Reichenbach's ε-Definition of Simultaneity in Historical and Philosophical Pespective

§ 1. It is well-known that in the special theory of relativity the simultaneity of distant events is frame-relative: two events simultaneous according to the standard Einstein criterion as applied in one inertial frame are not simultaneous according to the same criterion applied in a relatively moving inertial frame. Not so well known is whether, given a fixed inertial frame, there is a fact of the matter whether a pair of distant events are really simultaneous relative to that frame. Einstein thought not. Reichenbach, at least according to the lore philosophers of science are taught, thought not. And up until roughly a quarter century ago, the thesis that frame-relative simultaneity is a matter of convention was the prevailing view.

Not any more. There is now a widely espoused anti-conventionalist view inspired by a result published by David Malament in 1977: the standard Einstein criterion is the only candidate for a frame-relative simultaneity criterion definable from the causal structure of Minkowski space-time.[1] Hence, according to the neo-anti-conventionalist, if frame-relative simultaneity is conventional, then it is conventional only in some minimal and not very exciting or robust sense.

However, claims as to what is exiting or robust, if they are to be meaningful, must be claims about comparisons. I want to take this opportunity to compare and contrast two possible conventionalist positions on frame-relative simultaneity, one of them decidedly more robust than the other. The more robust of these is implicitly suggested by Einstein's 1905 paper.[2] The other, I shall suggest, is the most plausible reading of Reichenbach's conventionalism in connection with his celebrated "ε-definition" of distant simultaneity. This will give me the chance not only to engage in a bit of Reichenbach scholarship, but also to impress upon the neo-anti-conventionalist camp that there is more than a minimal conventionality thesis to take into account.

§ 2. In order to set up the conventionalist position suggested in Einstein's original 1905 paper, recall the procedure followed there. One establishes the fixed points of an inertial frame (presumably by testing for inertial forces), lays out a spatial coordinate grid using rigid rods and the laws of Euclidean geometry, and institutes a natural clock at each fixed point. There remains the further task of synchronizing these clocks if one is to be able to describe trajectories as a function of a system-wide time. Without such one cannot assign a three-velocity tangent vector to a point of the trajectory. So, one simply stipulates that the time it takes for light to propagate from point A to point B of the frame equals the time of propagation from B to A. From this follows the familiar Einstein

[1] David Malament, "Causal Theories of Time and the Conventionality of Simultaneity", in: *Noûs* 11, 1977, pp. 293–300.

[2] Albert Einstein, "Zur Elektrodynamik bewegter Körper", in: *Annalen der Physik* 17, 1905, pp. 891–921.

synchronization criterion, that a natural clock at A is in synchrony with natural clock at B if

$$t_2 = \frac{t_1 + t_3}{2}, \tag{1}$$

where t_1 is the time at A of light emission from A, t_2 the time at B of its reflection at B, and t_3 the time at A of its return to A. The derivation of the Lorentz transformations proceeds directly from this (and the validity of the light postulate) by assuming that the "moving" frame uses the same synchronization criterion from its point of view for co-moving clocks. In short, the procedure is this. Adopt a synchronization procedure consistent with the empirically testable fact that the average round trip speed of light in any frame is a fixed constant c. Then apply the Principle of Relativity to that procedure in order to derive the coordinate transformations to a relatively moving frame.

But if, as Einstein insists, the one-way speed of light assumption on which clock synchronization is based is really just a matter of stipulation, what happens if one replaces it with an alternative stipulation compatible with the constancy of round trip average speed, and then applies the Principle of Relativity to the induced non-standard synchronization procedure? Does this yield a consistent alternative 3+1-dimensional formulation of special relativity?

§ 3. With this question in mind, I began to scour the Reichenbach archives for calculations to see if Reichenbach had indeed thought along these lines in the course of formulating his famous ε-definition of clock synchronization,

$$t_2 = t_1 + \varepsilon(t_3 - t_1) \tag{2}$$

from which, the standard Einstein criterion falls out as the special case $\varepsilon = 1/2$. In 1921 Reichenbach published a preliminary sketch of his plan to develop an axiomatization of relativity theory in such a way that it would have the virtue of separating out the factual from the conventional components of the theory by instituting only directly testable proposition as axioms and introducing the conventional components as "coordinative" definitions. Bu this "Bericht über eine Axiomatik der Einsteinschen Raum-Zeit-Lehre" introduces only the standard Einstein synchronization criterion as a coordinative definition [Definition 5].[3] The ε-definition first appears in print only in 1924 as Definition 2 of the extensively reworked and vastly expanded culmination of that project, *Axiomatik der relativistischen Raum-Zeit-Lehre*.[4] Moreover, its introduction there appears to have been an eleventh-hour addition. There are two complete copies of a monograph-length draft, suggesting that Reichenbach thought it ready, or

[3]Hans Reichenbach, "Bericht über eine Axiomatik der Einsteinschen Raum-Zeit-Lehre", in: *Physikalische Zeitschrift* XXII, 1921, pp. 683–687.

[4]Hans Reichenbach, *Axiomatik der relativistischen Raum-Zeit-Lehre*. Braunschweig: Friedrich Vieweg & Sohn 1924. Definition 2 appears on p. 26. This corresponds to p. 35 of Maria Reichenbach's translation, *Axiomatization of the Theory of Relativity*. Berkeley: University of Califormia Press 1969.

nearly ready for press, but in which still only the standard Einstein definition appears. Only amongst scraps of loose handwritten pages did I find a precursor (see fig. 1) the the ε-definition in the published version.[5]

[Insert Fig. 1]

The portion of typed text is the whole of Definition 5 cut out from a reprint of the "Bericht" (see fig. 2) and the emmendations make it accord verbatim with Definition 2 of the *Axiomatik*.

[Insert Fig. 2]

The context of its introduction also accords, although not verbatim, with the published version — the middle of a proof of Proposition 6, which, in order to grasp its upshot, requires some exposition of Reichenbach's overall strategy.

§ 4. When Hermann Weyl reviewed Reichenbach's *Axiomatik*,[6] he confessed that he found it "less than satisfactory: overly tedious and too obscure" [wenig befriedigend, zu umständlich und zu undurchsichtig].[7] This, Reichenbach later complained, was based on a gross misunderstanding.[8] In the review, Weyl had characterized the *Axiomatik* as, in the main, not a philosophical, but a purely mathematical investigation, and had registered the above assessment "from a mathematical point of view" [nach mathematischen Gesichtpunkten].[9] Reichenbach was dismayed that someone of Weyl's rank could have missed his main thrust.

Weyl, though, may have well have appreciated Reichenbach's intentions more fully than Reichenbach realized. The execution of the *Axiomatik* is in fact more easily grasped from a mathematical than a philosophical point of view. Reichenbach begins with the class of all possible world-lines in space-time and then attempts to find conditions, involving only the conformal structure of the space-time, sufficient to select out the distinguished subsets which correspond to inertial frames. Using only the behavior of light rays, he defines separate temporal and spatial metrics for individual frames, which he calls the "light-geometry" [Lichtgeometrie], and then derives the Lorentz transformations as the isometry boosts for the light-geometry. (A defect of this procedure, and one certainly not lost on Weyl, is that the class of inertial frames cannot be singled out in this fashion, but only a wider class of frames related by conformal transformations. Reichenbach had sensed the problem, but probably at a stage

[5]Document HR-024-15-02 of the Hans Reichenbach Collection, Archives of Scientific Philosophy and General Manuscripts. Reproduced by permission of the University of Pittsburgh. All rights reserved.

[6]In: *Deutsche Literaturzeitung* 30, 1924, pp. 2122–2128.

[7]*Ibid.*, p. 2128.

[8]Hans Reichenbach, "Über die physikalischen Konsequenzen der relativistischen Axiomatik", in: *Zeitschrift für Physik* 34, 1925, pp. 32–48. See especially Section II, pp. 37–38.

[9]*Ibid.*

too late for him to make major revisions. Only in a later section does he alert the reader to the difficulty, and then only in an attempt to mitigate the extent to which it might be viewed as undermining his program.)

So far no mention is made of rigid rods and material clocks. These notions are introduced only after the development of the light-geometry, at which point Reichenbach sets out a series of "matter axioms" which collectively assert that these material structures behave in accordance with the light-geometry and thus the Lorentz transformations. The upshot of the entire work is that, apart from the prohibition on super-luminal causal propagation, it is only these latter assertions that separate relativity theory from classical space-time physics. The physical axioms governing the behavior of light do not depend on a principle of relative motion and are consistent with both the (relativistic) light-geometry and the geometry [or rather, kinematics] of classical optics. As Reichenbach explains:

> It is remarkable that the Lorentz transformation, conceived in light-geometrical terms, does not contain a new axiom but depends solely on definitions and axioms of the single system. With respect to light, there does not exist a special axiom of uniform translation. Our derivation of the Lorentz transformation via the Galilean transformation has shown that it is merely a *reorganization* of the relations of uniformly moving systems contained in the Galilean transformation *into a new metrical determination*. Physically new is Einstein's idea that material structures do not adjust to the Galilean metrical determination but to the light-geometrical one.[10]

Faintly visible under this thumbnail sketch, in the notion of the "adjustment" [Einstellung] of material structures to the light-geometry, is the spectre of Weyl — or more precisely, that of Weyl's generalization of Reimannian geometry in his attempted unified field theory — which provides a clue as to why Reichenbach pursued the strategy of developing a light-geometry in no way dependent on the use of material metrical standards. Noteworthy is the fact that the first two matter axioms are formulated in tune with the conceptual framework of gauge field theory, asserting the path independence of the behavior of material rods and clocks. Indeed, Reichenbach mentions in a footnote in the Introduction[11] that a hint that the construction of a light-geometry is possible occurs in the Appendix to the fourth edition of Weyl's *Raum-Zeit-Materie*.[12] The notion of "adjustment" [Einstellung] is also taken over directly from Weyl. In his later reply to Weyl's review of the *Axiomatik*, though, Reichenbach clarifies that he does not take this to be a concept with explanatory force so much as a short-

[10] Reichenbach, *Axiomatik der relativistischen Raum-Zeit-Lehre*, loc. cit., p. 58. I have used here Maria Reichenbach's translation, loc. cit., p. 76. Emphasis is in the original.

[11] Ibid., p. 10.

[12] Berlin, Springer 1921. There are in fact two appendices. Reichenbach obviously intends to refer to Appendix I.

hand for the problem of finding a fundamental theory of matter. It is worth mentioning that Reichenbach there indicates that he regards the matter axioms to hold only as a "first approximation" (even to the extent that they are locally valid according to the general theory of relativity) and even cites the positive result of Dayton C. Miller's repetition of the Michelson-Morley experiment as potential evidence (assuming the result is not spurious, as it turned out to be). In contrast to his attitude toward the matter axioms, Reichenbach regards the light axioms as completely secure and describes the light-geometry as the most natural description (from the point of view of descriptive simplicity) of the intrinsic geometry of the electromagnetic field.

§ 5. What role, then, does Reichenbach's ε-definition of simultaneity play in the construction of the light-geometry? The sad answer is, disappointingly little, at least from a formal point of view. It is introduce early on, primarily in order to show how little needs to be assumed in order to introduce a global time function in conformity with causality constraints. The method is to choose a central "clock" in the sense of an arbitrarily parameterized world line and then to export this parameterization to every other world line in space-time via ε-synchronization for an arbitrary, but fixed value of ε. Reichenbach quickly specializes to $\varepsilon = 1/2$ in order to define a time function for each "stationary spatial" frame [stationäres räumliches Koordinatensystem]. The class of inertial frames is later extracted from the class of all "stationary spatial" frames.

There is thus no inkling of the possibility of an invariant non-standard synchronization criterion for inertial frames. Rather, it is more likely that Reichenbach believed that the restriction to $\varepsilon = 1/2$ is in fact necessary for selecting out the inertial frames in the course of constructing the light-geometry. This surmise is reinforced by a comment four years later in his *Philosophie der Raum-Zeit-Lehre*,[13] that Einstein's definition is in fact *essential* for the special theory of relativity: "Diese Definition ist zwar fuer die spezielle Relativitaetstheorie *wesentlich* ...".[14] He does go on to say that, nonetheless, this definition is not epistemologically necessary — any choice between zero and one for the parameter ε would work and could not be said to be false. But work in what regard?

In the *Axiomatik*, after Reichenbach introduces the ε-definition and puts it to limited technical use, there follows what might appropriately be called a philosophical scholium, though not labeled as such. It begins by characterizing the light axioms so far introduced as the "topological axioms of time order" and proceeds to call attention to the "topological problem of simultaneity": in essence, does causal structure pick out at a given space-time point a unique hypersurface as causally neither prior nor posterior, or, as is the case if there is a limiting causal process, does there exist an indeterminate region corresponding to an entire family of distinct hypersurfaces? In the former case it is appropri-

[13]Berlin: Walter de Gruyter 1928. Translation by Maria Reichenbach, *The Philosophy of Space and Time*. New York: Dover 1957.

[14]*Ibid.*, p. 151. Emphasis mine.

ate to speak of absolute simultaneity; in the latter, simultaneity is "relative" insofar as the definition of simultaneity is not uniquely determined by causal structure. The word "relative" here, however, does mean "specific to the special theory of relativity." In the Introduction to the *Axiomatik*, Reichenbach carefully distinguishes between what he calls the epistemological and the physical relativity of simultaneity. The former refers merely to the underdetermination by causal structure. The latter is specific to Einstein's theories of special and general relativity.

> It says in the special theory that a particular definition of simultaneity [$\varepsilon = 1/2$] for uniformly moving systems gives rise to the complete equivalence of all measurement procedures and in consequence the laws of nature have the same form for all such systems. [Sie besagt in der speziellen Theorie, dass bei einer gewissen Definition der Gleichzeitigkeit (Definitition 8) für gleichförmig bewegte Systeme völlige Geichartigkeit aller Massverhältnisse enststeht und die Naturgesetze dann für solche Systeme die gleiche Form haben.][15]

To summarize in light of the results achieved in the *Axiomatik*: The epistemological relativity of simultaneity refers only to the fact that numerous distinct light-geometries (e.g., Galilean vs. Lorentzian) are consistent with the light axioms. The physical relativity of simultaneity speaks to the consequences of the further imposition of the matter axioms, to the effect that the standard practices of measurement using material rods and clocks agree with the choice of $\varepsilon = 1/2$ and the resulting Lorentzian geometry. Thus, as far as the *Axiomatik* is concerned, any adoption of a non-standard simultaneity criterion would necessitate the adoption of a different and highly irregular set of matter axioms appealing to "compensatory" factors and the like.

As for the position in his *Philosophie der Raum-Zeit-Lehre*, Reichenbach sends no signal that he now intends the ε-definition to pertain to any thesis bolder than than the epistemological relativity of simultaneity. The comment that "if the special theory of relativity prefers the ... definition [that] sets ε equal to 1/2, it does so on the ground that this definition leads to simpler relations"[16] need not be construed to indicate anything more than a belief that other choices would necessitate the adoption of a different set of more complicated matter axioms. It is true that, in contrast to the *Axiomatik*, he explicitly mentions directionally dependent choices of ε, but these are in connection with the price of adopting a classical (Galilean) light-geometry.[17] The only portion of text that even remotely suggests otherwise reads:

[15]Reichenbach, *Axiomatik der relativistischen Raum-Zeit-Lehre*, loc. cit., p. 8. The translation is mine. Maria Reichenbach's translation, loc. cit., p. 11, does not adequately convey the intent of the passage.

[16]Maria Reichenbach's translation, loc. cit., p. 127.

[17]See, for example, p. 204 of *Philosophie der Raum-Zeit-Lehre*, loc. cit. (p. 176 of Maria Reichenbach's translation, loc. cit.). Note that the example worked at length in § 26 is an explicit illustration of this.

> It was believed that the coupling of the space and time axes supplied by the Lorentz transformation, according to which every choice of the time axis determines a corresponding space axis as the conjugate diameter, signifies a more fundamental junction of space and time. This coupling, however, is relatively unimportant because it is based on an arbitrary additional requirement, introduced only for descriptive simplicity, for which there is actually no epistemological need.[18]

But the passage goes on:

> The mistake committed here is the one pointed out on page 146; it springs from the erroneous conception that there is a relation between the relativity of simultaneity and the relativity of motion.[19]

And indeed, if one goes back to consult page 146,[20] it is clear that what Reichenbach means is that the *epistemological* relativity of simultaneity "has nothing to do with the relativity of motion. It rests solely on the existence of a finite limiting velocity for causal propagation."[21]

§ 6. What I have argued is that Reichenbach nowhere suggests the possibility of using a non-standard simultaneity criterion in conjunction with the principle of relativity. Such a criterion would have the following properties. It would result in the same light-geometry as the standard Einstein criterion in the sense that it would yield the same temporal and spatial measures as the standard within each inertial frame. Only the Lorentz transformations would need to be replaced by a conjugate representation of the same group. But each of the matter axioms would remain satisfied without revision, since these in fact do not explicitly involve the Lorentz transformations. I further suggest that Reichenbach believed that this could not be done.

Many a reader may wonder whether indeed it can. Rather than formulating such in terms of ε directly, I'll simply state a one way speed of light rule that suffices. Using spherical coordinates, let θ be the angle from the azimuth. Then stipulate that the speed of light V in the direction θ satisfies the condition:

$$V(\theta) = \frac{c^2}{c + a\cos\theta}, \tag{3}$$

where c is the usual average round-trip speed of light and a is an arbitrary scalar less than c. As an exercise, one can verify that this satisfies the requirement of

[18] Maria Reichenbach's translation, loc, cit., p. 189.

[19] Ibid.

[20] This pagination refers to Maria Reichenbach's translation, loc. cit. In the original it is p. 172.

[21] Maria Reichenbach's translation, loc, cit., p. 146.

constancy of average round-trip, derive the corresponding conjugate representation of the Lorentz transformations, and develop a complete, non-standard 3+1-dimensional formulation of special relativity. Moreover, it is not too difficult to show that the most general category of frame invariant non-standard simultaneity criteria is given by letting the magnitude of the cosine term in the denominator represent the projection of an arbitrary irrotational vector field (of norm less than c) onto the azimuth.[22]

The intent here is not to denigrate Reichenbach for failing to realize a possibility that in fact exists. Rather it is to come to a clear understanding as to the range of possibilities that he in fact did recognize. Indeed, it is to his credit to have articulated the distinction between what he called the epistemological and the physical relativity of simultaneity. For that distinction, just slightly re-articulated, serves to demarcate two distinct levels of convention potentially inherent in the adoption of a simultaneity crierion. The epistemological level concerns the degree of fixity dictated by causal structure alone, entirely apart from constraints deriving from the standard deployment of material measuring rods and clocks for the determination of spatial distances and proper-time lapses, respectively. The "physical" level corresponds to the degree of freedom that remains with these latter constraints in place. Much of the debate on the conventionality of simultaneity has tended to suffer from not tracking carefully enough the difference between these two levels.

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Robert Rynasiewicz
Department of Philosophy
Johns Hopkins University

[22]It has recently come to my attention that this result is reported in R. Anderson, I. Vetharaniam, and G. E. Stedman, "Conventionality of Synchronization, Gauge Dependence and Test Theories of Relativity", in: *Physics Reports* 295, 1998, pp. 93–180, and can be traced back to section 9.16 of C. Møller, *The Theory of Relativity*, second edition. Oxford: Clarendon Press, 1972.