

On the Logic of Quantifier Variance

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Abstract

Eli Hirsch recently suggested the metaontological doctrine of so-called *quantifier variance*, according to which ontological disputes—e.g. concerning the question whether arbitrary, possibly scattered, mereological fusions exist, in the sense that these are recognised as objects in our ontology—can be defused as insubstantial. His view is that the meaning of the quantifier ‘there exists’ varies in such debates: according to one opponent in this dispute, some existential statement claiming the existence of, e.g., a scattered object is true, according to the other it is not. This paper argues that Hirsch’s proposal leads into inconsistency.

The term ‘metaontology’ has recently been coined for a well-established field in metaphysics. As Matti Eklund put it succinctly in the introductory remarks of his (Eklund, 2006a, 317): “Ontology is the study of what there

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is. *Metaontology* is the study of the nature of questions about what there is; that is, the nature of ontology.” Metaontology is concerned with methodological questions concerning ontology and with how we should go about solving ontological disputes or problems. Analytical metaphysics has engaged since its beginning in this reflective enterprise. Gottlob Frege used methodological considerations in his arguments against his various opponents, and the dispute between Rudolf Carnap and W.V. Quine obviously falls in the metaontological category.

Ontological disputes are about the questions whether certain putative objects are to be admitted into our ontology or not. A popular example, and the one that will be of particular importance for what follows, concerns what is often termed “unrestricted mereological composition”. Proponents of this position, like Nelson Goodman and David Lewis, for instance, hold that any collection of objects has a mereological fusion—often called a “mereological sum” if it is of only two objects—which itself is an object. This is said to hold no matter how diverse or widely scattered the objects are: think, for example, of the mereological fusion of your right ear and the stars that make up the Big Dipper. Another example for an ontological dispute is the question whether abstract objects exist. Of particular interest here has recently been the question of how we are to think of the “introduction” of abstracts objects by abstraction principles.¹

Eli Hirsch recently embarked on a quest to defuse ontological debates regarding the ontological category of an *object*. In what follows, we I focus on his metaontological proposal, dubbed “quantifier variance”.² Hirsch’s take on the matter of ontological disputes is to characterise them as being about which existential statements are true according to the opponents in

¹Authors involved in the metaontological discussion regarding abstraction principles include (Eklund, 2006b), (Sider, 2007) and (Hawley, 2007). Abstraction principles are employed in the Neo-Fregean philosophy of mathematics, see (?) and (?). For Neo-Fregean objections to the metaontological discussion of their tenets see (?) and (?).

²See, in particular, his (Hirsch, 2002) and (Hirsch, 2005).

such a dispute. His aim is to mark these disputes as purely verbal and thus insubstantial. The main example concerns the above mentioned issue of unrestricted mereological composition. According to Hirsch, there is no real disagreement between proponents of unrestricted mereological composition and their opponents who include, for instance, himself. (This might, initially, have the ring of a cognitive dissonance; it will become clear below, how it can be understood.) His idea is that the meaning of the quantifiers—in particular, “there exists”—varies from opponent to opponent, from one conceptual scheme to the next.

This idea is not new, of course, and reminiscent of sentiments expressed by Carnap or Goodman, amongst others. Hirsch readily acknowledges this: indeed, he takes the doctrine of quantifier variance to be an elaboration of Hilary Putnam’s conceptual relativism. Also Putnam states that

the logical primitives themselves, and in particular the notion of object and existence, have a multitude of different uses rather than one absolute ‘meaning’. (Putnam, 1987, 71)

Hirsch’s promise is that quantifier variance will allow one to accept the idea of relativism concerning conceptual schemes whilst preserving realism: it is the world that makes existential statements true or false. We have only to accept that ‘exists’ has different meanings in statements made on the background of different conceptual schemes. The only constraint that needs to be made, so Hirsch, is that the difference must not be empirically testable. Ontological disputes concerning what is empirically testable are naturally to be decided empirically.³

The criticism presented in this paper requires a somewhat detailed description of the doctrine of quantifier variance, in order to develop in full all the ingredients—five in total as explained below—that make quantifier

³Hirsch also does not want quantifier variance to apply to the question of abstract objects (Hirsch, 2002, 64). It thus seems that he would deem the discussion referenced in footnote 1 above to be misguided.

variance an interesting and powerful novel metaontological position. The discussion will lead to the conclusion that, in fact, the position is too powerful: it leads to a contradiction.

To start with the basic techniques involved: how does one make quantifiers vary in meaning? The first idea that suggests itself is to employ domain restrictions. Hirsch, however, is against this approach: to opt for domain restriction means, for all that matters, to side with the metaontological view of *Maximalism*⁴ which pleads for a principle of plentitude: everything that (consistently) can exist, does. As mentioned above, Hirsch himself prefers a rather sparse ontology, in particular, as regards mereological fusions. If domain restrictions were used in order to flesh out quantifier variance, any non-maximalist position would appear as a restriction of the truly all-comprising domain. This would hardly be in the spirit of Hirsch's proposal. Employing varying domains would run into similar difficulties. Different quantifiers might be construed as having different domains, without characterizing them as restrictions of an all-comprising domain. Existential claims made with one quantifier could thus differ in truth-value from the analogous claims made with a different quantifier, depending on whether the objects that are asserted to exist are included in the domain pertaining to the quantifier in question or not. However, it is usually assumed that domains can be joined. The union of all domains would then, again, be the all-comprising, maximal domain.⁵ *Modulo* the possibility of the existence of objects that are unproblematic if assumed separately, but inconsistent if assumed together,⁶ domain

⁴See (Eklund, 2006a) and (Eklund, 2006b).

⁵Sider uses domain restrictions and expansions in his attempt to provide a formal model for quantifier variance in what as he calls an "algebraic approach" (Sider, 2007, 213). For reasons given above, Sider's endeavour is probably best understood as a merely heuristic model from the viewpoint of the maximalist.

⁶Matti Eklund (Eklund, 2006b, §VI) and Katherine Hawley (Hawley, 2007, §IV) discuss such cases for the realm of abstract objects, which in abstractions' circles are known under the technical term '*nuisances*'; see (?) and (?) for the origin of nuisance principles and their discussion. It does not seem immediately obvious that similar nuisances cannot be constructed for the realm of concrete objects as far as non-empirical differences in ontology

restriction and domain variance thus should come down to essentially the same.

Neither domain restriction nor domain variation is thus available to the quantifier variantist. Hirsch's own proposal is to insist that the quantifier expressions vary their *meanings*, rather than straightforwardly their ranges. Hirsch aims to elucidate this proposal with a range of examples. Think, for instance, of a proponent of unrestricted mereological composition. According to such a view, arbitrary mereological fusions exist, for example the fusion of Hilary Clinton's nose and the Eiffel Tower. This, however, means that

- (A) There is an object that is the mereological fusion of Hilary Clinton's nose and the Eiffel Tower.

is true, according to the meaning of 'there is' as used by the proponent of unrestricted mereological composition. If the 'there is' in (A) is taken in the meaning that the existential quantifier has for someone who does not believe in the existence of scattered objects, then (A) is false. In both cases, however, the quantifiers are unrestricted, albeit tied to the ontology of the relevant conceptual scheme.

Hirsch's contention is that we can accommodate different views in ontology and remain *realists*. Different conceptual schemes carve up "the facts" in different ways, and thus different things exist according to different conceptual schemes. It is, nevertheless, correspondence with the facts that makes statements true. This is the ontological picture⁷ of the world that underlies quantifier variance: facts themselves are unstructured; their structure is imposed only by the conceptual schemes that are employed. Together with the conceptual schemes, the meaning of the quantifiers vary. As Hirsch puts it,

the basic idea of quantifier variance can be nicely formulated by saying that the same (unstructured) facts can be expressed using

are concerned.

⁷Or is this a metaontological picture? Presumably, a metaontological doctrine like quantifier variance should not depend on a specific ontological view.

different concepts of “the existence of a thing”, that statements involving different kinds of quantifiers can be equally true by virtue of the same (unstructured) facts in the world. [...]

I am inclined to agree with Putnam that, once we’ve accepted quantifier variance, there is no point in trying to hold onto language-shaped facts that are in the world independent of language. (Hirsch, 2002, 59).

Together with a privileged carving-up of the world, Hirsch also rejects the idea of a privileged metaphysical language which would contain *the real* existential quantifier and *the real* concept of a thing.

As mentioned above, Hirsch tries to square the claim that (empirically not decidable) ontological disputes are non-substantial with his own preference against unrestricted mereological composition.⁸ He takes the “ordinary meaning” of ‘exists’ as primary and claims that according to the meaning of ‘exists’ in ordinary English there does not exist an object that is composed of Clinton’s nose and the Eiffel Tower. Ontological positions like unrestricted mereological composition, or Peter van Inwagen’s denial of the existence of any composite objects except organisms, are considered metaphysical flights of fancy for which new meanings of the existential quantifier are invented. We can, however, understand these deviant meanings of ‘exists’ using our understand of the ordinary English quantifier ‘exists’.

To see how we perform this trick (without relying on domain restriction or variation) consider the following example. Following Hirsch, let us call proponents of unrestricted mereological composition (like Goodman or Lewis) ‘mereologists’ and the language that mereologist use ‘M-language’. Further, call speakers of ordinary English ‘anti-mereologists’ and their language (i.e., ordinary English) ‘A-language’. Anti-mereologist listen to claims stated in M-language and observe the similarities that the mereologists’ ‘exists’ has

⁸He admits: “I am myself an anti-mereologist” (Hirsch, 2002, 60).

with the ‘exists’ of A-language. Employing a principle of charity, the conclusion is meant to be that ‘exists’ performs the same role in M-language despite its having a somewhat different meaning:

we are relying on our shared sense of the analogy between our A-quantifiers and the M-quantifier. Starting with the A-language I teach someone the M-language *ostensively*, by giving a few representative examples of how the M-language works. (Hirsch, 2002, 58, emphasis in the original)⁹

The important feature is thus the *similarity* between the quantifiers in A-language and M-language.

In particular, the purely syntactic and formal logical properties of the expression [i.e., the existential quantifier] will not be changed at all (the formal principles of quantificational logic will be unaltered). (Hirsch, 2002, 53)

Vested with this understanding of the M-language, based on our mastery of A-language, Hirsch claims that we will always be able to translate between the two languages in the sense that for any M-language sentence we will be able to find an A-language sentence with the same truth conditions. Hirsch does not provide a translation manual, but he imagines examples like this: the M-language sentence

(B) Someone is touching the object that is the mereological fusion of Hilary Clinton’s nose and the Eiffel Tower.

can be translated into the A-language sentence

⁹One may wonder: how can someone be taught the meaning of some term *ostensively* in a manner fine-grained enough for Hirsch’s purpose? Presumably, an analogue would be ostensively to distinguish whether a term referred to rabbits, undetached rabbit-parts, rabbit stages, or even an environmental attribute: “it’s rabbiting”. We will bracket these concerns for the sake of argument.

(C) Someone is either touching Hilary Clinton’s nose or the Eiffel Tower.

which has the same truth-conditions as the former. Other examples are easily provided: take van Inwagen’s notorious “simples, arrange table-wise” as opposed to tables, for instance. The sameness of truth-conditions for such sentences will, of course, only hold if truth-conditions are defined the way Hirsch suggests: with respect to unstructured facts. Otherwise, the requirement of the existence of the fusion mentioned in (B) but not in (C) would obviously make a difference in truth-conditions.

Granting this, one may still wonder how cases that are less common to the debate will be handled. How, for instance, will modal truths be translated? All necessary truths presumably have the same truth conditions, but different conceptual schemes may differ regarding what is possible or necessary. To pick a somewhat contrived example, the “anti-mereologist” will deem situations where exactly two objects exist possible. The “mereologist” will have to deny this however: if there are two object, there is also a third one, the sum of the two. More generally, in an atomistic universe with n atoms, the ontology, according to the “mereologist”, has the cardinality $2^n - 1$; and ‘ $2^n - 1 = 2$ ’ does not have a solution in the integers. (Any gunky universe is infinite, and hence does not have cardinality 2 either.) Thus, the “mereologist” must deem it necessarily false that there are exactly two objects. Admittedly, resorting to a more creative translation, the A-language sentence ‘There are exactly two objects’ may be translated into M-language as ‘There are exactly two atoms’. But there does not appear to be any guarantee that we can, in all cases, hope for a translation that renders sentences that are necessary (possible, contingent) according to one conceptual scheme as sentences that are necessary (possible, contingent) in any other.

The doubt can be pressed further. Conceptual schemes may well differ in their respective expressive resources. (Hirsch, 2008) suggests that not all languages in which ontological claims can be or are expressed are equally

good. In particular, he is prepared to disqualify languages in ontological disputes which are expressively too weak, so that some ordinary English existential statements cannot be expressed in them, which would make a translation impossible. What, however, about languages that are expressively stronger than ordinary English, so that some of their existential claims cannot be translated into ordinary speak? Consider a language for a gunky ontology according to whose conceptual scheme gunk has the structure of the set-theoretic universe of ZFC. Fusions of bits of gunk exist according to this conceptual scheme without there being any obvious way of saying that they do in ordinary English (using the ordinary English ‘exists’). Think of the sentence:

- (D) There is a fusion of 2^{\aleph_0} discrete bits of gunk, all of which are colored red, and none of which overlap or have a common border.

It is doubtful that there is a sentence in ordinary English that could be considered a translation of (D), or that at least has the same truth conditions.

The real problem with Hirsch’s proposal, however, is revealed when we reflect upon the following problem. What is the language that ontological disputes are held in? And what is the language that Hirsch is using himself in his paper? Recall that there is not to be a privileged language of metaphysics that one can retreat to for this purpose.¹⁰ If we follow Hirsch, everyone starts out speaking ordinary English, a.k.a. A-language, and then appropriates other existential quantifiers with different meanings. The logical rules for all these quantifiers are supposed to remain the same, so Hirsch. This is how we understand the deviant existential quantifiers, and how we realise that existential statements which contain these quantifiers are true

¹⁰As a privileged language of metaphysics would also count a metalanguage which only mentions, rather than uses, the different existential quantifiers. Such a metalanguage would have to include its own quantifiers capable of interpreting all object-language quantifiers. There would thus have to be either one quantifier which would have to be maximal (or at least all-embracing); or there would have to be many quantifiers: the problem would then just be pushed one level up.

(relative to their meaning). The language of ontological disputes is thus ordinary English, amended by adding the deviant existential quantifiers.

This, however, leads into inconsistency. There is a well-know proof (provided by J.H. Harris¹¹) that any two logical constants are indistinguishable if they have the same introduction- and elimination-rules: these constants “collapse” in the sense that they are interchangeable in any sentence of the language (at least in extensional contexts). For the existential quantifier, this collapse proof runs like this:

1	(1)	$\exists_1 x \varphi(x)$	Premise
2	(2)	$\varphi(a)$	Assumption
2	(3)	$\exists_2 x \varphi(x)$	2, \exists_2 -introduction
1	(4)	$\exists_2 x \varphi(x)$	1, 2, 3, \exists_1 -elimination

The other direction is exactly analogous: just exchange the indices. The only rules used are the introduction- and elimination-rules for the existential quantifiers. What this means is that one cannot have two essentially different logical constants in a language that have the same operational rules.

The inconsistency arises thus: an A-language speaker denies, for instance, the existence of *shmrees*, i.e., objects that are the mereological fusion of the temporal parts of a (particular) whole tree by day, and only its trunk by night—“a brown, wooden object in the yard that loses its branches every night and regains it every morning” (Hirsch, 2002, 63). Using the existential quantifier of A-language, he thus asserts:

$$(0') \quad \neg \exists_A x \text{Shmree}(x)$$

On the other hand, understanding M-language, he also asserts, that according to the meaning of the existential quantifier of the M-language, *shmrees* do exist (exist_M):

¹¹(Harris, 1982).

(1') $\exists_M x \text{ Shmree}(x)$

It is a commonplace principle that logical constants are fully schematic in the sense that their rules continue to apply no matter what vocabulary is in the language—this is the hallmark of their logicity. So, when Hirsch's A-language speaker learns the M-quantifiers, the general collapse proof applies to them in the following way. Assume (for \exists_M -elimination) that a is a shmree:

(2') $\text{Shmree}(a)$

On the assumption that a is a shmree infer that there exists_A a shmree (surely, *if* a is a shmree, *then* there is a shmree):

(3') $\exists_A x \text{ Shmree}(x)$

Now discharge the assumption that a is a shmree in the inference step of \exists_M -elimination from (1') to the sentence of (3') as a conclusion:

(4') $\exists_A x \text{ Shmree}(x)$

(4') now only depends on (1'): if there exists_M shmrees, then there exist_A shmrees. But (0') says shmrees do not exist_A. Contradiction.

There are, of course, plenty of ways to avoid the contradiction, but I suggest that any of them will involve rejecting one of the principles that Hirsch explicitly endorses, and must endorse, in order for quantifier variance to be a genuine and stable position. In other words, since these principles lead to a contradiction, quantifier variance is *not* a stable position.

First note, though, that denying (2') is not an option. One might be tempted to think that one could find fault with giving the name ' a ' to an object (i.e., to a shmree) whose existence is not acknowledged by A-language speakers. Note that this is not what is happening in (2'), however. We merely *assume* that a is a shmree, and this assumption is discharged in the step to (4'). There are no centaurs. Assuming that Bob is a centaur (for *reductio*,

say) is innocuous. Indeed, we would have to make that assumption (in a formal calculus) in order to prove ‘ $\neg\text{Centaur}(\text{Bob})$ ’ from ‘ $\neg\exists x \text{Centaur}(x)$ ’.¹²

There are five principles that Hirsch holds—and, I think, has to hold—in order to characterise quantifier variance in the way he intends to:

- (i) Quantifiers vary their meaning according to the conceptual schemes they belong to.
- (ii) Ontological disputes are merely verbal and thus insubstantial.
- (iii) The (unstructured) facts determine which sentences are true (Hirsch’s version of realism).
- (iv) There is no privileged language for ontological disputes, i.e., no privileged language of metaphysics that contains “the real” existential quantifier.
- (v) The logical rules of the quantifiers remain the same.

(v) is most obviously contributing to the collapse of the quantifiers and thus to the proof of the inconsistency. But giving up this principle would leave Hirsch to explain how we understand speakers that use a different existential quantifier, without retreating to a metaphysically privileged language (iv), whilst acknowledging that their concept of existence is equally good (ii) and they speak truly in a realist sense (iii), given their meaning of ‘exists’ (i).

Could we, perhaps, avoid the interplay of the vocabulary of the different conceptual schemes and so block the proof? An obvious way to do this would be to index the relevant terms (names and variables) and only allow the existential quantifiers to operate on “their” terms.¹³ Even if I could convince myself that this was in keeping with (v), and to give up the principle that

¹²Compare also (Eklund, 2008): Eklund presents an argument against Hirsch’s view that relies on naming a scattered sum

¹³This is the way Sider sets up the language for his model of quantifier variance in order to avoid the quantifier collapse: see (Sider, 2007, 217–218), but see also footnote 5 above.

genuine logical constants should be schematic in the sense that they apply to any vocabulary of the right grammatical category, this practice would give immediate rise to the question: which of the quantifiers is “the real” one, which quantifier expresses real existence? In this framework it can also easily be shown that the the A-existential quantifier is a restriction of the M-existential quantifier. Not only does Hirsch reject quantifier restriction,¹⁴ and must do so for the reasons given above, but it is also not clear, what sense to make of this restriction. One of (ii), (iii), or (iv) seems to be at stake: one of the quantifiers does not express existence but a wider or narrower concept, *contra* (ii); the existential statements are *only* true according to the specifications of a conceptual scheme and not true *simpliciter* and determined to be so by the world, *contra* (iii); one of the quantifiers is “the real” existential quantifier and the other one is in some way parasitic on it, *contra* (iv).

Moreover, with the indexed vocabulary, it would be easy to define a “super-quantifier”, in effect as a union of all the existential quantifiers we have in the language. If we made this quantifier inclusive enough (say, as a union of all possible existential quantifiers) we would end up with the maximalist quantifier. This, on the original proposal, was meant to be just one of the many quantifier meanings, but now it starts to look like the quantifier of a privileged metaphysical language, the one that encompasses all possible meaning of ‘exists’. This violates (iv).

There are variants of the above strategies, but it seems more than likely that at least one of the principles will have to be abandoned in order to avoid the contradiction. Giving up any of the principles will leave either leave a massive explanatory gap (as for (v)), or make the proposal collapse into one of the rival positions.

¹⁴See (Hirsch, 2002, 64).

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