

Symmetries, Indexicality and the Perspectivist Stance

Abstract

I critically examine the assumption that the theoretical structure that varies under theoretical symmetries is redundant and should be eliminated from a meta-physical picture of the universe, following a “symmetry to reality” inference. I do so by analysing the status of coordinate change symmetries taking a pragmatic approach. I argue that coordinate systems function as indexical devices, and play an important pragmatic role for representing concrete physical systems. I examine the implications of considering this pragmatic role seriously, taking what I call a perspectivist stance. My conclusion is that under a perspectivist stance, all symmetries (including local gauge symmetries) potentially have a direct empirical status: they point to dynamical aspects that are invariant under changes of operationalisation, and they constitute a guide not to reality, but to nomology and kinship.

Keywords: symmetries;perspectivism;indexicality;scientific representation

1 Symmetries and Surplus Structure

Galileo noted that someone performing experiments confined to a ship could not tell whether the ship was stationary or moving at a uniform speed. Michelson and Morley observed that the speed of light was the same whether it is measured in the direction of motion of the Earth relative to the Sun or in a perpendicular direction. Everyone can see that a lot of phenomena behave in the same way irrespective of location and time. This kind of observations constitutes an important heuristics in science, and perhaps one could go as far as claiming that the aim of physics is to capture what remains invariant under certain changes. The structure of our physical theories reflects this idea: they do respect a number of *theoretical symmetries*, that is, transformations of theoretical models that preserve parts of their structures, and these symmetries often serve as a guide for developing new theories.

One could be satisfied with assuming that the theories of physics and their theoretical symmetries reflect symmetries in the world and that it is one of their main purposes. But some philosophers want to draw a different conclusion: they want to identify the invariant components of theories with reality—what I shall call, following Dasgupta (2016), a *symmetry-to-reality inference*. According to them, the theoretical elements that vary with symmetry transformations are redundant: they play no representational role. Some argue that we should present our theories without referring to this “surplus structure” (as in coordinate-free formulations of Newtonian mechanics in Galilean spacetime (North 2009)), or, more radically, that we should eliminate this surplus structure by “quotienting” our theories to turn them into what I shall call *reduced theories* (Belot 2001; Earman 2002; Baker 2010). This concerns, for example, the positions in space and time assigned to objects in a model: according to this latter point of view, different models related by a rigid translation or by a rotation in space are mere redescrptions of the same state of affairs, and we could as well describe the same system more parsimoniously in terms of spatiotemporal relations rather than absolute positions. This strong version of the symmetry-to-reality inference can lead to deep metaphysical conclusions, such as the alleged non-existence of change (Earman 2002).

However, this move is paradoxical, assuming that empirical symmetries do constitute an important heuristics for science, whose role would be to account for them: once our theories have been purged of their surplus structure with respect to some relevant symmetries, they do not respect these symmetries any more. How then can they still explain the empirical symmetries they were designed to account for? It seems (to paraphrase Wittgenstein) that these authors want us to throw away the ladder that we have just climbed up. The sole claim that variant structures are mere redundancies leads to a puzzle, which can be framed as follows (Teh 2016): if symmetry transformations relate representations of identical states of affairs, how can they have any observable consequence (including, presumably, the ones from which they were inferred)? The paradox is only stronger if, as Roberts (2008) claims, only quantities that are invariant under symmetry are measurable.

The traditional answer to this paradox starts from the assumption that a symmetry has a *direct empirical status* (DES) if one can observe that a transformation has taken place, and one observes the invariance of the relevant features under that

transformation (Kosso 2000). This can be the case if the symmetry is applied only to a subsystem of the universe, excluding our measuring instruments, and that at the level of the universe, the transformation that occurs (the symmetry transformation for the subsystem, combined with an identity transformation for the rest of the universe) is not a theoretical symmetry, because it relates distinguishable global states of affairs (Greaves and Wallace 2014). The quantities that vary with this transformation concern the relations between the subsystem and its environment (including our measuring instruments), but not the intrinsic properties of the subsystem. In sum, according to Greaves and Wallace, both aspects, the relation between empirical and theoretical symmetries on the one hand and the inference from symmetry to reality on the other, concern different kinds of symmetries, with different ranges of application: to subsystems in one case and to larger systems up to the whole universe in the other.

Note, however, that the authors concerned with the debate on DES present their case using the traditional formulation of physical theories, with all their “surplus structure”. And that for a good reason: a reduced theory has no symmetry, so trivially, no symmetry of this theory can have DES. Galileo’s ship would be represented by the same model in a reduced theory, or by the same sub-structure of a model, whether it is moving or not; a distinct part of a model encompassing the shore and the ship, representing the distance (or the region) between the two, would vary in the two situations, but the part that represents the ship would be identical. Arguably, reduced theories still *explain* why one cannot tell whether Galileo’s ship is moving from the inside, but without appealing to symmetries: the explanation is simply that Galileo’s ship is in the same state in both cases. Friederich (2015) draws the same conclusion with regards to local symmetries (in *non-reduced* theories), after arguing that they have no DES: “what appear to be two physically distinct yet empirically indistinguishable subsystem situations [...] turns out to be one single physical subsystem situation”. This conclusion comes out naturally with reduced theories.

However, scientists do not use reduced theories either when representing concrete physical systems. This could be because reduced theories are more difficult to handle. But most of the time, they do not even use coordinate-free formulations of non-reduced theories, except in foundational discussions (Wallace 2019b). As remarked by Belot (2018), the received view, among scientists, is that translations and rotations applied to a model do represent a genuine physical change (or at least they can represent such a change).

So, part of our original puzzle remains: if what is real is what is invariant under symmetry, and if the aim of science is to describe reality, why aren’t scientists and philosophers doing away with coordinate systems or gauges when representing physical systems, or the universe as a whole? And why should we assume that any symmetry has empirical significance if, ultimately, theories without symmetries are more faithful descriptions of reality?

In this article, I examine this issue by adopting a pragmatic approach. I focus on the case of coordinate transformations in particular, and analyse the role they play in scientific representation. I argue that coordinate systems are indexical devices (section 2), and that such devices play a central role in representation (section 3). From this observation, two options are available (section 4): one can maintain that

their role is *merely* pragmatic, and still claim that the associated surplus structure should be eliminated from a metaphysical picture of the world. Or one could take this pragmatic role more seriously. I examine the latter option, which implies what I call a perspectivist stance, and the role played by symmetries if one adopts this stance. My conclusion is that under this option, symmetries do have DES, including when they correspond to mere re-descriptions of the same system (section 5), and that it is likely that this applies not only to coordinate systems, but also to at least some local symmetries (section 6): the empirical significance of symmetries is that they indicate an invariance of nomological content under various operationalisations. They also allow us to identify *kinds* of systems that share the same nomology.

This means that the symmetry-to-reality inference should be replaced by a symmetry-to-nomology and a symmetry-to-kinship inference, where nomology corresponds to necessary connections between possible perspectives on objects of a given kind. If we accept this, there is no puzzle, because there is no incompatibility between the idea that reduced and coordinate-free theories describe the nomological structure of the world (but not concrete objects directly), and the idea that non-reduced theories with coordinate systems can be used to describe the world from particular perspectives.

2 Scientific Representation and Indexicality

My aim in the first two sections of this article is to examine the representational role of coordinate systems, taken as typical candidates for being “surplus structure”. I will argue that they play an important pragmatic role, which will serve as a basis to develop a perspectivist stance towards symmetries.

Let us start with a very general question: what is it to represent a physical system? It is now commonplace to assume that representation is (at least) a three-place relation between a model, a target and a user (Suárez 2003; Bailer-Jones 2003; van Fraassen 2008; Giere 2010a). Users ensure the directionality of representation by taking particular symbols of the vehicle to denote objects, properties or functions of the target system, that is, by interpreting the model (see Contessa (2007) for an account of interpretation). By analogy, when interpreting a metro map, a user will take certain symbols to denote metro stations and coloured lines to denote their connections by metro lines, and this is what allows the user to make inferences on the target of representation, concerning for example how to travel from one station to another. We can suspect that there are norms concerning how to interpret a scientific model (or a metro map) appropriately, which boils down to a correct interpretation of its symbols. A model is accurate if the inferences it allows lead to true conclusions.

One aspect is obscured by this notion of interpretation, and by the map analogy, however: most models in science do not denote particular concrete targets. For example, Bohr’s model of the hydrogen atom does not represent one specific atom in the universe. It does not represent all atoms in the universe either, because what is represented is not a collection of atom. The model rather seems to describe a *type* of system, or perhaps a *prototype*, and it can in principle be applied to any given atom, but outside of an experimental context of use, it has no concrete reference.

In Ruyant (2021), I have proposed to analyse this aspect by drawing an analogy

with a notion of philosophy of language: indexicality. An indexical term, such as “I”, “now”, “here” (or a demonstrative such as “this computer”) does not have a reference outside of a context of use. Indexical sentences, such as “I am reading”, have no intension (or truth-condition) outside of a context. However, the meaning of indexical terms and sentences can be interpreted in terms of a *character*, which is a function from context to content (Kaplan 1989). Every competent locutor knows that “I” will normally refer to the locutor, “here” to the place of locution, and “this computer” to the salient computer in the context of locution. The character of the sentence “I am reading” can thus be analysed as a function that assigns to every context where a locutor asserts the sentence an intension, corresponding to the proposition that this locutor is reading.

I believe that the best way to understand how abstract models are used in science, and what is their general representational status, is to take them to be indexical: they only acquire their reference and intension (conditions of accuracy) in particular contexts of use¹. Pursuing the analogy, it is quite plausible that the symbols of a scientific model have a *character*. This character takes the form of a function from context to concrete referent, and thus it fixes, or at least constrains, the way the model should be interpreted in context. Such characters could be expressed as follows: symbol “O” in the model of the simple pendulum refers to the centre of mass of the salient pendulum in context, symbol “x” refers to its position along the axis of rotation relative to “O”, etc.

Maybe not all symbols of a model can be interpreted in this indexical way, but coordinate systems certainly can. The origin of coordinates in, say, the model of the simple pendulum does not refer to a particular location in space and time in abstract contexts, but it does in particular experiments, when the model is applied to a concrete pendulum. The same goes for all other points of the coordinate system: once the origin and axes have been mapped to physical locations and directions, they all acquire a reference. Furthermore, the coordinate system as a whole can have a character that constrains appropriate applications of the model: in the case of Newtonian models, for example, it is often implicitly assumed that the mapped referential should be inertial.

Concretely, what does it mean for a point of the coordinate system to acquire a reference? Assuming that they refer to spacetime points, taken to be metaphysical entities, would be problematic (even a substantialist about spacetime would deny that our pendulum has a fixed position in absolute spacetime). They rather refer to specific locations relative to a concrete reference for position measurements in an experimental context. The function of the coordinate system, once interpreted, is to tell us how we should measure the positions of objects when we use the model (as their name indicates, they are used for coordination between the model and the world): if, for

¹van Fraassen (2008) has also applied the notion of indexicality to scientific representation, but in a different way. It is associated, for example, with self-location when using a map (“I am here”). However, maps are not indexical representations in the sense proposed here, since in general, the symbols on a map always refer to the same objects whatever the context of use. Van Fraassen’s idea is that an ineliminable indexical is involved when using a representation (even a non-indexical representation) in context. This is a “deep” but controversial idea (I will mention it again later). The idea that general scientific models are indexical because they only refer to concrete objects when applied in context is much more mundane, and should be less controversial.

example, the x axis in a model is set to denote a particular direction relative to our position on Earth in context (say, from South to North), then we will expect that x coordinates of objects in the model correspond to measures along this physical direction. If measures are performed with a ruler, its origin should be made to coincide with the origin of coordinates, and it should point towards the North. If such a measure cannot be performed directly for practical reasons, then we can use coordinate transformations to retrieve the correct value from indirect measures (using theoretical symmetries), but then, another model with a different coordinate system is implicitly used.

So, interpreting a coordinate system in context amounts to adopting a particular class of operationalisations for position measurements, associated with concrete referents (physical directions, etc.). Furthermore, as noted earlier, the interpretation of the coordinate system is constrained by its character. If “O” in the model corresponds to the rest position of the pendulum, then we should interpret it as such in context. This means that a general scientific model constrains, by means of its coordinate system, the class of operationalisations that should be used when applying the model.

This is not to deny that the application of these constraints to concrete experiments can be fairly complicated. The coordinate systems of theoretical models are often characterised in theoretical terms without any guarantee that concrete objects will satisfy these characterisations (for example, an “inertial coordinate system”). In this case, the mapping between the coordinate system and our measurements will be indirect. Yet in any application of a model with position measurements, the coordinates will be interpreted either directly or indirectly in terms of specific locations relative to concrete referents, such as measuring instruments.

I believe that this is the right way of understanding how coordinate systems are used in science: as indexical devices that refer in context and tell us how we should measure the quantities that the model describes. This is their “character”. This picture could perhaps be extended to other kinds of “redundant structures”, such as electrostatic potentials (the zero potential corresponding to where we should set the chassis ground of our measuring apparatus), and perhaps even to gauge structures. I will briefly explore this possibility later, but for now, let us keep focusing on coordinate systems.

3 The Pragmatics of Coordinate Systems

Having argued that coordinate systems are indexical devices, I now wish to argue that they play an important pragmatic role, and that they are not so easily eliminable from our representations of the world. They might even be considered indispensable. What I wish to examine in this section is whether, in light of the indexical role played by coordinate systems, it would make sense for scientists to work exclusively with a coordinate-free formulation of a theory, and I will argue that it would not make much sense.

Let us take as an illustration a very simple model: a harmonic oscillator, that is, a point mass which experiences a central force. When using the coordinate-based formulation of the theory, it is natural, in such a model, to take as the origin and direction

of coordinates the rest position and the axis of movement of the point mass. In this case, as is well known, the trajectory of the point mass will be a sinusoid, and its trajectory in phase space an ellipse. However, assuming that symmetries of Newtonian mechanics relate representations of the same state of affairs, this is only one way of representing the system: applying a boost or a simple translation to the trajectory along the axis in the model would yield another representation of the same physical system². There is therefore a class of equivalent models which describe various trajectories that are not sinusoids, or trajectories in phase-space that are not ellipses, but still represent our harmonic oscillator. These models are related by symmetry transformations.

What happens if we want to get rid of this surplus structure? We can first adopt a coordinate-free formulation of the theory using differential geometry, so that no reference is made to symmetry-variant quantities. This turns all models related by rigid transformations into isomorphic models. We can go further and adopt a Galilean spacetime formulation, so that models related by a boost transformation become isomorphic as well. The idea is to consider that these models are representationally equivalent.

Note that our theory still has the same symmetries as the original one. Distinct models can still be taken to represent the exact same state of affairs: all we did is make these models isomorphic (the theory still quantifies over points in spacetime, and the fact that an object is located at one point rather than another is somehow arbitrary, but it does not assign symmetry-variant properties to these points). A further step consists in actually removing the surplus structure and associated symmetries by quotienting the theory. This can be done either by considering a theory whose models correspond to equivalent classes of models of the original theory, or alternatively, a theory which only quantifies over invariant objects (e.g. spacetime relations instead of spacetime points). The two are roughly equivalent (although the second approach requires introducing holistic properties assigned to whole systems in order to preserve the content of the original theory (Belot 2001)). This second step leads to a more drastic reformulation of the original theory, and it is less often adopted in the literature (Wallace (2019a), for instance, rejects it explicitly (p. 8)). In any case, it will be enough for our purpose to consider coordinate-free formulations.

Now consider a coordinate-free model of our harmonic oscillator in Galilean spacetime, and imagine that in an experimental situation one wants to verify whether or not a given system is a harmonic oscillator, and then make predictions on the behaviour of this harmonic oscillator. It is not very clear how the model should be interpreted in terms of its concrete target. Our model describes a particular trajectory in a manifold, but it does not contain any position quantity. It does not contain any distance quantity either, because it only refers to one material object. Galilean spacetime still has a metric, from which we can compute the acceleration of our trajectory, an invariant quantity. In order to verify whether the model is accurate, one could measure the acceleration of the physical system and compare it to that of the model (the magnitude of the acceleration is insufficient to select harmonic oscillators; changes in its

²I neglect the fact that, strictly speaking, the existence of a central force breaks the symmetries of Newtonian mechanics: I assume that the transformation applies to the force as well.

orientation must be considered as well, but I will ignore this problem here). But how shall we do this?

The simplest way, it seems, is to measure the position of the system relative to an inertial reference frame at various times, and then evaluate the second derivative of the trajectory. These evaluations of the acceleration will constitute our data model, and the data model will be compared to the theoretical model. However, measuring positions in an inertial reference frame implies using a representation, if not a full theoretical representation of the target system, at least a *model of the experiment* (I borrow this notion from Suppes (1969)). This model of the experiment must have a coordinate system centred on our reference for position measurements. Furthermore, all concrete predictions concerning the future position of the target must be derived by using the mediation of the model of the experiment as well. In this context, we are not really doing away with coordinate systems.

An alternative approach, which is roughly equivalent, consists in interpreting part of the structure of the manifold on which the model is built (an inertial worldline, an event on this worldline for time origin, spatial directions, all appropriately chosen) in terms of our reference for measurement, and then compare spatial distances between this structure and the trajectory of the oscillator in the model to our measurements (this move is not available in reduced theories). However, “giving a name” to these elements of the manifold just amounts to reintroducing the structure of a coordinate system. So, again, we are not really doing away with coordinate systems.

If we were to renounce the use of any coordinate system or equivalent structures, we could use an accelerometer to measure the acceleration of the oscillator directly. Then we would be unable to predict anything more than the evolution of acceleration over time. This looks like a drastic limitation of the kinds of experiments one could do with an oscillator. Measuring positions, and not only accelerations, is certainly important in physics. So, it seems that coordinate systems are indispensable for most scientific uses of a model.

One could object that the problem is that we are only modelling the target, and not the reference for our measurements. Imagine that our oscillator is a pendulum (a simple pendulum approximates harmonic oscillators for small angles): we could decide to incorporate the physical support of the pendulum in our coordinate-free model as a solid object, thus extending the target system represented by our model. Then the position of the pendulum relative to its physical support could be directly inferred from the model, because it is an invariant quantity (in Newtonian mechanics). This relative position (its distance from its physical support along an axis specified by the support) could be compared to measurement results without any coordinate system performing the mediation. In the cases where the natural reference for coordinates in the model does not correspond to any physical body, for example, when modelling planetary motions, one could still incorporate our measuring instruments in the model and get a similar result. We could also extend this to time measurements, by modelling the clock used in the experiment (which might just be another harmonic oscillator!).

Incorporating a reference system into our model is not very different from using a model of experiment, and the previous remarks boil down to the observation that in principle, if the full experiment is represented and not only the original target, one

can possibly dispense with a coordinate system and associated structures for practical purpose. This shows that the use of coordinate-free, or even reduced models is not strictly impossible for making predictions, at least in principle. But this alternative does not correspond to what actual scientists do in practice.

I will examine the reasons why this is so in a moment, but let us first mention another worry: this move, consisting in incorporating physical references into the model in order to get rid of the surplus structure, could lead to an infinite regress. I have claimed that the main function of a coordinate system is to tell us how positions should be measured. The object+reference system needs to be measured as well to have any empirical import. Don't we need yet another coordinate system to tell us how it must be measured? Not really, because the way of measuring the whole system is no more contextual, since all contextual elements (the physical reference) have been incorporated into the model. Knowing how to measure distances between objects is just knowing what is the empirical interpretation of the theoretical vocabulary *in general*. However, this problem of infinite regress occurs if, in a purely structuralist spirit, we want the structure of the model alone to tell us what, in the model, counts as relative velocity and what counts as distance (for example), without using an interpreted vocabulary. Trying to settle these matters by modelling measuring apparatus for instance would lead to an infinite regress, because we will want this new model to tell us why this apparatus measures distances, and not velocity.

Firstly, note that this move is idealistic. What matters for scientists is the reliability of an instrument for measuring a certain quantity, and reliability is not in general justified by the fact that the instrument is entirely modelled by the experimenters. As Hacking (1983) observes, telescopes were used for centuries without a complete theory of their functioning. The use of apparatus rests not only on theoretical knowledge, but also on practical knowledge that is contextually implemented: for example, an instrument can be calibrated in a particular situation, so as to achieve stable outcomes. But we do not need to model the calibration procedure, or the whole experimental context (including remote stars if they are used as reference in the measurement procedure?), in order to interpret the measurement results as corresponding to a certain type of quantity. The idea of systematically modelling our measuring instruments and the whole environment is idealistic at best, but it is not really attainable, and it does not seem required for interpreting measurement results if outcomes are robust enough.

Secondly, this move could lead to an infinite regress because of a problem related to Putnam's model-theoretic argument, and also possibly to the "preferred basis problem" in quantum mechanics (see Wallace 2010). One can fix a "preferred basis" by considering the interaction between a system and measuring instruments, so as to settle "what is measured" on the system. This would solve the problem of interpretation: we could say that the preferred basis in such or such experimental situation corresponds to what we call "distance". The problem is that there is still no "preferred basis" for the object+instrument system, and therefore the apparatus-object cut that generates this preferred basis is arbitrary. Invoking a larger environment in order to fix this cut non-arbitrarily leads to an infinite regress.

The formal status of a "basis" in quantum mechanics is analogous to a coordinate system for the space of observables, and there might be interesting connections with

the indexical aspect discussed above. Perhaps the “basis”, or the interpretation of theoretical language (what counts as distance, etc.), has an indexical aspect as well (van Fraassen seems to go in this direction when he claims “A theory says nothing to us unless we can locate ourselves, in our own language, with respect to its content”, p. 235). But I won’t explore these themes any further here. In any case, the infinite regress problem might not occur if one assumes a realist interpretation for the theoretical vocabulary, for example in terms of real properties such as “distance”.

Let us now return to our original issue: if it is possible to eliminate coordinate systems, and even use reduced theories, by incorporating measuring instruments or references into our models, then why don’t scientists do that?

The answer, I believe, is that it would be a poor choice pragmatically speaking. Once we have modelled a reference system for our object within a model, we can extract from it a structure that will be isomorphic to a non-reduced model of the target alone located in a coordinate system. So, we haven’t gained anything in the procedure: a mechanical model, to be of any use, must have a structure that corresponds to a coordinate system for the “real” object of interest. There is thus no gain, but there is an important pragmatic loss: flexibility. Once the physical reference or measuring apparatus is integrated into the model, we are not free anymore to switch to a different experimental setup, or to operationalize our measurements differently, because our model now describes the experimental setup as well as the intended target.

These remarks point to what I take to be the main pragmatic virtue of coordinate systems (and, possibly, of “surplus structure” in general), which is their modularity. Having a theory with surplus structure facilitates the use of models for different purposes, the combination of different models into larger ones or the reverse procedure of splitting models into parts (this aspect is developed by Rovelli (2014)). This modularity is directly related to indexicality: a given model should be applicable to various possible targets of the same type in various contexts, using different types of apparatus. So, the model should not represent any elements of the context directly, but rather contain indexical structures that can be interpreted differently in any context.

Is modularity a mere pragmatic virtue of models though? Maybe it is a bit more important than this, and maybe coordinate systems are actually indispensable for science.

I observed earlier that scientific models rarely represent one particular concrete object. They more often represent types of objects. It seems to be part of the aim of science to produce representations that can be generalised, and applied in various contexts. Reduced models have huge drawbacks in this respect: they can be of any use only if they incorporate environmental features, such as measuring instruments or physical references, and then their scopes become very limited. For this reason, the symmetry-to-reality inference cannot be turned into a normative claim that we *should* use reduced models in science. As for coordinate-free models, they only do better in so far as their manifold can easily be complemented with a coordinate system or something equivalent.

This poses a dilemma for a scientific realist who would claim that the aim of science is to produce faithful representations of reality. If the symmetry-to-reality inference is valid, then reduced models are more faithful and parsimonious representations of reality than non-reduced models. However, scientific models are not, in

general, reduced, and it seems that reducing them systematically would impair the great empirical achievements of science, and in particular, the ability of scientific representations to be applicable in a variety of contexts (a feature typically associated with explanatory depth). The symmetry-to-reality inference could be incompatible with the typical arguments for scientific realism, which infer theoretical truth from empirical success, or at least this line of argumentation should be qualified.

4 The Universalist Stance and the Perspectivist Stance

In the previous section, I identified a pragmatic role for coordinate systems. I argued that they are hard to dispense with, and that eliminating them could be incompatible with the aim of science. But this does not mean that these structures are strictly indispensable for representation and predictions, so someone with metaphysical leanings could be unmoved by these considerations: surplus structure is useful, she would say, and perhaps even indispensable for science, but it is not indicative of what really exists, and eliminating this surplus structure is still the way to go as far as metaphysics is concerned. Perhaps scientific models and theories incorporate pragmatic as well as ontological aspects in their structure, but the metaphysician can tell them apart.

This attitude can be associated with what I will call a universalist stance (US), as opposed to what I shall call a perspectivist stance (PS). The difference between the two stances is the following: for US, correctly interpreting what a theory says about the world implies being able to represent accurately the whole universe independently of any particular perspective, while for PS, it only requires being able to represent accurately any object of interest from any possible perspective.

Taking stock from the previous sections, I understand a *perspective* as involving (at least) a class of possible operationalisations on a particular object, all associated with a common reference. For example, all possible position measurements in a given reference frame can be associated with one perspective on an object. In this sense, representations of physical systems as having particular positions in space and time in a coordinate system are perspectival, because (as argued earlier) this assignment of positions is associated with a class of operationalisations. The notion of operationalisation should be understood firstly as an epistemic notion: although not necessarily tied to a single epistemic agent, and therefore not subjective (a single perspective on an object can be shared by scientists working together on the same experiment, by means of ostentation among other things), it is associated with epistemic agents in general. A perspective associates its possible operationalisations in an experimental context with a theoretical vocabulary and a minimal structure (typically, that of a coordinate system). It acts as a mediator between a theoretical model and situated experience.

US can be described as pursuing what is sometimes called a “view from nowhere”. The idea would be that perspectival representations are not fundamental, but derived: any given perspective can be recovered from the “view from nowhere” in terms of relations between observers and targets, in the same way as we noted that the substructure of a reduced model incorporating our physical reference is isomorphic to a non-reduced model of the target system alone. This means that for US, coordinate

systems are redundant surplus structure.³

This “view from nowhere” is an idealisation that serves metaphysical (but not scientific) purposes. It is a fiction. By contrast, PS takes scientific representational activities at face value. When interpreted in experimental contexts, physical models always represent particular objects from particular perspectives (even in cosmology: in so far as it is interpreted in terms of measurement results, a model of the universe represents the universe as viewed from our perspective). For PS, invariant structures under symmetry transformation have a different status than for US: they do not represent all and only what is real, but rather what is constant across all possible perspectives related by the symmetry transformation. Since the same theoretical models can in general be used in various circumstances, these invariant aspects concern not only the possible perspectives on a particular object, but also on all possible objects of the same type. This does not exhaust what there is, since particular objects and perspectives also exist on top of this invariant structure.

Talking of *possible* perspectives implies a modal status for the invariant structures mentioned here: they correspond to what remains the case in all possible perspectives, or to “necessary connections” between perspectives. In the context of physics, the modality involved is naturally interpreted as being *nomological*. Thus when PS is adopted, the inference does not go from symmetry to reality, but rather from symmetry to *nomology*. A reduced model directly represents the nomology of a *type* of system (and it need not incorporate any physical reference, since these are specific to particular instances). A non-reduced, but coordinate-free model is perhaps more naturally interpreted as representing a prototype. A structural realist could say that such models describe the “modal structure” of a type of system. But if she adopts PS, she will not go as far as claiming that this structure is “all there is” or “all we can know”, since we can have knowledge of particular instances from particular perspectives that does not supervene on this modal structure (for example, and quite trivially, that *this* system has *this* location relative to us, now). So, she will adopt neither ontic nor epistemic structural realism in their traditional formulations.

Indeed, when representing a particular physical system, presenting a modal structure is not enough for PS: saying what remains constant in all possible perspectives doesn’t say, for instance, which perspectives are actual and which are not, or which correspond to ours and which do not (this aspect is particularly important assuming that operationalisations, and so, perspectives, are generally active: I return to this theme below). This is so because the defender of PS resists the idealistic move of incorporating the measuring instruments associated with a given perspective into her model: this would merely mean switching to a different perspective on a larger object (at most, doing so gives us insight into how the associated perspectives are related). Measuring instruments give us access to the target, but they are not the target of

³Strictly speaking, adopting this view from nowhere does not necessarily imply being able to represent the universe as a whole. So, an author such as Wallace (2019a) expresses doubts with regards to the idea that we should systematically interpret physical theories in cosmological terms. Yet he bases his defense of a version of the symmetry-to-reality inference on a consideration of models that incorporate measuring instruments as well as target systems. He remarks that no variant quantity of the target+instrument system can be measured by said instrument, and infers that variant quantities are unobservable. This reasoning amounts to eliminating perspectival aspects by invoking a larger model, taken to be more fundamental for interpretive purposes. As explained further, this is precisely the kind of move that PS resists.

representation. And if they are not represented, then we generally need an indexical device, for example, a coordinate system, on top of the modal structure of the target to describe the target from *our perspective*.

It follows that someone who adopts PS will take coordinate systems and their pragmatic role more seriously than the one who adopts US: an interpreted coordinate system is indeed associated with a particular perspective on a physical system, which corresponds to the choice of a particular reference and class of operationalisations. From a perspective, a physical system does have a precise position, and there is no representation without a perspective. In this respect, reduced and coordinate-free models can only be partial representations.

One could wonder in what sense perspectival aspects *exist* on top of invariant (nomological) aspects. Existence and reality are often characterised in terms of independence from epistemic agents, but the notion of perspective certainly brings a form of epistemic relativity in representation (which is *not* subjectivity). Nevertheless, it does not make much sense (in my view) to claim that the nomology of the world is real, but not its concrete instantiation in observed phenomena. One option to resolve this tension is to go full anti-realist, and claim that the nomology inferred from symmetries is itself epistemically relative rather than real in the strong sense, because it consists of relations between perspectives. A more conservative approach would consist in maintaining that the perspectival elements of representation are real, but metaphysically derived rather than fundamental, and yet indispensable for us. This would mean that PS is not incompatible with US and with a “symmetry-to-fundamentality” inference after all⁴. Perhaps there is a middle way: some authors have recently attempted to combine perspectival and realist elements in their epistemology, giving rise to a family of positions dubbed “perspectival realism” (Giere 2010b; Massimi 2018). They also reject the “view from nowhere”, but claim to be realists (However, see Ruyant (2020)). The notion of perspective used by these authors is often associated with research programs or conceptual schemes rather than with concrete, spatiotemporal perspectives, so it remains to be seen whether the same arguments could be used. Unfortunately, there is not enough space to say more about this topic here.

I have the impression that US is implicitly adopted in the literature on symmetries. Section 3 ended with the formulation of a problem for US: they cannot turn their position into a claim about what science aims at (a representation of the world free of perspectival aspects), or into a normative claim, since perspectives are pragmatically relevant. So, it seems that the perspectivist stance is more faithful to scientific practice. This is related to the problem highlighted at the end of the introduction: a symmetry-to-reality inference looks like a natural move for US, but this move eliminates symmetries from our representation of reality (or turns them into isomorphisms), and the empirical significance of these theoretical symmetries, as well as the heuristic role of empirical symmetries, become puzzling. All this plays in favour of PS. However, the pragmatic relevance of perspectives is not enough to show that they are metaphysically relevant, as PS implies.

One way of advocating PS is to argue for the indispensability of indexicality, as Lewis (1979), Perry (1979), Ismael (2007) and van Fraassen (2008) did (but see Cappelen

⁴As noted by an anonymous reviewer, this is a way of interpreting Baker (2010)’s proposal

and Dever 2013). However, doing so would lead us to considerations that run deep into the philosophy of language, the philosophy of action and the philosophy of mind, which is, obviously, far beyond the scope of this paper.

What I wish to do instead is to assume PS and examine what it implies for the empirical status of symmetries. I think that adopting this stance can lead to novel illuminating considerations on this topic that can dissolve the puzzling aspects mentioned above, which is an indirect way of arguing for the position.

5 Symmetries in the Perspectivist Stance

What is the role of coordinate change symmetries from the point of view of PS? Do they have DES? Take first the case of passive transformations, where the same object is described relative to various references. If we accept that coordinate systems are indexical devices, and that they have a character that constrains legitimate operationalisations, then such transformations also mean describing the same object in relation to various classes of operationalisation: for example, a given coordinate system stipulates that the position of objects should be measured with regards to a given reference. Adopting a different operationalisation requires applying coordinate transformations, which means switching to a different coordinate system.

In general, a given model is related to an infinite number of other models by symmetry transformations. But this does not necessarily mean that all corresponding perspectives are actual. Most of the time, only one perspective is adopted for measuring a given target, so only one is realised. For this reason, and as already noted, it is natural, from the point of view of PS, to understand symmetries in modal terms: in the case of passive transformations, symmetries relate *possible* perspectives on the same target system, or possible classes of operationalisations.

What coordinate change symmetries indicate, in this context, is that various possible operationalisations of our measurements would give the same results with respect to some aspects of the target. In Galileo's thought experiment, which is a traditional context for assigning DES, whether we observe the ship moving uniformly from the shore, or whether we observe it from the inside, we will measure the same relative distances between objects moving in the ship (assuming Newtonian mechanics), and the same acceleration for these objects.

The idea that various operationalisations associated with symmetry transformations would give the same results could seem trivial in some respects. Take the case of a translation symmetry: if we measure the position of an object A along an axis relative to a reference O (for example, putting the "0" of a ruler on O), and then the position of another object B on the same axis, we can calculate the distance between A and B : $d(A, B) = OB - OA$. Now doing the same operation relative to another reference O' on the same axis would yield the same result. One could think that this fact merely reflects the reliability of our instruments and the robustness of our measurements, and that it is not indicative of any empirical symmetry: it simply tells us that our ruler does measure positions accurately. But the fact that spatiotemporal symmetries depend on the theory shows that this is far from trivial: in the theory of relativity, if O and O' are moving relative to each other, the distance between A

and B will be different if measured from the perspective of O as compared to from that of O' . So, which kinds of operationalisations robustly yield equivalent results is not a trivial matter, and I think that this is enough to claim that passively interpreted symmetries have DES.

Let us now turn to active transformations. What if, instead of looking at Galileo's ship from a different perspective, we compare two possible experiments, one where the ship is moving relative to the shore and one where it is static, where the two experiments use the same reference for positions? If we are talking about the same object at the same time in two different possible worlds, assuming that this makes sense, the treatment needs not be very different: the two experiments are implementing different perspectives on the same object. Making the object move uniformly relative to us, and then measuring the distances of objects inside the object is just one kind of operationalisation, and as it happens, this gives the same results regarding invariant quantities. This counts as an operationalisation assuming that operationalisations are, in general, active interventions (more on this later). It seems that from the point of view of PS, active and passive transformations are quite similar: they are transformations from one perspective to another. The only difference is that in the case of passive transformations, the two operationalisations are mutually compatible, while in the case of active transformations, they are mutually exclusive, and only one must actually occur.

We can extend the rationale to active transformations between instances of the same object at different times in the actual world (two successive experiments with the same ship), or between different objects of the same type (two ships), which is also acceptable for establishing DES (Brading and Brown 2004, 647, note 5). In these cases, we can understand the two experiments as implementing different perspectives on the same *type* of object (or situation), which is just a generalisation. The instance of the type involved is no more the same in the two experiments, but this is not a difference that matters from the point of view of PS, since the inference is from symmetry to nomology, and nomology is not about particular instances.

This blurring of the distinction between passive and active transformations is perhaps counterintuitive. However, as noted earlier, operationalisation is not always a passive activity: in general, one has to interact with a system in order to measure it. There are lots of complex manipulations going on to observe the characteristics of fundamental particles for example, and most of the time, making an observation means creating a particular situation where the target is embedded. I don't think that there can be any principled distinctions, among the various kinds of operations that can be performed on a system in an experimental setup, between those that actively "change the system" and those that do not.

Surely, moving oneself while merely looking at a ship brings no change to the ship, while turning on its engines to make it move does. Visual observation is passive because photons will go in all directions anyway: the perspective of any moving observer is "already available", so to speak. All these perspectives are mutually compatible. But in general, whether measurements are perturbative or not is a matter of degree, and as quantum mechanics has taught us, not all measurements can be performed simultaneously. Assuming that passive symmetry transformations relate different perspectives (or classes of operationalisations) on a given object, mutually

compatible or not, it follows that the distinction between passive and active transformation cannot rest on the idea of changing or not the observed object. The only distinction that is not a matter of degree is that between performing various kinds of operations on the same situation at the same time (which is rarely done), or on different situations involving objects of the same type. Following this distinction, one could understand passive symmetry transformations as relating *mutually compatible* perspectives on a given situation, implying that all associated operations could be performed simultaneously. But with this understanding, some transformations that we would intuitively classify as passive would actually count as active in quantum mechanics, because the two corresponding measurements are incompatible. In any case, this distinction between passive and active transformations might not be very significant if science is in the business of producing generalities rather than describing particular situations.

So we can understand symmetry transformations in full generality, passive or active, as relating various possible perspectives on a given *type* of physical object.

Note that whether different represented objects are or are not of the same type can be informed by the symmetry itself. In physics, types of particles are characterised by symmetry groups. We could say that the still and the moving ships are physical systems of the same type *because* they are related by a symmetry of our theory, *because* some dynamical aspects are invariant for the two systems. This means that from the perspective of PS, we can make an inference from symmetry to nomology, but also from symmetry to kinship: systems sharing the same dynamics can be classified as systems of the same kind⁵. Invariant quantities, such as spacetime intervals between component parts of a represented situation in a relativistic model, or the conserved quantities involved in Noether's theorem, can be understood as characterising kinds of systems or situations.

In sum, if we adopt a perspectivist stance, all realisations of coordinate change symmetries in the world lead to DES, whether they correspond to redescrptions of the same target from different perspectives or not: in either case, complying with Kosso (2000)'s conditions for leading to DES, one can observe that a change has occurred (a change of operationalisation) and that some aspects are invariant under this change. This allows us to perform an inference from symmetry to nomology, and to a classification of objects into kinds, where these kinds precisely share nomological aspects. The aim of science is not to describe the whole universe in a theory, including its observers, but to identify relevant kinds and their nomology from the perspectives available to these observers.

It is certainly interesting, from a theoretical perspective, to analyse the structure of these kinds and laws, abstracting away from particular instances. A theory that avoids referring to the "surplus structure" (i.e. to the components of representation that only have contextual significance) can be used for this purpose. This explains why, for example, coordinate-free formulations of theories are often used when discussing foundational issues. But by no means this entails that all there is to reality is a set of kinds and laws: there are also instances, these are always described from par-

⁵This is close to an inference from symmetry to reality, but not quite the same. It could be interpreted in terms of the symmetry-to-fundamentality mentioned before.

ticular perspectives, and so, they are better represented in non-reduced, coordinate-based theories. Overlooking this aspect amounts to eliminating the empirical basis that theories are designed to account for in the first place.

6 Local Symmetries

Before concluding, let me briefly examine whether this account can be extended to other types of symmetries, such as local gauge symmetries.

A local symmetry is an invariance under a class of transformations that are function of space and time. Loosely speaking, this means that in order to fix a gauge, we have to choose a reference for every point of spacetime, and not only for the system as a whole. These symmetries are more often (Kosso 2000; Brading and Brown 2004; Healey 2009; Friederich 2015) but not always (Greaves and Wallace 2014; Teh 2016) interpreted as having no DES. Can we assign DES to them from the point of view of PS, as we did in the case of global symmetries?

A first step is to examine whether the references involved also have an indexical status and a character that constrain potential operationalisations. At least sometimes they do (for example, a zero potential corresponding to the chassis ground for electric potential measurements), in particular if we include practically unrealisable operationalisations that are conceivable in principle. It remains to be seen whether all kinds of gauges, for example quantum phases or diffeomorphisms in general relativity, can be associated in this way with operationalisations. But when they can, we can attempt to apply the same arguments: invariance under gauge choices would be indicative of invariance of dynamical aspects under various operationalisations, be it on the same object at the same time or on different objects of the same type at different times. But we can see that there is an important disanalogy: a global symmetry fixes only one reference for the represented system, while a local symmetry fixes an infinity of references, one for each spacetime point. Obviously, no concrete experiment will use all these references simultaneously.

This means that a concrete operationalisation is compatible with an infinity of gauge choices, whereas in the case of global symmetries, an operationalisation is associated with one preferred reference for the whole system, tied to the measuring apparatus. This disanalogy captures the intuition that local symmetries do not have the same empirical status as global ones: for a given operationalisation, many gauge choices are available, so they seem arbitrary. We can understand why physicists generally consider gauge symmetries to point to mere redundancies: this is what they are *relative to a given perspective* associated with a limited class of operationalisations.

Belot (2018) examines the case of asymptotically flat models in general relativity, which are used to represent isolated gravitational systems. He claims that physicists will consider that two models related by a diffeomorphism are representationally equivalent if they “agree at infinity” (that is, global shifts in space and time still correspond to distinct states of affairs). This agrees with our conclusions: in so far as we are generally located very far away from the represented systems, transformations that agree at infinity cannot make any operational difference from our perspective.

However, it is possible that given any two distinct gauge choices, there is always

a conceivable operationalisation that will be compatible with one of them, but not the other, if we permissively include practically unrealisable operationalisations (for example when the transformation only concerns regions too small for our instruments to measure⁶). This means that local symmetries can point to invariance under changes of operationalisation, if not for actual ones, at least for conceivable ones, and this means that, from the point of view of PS, they do have DES (in a permissive sense). Greaves and Wallace (2014) noted that it would be puzzling if local and global symmetries had a different empirical status, for global symmetry groups are usually a subset of local symmetry groups, and theories with global symmetries are often superseded by theories that account for the same phenomena with local symmetries. But from the point of view of PS (as for Greaves and Wallace), there is no puzzle: local and global symmetries do, indeed, have the same empirical status (at least the ones that can be associated with particular operationalisations). However, we can also understand why we have the intuition that local symmetries are more arbitrary, given that many are compatible with any given operationalisation.

It remains to be seen if this result can be generalised to all symmetries, and in particular to discrete global symmetries, such as time reversal symmetry, charge, parity, permutation or super-symmetry. An analysis in terms of perspectives on a single system does not seem available. An observer cannot occupy a time-reversed position with regard to an object, nor change an electron into a selectron by merely switching her perspective on it. However, we have seen that symmetry transformations relating distinct objects of the same type can receive a similar treatment under PS than passive transformations on a single object. So, there is no principled obstacle for an extension of these analyses to these cases, assuming that parity or time-reversal transformations (for example) relate *distinct objects of the same kind* (at least, again, when a particular operationalisation or preparation can be associated with the variant quantity: preparing a system that is the exact time-reverse of another one for instance). Of course, more analysis would be required to cover all cases.

7 Conclusion

I examined the status of symmetries under a perspectivist stance, taking the aim of science to represent accurately not the universe as a whole, but all possible objects under all possible perspectives. A perspective involves a class of possible operationalisations on a target system that share the same reference. My conclusions are the following: (i) the distinction between active and passive transformations is blurred and becomes irrelevant, assuming that operationalisation is an active process, and (ii) all theoretical symmetries can have DES, permissively understood, as far as they point to dynamical aspects that are invariant under conceivable changes of perspective.

From the point of view of the perspectivist stance, the structure that varies under a symmetry transformation is not redundant: it captures the fact that concrete physical

⁶In some cases, this will require understanding possibilities in a fairly permissive way, for example when the transformation only concerns the remote past in a cosmological model. If we were to rule out such possibilities, some symmetries would lose their empirical status because of practical limitations due to our situation in the universe, but not for reasons of principle.

objects are accessed from various perspectives. Symmetries point to invariant aspects under changes of operationalisation. These invariant aspects can be interpreted in terms of nomological characteristics shared by objects of the same kind. But by no means do they exhaust reality. This analysis applies at least to coordinate systems, and could potentially be extended to all theoretical symmetries, including local gauge symmetries.

The difference between a perspectivist and a universalist stance runs too deep into fundamental philosophical issues to be addressed here. But at least, the treatment of symmetries that the perspectivist stance allows is well connected to scientific practice (where local gauges and coordinate systems are not eliminated, but used when representing concrete objects). It makes sense of the continuity between global and local symmetries. It can also help us to understand why theoretical symmetries have an empirical status, as well as why empirical symmetries constitute an important heuristics in science.

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