim is the premiss given in line 12. On the other hand, if he took himself to establish the non-modal claim, which is perhaps more likely, the premiss in line 12 can still be justified. For in this case it is reasonable to think that this non-modal

[39] Modal claim: *οὔτε δὴ τὸ ἄπειρον οἷόν τε ἐν πεπερασμένῳ χρόνῳ διελθεῖν*, 6. 2, 233ª31–2; *οὐδ᾽ ἐν πεπερασμένῳ χρόνῳ ἄπειρον οἷόν τε κινεῖσθαι*, 6. 7, 238ª20–1. Non-modal claim: *οὐ δίεισιν ἐν πεπερασμένῳ χρόνῳ τὸ ἄπειρον*, 6. 7, 238ª29–30; *οὔτε . . . τὸ ἄπειρον ἐν πεπερασμένῳ χρόνῳ κινεῖται*, 6. 7, 238ᵇ17–19.

claim, having been proved, has the status of a theorem of Aristotle's physics. Scientific theorems are paradigmatic objects of knowledge, and according to the *Posterior Analytics*, such objects of knowledge are necessary.[40] So scientific theorems are true of necessity.[41] Hence it is true of necessity that no motion traverses an infinite distance in a finite time, and the premiss in line 12 is justified.

A second issue concerns the two statements labelled 'premiss' in lines 2 and 5 of [**P11**]. Each of them is introduced within a subordinate deduction. However, since these statements serve as premisses of the whole argument, they should strictly speaking have been introduced outside of any subordinate deduction, and then been iterated into the subordinate deductions in which they are used. We have not done this, for the sake of brevity. Nevertheless, we ought to consider whether the iteration of these premisses into their respective subordinate deductions would be justified. The iteration is unproblematic for the premiss in line 2, since iteration into *reductio* subordinate deductions is allowed without restriction. The statement in line 5, however, needs to be iterated into the modal subordinate deduction. As we discussed earlier, such iterations are not in general permissible, but they are permissible when there is a guarantee that the iterated statement is true of necessity. It is reasonable to think that for the purposes of his present argument, Aristotle treats the statement in line 5 as a theorem of his physics.[42] If so,

[40] *Post. An.* 1. 2, $71^b$15–16; see also 1. 4, $73^a$21–3; 1. 6, $74^b$6, $75^a$12–13.

[41] There are two passages in which Aristotle seems to admit the existence of scientific theorems which are true for the most part, and hence presumably not true of necessity (*Post. An.* 1. 30, $87^b$19–25; *Pr. An.* 1. 13, $32^b$18–21; cf. J. Barnes (trans. and comm.), *Aristotle's* Posterior Analytics, 2nd edn. [*Posterior Analytics*] (Oxford, 1994), 192–3, and id., *Truth, etc.: Six Lectures on Ancient Logic* [*Truth*] (Oxford, 2007), 486). Nevertheless, Aristotle's usual view is that scientific theorems are true of necessity, and we may assume that he takes advantage of this view in our passage.

[42] We have already briefly considered Aristotle's proof of this statement. The proof relies on the claims that in any chain of moved movers, (1) each individual motion occurs in a finite time, and (2) all the individual motions are simultaneous. There is good evidence that (1) has the status of a theorem in Aristotle's physics; see nn. 31 and 32 above. Aristotle's argument for (2) at 7. 1, $242^a$57–62, seems to rely on the following two premisses. (i) In a chain of moved movers, each member (except the first and last) imparts motion to its predecessor exactly while undergoing the motion imparted to it by its successor. (ii) Whenever one item imparts motion to another, the latter item undergoes this motion exactly while the former imparts it. Something close to premiss (ii) is asserted at *Phys.* 2. 3, $195^b$16–21. The source of (i) is less clear; in fact (i) seems to be in tension with *Phys.* 8. 10, $267^a$5–9. Moreover, (1) fits uneasily with Aristotle's commitment to eternal circular motions (*Phys.* 6. 10, $241^b$12–20). Despite these worries, we may assume that in the present context Aristotle treats (1) and (2) as theorems.

then this statement is treated as being true of necessity, and may therefore be iterated into the modal subordinate deduction. Thus, the presence of this statement in line 5 can be justified.

The third and final issue we want to discuss is more problematic. It concerns the iteration of the assumption for *reductio* in line 1, that S is an infinite chain of moved movers, into the modal subordinate deduction in line 8. Aristotle clearly does not regard the infinity of S as genuinely necessary, since his aim is to disprove it. Nor is there an indication that he took the assumption of its infinity in line 1 to yield the inference that it is necessarily infinite.[43] So the iteration of this statement in line 8 cannot be justified on the grounds that the iterated statement is true of necessity. Since there is also no other obvious justification for this iteration, the natural conclusion to draw is that, similarly to what we saw in *De caelo* 1. 12, Aristotle's argument is not valid.

Now that we have discussed Aristotle's argument in some detail, we may turn to a final remark he makes at the end of chapter 7. 1 about the structure of his argument:

> It makes no difference that the impossible results from a hypothesis. For the hypothesis that was supposed is possible, and if something possible is posited, nothing impossible should result through this. (*Phys.* 7. 1, $242^b72$–$243^a31$)

The phrase 'the impossible' in the first sentence refers, we think, to the conclusion of the modal subordinate deduction, namely that a motion traverses an infinite distance in a finite time (line 10 of [**P11**]). Accordingly, the 'hypothesis' referred to in this passage is the assumption for the possibility rule, namely that the chain of moved movers is non-decreasing (line 4 of [**P11**]).[44] Aristotle gives a concise restatement of the possibility rule in the second sentence of his remark, in terms similar to his discussion of this rule in *Prior*

[43] Such an inference could be carried out if it were assumed that the identity of a chain, like the identity of a sequence in mathematics, is determined by the identity and order of its members. On this view, given that a chain has certain members, it necessarily has precisely those members; and consequently, given that a chain has a certain number of members, it necessarily has precisely that number of members. Be that as it may, there is no indication that Aristotle would accept such an inference.

[44] Here we are in agreement with C. Prantl (ed., trans., comm.), *Aristoteles' acht Bücher Physik* (Leipzig, 1854), 518; P. H. Wicksteed and F. M. Cornford (ed. and trans.), *Aristotle:* The Physics, 2 vols. (Cambridge, Mass., 1934), 217; B. Manuwald, *Das Buch H der aristotelischen 'Physik': Eine Untersuchung zur Einheit und Echtheit* (Meisenheim am Glan, 1971), 35–6.

*Analytics* I. 15.[45] This reminder is called for because someone who is not aware that Aristotle's proof employs the possibility rule might be troubled by the appeal to the statement that the chain of moved movers is non-decreasing; such a person might perceive this statement as a further premiss which Aristotle has smuggled in without justification. In the present remark Aristotle therefore clarifies that this statement is an assumption for the possibility rule, and briefly explains the way in which the possibility rule is applied.

Some commentators, on the other hand, interpret Aristotle's remark in a rather different way. They take the 'hypothesis' in this remark to be not an assumption for the possibility rule, but rather the assumption for *reductio*, i.e. the assumption that there is an infinite chain of moved movers (line 1 of [**P11**]).[46] Aristotle would then be saying that something impossible follows from the assumption for *reductio*. Such a reading is correlated with interpretations on which Aristotle's argument does not make use of anything like the possibility rule. However, such interpretations face problems. Above all, it is difficult for them to explain whether and why Aristotle is justified in making use of the statement that the chain of moved movers is non-decreasing. Commentators have attempted to address this problem in various ways, but none of them is fully satisfactory.[47] Moreover, it is hard to see what the point of Aristotle's concluding remark would be on these interpretations: the remark would have to concern *reductio* arguments in general, but there is no need for Aristotle to explain the structure of proofs by *reductio* at this point in the *Physics*.

Robert Wardy offers the most worked-out version of an interpretation on which Aristotle's argument does not involve the possibility rule. On his interpretation, the statement that the chain of moved movers is non-decreasing can be inferred from the assumption for *reductio*, by means of a general principle according to which

[45] See *Pr. An.* I. 15, 34$^{a}$25–7, which was discussed in sect. 2 above.

[46] Simpl. *In Phys.* 1047. 16–29 Diels; W. D. Ross (ed. and comm.), *Aristotle's* Physics*: A Revised Text with Introduction and Commentary* [*Physics*] (Oxford, 1936), 671; H. Wagner (trans. and comm.), *Aristoteles: Physikvorlesung* (Berlin, 1967), 649–50; id., 'Über den Charakter des VII. Buches der Aristotelischen Physikvorlesung', *Archiv für Geschichte der Philosophie*, 56 (1974), 172–81 at 176–7; R. Wardy, *The Chain of Change: A Study of Aristotle's* Physics *VII* [*Chain of Change*] (Cambridge, 1990), 107.

[47] For example, Ross, *Physics*, 670, holds that Aristotle is simply 'ignoring the . . . possible case . . . in which the movements of *A*, *B*, *Γ*. . . are a series of movements decreasing in magnitude'.

effects cannot exceed their causes.[48] He takes this principle to imply that the motion undergone by any moved mover must be at least as great in size as the motion undergone by the thing it moves. However, it is not clear whether Aristotle would accept this latter claim, especially where the size of a motion is understood to be the distance traversed by the motion.[49] In any case, there is no indication that Aristotle is relying on the principle that Wardy attributes to him in the present argument.

Wardy's interpretation is linked with a textual issue concerning book 7 of the *Physics*. The first three chapters of this book have been transmitted in two different versions. Ross calls them the $\alpha$- and $\beta$-versions, and holds that the $\alpha$-version is superior.[50] His verdict is widely accepted, and we have been following the $\alpha$-version in this paper. In this version, Aristotle does not categorically assert that the chain of moved movers is non-decreasing. Instead, he presents this as something which is merely possible (242$^b$47–8, 66), and then goes on to assume as a hypothesis that it is the case (242$^b$49–50, 66–7). This is difficult to explain on Wardy's interpretation. In the $\beta$-version, on the other hand, the statement is asserted categorically without a modal qualification: 'the motion will either be equal to the motion of A or greater than it'.[51] This is in accordance with Wardy's interpretation, and Wardy takes this as a point in favour of the $\beta$-version over the $\alpha$-version.[52] However, doing away with an assumption for the possibility rule at this point in the text makes other remarks in the $\beta$-version look mysterious. For the $\beta$-version contains two remarks that are reminiscent of an application of the possibility rule: 'let that which is possible be supposed', and 'if something possible is posited, nothing absurd should follow'.[53] It is not easy to make sense of these remarks in the context provided in the $\beta$-version. They seem to confirm Ross's view that the $\beta$-version is a derivative and distorted one, perhaps written from memory by a student. If this is correct, then Wardy's interpretation should be rejected, since it only matches the $\beta$-version. Provided that a clear and plausible line of reasoning can be offered which closely matches

48 Wardy, *Chain of Change*, 108.

49 For example, it is plausible that a heavy mover traversing a short distance may cause a light item to traverse a long distance.

50 See Ross, *Physics*, 11–19.

51 *Phys*. 7. 1, $\beta$-version, 242$^b$17–18.

52 Wardy, *Chain of Change*, 107–8.

53 *Phys*. 7. 1, $\beta$-version, 242$^b$27–8 and 243$^a$1–2.

the $\alpha$-version, as we hope we have done, there is no reason to prefer the $\beta$-version of *Physics* 7. 1–3.

### (*b*) *Not everything moved is moved by something else that is moved* (8. 5, 256$^b$3–13)

*Physics* 8. 5 contains a series of arguments concerning the existence of self-movers and unmoved movers. Roughly the first half of the chapter is devoted to showing that the source of every motion can be traced back either to a self-mover or to an unmoved mover (256$^a$4–257$^a$27). This is followed by a detailed analysis of self-motion, one result of which is that every self-mover consists of a moved part and an unmoved part, with the latter imparting motion to the former (257$^a$27–258$^b$4). The unmoved part of a self-mover can be regarded as an unmoved mover. Hence the final lesson of the chapter is that everything that is moved is ultimately moved by an unmoved mover (258$^b$4–9).[54]

Within this broad line of reasoning, there is a complex argument establishing the following intermediate result: not everything moved is moved by something else that is moved (256$^b$3–257$^a$27). Since Aristotle has argued previously that everything moved is moved by something (*Phys*. 8. 4, 256$^a$2–3), the result entails that something is moved either by itself or by an unmoved mover. Hence, there exist self-movers or unmoved movers.

Aristotle's argument for this result begins as follows:

> [i] If everything moved is moved by something which is moved, [ii] either this applies to things accidentally, such that a thing imparts motion while being moved but not because it is itself being moved, [iii] or not accidentally but *per se*. (*Phys*. 8. 5, 256$^b$4–7)

In point [i] Aristotle introduces the claim to be refuted, namely that everything moved is moved by a moved mover. Aristotle does not here include a clause to the effect that the moved mover is distinct from the thing moved by it, but it seems clear from the context that it should be understood.[55] Thus, the claim to be refuted is that everything moved is moved by something else that is moved. In

[54] 'Is moved' translates the Greek κινεῖται, which has both an intransitive sense (undergoing motion) and a passive sense (being moved by something). Since Aristotle has argued at length in *Phys*. 8. 4, as well as in *Phys*. 7. 1, that whatever undergoes motion is moved by something, we will allow ourselves to translate κινεῖται always by 'is moved'.

[55] See the phrase 'by something else' (ὑπ' ἄλλου) in the conclusion of the complex

[ii] and [iii] Aristotle goes on to distinguish two ways in which this claim might be true, namely either 'accidentally' or '*per se*'. The distinction sets the stage for a pair of *reductio* arguments, showing that the claim leads to an unacceptable consequence in each case. We will only consider the first, accidental case, since it is there that Aristotle applies the possibility rule.

Aristotle's discussion of the accidental case proceeds as follows:

> [iv] First, if it applies accidentally, [v] it is not necessary for that which imparts motion to be moved. [vi] But in that case, clearly it is possible that at some time, none of the things that exist is moved. [vii] For an accident is not necessary, but it is possible for it not to be. [viii] If, then, we posit that which is capable of being, nothing impossible will follow, though perhaps something false will. [ix] But for there not to be motion is impossible: it has been proved earlier that it is necessary for there always to be motion. (*Phys*. 8. 5, 256$^b$7–13)

In point [iv] Aristotle indicates the assumption for *reductio*, namely that it applies to things accidentally that everything moved is moved by something else that is moved. It is not clear exactly what it means to say that this applies to things accidentally. It might be taken to mean that, although everything moved is moved by something else that is moved, each mover could impart motion without itself undergoing motion.[56] Whatever precisely the assumption for *reductio* amounts to, Aristotle goes on in [v] to infer from this assumption that it is not necessary for any mover to be moved.[57] From this he infers in [vi] that it is possible that at some time, nothing at all is being moved. In point [vii] Aristotle offers some justification for his inference from [iv] through [v] to [vi]. However, the justification is incomplete and does not fully explain the inference. It would take a great deal of effort to explore what Aristotle's full justification might be; and while the inference is crucial to Aristotle's argument,

argument, 257$^a$25. Without the 'something else' clause, the argument would establish the existence of unmoved movers. But Aristotle seems to take it to establish only the existence either of unmoved movers or of self-movers; see the further argument from the existence of self-movers to the existence of unmoved movers at 257$^a$27–258$^b$4.

[56] This kind of interpretation is adopted by Simpl. *In Phys*. 1225. 14–16 Diels, and is suggested by Aristotle's formulation of the second, non-accidental case at *Phys*. 8. 5, 256$^b$28–9.

[57] We accept, for present purposes, Ross's decision to print *κινοῦν* rather than *κινούμενον* in 256$^b$8.

it does not pertain directly to the specific aims of this paper. Thus we must leave it unexplained.

In point [viii] Aristotle proceeds by offering a reminder of the possibility rule. He appears to intend an application of the rule building on the statement of possibility in [vi]. Thus the assumption for the possibility rule, briefly indicated in [viii], would be that nothing is being moved at some time. In [ix] Aristotle specifies the conclusion of the modal subordinate deduction, namely that at some time there is no motion, and he states that this conclusion is impossible.

Aristotle's argument can thus be reconstructed as follows:

| | | | |
|---|---|---|---|
| 1. | It applies accidentally that everything moved is moved by something else that is moved | [assumption for *reductio*] | [**P12**] |
| 2. | It is possible that at some time, nothing is being moved | [from 1] | |
| 3. | At some time, nothing is being moved | [assumption for possibility rule] | |
| 4. | At some time, there is no motion | [from 3] | |
| 5. | It is possible that at some time, there is no motion | [possibility rule: 2, 3–4] | |
| 6. | It is not possible that at some time, there is no motion | [premiss] | |
| 7. | It does not apply accidentally that everything moved is moved by something else that is moved | [*reductio*: 1–5, 6] | |

There are two issues which we want to discuss further in connection with this reconstruction. The first issue concerns the justification of the premiss in line 6 that it is impossible for there to be no motion at some time. Aristotle says in point [ix] that he has proved this premiss earlier. He appears to be referring to a proof in *Physics* 8. 1 whose conclusion is that 'there neither was nor will be any time in which there was not or will not be motion' ($252^{b}5$–6). A worry might arise because this conclusion does not state that it is impossible for there to be some time without motion, only that there is no such time. However, since the conclusion has been proved, it has the status of a theorem of Aristotle's physics. As explained above (see pp. 208–9), Aristotle holds that scientific theorems are true of necessity, and so Aristotle's premiss in line 6 is justified by the proof from *Physics* 8. 1.

The second issue concerns the nature of the modal subordinate deduction in [**P12**]. This deduction consists only in a transition from 'at some time, nothing is being moved' to 'at some time, there is no motion'. Accordingly, the only function of the possibility rule here is to perform a transition from the possibility of the first statement to the possibility of the second. It is understandable that Aristotle wants to derive the possibility of the second statement, since it more straightforwardly contradicts the official conclusion of *Physics* 8. 1; as we have just seen, the wording of that conclusion refers to the existence of motion rather than of moved items ($252^b5$–6). However, it is not clear that the possibility rule is needed for this purpose. The two phrases 'there is no motion' and 'nothing is being moved' are very similar in meaning, and there is no indication in *Physics* 8. 1 or 8. 5 that replacing one with the other requires argument. Aristotle occasionally switches between them in a way which suggests that he took them to be freely interchangeable.[58] Thus, it might seem that the application of the possibility rule in the argument is superfluous, and that Aristotle could have inferred the statement in line 5 of [**P12**] from the statement in line 2 directly. Of course, none of this makes it impermissible for Aristotle to apply the possibility rule in his argument, and he may have had special reasons to do so. Still, it is not clear what these reasons might be, and we must leave this issue as an open question for further research.

## 5. Atomism and infinite divisibility: *De generatione et corruptione* 1. 2

Early in his treatise *De generatione and corruptione*, Aristotle states that the proper explanation of coming into being and perishing depends in important ways on whether or not the thesis of atomism is true (*GC* 1. 2, $315^b24$–8). The atomist thesis is that magnitudes have indivisible extended parts.[59] As is well known, Aristotle rejects this thesis and holds instead that magnitudes are infinitely divisible. In *GC* 1. 2 he presents a detailed argument in favour of the atomist thesis ($316^a14$–$^b34$), and then goes on to diagnose a fallacy in it ($317^a1$–12). Aristotle associates the argument with the name of

[58] See the two occurrences of 'nothing is being moved' ($250^b12$, $250^b23$) interspersed among talk of the existence of motion at *Phys*. 8. 1, $250^b11$–24. Elsewhere, however, Aristotle seems to regard it as not obvious and worth asserting that motion is present if and only if something is being moved; see *Phys*. 6. 1, $231^b25$–6.

[59] See *GC* 1. 2, $315^b26$–7, $316^b15$–16, $316^b32$–3. We use 'part' to mean both proper and improper parts.

Democritus (316ᵃ13), and the argument probably reflects Democritus' reasoning.

Aristotle's presentation of the argument is complex, and falls into two main parts. By the end of the first part (316ᵃ14–ᵇ16) he has already reached the atomist conclusion, but then he announces that it is necessary to restate the argument from the beginning. He does this in the second part, giving a somewhat different and more concise version of the argument (316ᵇ19–34). Both versions of the argument involve the possibility rule, and we will examine them in order.

### (*a*) *No magnitude is divisible everywhere, first argument* (316ᵃ14–ᵇ16)

The first argument for atomism begins with the assumption for *reductio* that some magnitude is divisible everywhere (316ᵃ14–16). The contradictory of this assumption is taken to entail the atomist thesis. Divisibility is a modal notion; something is divisible somewhere just in case it is possible for it to have been divided there. Because of this, the argument lends itself to an application of the possibility rule. Aristotle begins his presentation of the argument by indicating how this rule is going to be applied:

> [i] If it is divisible everywhere, and this is possible, [ii] then it could also have been divided everywhere simultaneously, even if it has not been divided simultaneously. [iii] And if this should come about, there would be nothing impossible. (*GC* 1. 2, 316ᵃ17–19)

In point [i] Aristotle states the assumption for *reductio* that some magnitude is divisible everywhere. In [ii] he infers from this assumption that it is possible for the magnitude to have been divided everywhere. This inference is problematic; the assumption for *reductio* appears to mean that for every 'somewhere' it is possible that the magnitude has been divided there, whereas according to the inferred claim, it is possible that for every 'somewhere' the magnitude has been divided there. The latter claim does not follow from the former, as is clear from the analogy of winning a game: typically, for every player it is possible that he or she wins, but it is not possible that every player wins. Aristotle is aware of the problems with this inference, and in fact his response to the argument as a whole will consist in blocking it (317ᵃ1–12). At the present stage, however, he does not call the inference into question.

Having derived a statement of possibility in point [ii], Aristotle invokes the possibility rule in [iii]. After this, he goes on to introduce an assumption for the possibility rule, and to begin exploring its consequences, as follows:

> [iv] Now since the body is such [i.e. divisible] everywhere, [v] let it have been divided. [vi] Then what will be left? [vii] A magnitude? This cannot be, since there would be something that has not been divided, whereas the body was divisible everywhere. (*GC* 1. 2, 316$^{a}$23–5)

In point [iv] Aristotle briefly recollects the assumption for *reductio* and the consequence derived from it in [ii]. He proceeds in [v] to introduce the assumption for the possibility rule that the magnitude has been divided everywhere. In order to proceed within the modal subordinate deduction, he raises in point [vi] the question of what is left after the magnitude has been divided everywhere. In [vii] he argues that no magnitude, i.e. nothing extended, is left; for if a magnitude were left, this would contradict the assumption for the possibility rule.[60]

Aristotle next considers two further alternatives as to what is left after the magnitude has been divided everywhere:

> [viii] But then, if there will be no body and no magnitude left, and there will be a division, [ix] then either the body will be composed out of points, and the things from which it is composed will be unextended, [x] or nothing at all will be left, [xi] with the consequence that the body could both come into being out of nothing and would be composed out of nothing, and the whole would be nothing but an appearance. (*GC* 1. 2, 316$^{a}$25–9)

In point [viii] Aristotle recollects the assumption for the possibility rule, and reiterates the consequence, drawn in [vii], that no magnitude is left. On the basis of this, he infers in [ix] and [x] that only two alternatives remain: either points are left or nothing at all. He introduces the first alternative by immediately stating a consequence of it: instead of saying that only points are left, Aristotle says in [ix] that the magnitude is composed out of points. Thus he seems to assume that whatever is left after the division of a magnitude is that out of which the magnitude was composed.

[60] If a magnitude is left, then this magnitude has not been divided. Being left over from division, this magnitude must have been a part of the original magnitude. It follows that some extended part of the original magnitude has not been divided. But it was assumed for the possibility rule that the original magnitude has been divided everywhere, and this presumably entails that every extended part of the original magnitude has been divided.

Aristotle will return shortly to this alternative, but for now he goes on to discuss the other alternative, introduced in [x], on which nothing at all is left. This implies that the magnitude has been divided into nothing. In [xi] Aristotle draws two consequences from this alternative. The first is that the magnitude could come into being out of nothing; the idea is that the process of division could be reversed into a process of composition, and hence of coming into being. The second consequence is that the magnitude would be composed out of nothing, and it seems to be inferred along similar lines to those we saw in [ix]. It is safe to assume that Aristotle regards both consequences derived in [xi] as impossible (for the impossibility of the first consequence see 316$^b$26–7).

Finally, Aristotle returns to the alternative introduced in [ix], on which only points are left:

> [xii] And similarly, if the magnitude is composed out of points, it will not be a quantity. [xiii] For when the points were touching and there was a single magnitude and they were together, they did not make the whole any bigger. For when something has been divided into two or more pieces, the entirety is no smaller or bigger than before. Consequently, when all the points have been put together, they will not make any magnitude. (*GC* 1. 2, 316$^a$29–34)

On the assumption that only points are left, Aristotle infers in [xii] that the magnitude is not a quantity. He justifies this inference in [xiii], based on the idea that since points individually have no extension, no amount of points taken together will have extension either. The details of the justification are complicated and can be left aside for present purposes. It is clear that Aristotle takes it to be impossible for a magnitude not to be a quantity.[61]

At this stage, Aristotle has finished presenting the core of the argument for atomism. From the assumption for the possibility rule that the magnitude has been divided everywhere, he takes it to follow that what is left after the division is either a magnitude, points, or nothing.[62] In other words, the magnitude has been divided either into a magnitude, or into points, or into nothing. The first of these alternatives is incompatible with the assumption for the possibility

[61] Cf. *Phys.* 1. 2, 185$^b$5; *Metaph.* Δ 13, 1020$^a$7–10.

[62] This tripartite disjunction might be derived by a series of two dichotomies: either something is left or nothing, and if something, then either something extended (i.e. a magnitude) or only unextended items are left. (It would need some further argument to show that these unextended items must be points.)

rule, and each of the other two is shown to imply an impossible consequence. The argument can thus be reconstructed as follows:

| | | | |
|---|---|---|---|
| 1. | Magnitude M is divisible everywhere | [assumption for *reductio*] | [**P13**] |
| 2. | It is possible that magnitude M has been divided everywhere | [from 1] | |
| 3. | Magnitude M has been divided everywhere | [assumption for possibility rule] | |
| 4. | Magnitude M has been divided either into a magnitude or into points or into nothing | [from 3] | |
| 5. | M has been divided into a magnitude | [assumption for *reductio*] | |
| 6. | M has not been divided everywhere | [from 5] | |
| 7. | M has not been divided into a magnitude | [*reductio*: 5–6, 3] | |
| 8. | Magnitude M has been divided either into points or into nothing | [from 4, 7] | |
| 9. | Magnitude M is not a quantity or is composed out of nothing | [from 8] | |
| 10. | It is possible that magnitude M is not a quantity or is composed out of nothing | [possibility rule: 2, 3–9] | |
| 11. | It is not possible that magnitude M is not a quantity or is composed out of nothing | [premiss] | |
| 12. | Magnitude M is not divisible everywhere | [*reductio*: 1–10, 11] | |

Note that this reconstruction does not involve an iteration into the modal subordinate deduction. Thus the present argument is not subject to the difficulties with unjustified iteration which we have encountered in some of the previous arguments.

In order to make the modal structure of Aristotle's argument clearer, some details are omitted in [**P13**]. For example, unlike what we find in Aristotle, lines 8–11 treat two alternatives simultaneously (namely that points are left and that nothing is left). Furthermore, the reconstruction leaves a number of substantive steps unanalysed, namely those performed in lines 2, 4, 6, and 9. As mentioned above, the first of these steps is problematic, and Aristotle himself will diagnose a fallacy in it when he rejects the argument for atomism. As to the steps in lines 4 and 6, we have already given some explanation of how they can be justified.[63] However, the inference from line 8 to line 9 involves two issues which we want to discuss further.

The first issue is that one might be puzzled by temporal aspects

[63] For the step in line 4, see n. 62 above; for the step in line 6, see n. 60 above.

of the inference. The statement in line 8 concerns a time after magnitude M has been divided everywhere, whereas the statement in line 9 seems to concern a time before the magnitude has been divided. This temporal difference between the two statements could have been represented through the addition of suitable temporal indices.[64] Since this does not pertain directly to the modal structure of the argument, we have omitted such indices in the above reconstruction.

The second issue concerns the validity of the inference from line 8 to line 9. This inference might be contested for various reasons. Aristotle himself can be taken to consider a potential objection to it at 316$^a$34–$^b$5, namely that the things into which a magnitude has been divided need not include all the things out of which it was previously composed. Rather, some of the things out of which the magnitude was composed might have altogether disappeared or gone away 'like sawdust' (316$^a$34). Aristotle develops this idea in a few ways but ultimately rejects it. From a modern perspective, the inference from line 8 to line 9 could also be blocked in a quite different way. One might maintain on the basis of Cantor's work that something which is composed out of points can after all be a magnitude and hence a quantity, provided that there are uncountably infinitely many of these points. Of course, this kind of objection would not be available to Aristotle. For him, it was reasonable to regard the inference from line 8 to line 9 as a compelling one.

As mentioned earlier, Aristotle associates the above argument for atomism with the name of Democritus. It is sometimes thought that the argument he presents is Democritus' own.[65] However, although

[64] For example, the statement in line 8 would be 'at $t_0$ M is a magnitude, and at $t_1$ M has been divided either into points or into nothing', whereas the statement in line 9 would be 'at $t_0$ M is a magnitude which is not a quantity or is composed out of nothing'. Corresponding adjustments would be required throughout the proof; for example, the premiss in line 11 would read 'it is not possible that at some time, M is a magnitude which is not a quantity or is composed out of nothing'.

[65] See Philop. *In GC* 27. 29–30 Vitelli; H. H. Joachim (ed. and comm.), *Aristotle:* On Coming-to-Be and Passing-Away (Oxford, 1926), 76; C. J. F. Williams (trans. and comm.), *Aristotle's* De generatione et corruptione (Oxford, 1982), 72; P. S. Hasper, 'The Foundations of Presocratic Atomism', *Oxford Studies in Ancient Philosophy*, 17 (1999), 1–14 at 5 n. 7; D. Sedley, '*On Generation and Corruption* I 2' ['*On GC* I 2'], in F. A. J. de Haas and J. Mansfeld (eds.), *Aristotle:* On Generation and Corruption, *Book 1* (Oxford, 2004), 65–89 at 67; G. Betegh, 'Epicurus' Argument for Atomism', *Oxford Studies in Ancient Philosophy*, 30 (2006), 261–84 at 266–9. *GC* 1. 2 is also taken as evidence about the views of Democritus by C. C. W. Taylor, *The Atomists: Leucippus and Democritus* (Toronto, 1999), 164–6.

it is likely that parts of the argument derive from him, it is not clear whether the entire argument does. In particular, we think it is unlikely that the possibility rule was available to Democritus. Aristotle's writings convey the impression that this rule was first discovered by himself, as the result of substantial theoretical work on his part. Given this, the argument for atomism presented by Aristotle can be regarded as a combination of Democritean and Aristotelian elements, with the application of the possibility rule being one of the latter.

### (*b*) *No magnitude is divisible everywhere, second argument* ($316^b19$–27)

Having presented the above argument for atomism, Aristotle claims that there are powerful opposing reasons not to accept the atomist doctrine ($316^b16$–18). He does not give these reasons here, but refers to other works of his, presumably to *Physics* 6.[66] Aristotle is persuaded by these reasons against atomism, and wishes to explain why the argument for atomism just presented is not successful. In order to do this, he says, he first needs to give a restatement of the argument ($316^b18$–19). The restated argument begins as follows:

> It would seem to be impossible [for a perceptible body ($316^b19$)] to be simultaneously divisible everywhere in potentiality. For if it is possible, then it could come about, not so as to be simultaneously both actually undivided and divided, but so as to be divided at any point whatsoever. (*GC* 1. 2, $316^b21$–5)

In the first sentence of this passage Aristotle gives the intended conclusion of the argument; he thereby also specifies the assumption for *reductio*, namely that a perceptible body 'is simultaneously divisible everywhere in potentiality'. This differs from the assumption for *reductio* in the first argument in that Aristotle now adds the qualification 'simultaneously' to the phrase 'divisible everywhere'. This addition might be taken to have no significance, so that the present assumption for *reductio* is equivalent to the earlier one.[67] Alternatively, the addition might indicate a strengthening from separate possibility to joint possibility. The assumption for *reductio* would then be that a magnitude can have been divided everywhere, rather than merely anywhere. Thus the assumption would be equivalent to

[66] See also *De caelo* 3. 4, $303^a3$–$^b8$.

[67] This view seems to be held, for example, by Sedley, '*On GC* I 2', 69, 73–4.

line 2 of [**P13**], rather than to line 1. The question which of these two interpretations should be preferred is important for understanding Aristotle's overall argumentative strategy in *GC* 1. 2; but for present purposes, we can leave the question open.

The second sentence of the passage indicates how the possibility rule is applied in the argument. The phrase 'it could come about' points towards an assumption for the possibility rule. The content of this assumption is that the magnitude has been 'divided at any point whatsoever', which presumably means that the magnitude has been divided at *every* point. After this, Aristotle infers the following consequences within the modal subordinate deduction:

> Then there will be nothing left, and the body will have perished into something bodiless, and it could come into being again either out of points or altogether out of nothing. But how is that possible? (*GC* 1. 2, 316$^{b}$25–7)

The first consequence is that nothing will be left; in the light of the rest of the sentence, this seems to mean that nothing bodily is left. The second consequence is that the magnitude has perished into something bodiless. This seems to be derived from the first consequence on the grounds that division entails perishing, and that what is left after something has been divided is that into which the thing has perished. As is clear from the rest of the sentence, Aristotle takes the 'something bodiless' into which the magnitude has perished to be either points or nothing at all. The third and last consequence drawn by Aristotle is that the magnitude could come into being again either out of points or out of nothing. This seems to be derived from the second consequence by way of a tacit premiss to the effect that processes of destruction are reversible into processes of coming into being. It is not clear whether Aristotle himself would accept such a premiss, but it may be a premiss which was accepted by the atomists, and which can therefore be relied upon in an argument for atomism.[68]

Aristotle thinks that the third consequence he has derived is impossible: it is impossible that a magnitude could come into being out of points or out of nothing. He makes this clear, at the end of the passage, through the rhetorical question 'but how is that possible?'. That it is impossible for a magnitude to come into being

[68] As Aristotle emphasizes in *GC* 1. 2 (315$^{b}$6–9, 316$^{b}$33–4), the atomists hold the view that perishing and coming into being consist in separation and aggregation respectively. This view may help to support the premiss under consideration, given that separation is reversible into aggregation.

out of nothing is guaranteed by a principle generally accepted in Aristotle's time, namely that nothing can come into being out of nothing.[69] And if it is impossible that a magnitude comes into being out of nothing, it is presumably also impossible that a magnitude *could* come into being out of nothing.[70] That it is impossible for a magnitude to come into being out of points can be taken to follow from the earlier argument, in points [xii]–[xiii], that it is impossible for a magnitude to be composed of points.[71] Given this, it is presumably also impossible that a magnitude *could* come into being out of points.

Aristotle's present argument can thus be reconstructed as follows:

| | | | |
|---|---|---|---|
| 1. | Magnitude M is simultaneously divisible everywhere | [assumption for *reductio*] | [**P14**] |
| 2. | It is possible that magnitude M has been divided everywhere | [from 1] | |
| 3. | Magnitude M has been divided everywhere | [assumption for possibility rule] | |
| 4. | Nothing bodily is left of magnitude M | [from 3] | |
| 5. | Magnitude M has perished into something bodiless, i.e. into points or into nothing | [from 3, 4, division is perishing] | |
| 6. | Magnitude M could come into being out of points or out of nothing | [from 5, reversibility of perishing] | |
| 7. | It is possible that magnitude M could come into being out of points or out of nothing | [possibility rule: 2, 3–6] | |
| 8. | It is not possible that magnitude M could come into being out of points or out of nothing | [premiss] | |
| 9. | Magnitude M is not simultaneously divisible everywhere | [*reductio*: 1–7, 8] | |

As mentioned earlier, the assumption for *reductio* in line 1 of this reconstruction is open to two different interpretations. If it is taken

[69] For this principle see *GC* 1. 3, $317^{b}$28–31; *Metaph. K* 6, $1062^{b}$24–6; *B* 4, $999^{b}$8; *Phys.* 1. 8, $191^{a}$30–1; *De caelo* 3. 2, $302^{a}$3–6.

[70] The latter statement involves a doubling of modalities, indicated by the phrases 'impossible' and 'could'; it is equivalent to 'it is impossible that it is possible that a magnitude comes into being out of nothing'. Aristotle does not discuss the issue of double modality in his works; cf. Barnes, *Truth*, 486. Nevertheless, it is plausible that he would agree that 'it is impossible that . . .' implies 'it is impossible that it is possible that . . .'.

[71] According to the atomists, all generation consists in aggregation (see n. 68). If something has come into being out of some items by aggregration, then it is composed out of these items. Hence, if a magnitude has come into being out of points, then the magnitude is composed out of points. Since the latter is impossible, so is the former.

to be equivalent to the assumption for *reductio* in the first argument (line 1 of [**P13**]), then, as explained above, the inference from line 1 to line 2 in [**P14**] is problematic. Aristotle's response in 317$^a$1–12 to the argument can then be understood as blocking this inference. On the other hand, if the assumption in line 1 of [**P14**] is taken to be already equivalent to the statement in line 2, then the present argument may well be acceptable to Aristotle, as far as it goes. His response to it can then be understood as an explanation of why the conclusion in line 9, being the contradictory of the assumption for *reductio*, does not establish the atomist thesis.[72]

## 6. *Metaphysics Θ* 4

Book *Θ* of the *Metaphysics* is concerned with various aspects of modality. It contains two applications of the possibility rule, both of them in chapter 4. The first occurs within a refutation of certain views about modality which are exemplified in the claim 'it is possible for the diagonal to have been measured but it will not be measured'. The second application occurs in a proof of the possibility principle (for this principle see Section 2 above).

Both applications of the possibility rule in *Θ* 4 build on a passage from *Θ* 3 in which Aristotle characterizes the notion of being capable, as follows:

> Something is capable [of something] if nothing impossible will obtain if the actuality of that of which it is said to have the capacity belongs to it. (*Metaph*. *Θ* 3, 1047$^a$24–6)

In this passage Aristotle characterizes being capable as opposed to being possible. Still, although the notions of capacity and possibility are not strictly the same, it seems clear that the passage bears on the latter notion as well as the former.[73] As we will see, Aristotle will appeal to this passage in order to justify his applications of the

[72] So the second argument for atomism either involves a fallacy or it does not establish the intended conclusion; see Philop. *In GC* 34. 12–35. 12 Vitelli (especially 34. 31–2 and 35. 10–12); P. S. Hasper, 'Aristotle's Diagnosis of Atomism', *Apeiron*, 39 (2006), 121–55 at 126.

[73] Compare the parallel passage at *Pr. An*. 1. 13, 32$^a$18–20, which clearly concerns possibility. Accordingly, Ross takes the passage at 1047$^a$24–6 to give 'a criterion for the determination of possibility' (W. D. Ross (ed. and comm.), *Aristotle's* Metaphysics*: A Revised Text with Introduction and Commentary* [*Metaphysics*], 2 vols. (Oxford, 1924), ii. 245). Beere also takes the passage to give a 'criterion of possibi-

possibility rule in $\Theta$ 4, and this shows that the passage is intended to cover the notion of possibility which figures in that rule. In the present passage, the notion of capacity can be taken to be used in such a way that something is capable of $\phi$ing just in case it is possible for the thing to $\phi$.[74] If so, then Aristotle would in effect be saying that it is possible for something to $\phi$ just in case nothing impossible will follow if it $\phi$s. This comes close to a statement of the possibility rule.

### (*a*) *It is not possible to measure the diagonal* (1047$^b$3–14)

Chapter $\Theta$ 4 opens as follows:

> [i] If what we have stated is the possible or follows from it, [ii] it is clear that it cannot be true to say that this is possible but will not be, [iii] in such a way that what is incapable of being thereby disappears. (*Metaph.* $\Theta$ 4, 1047$^b$3–6)

Aristotle begins, in point [i], by referring back to the passage from $\Theta$ 3 just discussed.[75] He takes this passage to justify him in denying, in [ii], the truth of certain claims about possibility. Commentators disagree about precisely which claims these are. One prominent position is that Aristotle is denying the truth of any claim of the form 'A is possible but will never be'. Adherents of this position fall into two main groups. One group holds that in [ii] Aristotle is asserting the so-called 'principle of plenitude', namely that every possibility is realized at some time.[76] The other group thinks instead that Aristotle's statement in [ii] is relying on a certain view

lity', while he translates δυνατόν by 'able' in it (J. Beere, *Doing and Being: An Interpretation of Aristotle's* Metaphysics *Theta* [*Doing and Being*] (Oxford, 2009), 119). Makin even translates δυνατόν by 'possible' in the passage, although this is problematic for linguistic reasons (S. Makin (trans. and comm.), *Aristotle:* Metaphysics *Book Θ* [*Metaphysics Θ*] (Oxford, 2006), 4 and 72–4).

[74] This is not to claim that Aristotle always uses the notion of capacity in this way. In some contexts he might hold that someone is capable of acting generously although circumstances make it impossible for him to act generously, or conversely that it is possible for a man to speak Persian although he has not yet learnt Persian and therefore does not have a capacity to speak it. See Beere, *Doing and Being*, 50 and 241.

[75] See E. Zeller, 'Ueber den κυριεύων des Megarikers Diodorus', *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin* (1882), 151–9 at 156; Ross, *Metaphysics*, ii. 247; Makin, *Metaphysics Θ*, 83.

[76] Ps.-Alex. Aphr. *In Met.* 575. 20–3 Hayduck; Ross, *Metaphysics*, ii. 246–7; Hintikka, *Time & Necessity*, 107–9; G. Seel, *Die Aristotelische Modaltheorie* (Berlin and New York, 1982), 337–8; W. Detel (trans. and comm.), *Aristoteles: Analytica Posteriora*, 2 vols. (Berlin, 1993), i. 186.

about truth and tense, according to which a future-tense sentence is neither true nor false at a given time unless it is already necessary or impossible at that time.[77] On this view, if it is true at some time that A will not be, then it is impossible at that time that A will be; this would explain why it cannot simultaneously be true that A is possible and be true that A will not be. According to both groups of commentators, the clause 'what is incapable of being thereby disappears' in point [iii] states an unacceptable consequence of asserting any claim of the form 'A is possible but will never be'. This interpretation is evidenced, for example, in Barnes's revised Oxford translation of our passage:

> It cannot be true to say 'this is capable of being but will not be',—a view which leads to the conclusion that there is nothing incapable of being.[78]

There are a number of problems with the interpretation on which Aristotle asserts the principle of plenitude in [ii].[79] In general, it is questionable whether he would endorse such a strong principle in an unqualified way. A more specific difficulty concerns the context in which points [i]–[iii] occur. Aristotle will give only one example of the kind of claim whose truth he denies in [ii]. As we will see, he chooses a mathematical example, namely a claim to the effect that the measurement of a square's diagonal is possible but will never occur (1047$^b$6–7). Aristotle will show why this claim cannot be true, but showing this does not go far towards establishing the principle of plenitude; for it is obvious, and would be agreed even by opponents of the principle, that it is impossible for the diagonal of a square to be measured. If Aristotle were concerned to establish

[77] H. Weidemann, 'Das sogenannte Meisterargument des Diodoros Kronos und der Aristotelische Möglichkeitsbegriff' ['Möglichkeitsbegriff'], *Archiv für Geschichte der Philosophie*, 69 (1987), 18–53 at 35–6; Beere, *Doing and Being*, 126–7. Similarly U. Wolf, *Möglichkeit und Notwendigkeit bei Aristoteles und heute* [*Möglichkeit und Notwendigkeit*] (Munich, 1979), 119–20.

[78] Barnes (ed.), *Complete Works*, ii. 1654.

[79] Commentators who reject this interpretation include Owen and Kneale (cf. n. 81 below); J. Kung, '*Metaphysics* 8. 4: Can Be But Will Not Be' ['Can Be But Will Not Be'], *Apeiron*, 12 (1978), 32–6; Wolf, *Möglichkeit und Notwendigkeit*, 118–20; Sorabji, *Necessity, Cause and Blame*, 136; R. T. McClelland, 'Time and Modality in Aristotle, *Metaphysics* IX. 3–4' ['Time and Modality'], *Archiv für Geschichte der Philosophie*, 63 (1981), 130–49 at 146–8; Weidemann, 'Möglichkeitsbegriff', 35 n. 44; van Rijen, *Logic of Modalities*, 67–9; M.-T. Liske, 'In welcher Weise hängen Modalbegriffe und Zeitbegriffe bei Aristoteles zusammen?', *Zeitschrift für philosophische Forschung*, 49 (1995), 351–77 at 368–70; R. Gaskin, *The Sea Battle and the Master Argument* (Berlin and New York, 1995), 76; Beere, *Doing and Being*, 125–6.

the principle of plenitude, he should rather have chosen an example concerning some apparently possible state of affairs, for instance, a claim to the effect that a given cloak's being cut up is possible but will never occur. Showing that such a claim cannot be true would go further towards establishing that every possibility is realized at some time. Thus, as Joan Kung has pointed out, Aristotle's choice of example speaks against the interpretation on which he is seeking to establish the principle of plenitude in *Θ* 4.[80] Kung also offers a series of further effective objections to this interpretation.

There are also problems with the other interpretation, according to which Aristotle's statement in [ii] relies on the view that contingent future-tense sentences lack a truth-value. Aristotle seems to express such a view in *De interpretatione* 9, but he does not refer to it elsewhere in his writings, and it seems unlikely that he would appeal to this rather technical thesis in *Metaphysics Θ* 4 without indicating it more explicitly. A further problem for this interpretation is to explain how the appeal in point [i] to the above passage from *Θ* 3 is relevant to Aristotle's assertion in point [ii].

In sum, then, it is difficult to maintain the position that Aristotle is denying in [ii] the truth of all claims of the form 'A is possible but will never be'.

There is another prominent position, which we prefer, according to which Aristotle in [ii] is not denying the truth of all claims of this form, but only of a certain restricted subclass of them. The appropriate restriction is given in [iii]: Aristotle is denying the truth only of those claims of the form 'A is possible but will never be' which lead to the consequence that, as he puts it, 'what is incapable of being disappears'.[81] Aristotle does not explain exactly what this consequence is, but he evidently regards it as undesirable. The consequence seems to consist in denying the impossibility of things which Aristotle regards as obviously impossible. Some claims of the form 'A is possible but will never be' do not lead to this kind of consequence, for example, 'this cloak's being cut up is possible but will never occur'. Other claims do lead to this kind of consequence, and Aristotle goes on to give an example of such a claim:

[80] Kung, 'Can Be But Will Not Be', 33–4.

[81] This interpretation is presupposed by our translation of point [iii]. It is endorsed by G. E. L. Owen and M. Kneale (as reported in Hintikka, *Time & Necessity*, 107–8); Kung, 'Can Be But Will Not Be'; McClelland, 'Time and Modality', 146–7.

[iv] I mean, for example, if someone should claim that it is possible for the diagonal to have been measured [μετρηθῆναι] but that it will not be measured. (*Metaph.* Θ 4, 1047$^{b}$6–7)

This example concerns the diagonal of a square. Saying in this context that the diagonal is measured means that it is measured by a magnitude which also measures the side of the square, i.e. that the diagonal and the side are commensurate. In Aristotle's time it was already a well-known theorem that they are not commensurate. Indeed, for the diagonal to be commensurate with the side is one of Aristotle's standard examples of something which is impossible.[82] Hence, the claim that the diagonal's having been measured is possible but will never occur denies the impossibility of something which for Aristotle is obviously impossible. The claim thereby has the consequence described in [iii], that 'what is incapable of being disappears'.

Now, when he formulates the claim in point [iv], Aristotle uses the aorist form of 'measure' (μετρηθῆναι). The aorist forms of this verb are unusual in mathematical contexts. Aristotle's use of the form here seems to have some significance, and we will shortly consider the question of what its significance may be. In order to make the aorist form recognizable in our translation, in distinction from the present-tense form to be encountered in point [viii] below, we have chosen 'to have been measured' as a reasonable approximation.

Aristotle's refutation of the claim introduced in point [iv] consists in showing that it is not possible for the diagonal to have been measured. His argument contains an application of the possibility rule, as follows:

[v] But the following is necessary given what has been laid down: [vi] that if we assume that something which is not, but is possible, is or has come to be, then there will be nothing impossible. [vii] But something impossible will indeed result, [viii] since it is impossible for the diagonal to be measured [μετρεῖσθαι]. (*Metaph.* Θ 4, 1047$^{b}$9–12)

This argument proceeds by *reductio*, beginning from the assumption that it is possible for the diagonal to have been measured. In [v] Aristotle refers back to the characterization of 'being capable' from Θ 3 discussed above.[83] Based on that characterization, he affirms

[82] See e.g. *Phys.* 4. 12, 221$^{b}$23–5; *De caelo* 1. 11, 281$^{a}$4–7; 1. 12, 281$^{b}$12–14; *Metaph.* Δ 12, 1019$^{b}$24–7; *Rhet.* 2. 19, 1392$^{a}$16–18.

[83] Θ 3, 1047$^{a}$24–6; see Makin, *Metaphysics Θ*, 83.

the possibility rule in point [vi]; the phrase 'we assume' (ὑποθοίμεθα) indicates an assumption for the possibility rule. Given what the assumption for *reductio* was, the assumption for the possibility rule should be that the diagonal has been measured.

In [vii] Aristotle indicates that the conclusion of the modal subordinate deduction is impossible. Point [viii] may be read as specifying what this conclusion is. On this reading, the conclusion of the modal subordinate deduction is the statement that the diagonal is measured, formulated by means of the present tense (μετρεῖσθαι). (We will consider an alternative reading of [viii] later.) Aristotle's argument in points [iv]–[viii] can then be reconstructed as follows:

| | | | |
|---|---|---|---|
| 1. | It is possible that the diagonal has been measured | [assumption for *reductio*] | [**P15**] |
| 2. | The diagonal has been measured | [assumption for possibility rule] | |
| 3. | The diagonal is measured | [from 2] | |
| 4. | It is possible that the diagonal is measured | [possibility rule: 1, 2–3] | |
| 5. | It is not possible that the diagonal is measured | [premiss] | |
| 6. | It is not possible that the diagonal has been measured | [*reductio*: 1–4, 5] | |

There are three closely related questions which we need to address in connection with this reconstruction. First, what is the relevant difference between the assumption for the possibility rule in line 2 and the conclusion of the modal subordinate deduction in line 3? Second, how is the latter inferred from the former? And third, how is the premiss in line 5 justified?

Let us begin with the first question. Line 2 contains the phrase 'has been measured', representing Aristotle's aorist form, while line 3 contains the phrase 'is measured', representing his present-tense form. Our proposal in brief will be that 'is measured' expresses a mathematical state of affairs, in which two things are commensurate, whereas 'has been measured' expresses the complete occurrence of a process in which these things have become commensurate (or become known to be commensurate). The proposal requires some justification.

In mathematical contexts, such as in Euclid's *Elements*, the verb 'measure' is typically used in present- and future-tense forms. In the *Elements* these forms express claims to the effect that one magnitude measures another magnitude. This means that the two magnitudes stand in a certain mathematical relation, roughly the relation of the one's being divisible by the other without remainder. Accordingly, Aristotle's present-tense statement that the diagonal is measured, in line 3, seems to assert that there is a magnitude which stands in the mathematical relation of measuring to the diagonal. More particularly, the intended assertion is that some magnitude stands in this relation both to the diagonal and to the side.

How does the aorist form 'has been measured' relate to such a present-tense statement? Aristotle elsewhere discusses two sorts of pairing between aorist and present-tense verbs: one is exemplified by the pair 'has become F'—'is becoming F' (*γενέσθαι—γίγνεσθαι*), the other by the pair 'has become F'—'is F' (*γενέσθαι—εἶναι*).[84] In both pairings the aorist form expresses something like the complete occurrence of a process; this is paired either with a form expressing the current going-on of the process (*γίγνεσθαι*), or with a form expressing that the state of affairs resulting from the process obtains (*εἶναι*). We have just seen reason to think that the present-tense form 'is measured' in *Θ* 4, like the present-tense forms in Euclid, expresses the obtaining of a state of affairs, rather than the going-on of a process. It expresses the existence of a magnitude standing in the relation of measuring to diagonal and to side. Accordingly, it seems natural to understand the aorist form 'has been measured' as expressing the complete occurrence of a process resulting in the existence of such a magnitude. Thus, where 'be measured' is equivalent to 'be commensurate', 'have been measured' can be regarded as equivalent to 'have become commensurate' (*μετρεῖσθαι* is equivalent to *σύμμετρον εἶναι*, and *μετρηθῆναι* to *σύμμετρον γενέσθαι*).

Before turning to the second question, we should add a qualification concerning the nature of the process referred to by the aorist form 'has been measured'. The view we take of this process will depend on our views about the nature of mathematical objects. If mathematical magnitudes do not exist always, but only when they have been constructed, the aorist form can truly refer to the coming into being of a common measure. On the other hand, if magnitudes exist always or atemporally, and can be discovered but not created,

[84] Aristotle subjects the first pair to the principle that if becoming F is possible, then having become F is possible; see *Phys.* 6. 10, 241b3–11, *Metaph.* *B* 4, 999b11. The second pair is subject to the principle that when something has become F, then it is F; see *Pr. An.* 1. 15, 34a11–12.

then it is not possible for such a common measure to come into being. In this case, the aorist form might instead be taken to describe a process by which we gain knowledge of a common measure's existence. Regardless of which interpretation is preferred, the process in question can be thought of as one of construction, directed at either the creation or the discovery of a common measure. An example would be a process of successive approximation, in which a series of magnitudes is constructed which come ever closer to measuring both the diagonal and the side. Given that the diagonal and the side are in fact incommensurate, any such process would go on to infinity without ever being completed. Aristotle's argument in points [iv]–[viii] would concern a person who nevertheless claims that it is possible for this process to occur completely, although she agrees that it never will.

We are now in a position to answer the second question, how line 3 of [**P15**] is inferred from line 2. It is obvious that as soon as a process has occurred in its entirety, the state of affairs which results from it obtains.[85] So, as soon as the aorist statement in line 2 is true, the present-tense statement in line 3 is also true.[86] This explains how the latter statement is inferred from the former.[87] Claims using the present-tense form of 'measure' are typically dealt with by mathematicians and are the subject of mathematical theorems, whereas this is not true for claims using the aorist form. Aristotle's inference from line 2 to line 3 effects a transition from the one kind of claim to the other.

The third question we want to discuss concerns the premiss in line 5 of [**P15**], that the present-tense statement 'the diagonal is measured' is impossible. How is this premiss justified? Aristotle was aware of a theorem to the effect that the diagonal and the side are not commensurate:

[85] See *Pr. An.* 1. 15, 34$^a$11–12; *Phys.* 6. 5, 235$^b$7–8, 30–2.

[86] If the process results in knowledge of the existence of a common measure, then as soon as the aorist statement is true, the present tense statement is *known* to be true, and, since knowledge is factive, is true.

[87] To show properly that the statement in line 3 follows from the statement in line 2, one would need to show that the former is true *whenever* the latter is true. We have not done this. However, we could construe Aristotle's argument in such a way that line 3 contains the statement 'the diagonal is measured at some time' instead of 'the diagonal is measured', and we have shown that this follows. The premiss in line 5 would then need to be modified to read 'it is not possible that the diagonal is measured at some time'. The temporal qualification 'at some time' could also be added throughout the proof; cf. n. 64 above.

For example, we prove that the diagonal is incommensurate through its resulting that odd numbers are equal to even numbers when the diagonal is posited to be commensurate. (*Pr. An.* 1. 23, $41^a26$–7)

In this passage Aristotle uses the adjectives 'commensurate' and 'incommensurate' rather than the present-tense form 'is measured'. However, as explained above, the present tense statement 'A is measured by something by which B is measured' is equivalent to the statement 'A and B are commensurate'.[88] Thus, the statement that the diagonal and the side are not commensurate is equivalent to the statement that the diagonal is not measured (sc. by a magnitude by which the side is also measured). The first of these two statements has for Aristotle the status of a theorem. Since theorems are true of necessity, it follows that the first, and hence also the second, statement is true of necessity.[89] Thus, it is necessary that the diagonal is not measured, and this justifies Aristotle's premiss in line 5.

We have now finished discussing the above reconstruction of Aristotle's argument. Before we move on to the second part of *Metaphysics Θ* 4, we should mention the availability of a slightly different reconstruction of this argument. The alternative reconstruction is based on a different reading of point [viii], 'since it is impossible for the diagonal to be measured'. Until now we have been taking this to specify the conclusion of the modal subordinate deduction, and to state that this conclusion is impossible. However, one could instead take point [viii] merely to indicate a reason for thinking that an impossible conclusion can be derived within the modal subordinate deduction, without specifying what this conclusion is. The conclusion of the modal subordinate deduction might then be taken to be the consequence indicated in the above passage from *Prior Analytics* 1. 23, namely that odd numbers are equal to even numbers. This presumably means that some number is both odd and even, and Aristotle's argument would then rely on the premiss that it is impossible for a number to be both odd and even.[90] This yields the following reconstruction:

[88] This equivalence is confirmed by Euclid's usage. For example, he moves freely between the phrases 'some magnitude measures A and B' and 'A and B are commensurate'; see Euclid, *Elements* 10. 4, 14–18. By contrast, the aorist statement 'A has been measured by something by which B has been measured' would be equivalent to 'A and B have become commensurate'.

[89] See pp. 208–9 above.

[90] Euclid, *Elements* 7, definitions 6–7, defines an even number as one which is divisible into two equal parts, and an odd number as one which is not divisible into two

| | | | |
|---|---|---|---|
| 1. | It is possible that the diagonal has been measured | [assumption for *reductio*] | [**P16**] |
| 2. | The diagonal has been measured | [assumption for possibility rule] | |
| 3. | The diagonal is measured | [from 2] | |
| 4. | Odd numbers are equal to even numbers | [from 3, by mathematics] | |
| 5. | It is possible that odd numbers are equal to even numbers | [possibility rule: 1, 2–4] | |
| 6. | It is not possible that odd numbers are equal to even numbers | [premiss] | |
| 7. | It is not possible that the diagonal has been measured | [*reductio*: 1–5, 6] | |

The inference from line 3 to line 4 relies on several axioms and theorems of mathematics.[91] Each of them could be taken to be introduced as a premiss before the *reductio* begins in line 1, and then to be iterated into the modal subordinate deduction. Since mathematical theorems and axioms are true of necessity, these iterations would be permissible. So, whether we prefer the present reconstruction or the earlier reconstruction given in [**P15**], Aristotle's argument is free from the difficulties with unjustified iteration which we encountered in *De caelo* 1. 12 and *Physics* 7. 1.

### (*b*) *A proof of the possibility principle* (1047$^{b}$14–26)

In the second part of *Metaphysics Θ* 4 Aristotle is concerned with what we have called the possibility principle. As we saw above in Section 2, Aristotle uses the possibility principle in *Prior Analytics* 1. 15 to justify the possibility rule. His argument in *Metaphysics Θ* 4 is exactly the other way around, using the rule to prove the principle. Aristotle's proof is rather complicated. Before examining the text of the proof, it will therefore be helpful to discuss in general how the possibility rule can be used to prove the principle, independently of Aristotle's text. After that, we will be in a better position to understand how Aristotle's own proof proceeds.

First, recall that the possibility principle is the following:

If $A \Rightarrow B$ then $\text{Poss}(A) \Rightarrow \text{Poss}(B)$

equal parts. Since the one *definiens* is contradictory to the other, the principle of non-contradiction guarantees that it is impossible for a number to be both odd and even.

[91] For a justification of this inference, based on the Pythagorean theorem, see Euclid, *Elements* (*demonstrationes alterae*) 10. 27.

The conditional $A \Rightarrow B$ corresponds to such phrases in Aristotle as 'it is necessary for B to be when A is'. It expresses that B is a necessary consequence of A, or, for short, that B follows from A. Aristotle's indicative conditional 'if . . . then . . .' in the possibility principle might be taken also to convey necessary consequence; or alternatively it may be understood as a weaker kind of conditional. For our purposes it is not necessary to decide the question; we will assume no more than that 'if . . . then . . .' is implied by '$\Rightarrow$'.

In order to construct a proof of the possibility principle, the following rule of conditionalization will be helpful:

> Conditionalization rule: Given a subordinate deduction of B from A, you may infer $A \Rightarrow B$.

As in the case of the possibility rule, there are restrictions on the kinds of statement that may be iterated into the subordinate deduction of B from A. Unrestricted iteration is not allowed, but iteration is permitted if there is a guarantee that the statement iterated is true of necessity. For if B is deduced from A combined with contingent statements, this does not show that B is a necessary consequence of A; whereas if A is combined only with statements that are true of necessity, B is shown to be a necessary consequence of A.

With the rule of conditionalization in place, it is not difficult to prove the possibility principle, as follows:

| | | | |
|---|---|---|---|
| 1. | $A \Rightarrow B$ | [assumption for conditionalization] | **[P17]** |
| 2. | Poss(A) | [assumption for conditionalization] | |
| 3. | A | [assumption for possibility rule] | |
| 4. | $A \Rightarrow B$ | [iterated from 1] | |
| 5. | B | [from 3, 4] | |
| 6. | Poss(B) | [possibility rule: 2, 3–5] | |
| 7. | Poss(A) $\Rightarrow$ Poss(B) | [conditionalization: 2–6] | |
| 8. | If $A \Rightarrow B$ then Poss(A) $\Rightarrow$ Poss(B) | [conditionalization: 1–7] | |

In line 8 of this proof the conditionalization rule is used to introduce the connective 'if . . . then . . .' rather than '$\Rightarrow$'. This is justified given our assumption that the former is implied by the latter.

We must also briefly comment on the iteration of $A \Rightarrow B$ in line 4. This move should be thought of as consisting of two iterations, the first into the subordinate deduction that extends from line 2 to line 6, and the second from there into the modal subordinate deduction. Each of these two iterations is justified if there is a guarantee that the statement iterated is true of necessity. Now it is reasonable to think that if B is a necessary consequence of A, then it is necessary that B is a necessary consequence of A.[92] This, combined with the presence of $A \Rightarrow B$ in line 1, serves as a guarantee that $A \Rightarrow B$ is true of necessity. Hence, both iterations can be justified.

Now, instead of giving a direct proof such as the one given in [**P17**], Aristotle proves the possibility principle by *reductio*, in accordance with his general practice of always embedding the possibility rule within a *reductio*. A proof by *reductio* of the possibility principle naturally begins as follows:

| | | | |
|---|---|---|---|
| 1. | │ $A \Rightarrow B$ | [assumption for conditionalization] | [**P18**] |
| 2. | │ │ Poss(A) | [assumption for conditionalization] | |
| 3. | │ │ │ Not Poss(B) | [assumption for *reductio*] | |

In order to complete the *reductio* initiated in line 3, it suffices to derive 'Not Poss(A)' within the *reductio* subordinate deduction. The easiest way to derive this, in turn, is to embed a second *reductio* initiated by the assumption Poss(A). This leads to the following deeply nested proof:

| | | | |
|---|---|---|---|
| 1. | │ $A \Rightarrow B$ | [assumption for conditionalization] | [**P19**] |
| 2. | │ │ Poss(A) | [assumption for conditionalization] | |
| 3. | │ │ │ Not Poss(B) | [assumption for *reductio*] | |
| 4. | │ │ │ │ Poss(A) | [assumption for *reductio*] | |
| 5. | │ │ │ │ │ A | [assumption for possibility rule] | |
| 6. | │ │ │ │ │ $A \Rightarrow B$ | [iterated from 1] | |
| 7. | │ │ │ │ │ B | [from 5, 6] | |
| 8. | │ │ │ │ Poss(B) | [possibility rule: 4, 5–7] | |
| 9. | │ │ │ Not Poss(A) | [*reductio*: 3, 4–8] | |
| 10. | │ │ Poss(B) | [*reductio*: 2, 3–9] | |
| 11. | │ Poss(A) $\Rightarrow$ Poss(B) | [conditionalization: 2–10] | |
| 12. | If $A \Rightarrow B$ then Poss(A) $\Rightarrow$ Poss(B) | [conditionalization: 1–11] | |

Although [**P19**] is a rather complex proof, it is a natural outcome of the decision to employ *reductio* in proving the possibility principle by means of the possibility rule. Moreover, when we turn to Aristotle's own proof of the possibility principle, we will see that his presentation matches [**P19**] quite well. So let us finally turn to Aristotle's text:

[i] At the same time it is clear that if it is necessary for B to be when A is,

[92] It is reasonable to think that Aristotle would agree that if something is necessary, then it is necessary that it is necessary; see n. 70 above.

then it is also necessary for B to be possible when A is possible. [ii] For if it is not necessary for B to be possible, nothing prevents its not being possible. [iii] Let A be possible. [iv] Then, when A is possible, if A should be posited, nothing impossible was to follow; [v] but it would be necessary for B to be. [vi] But B was impossible. [vii] Let it be impossible. [viii] If B is impossible, it is necessary for A also to be impossible.[93] [ix] But the first was impossible; [x] therefore the second also. [xi] If, then, A is possible, B also will be possible, [xii] if indeed they were so related that it is necessary for B to be when A is. [xiii] If, A and B being thus related, B is not possible on this condition, then neither will A and B be related as was posited. (*Metaph.* *Θ* 4, 1047$^{b}$14–26)

This text has presented commentators with difficulties. It is not easy to understand the individual steps made by Aristotle as forming a single coherent argument. For example, Makin points out that Aristotle's proof seems to be completed already in point [vi], and that it is difficult to see why he continues his presentation beyond this point.[94] Burnyeat *et al*. find the argument as a whole 'of little value as a proof' because it appears to them to be circular.[95] Brennan likewise suspects that 'no interpretation can render it non-circular and valid'.[96] However, Aristotle's argument can be defended against these charges. The charge of circularity has been answered by Kit Fine, who explains that Aristotle does not here establish the possibility principle by means of itself but by means of the possibility *rule*, and that his proof therefore is not circular.[97] Furthermore, as we will try to show now, the whole of Aristotle's text can be interpreted in such a way that each individual step in it contributes to the presentation of a single, valid argument. In showing this, we will at the same time show that the formal proof given in [**P19**] constitutes a satisfactory reconstruction of Aristotle's argument.

Aristotle begins in point [i] by stating the possibility principle,

[93] Point [viii] incorporates a widely accepted emendation due to H. Bonitz.

[94] Makin, *Metaphysics Θ*, 94.

[95] M. Burnyeat *et al*., *Notes on Eta and Theta of Aristotle's* Metaphysics (Oxford, 1984), 110.

[96] T. Brennan, 'Two Modal Theses in the Second Half of *Metaphysics* Theta. 4', *Phronesis*, 39 (1994), 160–73 at 161 n. 5.

[97] Fine, 'Megarian Maneuvers', 17. As we saw in Section 2, Aristotle uses the possibility principle to jusify the possibility rule in *Pr. An*. 1. 15, whereas his argument in *Metaph*. *Θ* 4 proceeds the other way round. The presence of these two arguments running in opposite directions may leave us wondering whether the principle or the rule is more basic; but this does not render either argument circular on its own.

which is the intended conclusion of his proof. In [ii] he introduces an assumption for *reductio*, namely that B is not possible. This assumption should be taken as being made under the assumptions that B follows from A and that A is possible. Hence we take point [ii] to correspond to the assumption for *reductio* in line 3 of our reconstruction in [**P19**], and implicitly to contain also lines 1 and 2.

The conclusion of the *reductio* subordinate deduction initiated in [ii] will be that A is not possible. As explained above, the easiest way to reach this conclusion is to make a second assumption for *reductio* to the effect that A is possible. Aristotle can be seen to make this assumption in point [iii], which thus corresponds to line 4 in our reconstruction. Next, in point [iv], Aristotle reminds us of the possibility rule.[98] He thereby also signals that he is making an assumption for the possibility rule to the effect that A is the case, corresponding to line 5 in the reconstruction. This assumption marks the beginning of the modal subordinate deduction. In [v] Aristotle infers within the modal subordinate deduction that B is the case, corresponding to line 7. This inference obviously relies on the earlier assumption that B follows from A. Thus Aristotle uses this assumption within the modal subordinate deduction; in our reconstruction, this is represented by the iteration in line 6 of $A \Rightarrow B$ from line 1.

The possibility rule now allows us to infer in line 8 that B is possible. As in his other applications of the possibility rule, Aristotle does not perform this step explicitly. Instead, he immediately asserts that the conclusion of the modal subordinate deduction is impossible. Thus, he asserts in point [vi] that B is impossible, which is a restatement of the assumption for *reductio* in line 3. This puts him in a position to conclude the inner *reductio*, initiated in line 4, by inferring in line 9 that A is not possible. Aristotle does not explicitly perform this inference, but we can understand point [vi] as saying 'but B is impossible, therefore A also is impossible'.

We have now traced Aristotle's argument up to point [vi] of the text, corresponding to lines 1–9 of [**P19**]. Aristotle's text from now on becomes less straightforward. In point [vii] he appears to make

[98] The past tense 'was to follow' (*συνέβαινεν*) in [iv] indicates that Aristotle is referring back to something previously established; see Ross's translation of this passage in W. D. Ross (trans.), *Metaphysica*, 2nd edn. (The Works of Aristotle, 8; Oxford, 1928): 'Then, when A was possible, we agreed that nothing impossible followed if A were supposed to be real.' The reference is to the statement of the possibility rule in $\Theta$ 4, 1047$^b$9–11 (which in turn refers back to $\Theta$ 3, 1047$^a$24–6; see p. 229 above).

an assumption to the effect that B is impossible. However, it is not easy to see what the function of such an assumption would be; according to our reconstruction, there is no need to introduce any new assumptions at this stage. Therefore we propose that in point [vii] Aristotle is not introducing a new assumption, but is merely restating the earlier assumption for *reductio* made in line 3, that B is impossible. This restatement allows him, in [viii], to summarize the *reductio* subordinate deduction from Not Poss(B) to Not Poss(A) in lines 3–9. He summarizes this deduction by way of the conditional statement 'if B is impossible, it is necessary for A also to be impossible'. Then, in [ix], he emphasizes that the antecedent of this conditional has actually been assumed in the course of his proof, namely in line 3. Correspondingly, as he reminds us in point [x], we have also derived the consequent of the conditional, namely in line 9. This is the first time that Aristotle explicitly mentions the statement in line 9 that A is not possible. So, in points [vii]–[x], Aristotle is reviewing the course of argument which ran from the assumption for *reductio* in line 3 to the conclusion of the *reductio* subordinate deduction in line 9.

The statement in line 9 contradicts the assumption for conditionalization in line 2. Thus, in line 10 we are in a position to infer by *reductio* that B is possible. By conditionalization, this yields the statement Poss(A)$\Rightarrow$Poss(B) in line 11. Aristotle's remark in point [xi], 'if A is possible, B also will be possible', expresses this statement. At the same time, point [xi] can be understood as a summary of the subordinate deduction extending from the assumption in line 2 that A is possible to the conclusion in line 10 that B is possible. Now, in point [xii], Aristotle reminds us of the fact that the statement in line 11 relies on the assumption made in line 1 that B follows from A. Hence points [xi] and [xii] taken together serve as a summary of the subordinate deduction extending from line 1 to line 11.

Finally, given the deduction extending from line 1 to line 11, the rule of conditionalization can be used to infer the possibility principle in line 12. Aristotle has already announced this principle in point [i], and can also be taken to state it, although in a somewhat complex way, in point [xiii].[99]

[99] The phrase 'A and B being thus related' in point [xiii] means that B follows from A. The phrase 'on this condition' (*οὕτως*) in [xiii] means 'on the condition that A is possible'; see Ross, *Metaphysics*, ii. 248. Thus 'B is not possible on this condition' can be taken to mean that the possibility of B does not follow from the possibility of

Aristotle's presentation from point [vii] onwards does not follow the order of lines in our reconstruction of his proof. Nevertheless, his presentation is not haphazard. Rather, we can see him working his way outwards through the nested layers of the proof given in [**P19**]. In points [vii]–[x] he summarizes the subordinate deduction in lines 3–9; in point [xi] he moves one layer out, summarizing the subordinate deduction in lines 2–10; and in points [xi] and [xii] taken together he summarizes the outermost subordinate deduction, in lines 1–11. In each case he summarizes the subordinate deduction by reminding us of its first and last lines. Hence, we obtain a satisfying correspondence between Aristotle's text and the reconstruction given in [**P19**].

This reconstruction is indebted to the interpretation of Aristotle's proof given by Kit Fine.[100] We would like to conclude our discussion of *Metaphysics Θ* 4 by comparing our reconstruction with the one that Fine offers. Setting aside some inessential differences between his logical framework and ours, Fine's reconstruction can be presented as in proof [**P20**] below. This proof is longer, but essentially similar to our proof in [**P19**]. The similarity can be exhibited in terms of a process of simplification by which Fine's proof could be transformed into ours. As a first step, one could do away with the outer *reductio* by removing line 2 of [**P20**] and line 16 in Fine's proof, and shifting lines 3–15 to the left. The resulting proof is still valid. Thus the assumption for *reductio* in line 2 is superfluous, and Fine probably included it only because Aristotle's phrasing in point [ii] of his presentation suggests that the argument involves an assumption for *reductio* at this stage.

As a second step, one could remove line 9 in Fine's proof, in which 'Not Poss(B)' is iterated from line 3. Fine works with a *reductio* rule differing slightly from ours (we label it '*reductio**'). His rule requires a pair of incompatible statements to appear within the *reductio* subordinate deduction. Ours requires instead an incompatibility between the conclusion of the *reductio* subordinate deduction and a statement occurring outside of this subordinate deduction.

A. Finally, 'neither will A and B be related as was assumed' means that B does not follow from A. So, the sentence in point [xiii] can be taken to have the form 'if p and not q, then not p', with 'p' standing for 'B follows from A', and 'q' standing for 'the possibility of B follows from the possibility of A'. This is equivalent to 'if p then q', which is the possibility principle.

[100] Fine, 'Megarian Maneuvers', 16–20.

| | | | |
|---|---|---|---|
| 1. | A ⇒ B | [assumption for conditionalization] | [**P20**] |
| 2. | Not (Poss(A) ⇒ Poss(B)) | [assumption for *reductio**] | |
| 3. | Not Poss(B) | [assumption for conditionalization] | |
| 4. | Poss(A) | [assumption for *reductio**] | |
| 5. | A | [assumption for possibility rule] | |
| 6. | A ⇒ B | [iterated from 1] | |
| 7. | B | [from 5, 6] | |
| 8. | Poss(B) | [possibility rule: 4, 5–7] | |
| 9. | Not Poss(B) | [iterated from 3] | |
| 10. | Not Poss(A) | [*reductio**: 4, 8, 9] | |
| 11. | Not Poss(B) ⇒ Not Poss(A) | [conditionalization: 3–10] | |
| 12. | Poss(A) | [assumption for conditionalization] | |
| 13. | Not Poss(B) ⇒ Not Poss(A) | [iterated from 11] | |
| 14. | Poss(B) | [from 12, 13] | |
| 15. | Poss(A) ⇒ Poss(B) | [conditionalization: 12–14] | |
| 16. | Poss(A) ⇒ Poss(B) | [*reductio**: 2, 15] | |
| 17. | If A ⇒ B then Poss(A) ⇒ Poss(B) | [conditionalization: 1–16] | |

Hence, if we use our *reductio* rule, we can invoke the contradiction between lines 8 and 3 in Fine's proof to infer line 10. Line 9 can then be omitted.

Three lines have now been removed from Fine's proof (lines 2, 9, 16), leaving it only two lines longer than ours. Now, lines 11 and 13 in Fine's proof contain the statement 'Not Poss(B) ⇒ Not Poss(A)', which does not occur in our proof. Fine derives this statement in line 11 based on the subordinate deduction in lines 3–10, and he goes on to use it (iterated in line 13) to effect the inference from line 12 to line 14. But we could instead include lines 3–10 themselves as a subordinate deduction between lines 12 and 14. In this case, the statement 'Not Poss(B)' in line 3 would need to be an assumption for *reductio* instead of an assumption for conditionalization. Lines 11 and 13 can then be removed. The result of this final transformation is our reconstruction in [**P19**].

In essence, then, Fine's reconstruction and ours are very similar. Still, we think our reconstruction avoids two difficulties faced by Fine's as an interpretation of Aristotle's text. One of them concerns his *reductio* inference of Poss(A) ⇒ Poss(B) in line 16 of [**P20**]. We have already mentioned that this *reductio* is superfluous. Of course, the two applications of *reductio* in [**P19**] are also superfluous, inasmuch as one could remove them both (i.e. remove lines 3, 4, 9, and 10) and be left with a valid proof—namely, the direct proof we gave at the very beginning in [**P17**]. However, Fine's *reductio* is superfluous in an especially troubling way. The assumption for *reductio* in his line 2 plays no role in deriving the conclusion of the *reductio* subordinate deduction in line 15, and this conclusion is the very same as what is then inferred by *reductio* in line 16. Such a *reductio* would be highly unusual for Aristotle.[101]

The second difficulty concerns what Aristotle says in points [ix] and [x]: 'but the first was impossible; therefore the second also'. Fine takes this to be an assertion of the conditional statement in his line 13.[102] However, Aristotle's language does not take the form of a conditional connecting an antecedent clause with a consequent clause; rather he affirms each clause individually.[103]

Neither of these difficulties is decisive evidence against Fine's reconstruction, but they do provide some reason to prefer the reconstruction we have given.

## 7. Demonstrative knowledge: *Posterior Analytics* 1. 6

The first book of Aristotle's *Posterior Analytics* is primarily concerned with demonstrations and the role they play in science. Demonstrations are deductions of a certain kind, namely deductions which confer knowledge of their conclusions. Aristotle argues that in order to confer this knowledge, demonstrations must possess a number of specific features which distinguish them from deductions in general. One of the features which all demonstrations must possess, Aristotle claims, is that their premisses are true of necessity. Aristotle offers a number of arguments for this claim in *Posterior Analytics* 1. 6, and one of his arguments makes use of the possibility rule.

### (*a*) *Premisses of demonstrations are true of necessity* ($74^b32$–9)

The argument we want to discuss turns on the point that a demonstration, when grasped, confers knowledge of its conclusion. Aris-

[101] Aristotle might even have reason to reject such a *reductio* as a case of the fallacy of the false cause; see *Pr. An.* 2. 17, and *SE* 5.

[102] Fine, 'Megarian Maneuvers', 19.

[103] It would be open to Fine to relate points [ix] and [x] to his lines 3 and 10. However, Fine also wants to take point [viii] as an assertion of his line 11. It would then be difficult to see why Aristotle in points [ix]–[x] was reviewing the subordinate deduction (Fine's lines 3–10) after he had already in point [viii] inferred the conditional statement in line 11 from it.

totle begins from a principle to the effect that a person's state of knowledge will not change so long as certain factors remain constant:

> Further, if someone does not know now, while he has the account and is preserved, and while the thing is preserved, and he has not forgotten, then he did not know earlier either. (*Post. An.* 1. 6, 74$^{b}$32–4)

This principle states that if a person does not have knowledge at a later time, and certain conditions are satisfied at this later time, then the person also did not have knowledge at an earlier time. Equivalently, if a person did have knowledge at an earlier time, and certain conditions are satisfied at a later time, then the person also has knowledge at this later time. What are these conditions? The first of them is described as the person's still 'having the account', where the account is presumably the reason on the basis of which the person believes the proposition in question at the earlier time.[104] Such a reason would typically be a certain deduction whose conclusion is the proposition believed. The second and fourth conditions, namely that the person 'is preserved' and 'has not forgotten', seem to add little beyond a general assurance that the person is still alive and of sound mind. All together, these three conditions can be taken to mean that the person still understands the deduction, believes its premisses, and accepts its conclusion on the basis of its premisses. We will express this by saying that the person still *endorses* the deduction in question. The remaining condition is that 'the thing is preserved', which appears to mean that the proposition believed at the earlier time is still true at the later time. With 'Callias' serving as stand-in for an arbitrary person, the whole principle may then be stated as follows:

> PRESERVATION OF KNOWLEDGE: If at time $t$ Callias knows proposition P through deduction D, and at time $u$ after $t$, P is true and Callias endorses D, then at $u$ Callias knows P through D.

With this principle in place, Aristotle's argument then proceeds in two stages. In the first stage he uses the principle to prove that if a deduction confers knowledge of its conclusion at some time, then none of its premisses will actually become false at a later time (more accurately, at a later time at which the deduction is still endorsed). In the second stage he will use this result, along with the

[104] Barnes, *Posterior Analytics*, 127.

possibility rule, to show that if a deduction confers knowledge of its conclusion at some time, then all of its premisses are *true of necessity*. Since demonstrations are deductions which confer knowledge of their conclusions, Aristotle will then be in a position to assert that all premisses of demonstrations are true of necessity. The first stage of the argument appears as follows:

> [i] The middle term could perish, if it is not necessary, [ii] so that he will have the account and be preserved while the thing is preserved—yet he does not know. [iii] Therefore he did not know earlier either. (*Post. An.* 1. 6, $74^b34$–6)

In this passage Aristotle envisages someone who knows a proposition through a certain deduction. When in point [i] he speaks of the perishing of the middle term, he means that at least one of the premisses of the deduction becomes false. Thus, point [i] states that if the premisses of the deduction are not both true of necessity, then it is possible that at least one of them will become false. This leads into a consideration, in [ii]–[iii], of the case in which one of the premisses will in fact become false. Under the supposition that the person knows the proposition through the deduction at the earlier time, Aristotle rules out this case by means of a *reductio* in [ii]–[iii]. Thus, point [ii] is governed by an implicit assumption for *reductio* to the effect that one of the premisses of the deduction will be false at a later time at which the person still endorses the deduction.

Aristotle wishes to apply the principle of preservation of knowledge within the *reductio*. To this end, he states in [ii] that 'the thing is preserved', i.e. that the proposition known by the person is still true at the later time. This can be justified by Aristotle's view that objects of knowledge are true of necessity.[105] Since the person knows the proposition at the earlier time, the proposition is true of necessity and hence also true at the later time. Aristotle also states in [ii] that the person 'does not know', meaning that the person does not know the proposition through the deduction at the later time.[106] This can be justified by way of Aristotle's view that nothing can be known on the basis of false premisses, since according to the assumption for *reductio* some premiss of the deduction

[105] For this view, see n. 40 above.

[106] We add the qualification 'through the deduction' because, in principle, the proposition might be known through some other means at the later time.

is false at the later time.[107] In [iii] Aristotle applies the principle of preservation of knowledge to infer that the person does not know the proposition through the deduction at the earlier time. This is the conclusion of the *reductio* subordinate deduction, and it contradicts the previous supposition that the person does know at the earlier time. Aristotle's argument in points [ii]–[iii] can then be reconstructed as follows:

| | | | |
|---|---|---|---|
| 1. | At time *t*, Callias knows proposition P through deduction D | [assumption for conditionalization] | [**P21**] |
| 2. | P is true of necessity | [from 1, necessity of what is known] | |
| 3. | At time *u* after *t*, some premiss of D is false while Callias endorses D | [assumption for *reductio*] | |
| 4. | At time *u*, Callias does not know P through D | [from 3, no knowledge through false premisses] | |
| 5. | At time *u*, Callias endorses D | [from 3] | |
| 6. | P is true of necessity | [iterated from 2] | |
| 7. | At time *u*, P is true | [from 6] | |
| 8. | At time *t*, Callias does not know P through D | [from 4, 5, 7, preservation of knowledge] | |
| 9. | Not: at time *u* after *t*, some premiss of D is false while Callias endorses D | [*reductio*: 1, 3–8] | |
| 10. | If at time *t* Callias knows proposition P through deduction D, then there is no time after *t* such that some premiss of D is false while Callias endorses D | [conditionalization: 1–9, generalization over *u*] | |

With this result in hand, Aristotle proceeds to the second stage of his argument, in which he invokes the possibility rule:[108]

> [iv] And if the middle term has not perished, but it is possible for it to perish, [v] then what results would be possible. [vi] But it is impossible for someone in such a condition to know. (*Post. An.* 1. 6, 74$^{b}$36–9)

In point [iv] Aristotle indicates an assumption for *reductio* to the effect that a person has knowledge through a deduction whose premisses are not all true of necessity. It follows that it is possible that some premiss of the deduction is false at a later time. In [v] he

[107] For the view that there is no knowledge through false premisses, see *Post. An.* 1. 2, 71$^{b}$19–21.

[108] That the possibility rule, or the possibility principle, is invoked here is observed by Barnes, *Posterior Analytics*, 128, and by Nortmann, *Modale Syllogismen*, 231.

signals an application of the possibility rule, where the assumption for the possibility rule is that some premiss of the deduction is false at a later time. In the light of the first stage of the argument in points [ii]–[iii], this seems to mean more specifically that some premiss is false at a time at which the person still endorses the deduction. (We will return to this.)

The phrase 'what results' in [v] refers to the conclusion of the modal subordinate deduction. In [vi] Aristotle briefly specifies what this conclusion is, and states that it is impossible. The phrase 'such a condition' in [vi] can be taken to mean the condition of endorsing a deduction one of whose premisses will be false at a later time. That it is impossible for someone in such a condition to know is guaranteed by the first stage of Aristotle's argument in points [ii]–[iii], as we will explain in more detail shortly. Thus, Aristotle's application of the possibility rule in points [iv]–[vi] can be reconstructed as in proof [**P22**] below.[109] Many aspects of this reconstruction could stand to receive more detailed discussion; we will only touch on some of them.

First, line 8 of [**P22**] derives from the proof given in the first stage of Aristotle's argument in [ii]–[iii]. Instead of simply repeating the conclusion of this proof (line 10 of [**P21**]), however, line 8 says in effect that that conclusion is true of necessity. It is reasonable to think that the proof given in [ii]–[iii] is itself a demonstration, and that its conclusion therefore has the status of a theorem of Aristotle's theory of science. Since for Aristotle theorems are true of necessity, he is justified in taking this conclusion to be true of necessity.

Second, in line 3 of [**P22**] we take Aristotle to perform an inference from the possibility that some premiss of D is false to the possibility that some premiss of D is false while Callias still endorses D. It is not clear how Aristotle would justify this inference. Perhaps he holds that there are no necessary connections between beliefs and contingent states of affairs. If so, then there would also be no necessary connection between Callias' belief in the premisses of D on the one hand and the truth of those premisses on the other, given that

[109] The conclusion of [**P22**] means that if a demonstration confers knowledge at some time, then its premisses are true of necessity. But what Aristotle wishes to establish in *Post. An.* 1. 6 is somewhat stronger, namely that the premisses of every demonstration are true of necessity. The latter could be inferred from the former if it is held that every demonstration in fact confers knowledge at some time. Alternatively, given that line 9 is a theorem and therefore true of necessity, it would also suffice to hold that every demonstration *possibly* confers knowledge at some time.

[**P22**]

| | | |
|---|---|---|
| 1. | At time *t*, Callias knows proposition P through deduction D and not all premisses of D are true of necessity | [assumption for *reductio*] |
| 2. | At *t*, Callias knows P through D | [from 1] |
| 3. | It is possible that at some time after *t*, some premiss of D is false while Callias endorses D | [from 1] |
| 4. | At some time after *t*, some premiss of D is false while Callias endorses D | [assumption for possibility rule] |
| 5. | At *t*, Callias knows P through D | [iterated from 2] |
| 6. | At *t*, Callias knows P through D, and at some time after *t*, some premiss of D is false while Callias endorses D | [from 4, 5] |
| 7. | It is possible that: at *t*, Callias knows P through D, and at some time after *t*, some premiss of D is false while Callias endorses D | [possibility rule: 3, 4–6] |
| 8. | It is not possible that: at *t*, Callias knows P through D, and at some time after *t*, some premiss of D is false while Callias endorses D | [premiss, from [ii]–[iii]] |
| 9. | If at time *t*, Callias knows proposition P through deduction D, then all premisses of D are true of necessity | [*reductio*: 1–7, 8] |

the premisses are contingently true. Hence it would be possible for the premisses to become false without any corresponding change in Callias' beliefs, and hence without his ceasing to endorse D.

The third issue concerns the iteration into the modal subordinate deduction in line 5 of the statement that at *t*, Callias knows P through D. The presence of this statement in the modal subordinate deduction is required by Aristotle's text, since it is needed to derive the conclusion of the modal subordinate deduction stated in [vi], namely that 'someone in such a condition *knows*'. But is the iteration of this statement justified? The statement that at *t*, Callias knows P through D should presumably be regarded as being contingently true, so the iteration cannot be justified on the grounds that the iterated statement is true of necessity. Nor is it obvious whether the iteration can be justified in another way. If it cannot, then Aristotle's argument in [iv]–[vi] is invalid.

The fourth and final issue we want to discuss concerns the conclusion of the modal subordinate deduction. In the reconstruction given in [**P22**], this conclusion is that Callias knows something through a deduction one of whose premisses will be false while the deduction is still endorsed (line 6). However, Jonathan Barnes offers a different interpretation, according to which the conclusion of the modal subordinate deduction is that a person knows and does not know the same thing at the same time.[110] But this does not seem to match Aristotle's wording. In point [vi] Aristotle says 'it is impossible for someone in such a condition to know', which indicates that the conclusion of the modal subordinate deduction can be expressed by the clause 'someone in such a condition knows'. This expression would be awkward if Aristotle had in mind a straightforward contradiction between knowing and not knowing; for it would be odd to describe one side of this contradiction, namely not knowing, by the indeterminate phrase 'such a condition'. On our interpretation, this phrase describes the condition of endorsing a deduction one of whose premisses will be false while the deduction is still endorsed. This yields a more satisfactory interpretation of point [vi], and thereby provides some confirmation for our reconstruction of Aristotle's argument.

## 8. Three borderline cases: *Metaphysics* *Λ* 6, *Physics* 6. 3, *De motu animalium* 4

We have now considered nearly all of Aristotle's applications of the possibility rule. Outside of *Prior Analytics* 1. 15, these are in fact all the cases of which we are aware in which Aristotle clearly indicates that the possibility rule is being applied. We now want briefly to consider three passages in which it is less obvious whether or not Aristotle is applying the possibility rule. The first comes from *Metaphysics Λ* 6, and we will suggest that Aristotle does indeed apply the possibility rule in it. The second passage, from *Physics* 6. 3, admits of two interpretations, one on which the rule is applied and one on which it is not. In the third passage, from *De motu animalium* 4, we will argue that Aristotle does not apply the possibility rule, although he appeals to the possibility principle.

### (*a*) *The essence of the first mover is not a capacity* (*Metaph. Λ* 6, 1071$^{b}$12–20)

According to Aristotle, motion is eternal, and there is an eternal substance which is always causing motion. In *Metaphysics Λ* 6 he

[110] Barnes, *Posterior Analytics*, 128, although Barnes does not employ the notion of a modal subordinate deduction.

claims that the essence of this substance is not a capacity but an activity, i.e. that it is essentially active in such a way as to cause motion. He undertakes to prove this claim as follows:

> [i] But if [the eternal unmoved substance] is something capable of imparting motion or of affecting, but is not active, there will not be motion. . . . [ii] Further, it is not enough if it will act but its essence is a capacity; [iii] for motion will not be eternal; [iv] for what potentially is, possibly is not. [v] Therefore, there must be such a principle whose essence is activity. (*Metaph.* *Λ* 6, 1071$^{b}$12–20)

A difficulty in the interpretation of this argument is that it is unclear how point [iii] is supposed to be derived from the modal claim in [ii]. Point [ii] appears to imply that it is possible for motion not to be eternal, but not that motion in fact is not eternal. The difficulty can be solved by invoking the possibility rule and taking point [iii] to be the conclusion of a modal subordinate deduction:[111]

| | | | |
|---|---|---|---|
| 1. | \| The essence of the eternal substance is a capacity | [assumption for *reductio*] | [**P23**] |
| 2. | \| It is possible that the eternal substance does not impart motion at some time | [from 1] | |
| 3. | \| \| The eternal substance does not impart motion at some time | [assumption for possibility rule] | |
| 4. | \| \| At some time there is no motion | [from 3] | |
| 5. | \| \| Motion is not eternal | [from 4] | |
| 6. | \| It is possible that motion is not eternal | [possibility rule: 2, 3–5] | |
| 7. | It is not possible that motion is not eternal | [premiss] | |
| 8. | The essence of the eternal substance is not a capacity | [*reductio*: 1–6, 7] | |

This is, we think, a plausible way of reconstructing Aristotle's argument, even if his presentation is too compressed to be interpreted with certainty.[112]

### (*b*) *Nothing moves in an instant of time* (*Phys.* 6. 3, 234$^{a}$24–31)

In *Physics* 6. 3 Aristotle argues that every motion is temporally extended, i.e. that no motion occurs in an instant of time:

[111] For the use of the future tense *οὐ γὰρ ἔσται* in [iii], cf. *Phys.* 7. 1, 242$^{b}$47, discussed in sect. 4 above. There too, Aristotle states the conclusion of the modal subordinate deduction without having first introduced the subordinate deduction.

[112] For the premiss in line 7 of [**P23**] see *Λ* 6, 1071$^{b}$6–7; *Phys.* 8. 5, 256$^{b}$7–13 (discussed in sect. 4 above).

[i] That nothing moves in an instant is clear from the following. [ii] If something does, [iii] then it is also possible to move faster and slower. [iv] Then let N be the instant, and let AB be the distance traversed in it by the faster thing. Hence the slower thing will traverse a distance smaller than AB in the same instant. Let AC be this smaller distance. [v] Since the slower thing has traversed AC in the whole instant, the faster thing will traverse AC in something smaller than this instant. [vi] Consequently, the instant will be divided. [vii] But it was indivisible. [viii] Therefore, it is not possible to move in an instant. (*Phys*. 6. 3, $234^{\mathrm{a}}24$–31)

In [iv] Aristotle introduces a situation in which two objects move at different speeds in the same instant. The actual existence of such a situation does not follow from what he has said in [i]–[iii], although the possibility of such a situation does. Because of this, one might take point [iv] to introduce an assumption for the possibility rule, as follows:

| | | | |
|---|---|---|---|
| 1. | Something moves in an instant | [assumption for *reductio*] | [**P24**] |
| 2. | It is possible that two objects move at different speeds in the same instant | [from 1; cf. $232^{\mathrm{b}}21$–2] | |
| 3. | Two objects move at different speeds in the same instant | [assumption for possibility rule] | |
| 4. | The distance traversed by the slower object in the instant is traversed by the faster object in a proper part of the instant | [from 3; cf. $232^{\mathrm{a}}25$–6] | |
| 5. | There is a proper part of an instant | [from 4] | |
| 6. | An instant is divided | [from 5] | |
| 7. | It is possible that an instant is divided | [possibility rule: 2, 3–6] | |
| 8. | It is not possible that an instant is divided | [premiss; cf. $233^{\mathrm{b}}33$–$234^{\mathrm{a}}24$] | |
| 9. | Nothing moves in an instant | [*reductio*: 1–7, 8] | |

Although this seems to us a plausible reconstruction of Aristotle's argument, it must be acknowledged that Aristotle himself gives no indication that he is applying the possibility rule here. It is therefore worth outlining an alternative interpretation on which the argument does not involve this rule. Instead, points [iv]–[vii] could be taken to constitute an auxiliary proof by *reductio* such as that given in proof [**P25**] below.

The premiss of [**P25**], in line 5, can be regarded as a theorem of Aristotle's physics. If so, then the conclusion of the proof in line 6

| | | | |
|---|---|---|---|
| 1. | Two objects move at different speeds in the same instant | [assumption for *reductio*] | [**P25**] |
| 2. | The distance traversed by the slower object in the instant is traversed by the faster object in a proper part of the instant | [from 1] | |
| 3. | There is a proper part of an instant | [from 2] | |
| 4. | An instant is divided | [from 3] | |
| 5. | No instant is divided | [premiss] | |
| 6. | No two objects move at different speeds in the same instant | [*reductio*: 1–4, 5] | |

can also be regarded as a theorem, and hence as a necessary truth. Given this, it is impossible that two objects move at different speeds in the same instant. With this result in hand, Aristotle's main proof could then be concluded as follows (corresponding to points [i]–[iii]):

| | | | |
|---|---|---|---|
| 1. | Something moves in an instant | [assumption for *reductio*] | [**P26**] |
| 2. | It is possible that two objects move at different speeds in the same instant | [from 1] | |
| 3. | It is not possible that two objects move at different speeds in the same instant | [premiss, from auxiliary proof] | |
| 4. | Nothing moves in an instant | [*reductio*: 1–2, 3] | |

We do not see decisive evidence for favouring one interpretation over the other. On both interpretations Aristotle's argument contains the same core material, albeit arranged in different ways. Hence, his argument can be interpreted as making use of the possibility rule, but can equally well be interpreted as not making use of it. It is to be expected that there are more such passages in Aristotle's writings, passages which could but need not be interpreted as involving an application of the possibility rule.

### (*c*) *The indestructibility of the cosmos* (*MA* 4, 699$^{\mathrm{b}}$23–9)

In *De motu animalium* 4 Aristotle lays out an *aporia* concerning the eternal existence of the cosmos. On the one hand, he is firmly committed to the view that it is not possible for the cosmos to be destroyed (699$^{\mathrm{b}}$21–2). On the other hand, he describes an argument to the effect that its destruction is after all possible. The argument begins as follows:

[i] If some motion exceeds, in power of motion, the earth's rest, it is clear that it will move the earth away from the middle. (*MA* 4, 699$^{b}$14–15)

Aristotle is assuming that when something undergoes motion or remains at rest, its motion or rest has some amount of power. He considers the idea of a motion whose power is greater than the power of the earth's rest. If such a motion is applied to the earth, then the earth will be moved away from its place in the middle of the cosmos. Similar considerations apply to other constituents of the cosmos such as the heavenly spheres: they too will be moved away from their places if an excessively powerful motion is applied to them. Since such displacements would amount to a destruction of the cosmos, we arrive at the following argument:

[ii] It is possible for there to be a motion greater than the power with which the earth rests and greater than the power with which the fire and upper body undergo motion. [iii] Now if there are exceeding motions, then these bodies will be dispersed away from one another. [iv] And if there are not such motions, but it is possible for them to exist . . ., then it would be possible for the cosmos to be dispersed. (*MA* 4, 699$^{b}$23–9)

Point [ii] asserts that it is possible for there to be motions whose power exceeds the power of the earth's rest and of the motions of other major constituents of the cosmos. Presumably this also means that it is possible for such motions to be applied to the relevant objects. For short, 'Poss(exceeding motions are applied)'. Point [iii] continues with the conditional statement that if exceeding motions are applied, then the constituents of the cosmos will be dispersed, so that the cosmos is destroyed. This can be read as a conditional of necessary consequence: 'exceeding motions are applied ⇒ cosmos is destroyed'.[113] Combining this with the possibility principle, we can reconstruct the argument as follows:

| | | |
|---|---|---|
| 1. Poss(exceeding motions are applied) | [premiss] = point [ii] | [**P27**] |
| 2. Exceeding motions are applied ⇒ cosmos is destroyed | [premiss] = point [iii] | |
| 3. Poss(exceeding motions are applied) ⇒ Poss(cosmos is destroyed) | [from 2, possibility principle] | |
| 4. Poss(cosmos is destroyed) | [from 1, 3] = point [iv] | |

113 Alternatively, the conditional might be weaker, though a bare indicative conditional might not suffice. In order for our reconstruction to succeed, the conditional in question (label it '→') need only support *modus ponens* and satisfy an analogue of the possibility principle: if (A → B) then (Poss(A) → Poss(B)).

This reconstruction appeals to the possibility principle but does not apply the possibility rule. Alternatively, one could frame an argument in which the same conclusion is derived by means of the possibility rule, as follows:

| | | | |
|---|---|---|---|
| 1. | Poss(exceeding motions are applied) | [premiss] | [**P28**] |
| 2. | Exceeding motions are applied ⇒ cosmos is destroyed | [premiss] | |
| 3. | \| Exceeding motions are applied | [assumption for possibility rule] | |
| 4. | \| Exceeding motions are applied ⇒ cosmos is destroyed | [iterated from 2] | |
| 5. | \| Cosmos is destroyed | [from 3, 4] | |
| 6. | Poss(cosmos is destroyed) | [possibility rule: 1, 3–5] | |

However, Aristotle does not seem to be envisaging this kind of argument in [i]–[iv]. He does not speak of 'assuming' so as to suggest an assumption for the possibility rule, nor does he speak of 'resulting', as he typically does when he indicates the conclusion of a modal subordinate deduction. Moreover, all of Aristotle's applications of the possibility rule which we have seen occur within a proof by *reductio*; but the present argument does not involve a *reductio*. These considerations suggest that the argument does not rely on the possibility rule but rather on the possibility principle.

## 9. Conclusion

We have now discussed all applications of the possibility rule in Aristotle's writings of which we are aware, with the sole exception of those in the modal syllogistic in *Prior Analytics* 1. 15. By way of conclusion, we want to synthesize our findings by reviewing two basic issues. First, we will reflect on how it can be determined whether or not Aristotle applies the possibility rule in a given passage. As we have seen, the answer to this question is often not obvious, and can be crucial for an adequate understanding of Aristotle's text. Second, we will confirm that all of Aristotle's applications of the possibility rule obey the general pattern given at the beginning of this paper.

### (*a*) *Recognizing applications of the possibility rule*

Aristotle does not have a perfectly uniform way of presenting applications of the possibility rule. To determine whether this rule is applied by him in a given argument requires not only attention to his explicit pronouncements, but also reflection on the logical and philosophical grounds of his reasoning. There are, however, two sorts of conspicuous signal whose presence will strongly suggest that Aristotle is invoking the possibility rule. One is when he speaks, in some way or other, of assuming something which is merely possible. Thus we find him saying, for example, 'let that of which it is capable obtain', or 'for we suppose what is possible'.[114] Such phrases mark an assumption for the possibility rule. The other signal is when Aristotle emphasizes the difference between falsehood and impossibility. He may do this to describe the modal status of the assumption for the possibility rule; for example, 'something false but not impossible was posited'.[115] Alternatively, he may do this in connection with the conclusion of the modal subordinate deduction; for example, 'what follows because of the hypothesis will be false but not impossible'.[116]

When Aristotle has completed a modal subordinate deduction, he may emphasize that its conclusion is 'not only false but impossible', or alternatively he may simply say: 'but this is impossible'.[117] The latter kind of phrase on its own, however, is not a clear signal that the possibility rule is being invoked. The reason is that the qualification 'impossible' is also used by Aristotle in connection with *reductio* arguments which do not involve an application of the possibility rule. Indeed, Aristotle's standard way of referring to

[114] *De caelo* 1. 12, 281$^{b}$21–2; *Phys.* 7. 1, 242$^{b}$49–50. Other examples are: 'we posit that that of which it is capable obtains', *De caelo* 1. 12, 283$^{a}$16–17; 'let that for which it has the capacity obtain actually', 283$^{b}$11; 'something possible is posited', *Phys.* 7. 1, 243$^{a}$30–1; 'we posit that which is capable of being', *Phys.* 8. 5, 256$^{b}$10–11; 'we assume that something which is not, but is possible, is or has come to be', *Metaph.* *Θ* 4, 1047$^{b}$10–11; 'when A is possible, if A should be assumed', 1047$^{b}$18–19; 'something false but not impossible is hypothesized', *Pr. An.* 1. 15, 34$^{a}$25–6; 'posit that B belongs to all C', 34$^{a}$36–7; 'positing that B belongs to C', 34$^{b}$3–4; 'let it be posited that B belongs to C', 34$^{b}$23; and 'let it have been divided', *GC* 1. 2, 316$^{a}$24.

[115] *Pr. An.* 1. 15, 34$^{b}$1. Other examples are: 'this is false but not impossible', *Pr. An.* 1. 15, 34$^{a}$37–8; 'it is not the same to hypothesize something false and something impossible', *De caelo* 1. 12, 281$^{b}$14. See also *Metaph.* *Θ* 4, 1047$^{b}$10–11.

[116] *Pr. An.* 1. 15, 34$^{a}$26–7. Another example is: 'nothing impossible will follow, though perhaps something false will', *Phys.* 8. 5, 256$^{b}$11–12. See also *De caelo* 1. 12, 281$^{b}$23–5; *Metaph.* *Θ* 4, 1047$^{b}$12–14.

[117] For examples of the latter phrase, see *De caelo* 1. 12, 283$^{b}$12–13; *Phys.* 7. 1, 242$^{b}$53; *Pr. An.* 1. 15, 34$^{b}$25. Similar examples are: 'but for there not to be motion is impossible', *Phys.* 8. 5, 256$^{b}$12; 'but B was impossible', *Metaph.* *Θ* 4, 1047$^{b}$20; 'the result is impossible', *Pr. An.* 1. 15, 34$^{b}$2; 'but it is impossible for someone in such a condition to know', *Post. An.* 1. 6, 74$^{b}$38–9.

*reductio* arguments is by phrases such as 'through the impossible' or 'leading to the impossible'. He says that in every successful *reductio* argument something 'impossible' is shown to result when the contradictory of the intended conclusion has been assumed.[118] Accordingly, he sometimes concludes a *reductio* argument with the phrase 'but this is impossible', even if the argument did not involve an application of the possibility rule.[119] In these contexts, the phrase is used in connection with the conclusion of a *reductio* subordinate deduction, not with the conclusion of a modal subordinate deduction. To see this difference more clearly, consider the following proof by *reductio* of the syllogism Barbara:

> If A belongs to all B, and C is the middle term, if it is hypothesized that A does not belong to all B . . ., and A belongs to all C, which was true, then it is necessary for C . . . not to belong to all B. But this is impossible. (*Pr. An.* 2. 11, 61$^{a}$27–30)

This proof of Barbara can be represented as follows:

| | | | | |
|---|---|---|---|---|
| 1. | AaC | | [major premiss] | [**P29**] |
| 2. | CaB | | [minor premiss] | |
| 3. | | AoB | [assumption for *reductio*] | |
| 4. | | AaC | [iterated from 1] | |
| 5. | | CoB | [from 3, 4, Baroco] | |
| 6. | AaB | | [*reductio*: 2, 3–5] | |

When Aristotle says, at the end of the passage, 'but this is impossible', the pronoun 'this' seems to refer to the conclusion of the *reductio* subordinate deduction in line 5 of [**P29**]. Thus Aristotle calls the statement CoB in line 5 impossible. However, there is no need to take him to be asserting 'Not Poss(CoB)' in the sense in which we have seen such statements appear in arguments using the possibility rule. Rather, he is calling CoB impossible simply in virtue of the fact that its contradictory, CaB, is present in line 2. Thus, the phrase 'but this is impossible' does not indicate an application of the possibility rule in the present passage.[120]

[118] 'For this is what deducing through the impossible was, namely proving something impossible by means of the initial assumption', *Pr. An.* 1. 23, 41$^{a}$31–2; see also 41$^{a}$25–6. Accordingly, the failure of a *reductio* argument can be expressed by phrases such as 'nothing impossible results'; cf. *Pr. An.* 1. 9, 30$^{b}$4; 1. 16, 36$^{a}$24–5; 1. 17, 37$^{a}$29–30.

[119] *Pr. An.* 1. 17, 37$^{a}$14; 2. 11, 61$^{a}$30, 61$^{b}$13, 61$^{b}$28; 2. 12, 62$^{a}$26, 62$^{a}$31, 62$^{b}$1–2.

[120] Alternatively, the pronoun 'this' in the present passage might be taken to refer

Phrases such as ‘this is impossible’, then, are used in two ways by Aristotle: in one way in the context of the possibility rule, and in another way in the context of straight *reductio* arguments which do not involve the possibility rule. The difference between the two uses is also evident from the fact that Aristotle emphasizes the difference between falsehood and impossibility in the former sort of context, but not in the latter. In fact, in straight *reductio* arguments he sometimes uses ‘false’ instead of ‘impossible’, saying, for example, that ‘something false’ results instead of that ‘something impossible’ results.[121] In some discussions of straight *reductio* arguments he uses the terms ‘false’ and ‘impossible’ interchangeably, going back and forth from one to the other.[122] Thus, when impossibility is emphatically contrasted with falsehood, this is a clear signal of the possibility rule, but the mere presence of phrases such as ‘this is impossible’ need not, and usually does not, mean that the rule is in play.

### (*b*) *The general pattern*

In closing, let us review the general structure of reasoning exhibited in Aristotle’s applications of the possibility rule. At the beginning of this paper we announced that all of his applications of the rule can be reconstructed as instances of the following pattern (Section 2, pp. 194–5):

| | | |
|---|---|---|
| │ Not C | [assumption for *reductio*] | [**P7**] |
| │ . . . | | |
| │ Poss(A) | [. . .] | |
| │ │ A | [assumption for possibility rule] | |
| │ │ . . . | | |
| │ │ B | [deduced from A and, perhaps, iterated statements] | |
| │ Poss(B) | [by possibility rule] | |
| . . . | | |
| Not Poss(B) | [. . .] | |
| C | [by *reductio*] | |

to the pair of statements CaB in line 2 and CoB in line 5. Then the phrase ‘but this is impossible’ would assert that these two statements are not jointly possible, i.e. ‘Not Poss(CaB, CoB)’. Still, the phrase would not indicate an application of the possibility rule.

[121] Cf. *Pr. An.* 1. 23, $41^{a}24$, $41^{a}30$, $41^{a}32$; 1. 29, $45^{b}5$–6. Accordingly, the failure of a *reductio* argument can be expressed by saying that ‘nothing false results’, or that ‘the false’ does not result through the assumption for *reductio*: *Pr. An.* 1. 17, $37^{a}36$; 2. 17, $65^{a}38$, $66^{a}2$–3.

[122] Compare *Pr. An.* 1. 23, $41^{a}25$ with $41^{a}30$; 2. 17, $65^{a}38$ with $65^{b}12$; $65^{b}20$ with $65^{b}21$; $66^{a}3$ with $66^{a}7$; $66^{a}9$–10 with $66^{a}11$–12.

For most of the reconstructions presented in the course of this paper, it is obvious how they instantiate this pattern. The only case which might seem problematic is the proof of the possibility principle in *Metaphysics Θ* 4 (Section 6, pp. 234–42). But even here the only difficulty lies in the order of certain elements in the reconstruction, and the reconstruction can easily be brought into line with the pattern.[123] It is also worth noting that in some cases the assumption for *reductio*, 'Not C', and the statement Poss(A), which serves as a premiss for the possibility rule, are identical. This occurs in the two proofs we discussed from *Metaphysics Θ* 4.

As mentioned above, the only applications of the possibility rule in Aristotle which have not been treated in this paper are the ones in the modal syllogistic, in *Prior Analytics* 1. 15. We discuss these in a separate paper, and show that they too conform to the general pattern given in [**P7**]. Thus we are in a position to affirm one of our main conclusions, namely that all of Aristotle's applications of the possibility rule can be reconstructed as instances of this pattern.

It is striking that Aristotle always applies the possibility rule within the context of a *reductio*. More specifically, he always applies it in such a way that the output of the possibility rule, Poss(B) in [**P7**], is the conclusion of the *reductio* subordinate deduction. Aristotle himself never explicitly expresses the output of the possibility rule. Rather, he states the conclusion of the modal subordinate deduction, B, and then states directly that this is impossible, 'Not Poss(B)'. Thus he does not present his procedure as being clearly articulated into an application of the possibility rule and a separate application of the *reductio* rule. The separation of these two steps in our reconstructions, with the output of the possibility rule being included as a separate line, is a piece of logical analysis intended to make the structure of Aristotle's arguments clearer.

An issue we have often encountered in this paper is the iteration of statements into modal subordinate deductions. In some cases Aristotle's iterations can be justified by means of a guarantee that the statement iterated is true of necessity. In other cases, however, his

[123] In the pattern, 'Not Poss(B)' occurs after the *reductio* subordinate deduction; but in our reconstruction of Aristotle's proof ([**P19**], p. 236), 'Not Poss(B)' occurs before the relevant *reductio* subordinate deduction (i.e. the one extending from line 4 to line 8). Strictly speaking, then, the reconstruction does not instantiate the general pattern. However, the statement 'Not Poss(B)' in line 3 of [**P19**] could simply be repeated between line 8 and line 9, and the resulting modified version of the reconstruction would instantiate the pattern.

iterations cannot be justified in any obvious way, so that they seem to render his argument invalid. This problem occurs in *De caelo* 1. 12, *Physics* 7. 1, and *Posterior Analytics* 1. 6. It would be desirable to have an explanation of why Aristotle was willing to perform these questionable iterations. Perhaps they are simply due to an oversight on his part. Such an oversight would not imply that Aristotle was generally unaware of the restrictions on valid iteration into modal subordinate deductions, but only that he lost sight of them in particular cases. On the other hand, he may have thought that in some cases there are alternative ways to justify those problematic iterations. We must leave this as an open question for further research. Fortunately, many of Aristotle's applications of the possibility rule do not suffer from problematic iterations. In these cases, his way of using the possibility rule is perfectly unobjectionable, even if, as in any ambitious philosophical reasoning, his arguments are open to challenge for other reasons.

Aristotle's formulation and use of the possibility rule was a major step in the origins of modal logic. Unlike other logical achievements of his, such as the *Prior Analytics*' theory of syllogisms, this rule is put to conspicuous and wide-ranging use outside of the *Organon*, in arguments which are of significant philosophical interest. Thus the possibility rule constitutes one of the very few examples in which Aristotle's logical theory and his philosophical practice are combined in a fruitful and ingenious, sometimes perhaps too ingenious, way.

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