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On the ‘Indispensable Explanatory Role’ of Mathematics

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Abstract

The literature on the indispensability argument for mathematical realism often refers to the ‘indispensable explanatory role’ of mathematics. I argue that we should examine the notion of explanatory indispensability from the point of view of specific conceptions of scientific explanation. The reason is that explanatory indispensability in and of itself turns out to be insufficient for justifying the ontological conclusions at stake. To show this I introduce a distinction between different kinds of explanatory roles—some ‘thick’ and ontologically committing, others ‘thin’ and ontologically peripheral—and examine this distinction in relation to some notable ‘ontic’ accounts of explanation. I also discuss the issue in the broader context of other ‘explanationist’ realist arguments.

1 Introduction

Much of the recent literature on the indispensability argument for mathematical realism discusses the ‘indispensable explanatory role’ of mathematics. It is commonplace to articulate the Quine–Putnam defence of mathematical realism in explicitly explanatory terms, and for many the prospects of platonism hang on the indispensability or otherwise of mathematics to empirical explanations. This paper identifies and addresses a clear lacuna in this prominent line of research: examining explanatory indispensability from the point of view of specific conceptions of scientific explanation. I will incentivize the task of filling out this critical lacuna and also take some initial steps towards it by introducing a pertinent distinction regarding the way in which mathematics contributes to explanation. Drawing this distinction is critical especially for the ontological conclusion of the indispensability argument.

To set the context I will review the significance of the notion of 'explanatory role' for the mathematical indispensability argument, first in general terms (Sect. 2), and then with reference to a particular mathematical explanation (Sect. 3). After drawing a distinction between ontologically committing 'thick' and ontologically peripheral 'thin' explanatory roles in general terms (Sect. 4), I will further examine this distinction in relation to three notable 'ontic' accounts of explanation: Jackson and Pettit's program explanation (Sect. 5); Woodward's counterfactual account (Sect. 6); and Strevens's kairetic account (Sect. 7). The connection between explanation and ontology is prima facie most plausible in the context of such ontic accounts of explanation, as opposed to epistemic or modal accounts, as I will explain (Sect. 4). In each case we will find ample room for regarding mathematics as playing only a thin, ontologically peripheral role in mathematical explanations of empirical phenomena. In conclusion I will discuss the take-home message in the broader context of other explanationist realist arguments (Sect. 8).

2 Realism and Explanatory Indispensability

Many realist arguments support ontological commitment to X by pointing to X's explanatory indispensability. This is exemplified by Baker's 'Enhanced Indispensability Argument' for mathematical realism:

(1) We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.

(2) Mathematical objects play an indispensable explanatory role in

science.

(3) Hence, we ought rationally to believe in the existence of mathematical objects.

(Baker 2009, p. 613)

This is, of course, just the familiar Quine–Putnam defence of platonism cast explicitly in terms of the 'indispensable explanatory role' of mathematics.[1] The key idea is that the indispensability argument should really turn on the specific kind of indispensability that commits a scientific realist to paradigmatic (non-mathematical) theoretical posits, such as electrons, quarks, and black holes; arguably the scientific realist is committed to such unobservables precisely because these 'theoretical posits' play an indispensable explanatory role in our best science.

This focus on mathematics' indispensable explanatory role is widespread in the current debates on the indispensability argument. The connection between a posit's explanatory role and ontological commitment to it is endorsed by some leading mathematical fictionalists alike. Field (1989), for example, states that:

> If a belief plays an ineliminable role in explanations of our observations, then other things being equal we should believe it, regardless of whether that belief is itself observational, and regardless of whether the entities it is about are observable. (Field 1989, p. 15)[2]

Both sides of the debate largely agree that one cannot respond to the explanation-focused indispensability argument by insisting that there cannot be mathematical

[1] The indispensability argument in the Quine–Putnam tradition turns on a broader notion of indispensability, typically associated more closely with predictions and confirmation. In the more recent literature there has been a distinct shift towards the explanationist construal of indispensability.

[2] It is clear from the quote's context that 'unobservable entities' for Field can include mathematical abstracta, and an important part of Field's (1989) defence of fictionalism is his denial that mathematics ever plays such an 'ineliminable' explanatory role.

explanations of empirical phenomena since all scientific explanation is causal.[3] The disagreement is rather about whether or not it is the case that 'mathematics can play non-causal explanatory roles in science and that this is a real role' (Colyvan 2012, p. 1033). Colyvan aptly sums up the gist of the debate:

> [A] great deal hangs on the role of mathematics in scientific explanations. If mathematical entities do not play the right sort of role in scientific explanations, then this needs to be spelled out in a way that distinguishes mathematical entities from other entities quantified over in our best scientific theories. (Colyvan 2012, p. 1043)

Given the state of debate, the key notion of 'explanatory role' is in dire need of clear analysis. But while there have been some efforts to spell out contrasting intuitions about whether or not mathematics is genuinely explanatory of empirical phenomena, remarkably the literature has made almost no contact with philosophical accounts of explanation.[4] In the current state of debate it is very much unclear exactly what 'mathematics' explanatory role' amounts to, and it is difficult to say who in the debate gets the upper hand regarding the contested claim that mathematics really does play such a role in science. What we have on the table goes little beyond largely unregimented interpretations of a heterogeneous assortment

[3]This both begs the question and is in tension with the actual science which is seemingly teeming with non-causal explanations.

[4]Baker 2005 contains a brief discussion of three broad accounts of explanation. Lyon 2012 is a much more notable exception, which I will presently examine (Sect. 5). See also Batterman 2010 and Pincock 2011 for a useful analysis of the explanatory role of mathematics in the context of the so-called mapping account of applied mathematics. Apart from these few exceptions the growing literature on the indispensability argument just floats freely of theories of explanation. For example, while Baker 2009 ably analyses (and defends) the notion that mathematics is indispensable for some of our best scientific explanations, he says little in general about what it is to play an explanatory role, and none of his analysis relates to a considered theory of scientific explanation. Similarly—on the other side of the debate—Yablo 2012 has recently argued that mathematics' indispensability in scientific explanations can be interpreted in ways that do not support the indispensability arguments, since the explanatory role need not be played by mathematical *objects*. Yablo's point is in my view well-taken, but again it is worth pointing out that his analysis of mathematics' explanatory roleis given in general terms and not in relation to any specific account of scientific explanation.

of examples of scientific explanations that *somehow* involve mathematics in an important way.

How do we best push the debate forward? Common sense suggests that we should aim to bring our best understanding of scientific explanation to bear on the debate and examine the notion of '(mathematics') explanatory role' in relation to different analyses and conceptions of explanation. In this way we can hope to gain a much closer grip on 'explanatory role' so as to be better placed to judge whether mathematics plays *the kind of* explanatory role that actually matters for the ontological debate in question.

In the rest of this paper I will take some initial steps in this direction, by first looking into the notion of 'explanatory role' in general terms, before examining it in more detail in relation to three specific accounts of explanation. The premise (1) of the Enhanced Indispensability Argument above is undermined as a result.

3 Dissecting 'explanatory role'

Let us briefly recall a central example of allegedly explanatory mathematics to begin with. (The rest of our discussion will benefit from having this staple example clearly in mind. Although I regard the example as problematic on multiple counts (see e.g. Saatsi 2011), for all its shortcomings and simplicity it functions well to illustrate the points I wish to make, and it will also conveniently allow us to relate the present point of view to earlier literature.)[5]

Explanandum: why is the life-cycle of the North-American cicada 13 or 17 years (depending on sub-species)?

Stylized explanation of the 17-year pediod:[6]

[5]Lange 2013 refines the conception of the cicada explanation as a mathematical explanation. I will stick to Baker's original presentation to better reflect the evolving dialectic around it.

[6]Quoted from Baker 2009 (p. 614).

(4) Having a life-cycle period which minimizes intersection with other (nearby / lower) periods is evolutionarily advantageous. [biological law]

(5) Prime periods minimize intersection (compared to non-prime periods). [number theoretic theorem]

(6) Hence organisms with periodic life-cycles are likely to evolve periods that are prime. ['mixed' biological / mathematical law]

(7) Cicadas in ecosystem-type, E, are limited by biological constraints to periods from 14 to 18 years. [ecological constraint]

(8) Hence cicadas in ecosystem-type, E, are likely to evolve 17-year periods.

We can similarly explain the 13-year pediod by plugging in appropriate ecological constraints. The key biological premise (4) can be supported in evolutionary terms, based e.g. on the relative fitness advantage of individuals whose offspring are more likely to avoid predators that are themselves periodical, and less likely to breed with other cicada species.

In this DN-type explanatory argument mathematics can seem to play an explanatory role by virtue of being employed in the deduction of the explanandum (8). For someone operating with a Hempelian DN-model in mind, the mathematical parts (5) and (6) above seem just as explanatory as the other parts, especially given the sense in which the universal 'law' (6) 'covers' the specific, ecologically constrained 17-year case stated in the explanandum.

The DN-model has long been obsolete, however, and it is furthermore unclear how 'explanatory role' should be linked to ontological commitment in something like the DN-account.[7] But once we renounce the DN-account it is an open question

[7]Although it is part of the 'adequacy conditions' in Hempel's DN-model that for any actual

how exactly explanatory contributions should be attributed to different parts of an explanatory story or argument (such as Baker's stylized explanation above). It is thus very much legitimate for us to ask: are the mathematics-laden elements (5) and (6) really contributing to the *explanatoriness* of the explanation of (8) above?

How do we begin to answer this question? Naturally, by considering it in relation to different accounts of scientific explanation. (How else?) Before we get started on this, let us motivate this approach further by recalling some earlier commentaries on the cicada example.

I have argued elsewhere that mathematics in the cicada case need not be viewed as playing an indispensable explanatory role at all (Saatsi 2011). Mathematics is not indispensable, since we could replace (5) and (6) in the above argument by:

(5/6)* For periods in the range 14–18 years the intersection minimizing period is 17. [Fact about time]

This yields an alternative deduction of (8) from the relevant biological assumptions without the mathematics, and I argued that this deduction is as explanatory as the original deduction (4)-(8) above.[8] Furthermore, I suggested that a seemingly indispensable employment of mathematics in scientific explanations could be interpreted as having a *representational* function: in the cicada case mathematics represents those features of time (e.g. its linearity) that are doing all the explanatory work. Mathematics can well be advantageous for empirical explanations by virtue of playing such representational role, for example by *justifying* non-mathematical

explanation the explanans must be *true*, the problem is that the DN-account is an 'epistemic' (as opposed to 'ontic' or 'modal') account of explanation. (I will recall the distinction presently.) According to the DN-model the function of an explanation is to give *understanding*. (Hempel equated understanding with *expectability*, but other conceptions of understanding could be conjoined with the central idea that explanations are arguments that provide understanding.) However understanding is construed, it is not clear why mathematics, fictional models, or even things like diagrams would have to be *true* in order to play an explanatory role in the epistemic sense of providing understanding.

[8] More specifically, I argued that the inclusion of mathematics in the cicada case is not a source of any further explanatory power, and that the prime number explanation does not enjoy any advantage of generality since (5/6)* can be naturally extended to more general facts about time.

explanatory assumptions (such as (5/6)*, above) that arguably provide all the explanatory power.

This line of thought can be accused of begging the question: what justification is there for recasting the mathematics involved as having a 'merely' representational role, if scientists themselves do not appeal to any kind of explanatory/representational dichotomy? Baker's thesis that mathematics is indispensable for explaining can be directly motivated, by contrast, by taking scientists' pronouncements to this effect at face value. This is in essence how Baker and Colyvan (2011) respond to the 'indexing strategy' (as executed in Daly and Langford 2009), according to which mathematics should only be viewed as 'indexing' (viz. representing) non-mathematical facts about the world. Baker and Colyvan maintain that the indexing strategy is at severe odds with the scientific practice which arguably indispensably involves number theory, for example, in connection with the periodical cicada. In a similar manner Pincock (2012) defends the existence of mathematical explanations (viz. explanations for which mathematics 'makes an explanatory contribution') via a 'comparison test', which considers available explanations of (say) the cicada phenomenon to see whether or not the 'best explanation' (as judged by scientists) makes use of a mathematical claim.[9]

If we follow these authors to accept the lead from scientific practice and admit that mathematics can indeed be indispensable to some of our best scientific explanations, is there any room left for me manoeuvre? Is it not simply incoherent to now deny that mathematics plays an indispensable explanatory role?

Well, answer to this question turns out on closer reflection to depend on what we mean by 'explanatory role'. It now becomes critical for us to clarify this notion in order to zero in on the key point of contention. For while there undeniably is *a*

[9]Note that Pincock does not endorse the inference from explanatory indispensability to mathematical realism, however, since he denies the principle of inference to the best explanation in a form required for this inference.

sense in which we can construe mathematics in the cicada and other examples as 'explanatory', or as 'playing an explanatory role', what really matters for the indispensability argument—all that matters!—is whether or not mathematics plays *the kind of explanatory role that we should take as ontologically committing.* A properly informed analysis of scientific explanation is required in order to see whether a face value reading of scientific practice (and the admission that mathematics is indispensable for explanations) implies that mathematics plays an explanatory role that is ontologically committing.

Instead of bluntly denying that mathematics is in no sense explanatory at all (or plays no explanatory role whatsoever), a critic of the explanatory indispensability argument is much better off insisting that mathematics does not play the *right kind* of explanatory role. This strategy, of course, requires that we can somehow distinguish between different types of explanatory roles, some 'thick' and ontologically committing, others 'thin' and ontologically peripheral. The next section will discuss this distinction in the abstract, along with some other useful distinctions. After that I will show how this kind of distinction can be drawn in the context of some specific ontic accounts of explanation.

4 Useful distinctions

Salmon (1985) introduced the notion of *ontic* account of explanation, distinguishing it from *epistemic* accounts, on the one hand, and *modal* accounts, on the other. This tripartite classification is well known in the philosophy of scientific explanation. It is worth recalling it here as its relevance has never been noted in the present context. For our discussion a useful way of characterising the classification is as follows.

The basic idea behind an *ontic* conception of explanation is that explanation is a matter of situating the explanandum within a broader ontic structure of the world. The explanatory power derives from stating some relevant worldly facts: objective causal or mechanistic facts, or nomological facts, or statistical relevance relations, or symmetries, or whatever ontic structures can bear an objective relationship of explanatory relevance to the explanandum. (Causal and causal-mechanical accounts of explanation are paradigmatic examples of the ontic conception, and also what Salmon primarily had in mind in characterising this conception, but we should construe 'ontic structure of the world' in broader terms to make room for mathematical and non-causal explanations.) Typically explanatory relevance is a matter of exhibiting some kind of dependence of the explanandum on the explanans, in the way 'difference-making' relations do in the paradigmatic case of causal explanation, for instance. But not all explanatory dependence is causal. A law can depend on other laws in an explanatory way, but laws do not cause other laws. An explanandum can depend on structural constitution in an explanatory way that is not causal, as in the case of glass's fragility being explained by its molecular structure. An explanandum can depend on more abstract (yet still real) structural features of the world, as in the case of a Lorentz-contraction being explained in relativity in terms of the fundamental kinematic structure of reality. We can also envisage the possibility of some kind of sui generis dependence of certain physical facts on some mathematical facts so as to make conceptual room within the ontic conception for ontologically committing mathematical explanations.

The basic idea behind an *epistemic* conception of explanation, by contrast, is that an explanation is whatever we give in order to explain, where *explaining* is an epistemic activity of *providing understanding*. Following Hempel and many others, providing understanding can be viewed as a matter of showing that the explanandum was to be expected (given the explanans), but different conceptions of understanding are possible. According to an epistemic conception the explanatory power of an explanation derives from the fact that the explanatory information suitably entails the explanandum so as to provide understanding. It does not derive from the explanation pointing to features of reality that stand in a worldly relationship of explanatory relevance to the explanandum.

Finally, according to Salmon the *modal* conception a scientific explanations does its job by *'showing that what did happen had to happen'* (Salmon 1985, p. 293). This is quite ambiguous by itself, and it is difficult to see any clear difference to the ontic conception: after all, situating the explanandum within a broader ontic structure of the world (as per the ontic conception) can show why the explanandum had to happen *given* that structure. Salmon thought the distinction between ontic and modal conceptions is a substantial one, but only in the context of indeterministic physics. Since this isn't our present context, Salmon's (somewhat inchoate) remarks on modal conception (mostly made in the context of probabilistic and statistical explanations) are of little interest to us here.

What is of interest, however, is a recent characterisation of the modal conception by Lange, according to whom 'the modal conception, properly understood, applies at least to distinctively mathematical explanation in science, whereas the ontic conception does not' (Lange 2013, p. 510). The modal conception, as Lange understands it, takes explanation to be a matter of showing that the explanandum is inevitable in the sense that it holds independently of any contingent ontic structure at stake.

To illustrate: Mother failed to distribute twenty-three whole strawberries evenly among her three children. Why? Because twenty-three cannot be evenly divided by three. According to Lange the explanatory power here stems from modal information provided by the mathematics, transcending the modal information provided by any (actual or possible) causal or nomic structures involved in the process.

> [T]hat twenty-three cannot be divided evenly by three supplies inform-

ation about the world's network of causal relations: it entails [as a matter of mathematical necessity] that there are no [counterlegally possible] causal processes by which twenty-three things are distributed evenly (without being cut) into three groups. (Lange 2013, p. 496)

That is, according to the modal conception (thus understood) we can explain by showing, by reference to mathematical facts, that the explanandum had to take place, or be the way it is, *regardless* of what the ontic structure of the world is like (as long as the world satisfies the description of the system at stake, involving a mother, and so on).[10]

Let us now consider the issue of ontological commitment in relation to this rough-and-ready distinction between ontic, epistemic, and modal conceptions. *Prima facie*, the connection between explanatory mathematics and ontological commitment to mathematics is plausible in the context of an ontic account of explanation, since according to the ontic conception explanatory power is derivative only from stating explanatorily relevant worldly facts. Given the characterisation of ontic explanation it is natural to think, prima facie, that if (purported) reference to X is indispensable for explaining, then X is real, as dispensing with X results in a loss of explanatory power, which is (purportedly) due to leaving out information about an ontic relationship of explanatory relevance between X and the explanandum.

If mathematical explanations are instead best construed in the fashion of either the epistemic or the modal conception, it is much less clear what the connection between explanation and ontological commitment is meant to be, and as a result explanatory indispensability arguments become immediately more contentious.

Consider the *modal* conception first. Here mathematics can play an explan-

[10] Note that an explanation conforms to the modal conception only if it provides such explanatory modal information without simultaneously providing ontic information about some kind of dependence relation connecting the explanandum to the relevant mathematical facts or properties. If an explanation provides such modal information *by virtue of* showing, for example, how the explanandum only depends on the system instantiating certain abstract mathematical properties, then it should be viewed as ontic.

atory role by virtue of being part of a derivation or deduction that shows that the explanandum was inevitable to a stronger degree than results from any assumptions regarding the ontic structure in place. Lange (2013) argues in a fascinating recent paper that this conception best captures (some) mathematical explanations in science. But Lange thinks, rightly in my view, that his modal account of mathematical explanations is largely orthogonal to issues of ontological commitment. In Lange's analysis of 'distinctively mathematical explanations' mathematics plays an indispensable role in providing explanatory *modal* knowledge, namely knowledge of the inevitability of the explanandum, in the sense of it not depending on the actual laws of nature. It is far from clear why this kind of explanatory role would give us grounds to construe mathematics realistically.

Assume for the sake of the argument that a given mathematical explanation—e.g. the strawberry example above—explains by providing knowledge of the independence of an explanandum of the actual laws of nature, so as to show that 'what did happen had to happen' in a strong counter-legal sense of necessity. It is clearly a non sequitur to conclude from this that the mathematics affording us this knowledge must itself be given a realist interpretation. Admittedly Lange writes in a way that lends itself to a realist reading, suggesting that mathematical *facts* are somehow responsible for the strong degree of necessity at stake. For example, 'the mathematical fact entails that even a pseudoprocess rather than a causal process [...] cannot involve such a division of twenty-three things' (Lange 2013, p. 496). But those who balk at a realist interpretation of 'mathematical necessity' and 'mathematical fact' can happily accept Lange's idea that mathematical explanations explain by showing that the explanandum is inevitable by virtue of being completely independent of some pertinent feature of the ontic structure of the world. According to Lange the explanatory power resides in this modal information, so what really matters are the explanatory modal facts, and there is no clear prima facie connection between explanatorily indispensable mathematics and ontological commitment. The platonist may been feel that an anti-realist will not be able to account for the indispensability of mathematics for obtaining such modal information. But it is not like the platonist has at this point an explanatory upper hand by being able to say something informative about the connection between mathematical facts and modal facts. And I do not see any reason to think that mathematics as construed by a fictionalist, for example, would be unable to provide the kind of modal knowledge that matters according to Lange.[11]

Now, let us move on to briefly consider the *epistemic* conception. Here an element of an explanation can be a source of explanatory power by virtue of its function in providing understanding. For example, if understanding is equated with nomic expectability, an advocate of the epistemic conception can take mathematics as explanatory if it is employed in an explanatory argument to show that (non-mathematical) laws and initial conditions entail the explanandum so as to demonstrate that it was to be expected. Playing an indispensable explanatory role of this sort could be a matter of calculational support, say; perhaps nomic expectability of the explanandum would otherwise be only apparent to a Laplacean demon, for instance. It is immediately far from clear why this conception of understanding—or other conceptions, for that matter—would give us grounds to construe mathematics realistically.[12]

So, to sum up, the ontic conception of explanation affords the most plausible connection between explanatory indispensability and ontological commitment, *prima facie*. In the rest of the paper I will focus on some of the best developed and most influential contemporary accounts of explanation, all of which are ontic ac-

[11]Note also that mathematics can also be used to obtain explanatory modal information even when it is not indispensable. The strawberry example is a case in point, since it can be paraphrased into an argument in first-order logic entailing the explanatory modal fact without referring to numbers.

[12]There is an interesting connection here to issues concerning the question whether understanding is factive: several philosophers have argued that falsehoods (such as idealized models) can function to provide understanding and even be indispensable for it. See e.g. Elgin 2004.

counts, to show that *even in the context of the ontic conception* the connection between explanation and ontology is much less direct than has been appreciated in the indispensability debate.[13] The crux of the matter is to recognise that within the ontic conception there is scope for critical, more fine-grained distinctions between different types of explanatory roles. We can start by conceiving the following dichotomy in the abstract:

'Thick explanatory role' is played by a fact that bears an ontic relation of explanatory relevance to the explanandum in question.

'Thin explanatory role' is played by something that allows us to grasp, or (re)present, whatever plays a 'thick' explanatory role.

In the following sections I will affirm the prospects of drawing this broad dichotomy in the context of some specific accounts of explanation. Recognising the possibility of this kind of dichotomy is significant because it points to the fact that even within the ontic conception of explanation not every explanatory role is necessarily ontologically committing (or at least is not obviously so). Only a thick explanatory role can be related to ontological commitment in a fairly straightforward way, and the critical question regarding the indispensability argument is, of course: Does mathematics ever play a thick explanatory role, or does it only play a thin one?

This question, I submit, can be properly answered only in the context of specific accounts of explanation, which also allow us to make clearer and more precise the distinction between thick and thin explanatory roles. I will now turn to these accounts.

[13] I do not pretend to have shown that the debate is a nonstarter in the context of modal or epistemic conceptions. But while further work remains to be done in this context, we can appreciate the corresponding lacuna in the indispensability debate just on the basis of our broad characterisation of these two conceptions.

5 Program explanation

Lyon (2012) is one of the few to analyse the explanatory role of mathematics in terms of a specific account of explanation. Lyon argues, in effect, that mathematics can be seen to play (what I have now called) a *thick* explanatory role: it plays a 'programming' role as characterised by Jackson and Pettit as part of their model of program explanation. In examples like the one involving periodical cicadas 'mathematics is indispensable to the programming of the efficacious properties' (Lyon 2012, p. 568).

I have elsewhere criticised Lyon's attempt to cast mathematical explanations as program explanations (Saatsi 2012). In the contest of our current discussion the key point of this criticism can be expressed as follows: Lyon has not established that mathematics indeed plays a thick explanatory role in the context of the program explanation account. That is, Lyon's account leaves fully open the possibility that the mathematics involved in empirical explanations, while indispensable, could be construed as playing merely a thin explanatory role.

To see this problem clearly, let us briefly recall Lyon's argument. Jackson and Pettit (1990) introduce the notion of program explanation by contrasting it with a *process explanation*, which is a fine-grained, straightforwardly causal explanation. A program explanation is more abstract, and is causal only in a derivative sense: it cites a property which *guarantees* the instantiation of some causally efficacious property involved in a process explanation of the same explanandum. According to Jackson and Pettit, a liquid's temperature T, for example, can (program-)explain the cracking of a glass receptacle by virtue of playing a programming role with respect to a lower-level property also instantiated by the liquid: some particular molecules hitting the receptacle wall with a high-enough collective momentum to cause the cracking.[14] Jackson and Pettit use examples like this to depict a sense

[14]See Jackson and Pettit 1990 for details.

in which a higher-level property (such as T), which is not causally efficacious itself, can nevertheless be explanatorily (and causally) relevant by virtue of standing in a suitable 'programming' relationship to some causally efficacious lower-level property.

The program explanation account is an *ontic* one: a successful explanation of an event E requires that we point to a feature of the world that is explanatorily relevant by virtue of there being a sense in which E *depends* on that feature of the world. The properties that are explanatorily relevant in this sense, by virtue of 'programming', play a thick explanatory role characterised by an intimate modal relation between the higher-level programming facts and the lower-level causal facts. (Basically, it is not possible to have an explanatory higher-level fact without also having one or another causally efficacious lower-level fact. In many examples this relation between the properties is some kind of 'realization' relation.) Jackson and Pettit capitalise on this modal relation to argue that a higher-level property provides information about what the explanandum depends on, going beyond the information provided by the corresponding lower-level process explanation.[15]

Lyon argues that mathematical explanations can also be viewed as program explanations.

> They cite properties and/or entities which are not causally efficacious but nevertheless program the instantiation of causally efficacious properties and/or entities that causally produce the explanandum. And importantly, they cite mathematical properties and/or entities that are doing (at least part of) this programming work. (Lyon 2012, p. 567)

The thought is that mathematical explanations can be similarly understood as pointing to mathematical facts on which the empirical explanandum in question *de-*

[15]For example, arguably the dependence of the cracking of the container on the liquid's temperature goes over and above of the information about the way in which the cracking depended on some particular set of molecules having specific momenta.

pends. Lyon argues, in reference to the cicada case, for example, a detailed (historico-ecological) process explanation fails to exhaustively explain the cicada phenomenon, because it 'misses the fact that the final evolutionary outcome, the convergence on 13 and 17, is *robust* with respect to the historico-ecological details' (Lyon 2012, p. 567). Arguably a program explanation is needed in order to explain this robustness—the fact that the convergence to these specific periods only depends on them being co-primes to every alternative period in the relevant range of possibilities—and here mathematics becomes indispensable:

> [I]f we take away any mention of primeness from the cicada explanation, the explanation falls apart, and there doesn't seem to be anything that would put it back together. (Lyon 2012, p. 568)

In this way mathematics arguably plays a thick explanatory role of programming.

There are various difficulties in construing mathematics as playing a programming role (cf. Saatsi 2012). A key problem is the following. For Jackson and Pettit 'programming' is a matter of a modal relation holding between higher-level ('programming') and lower-level (causally efficacious 'process') properties.[16] It is critical to have a grip on this modal relation in order to grasp the sui generis explanatory dependence at stake: program explanations are explanatory by virtue of providing information about a higher-level dependence rooted in the modal relation between higher- and lower-level properties. But we have *no such grip at all* for the alleged modal relation that is meant to hold between a mathematical fact and the lower-level causal facts.

An alternative and perhaps better way to express the problem is this. An advocate of the enhanced indispensability argument needs to establish that mathematics

[16]For the paradigmatic examples the programming relation is one of 'realization' (broadly construed): temperature, as mean kinetic energy, is realised in different molecular configurations. A determinable property is realised in different determinates, and so on.

is playing a *thick*, ontologically committing explanatory role; establishing indispensability simpliciter is not enough. In the context of the program explanation idea, this can only be done by providing a substantial account of the modal relationship between the mathematical features that allegedly ‘program’, on the one hand, and the properties that feature in the corresponding process explanations. Lyon does not provide anything to this effect, however; rather, he just stresses mathematics’ explanatory indispensability in explanations that he construes as program explanations (see, for example, the quote above). But this is not enough, as indispensable mathematics here could instead be interpreted as playing a thin, ontologically peripheral role.

Lyon’s appeal to the de facto indispensability of mathematics is similar in spirit to Baker and Colyvan’s appeal to scientific practice. Keeping in mind the need for a thick explanatory role and the details of the program explanation account we can see why they all achieve little by such an appeal; the explanatory practice of science does not wear its ontological commitment on its sleeve.

If mathematics does not play a programming role, but is nevertheless explanatorily indispensable, what other roles are there? A natural alternative is to argue that mathematics only plays the role of *representing* some non-mathematical features of the world that themselves play the programming role (as I have suggested in Saatsi 2012). The challenges of spelling out this suggestion depend on the case at hand, and one should not treat lightly the complexities involved in some important cases (see e.g. Batterman 2010 and Wilson 2013).[17] But in the cicada case, at least, mathematics arguably *can* be construed as playing such a thin representational role, representing features of time (such as that stated in (5/6)*, above, and natural generalisations thereof) that play a thick explanatory role (Saatsi 2011).

[17]But note that neither Batterman nor Wilson explicate the non-representational explanatory role of mathematics in terms of any account or conception of explanation. It is an open question how the details of these cases will look like in the context of an appropriate account of explanation.

Jackson and Pettit's program explanation model is not a full-blown theory of explanation. Rather, it is an attempt to make sense of higher-level explanations that do not feature causally efficacious properties. Its core idea of explanatory modal dependence can arguably be incorporated into a broader account of explanation due to Woodward, so it is worth examining mathematics' explanatory role in the context of this account.[18]

6 Counterfactual account of explanation

Woodward's counterfactual theory of explanation is a prominent contemporary account in the ontic tradition. Woodward has primarily intended his account to capture *causal* explanations, but various people have argued that its central idea can also be applied to non-causal (e.g. geometrical) explanations (e.g. Bokulich 2008, 2011; Saatsi and Pexton 2013; Ylikoski and Kuorikoski 2010).[19] Hence, there is no reason to dismiss it as irrelevant in relation to (prima facie) non-causal explanations e.g. of cicada periods.

The key idea in Woodward's account is that *counterfactual truths* are the source of 'explanatory power': to explain is to provide information that answers *what-if-things-had-been-different questions* ('w-questions').

> [A]n explanation ought to be such that [it enables] us to see what sort of difference it would have made for the explanandum if the factors cited in the explanans had been different in various possible ways. (Woodward 2003b, p. 11)

(The causal dimension of Woodward's account has to do with the fact that much of counterfactual, explanatory information can be causally interpreted. There are

[18] Ylikoski (2001) argues in detail that Woodward's account (or something very much like it) can do all the program explanation model can. See also Woodward 2009.

[19] Woodward's sympathies to this possibility are evident in section 5.9 of Woodward 2003b.

cases where such causal interpretation is not available, however. Cf. references above.[20])

For Woodward a DN-type deduction is only explanatory to the extent it provides such modal information. The explanatoriness (or 'explanatory power') of an argument or derivation springs from correctly representing the relevant *objective dependency relations* in the world, grounded in the world's actual (causal-)nomological structure.[21] The account thus accords with the ontic conception of explanation. The explanatory dependency relations can furthermore be found at different 'levels'. For example, there is a dependency relation that connects the cracking of the glass receptacle (in the section above) to the liquid's temperature, allowing Woodward to conceive the temperature as a cause of the cracking (in Woodward's precise sense of causation).

Woodward's account has the following feature that is of great interest to us here. While it is explicitly an *ontic* account, it allows at the same time for a kind of *instrumentalism* about genuinely explanatory *theories*: there is no need to accept the 'widely shared idea' that 'successful explanation is at bottom a matter of getting fundamental ontology right' (Woodward 2003b, p. 232). This instrumentalism is furthermore directly motivated by the scientific practice.

> [My account] is less demanding about the need for explanatory theories to have a defensible 'realistic' interpretation (in the sense of postulating only 'real' entities) and assigns a more prominent role to 'instrumental' success than many competing theories of explanation. . . . [S]uch an account fits explanatory practice in many areas of science

[20]Woodward (2003b, Sect. 5.9) gives the example of explaining the stability of planetary orbits in terms of its counterfactual dependence on the dimensionality of space.

[21]As Woodward puts it: 'It is physical dependency relations . . . that are primary or fundamental in causal explanation; derivational relations do not have a role to play in explanation that is independent or prior to such dependency relations, but rather matter only insofar as (or to the extent that) they correctly represent such relationships' (Woodward 2003b, p. 211).

much better than more ontologically oriented alternatives. (Woodward 2003b, p. 232)

For example, Woodward regards Newton's theory of gravity (within its domain of applicability) as (genuinely) explanatory, since it correctly latches onto explanatory modal relations so as to answer a range of *what-if-things-had-been-different* questions. The fact that the fundamental ontology of Newton's theory is false—nothing in the world corresponds to the notion of *gravitational force*, for example—is of little consequence to the theory's explanatory status. One way to capture this (qualified) instrumentalism is to say that in Woodward's account 'gravitational force' (as a theoretical posit) plays a thin explanatory role, while the corresponding thick role is being played by the objective dependency relations correctly captured by the theory.[22]

In Woodward's account there is scope for regarding mathematics also as playing a thin explanatory role, on a par with Newtonian gravity. Whether or not mathematics employed in scientific explanations is true in a realist sense, mathematics can be explanatory by virtue of allowing us to grasp physical dependency relations that play a thick explanatory role and are ultimately fully responsible for the *explanatoriness* of an explanation.

Consider the cicadas again. Explaining this phenomenon in a Woodwardian framework is a matter of grasping how the fitness-maximizing life-cycle of a given cicada species—construed as a variable that can take different values—depends on other biologically relevant variables, such as other cicada species' and predator species' life-cycles. Even if mathematics is indispensably involved in grasping, or representing these explanatory modal facts, in Woodward's account it is only the modal facts themselves that are ultimately the source of explanatory power.

[22] See Woodward 2003a. Harper (2011) explores in great detail the surprising richness of the modal relations encapsulated in Newton's theory.

An advocate of this account could thus agree with Baker, Colyvan, and Lyon that number theory is indispensable for a maximally 'robust' or 'general' explanation of the cicada periods, without viewing the numbers themselves as playing a *thick* explanatory role. Rather, number theory would be viewed as an indispensable vehicle for grasping the relevant modal facts: roughly speaking, in the long run the evolution of a prime-numbered period of p years depends *only* on there being competing/predator species of nearby periods and 'ecological constraints' that appropriately limit the range of viable possibilities (cf. Sect. 3).[23]

One may question the depth of the analogy between Newtonian gravity and mathematical explanations: Newtonian gravity of course is not really *indispensable* for explaining gravitational phenomena, given that we can (a) formulate Newtonian gravity geometrically (as the Newton-Cartan theory), so that the explanatory modal relations are captured by the curvature of 4-dimensional spacetime instead of gravitational force, and (b) supplant the theory by general theory of relativity that is explanatorily deeper by virtue of encompassing a larger set of explanatory dependence relations.[24]

In response, it can be argued that the indispensability or otherwise of a theoretical posit is neither here nor there regarding the key issue at stake: in Woodward's account a theory can be truly explanatory even if its 'fundamental ontology' is only playing a thin explanatory role. A right-minded advocate of the account would not infer the reality of gravitational force from its explanatory indispensability, *even if* Newton's theory had not been supplanted by general relativity, and even if the practising scientists preferred gravitational force explanations over a cumbersome Newton-Cartan formulation.

[23]An explanation that captures this 'robustness' or 'generality' of the prime numbered periods with respect to historico-ecological details can be viewed in Woodward's account as maximizing a particular *dimension* of 'explanatory depth', measured by the number of what-if-things-had-been-different questions. (See Hitchcock and Woodward 2003)

[24]This theory captures a larger set of explanatory dependencies since it is applicable to a wider range of situations, involving e.g. very large masses and velocities.

What must be admitted, however, is that this instrumentalist aspect of Woodward's account is a double-edged sword for a scientific realist who wishes to link explanatory indispensability to ontological commitment for quarks, electrons, and other non-mathematical theoretical posits: these of course can similarly be construed as playing only a thin explanatory role. Therefore any 'explanationist' argument for realism about quarks, electrons etc. must incorporate more than mere appeal to explanatory indispensability. (I will reflect on this issue further in the concluding section 8.)

Woodward's account of explanation is not the only game in town; there are of course competing theories in the ontic spirit. While not aiming to offer a comprehensive review, I will briefly consider one prominent alternative next: Strevens's (2008) kairetic account.

7 Kairetic account of explanation

Strevens's (2008) theory of explanation provides a criterion of explanatory relevance as a matter of *difference-making*: to explain an explanandum is (roughly speaking) to say what made a difference to it taking place or having the features it has. I am not going to delve deeply into the rich details of Strevens' 'kairetic' account of difference-making. My objective is rather to make the point that also in this ontic account a distinction between thick and thin explanatory roles can be drawn so as to enable us to divorce explanatory indispensability from ontological commitments.

At the heart of Strevens's account is the idea that explanatory difference-makers can be specified by *abstraction* from the specific details of fundamental causal relations (as well as laws and background conditions).[25] The abstracted difference-

[25]The intuitive idea can be illustrated as follows. A hooligan throws a rock into a window. What made a difference to the complete shattering of the window is a complex combination of the projectile's hardness, manner of impact, and momentum being together 'enough.' More specific facts

making facts are facts about nomological dependence (that Strevens understands as causation); thus, Strevens's is an *ontic* account. As with Woodward, a DN-type explanatory deduction is only explanatory to the extent it provides information about such worldly dependence relations. (Strevens is metaphysically more committed than Woodward: an explanatory deduction must 'mirror' worldly relations of explanatory relevance *grounded in* fundamental causal relations.)

Specifying difference-makers through abstraction has some remarkable payoffs. For example, Strevens uses it to give an account of the explanatory role of deliberate falsehoods involved in idealisations (e.g. the ideal gas model explanation of Boyles's law). Of particular interest to us is the way in which abstraction can also naturally lead to mathematical explanations in science, explanations in which mathematics is more than a derivational handmaiden by virtue of conveying 'understanding that turns on the appreciation of a central fact that is a matter of mathematics alone' (Strevens 2008, p. 301). That is, through abstraction mathematics can acquire a bona fide explanatory role. But at the same time Strevens explicitly states that '[one does not need] metaphysical assumptions to spell out its additional contribution to explanatory goodness' (Strevens 2008, p. 304). In other words, Strevens denies that mathematics, even when indispensable, gets to play a thick, ontologically committing explanatory role.

According to Strevens mathematics can be indispensably involved in explanation in two ways.

> First, the mathematical structure of an explanatory derivation must reflect the corresponding relations of causal production in the world.

regarding the missile's velocity or mass, say, should not be construed as difference-makers in and of themselves, since we could vary any of those precise facts without altering the outcome (as specified by the explanandum). There is an unduly complicated story of fundamental physics about the precise causal mechanism involved. The explanation of breaking does not explicitly involve the details of this story; the difference-maker is rather an abstract amalgamation of the object's hardness, momentum, and manner of impact.

> Only by grasping the mathematical structure do you follow the process of production from beginning to end. Second, the mathematics of the derivation tells you implicitly, and sometimes explicitly, what makes a difference to the causal production and what does not, and why it does or does not. (Strevens 2008, p. 329)

The first is a matter of mathematics being indispensable to (our best way of) deducing the explanandum from the explanans so as to mirror relations of causal dependence. Here a thick explanatory role is played by facts about causal dependence, and mathematics (quite obviously) only plays a thin role of representing the relevant laws, properties, and causal dependencies. Most explanatory derivations in physics and mathematical sciences are of this sort, and not 'distinctly mathematical' in the sense of Lange 2013.

The second, more interesting way for mathematics to contribute is exemplified by the cicada example. A detailed reconstruction and analysis of the cicada case in the context of Strevens's account is beyond the confines of this paper, but we can capture the gist of the matter by saying that in a Strevensian framework the mathematics functions to show how the explanandum is *independent* from the low-level causal details.[26] Namely, in the stylized cicada explanation (cf. Sect. 3) the derivation of (6) employes number theory so as to allow us to grasp the fact that almost nothing about the causal trajectories of individual cicadas makes a relevant difference to the long-run tendency to evolve a period that is co-prime to every ecologically available alternative.

Altogether, Strevens maintains that only facts about causal dependence play a thick explanatory role, and he is adamant that mathematics, while indispensable in this way, is only playing a thin explanatory role:

[26] Strevens does not consider the cicada example, but for comparison one can look at his evolutionary example of homozygous elephant seals which has some of the same features.

> No philosophy of mathematics, note, is invoked. I assume only that mathematics is somehow able to serve as a *representer* of things in the world; this is consistent both with the view that the world is inherently mathematical and with the contrary view. (Strevens 2008, p. 330)

8 Broader reflections

The principal aim of my discussion has been to make a simple but crucial methodological point: debates about the 'explanatory role' of mathematics (in the context of the indispensability argument) should be conducted much more closely in relation to specific accounts and conceptions of explanation. There are fruitful, more fine-grained discussions to be had about the various interesting cases of explanatory mathematics in the context of the recent ontic accounts of explanation developed by Strevens and Woodward, for example, and in the context of the modal account developed by Lange. In relation to these specific accounts I have argued that the advocates of the enhanced indispensability argument have fallen much short of having shown that mathematics, even when indispensable, plays the kind of thick explanatory role that is ontologically committing. (I believe there are further distinctions between ontologically committing and ontologically peripheral explanatory roles to be drawn in connection with various epistemic and modal accounts of explanation as well, and there is therefore much further work to be done to defend the indispensability argument, whichever account of explanation is preferred.)

I would now like to suggest more generally that also in other philosophical contexts arguments from explanatory indispensability would considerably benefit from precisifying the intuitive notion of explanatory indispensability in terms of a philosophical conception of explanation. My argument has taken place purely in the context of the indispensability argument for mathematical realism, but there is an obvious broader context for it. Consider the debate on scientific realism to begin with. The (explanatory) indispensability argument has been designed to mimic similar arguments for scientific realism, and there is indeed a long tradition in philosophy of science, going back to the very architects of the indispensability argument, to argue for scientific realism by attuning to the explanatory role that theoretical posits like quarks and electrons play in science. *If* an argument for scientific realism about quarks or electrons is *nothing but* an explanatory indispensability argument applied to these theoretical posits, it should clearly be supplemented with a justification for thinking that the relevant theoretical posits are playing a *thick* explanatory role. It is an interesting question exactly when this supplement can be provided, and what role (if any) causation should play.[27] Not all realist arguments can be readily assimilated with the explanatory indispensability argument, however, and even in the case of the realist's most famous inference to the best explanation—the no-miracles argument—it has been argued that it can be naturally furnished with fine-structure that critically differentiates it from the abstract explanatory indispensability argument (see e.g. Saatsi 2007).

There is also a (related) tradition in the philosophy of science, going back to Sellars 1963, to accept an 'explanatory criterion of reality':

> The only workable criterion of reality is the explanatory criterion: something is real if its positing plays an indispensable role in the explanation of well-founded phenomena. (Psillos 2011, p. 15)

Psillos is a scientific realist who takes his cue from Sellars, and in the spirit and letter of the above criterion argues that the scientific realist should be committed not only to the reality quarks, electrons, and such, but also to abstract models,

[27] It is not difficult to conceive of potentially relevant differences between mathematical and non-mathematical theoretical posits. There are many thinkable differences in the explanatory roles played by imaginary or prime numbers and quarks, for example, that do not boil down to the non-causal/causal contrast. The simple idea that ordinary observable matter is in some sense *composed of* quarks, for example, could perhaps serve as a starting point.

mathematics, and a host of other abstracta, because they all play an indispensable role in explaining empirical phenomena (Psillos 2010, 2011). In my view Psillos's brief shares the indispensability argument's shortcoming explored in this paper: the notion that idealized models and various other kinds of abstracta are indispensable to scientific explanations does not suffice to establish a realist commitment to them. What is needed is an argument that these objects play a *thick* explanatory role.[28]

It is easy to find other explanation-driven arguments in other areas of metaphysics that bear a close resemblance to these examples from philosophy of science. Some Australasian metaphysicians, like Armstrong and Sankey, are well known to frame their arguments for universals and natural kinds in terms of (something like) explanatory indispensability. As far as I am concerned, such arguments have little probative force without a proper analysis of the sense of explanation in play. Going even further afield, we find instances of (something very much like) the explanatory indispensability argument in areas such as meta-ethics, where we perhaps have even less of an idea what explanation exactly amounts to.[29]

There is a general, commonsensical point to be made about all such arguments: ultimately a handle is needed on the concept of explanation involved, for how else can we really assess the epistemic and ontological import of explanatory indispensability, if not by first spelling out what it takes to explain something?

[28]For example, Psillos argues that a scientific realist should countenance the reality of (non-mathematical) abstract objects like a system's *centre of mass* because it can play an indispensable explanatory role (despite being causally inert). Now, if one examines such an explanation in Strevens's account, for instance, the centre of mass naturally comes out as an explanatory difference-maker. But given the details of the account, it does so in a way that need not elicit a realist reification of it over and above of the more fundamental facts by virtue of which it is a difference-maker.

[29]See e.g. Sturgeon 2006 and Enoch 2013 for discussions of explanation as a guide to epistemology and metaphysics of moral realism.

References

Baker, Alan 2005: 'Are There Genuine Mathematical Explanations of Physical Phenomena?' *Mind,* 114, pp. 223–38.

——2009: 'Mathematical Explanation in Science'. *British Journal for the Philosophy of Science,* 60, pp. 611–33.

Baker, Alan and Mark Colyvan 2011: 'Indexing and Mathematical Explanation.' *Philosophia Mathematica,* 19, pp. 323–34.

Batterman, Robert 2010: 'On the Explanatory Role of Mathematics in Empirical Science.' *British Journal for the Philosophy of Science,* 61, pp. 1–25.

Bokulich, Alisa 2008: 'Can Classical Structures Explain Quantum Phenomena?' *British Journal for the Philosophy of Science,* 59, pp. 217–35.

——2011: 'How Scientific Models Can Explain'. *Synthese,* 180, pp. 33–45.

Colyvan, Mark 2012: 'Road Work Ahead: Heavy Machinery on the Easy Road'. *Mind,* 121, pp. 1031–46.

Daly, Christopher and Simon Langford 2009: 'Mathematical Explanation and Indispensability Arguments'. *The Philosophical Quarterly,* 59, pp. 641–58.

Dreier, James (ed.) 2006: *Contemporary Debates in Moral Theory.* New York: Wiley-Blackwell.

Elgin, Catherine 2004: 'True Enough'. *Philosophical Issues,* 14, pp. 113–31.

Enoch, David 2011: *Taking Morality Seriously: A Defense of Robust Realism*. Oxford: Oxford University Press.

Field, Hartry 1989: *Realism, Mathematics, and Modality*. Oxford: Blackwell.

Harper, William 2011: *Isaac Newton's Scientific Method: Turning Data Into Evidence about Gravity and Cosmology*. New York: Oxford University Press.

Hitchcock, Chris and James Woodward 2003: 'Explanatory Generalizations, Part 2: Plumbing Explanatory Depth'. *Noûs,* 37, pp. 181–99.

Jackson, Frank and Philip Pettit 1990: 'Program Explanation: A General Perspective'. *Analysis,* 50, pp. 107–17.

Lange, Marc 2013: 'What Makes a Scientific Explanation Distinctively Mathematical?' *The British Journal for the Philosophy of Science,* 64, pp. 485–11.

Lyon, Aidan 2012: 'Mathematical Explanations of Empirical Facts, and Mathematical Realism'. *Australasian Journal of Philosophy,* 90, pp. 559–78.

Lyon, Aidan and Mark Colyvan 2008: 'The Explanatory Power of Phase Spaces'. *Philosophia Mathematica,* 16, pp. 227–43.

Pincock, Christopher 2011: 'On Batterman's "On the Explanatory Role of Mathematics in Empirical Science"'. *The British Journal for the Philosophy of Science,* 62, pp. 211–17.

——2012: *Mathematics and Scientific Representation*. Oxford: Oxford University Press.

Psillos, Stathis 2010: 'Scientific Realism: Beyond Platonism and Nominalism'. *Philosophy of Science,* 77, pp. 947–58.

——2011: 'Living with the Abstract: Realism and Models'. *Synthese,* 180, pp. 3–17.

Radder, Hans (ed.) 2003: *The Philosophy of Scientific Experimentation*. Pittsburgh: University of Pittsburgh Press.

Ross, Don and James Ladyman (eds.) 2013: *Scientific Metaphysics.* Oxford: Oxford University Press.

Saatsi, Juha 2007: 'Living in Harmony: Nominalism and the Explanationist Argument for Realism'. *International Studies in the Philosophy of Science,* 21, pp. 19–33.

——2011: 'The Enhanced Indispensability Argument: Representational versus Explanatory Role of Mathematics in Science'. *The British Journal for the Philosophy of Science,* 62, pp. 143–54.

——2012: 'Mathematics and Program Explanations'. *Australasian Journal of Philosophy,* 90, pp. 579–84.

Saatsi, Juha and Mark Pexton 2013: 'Reassessing Woodward's Account of Explanation: Regularities, Counterfactuals, and Noncausal Explanations'. *Philosophy of Science,* 80, pp. 613–24.

Salmon, Wesley 1985: 'Scientific Explanation: Three Basic Conceptions'. *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association, Volume Two: Symposia and Invited Papers*, pp. 293–305.

Sellars, Wilfrid 1963: *Science, Perception and Reality*. Atascadero, CA: Ridgeview, rpnt 1991.

Strevens, Michael 2008: *Depth: An Account of Scientific Explanations*. Harvard: Harvard University Press.

Sturgeon, Nicholas 2006: 'Moral Explanations Defended'. In Dreier 2006, pp. 241–62.

Wilson, Mark 2013: 'Some Remarks on 'Naturalism' as We Now Have It'. In Ross et al. 2013, pp. 198–207.

Woodward, James 2003a: ‘Experimentation, Causal Inference, and Instrumental Realism’. In Radder 2003, pp. 87–118.

——2003b: *Making Things Happen: A Theory of Causal Explanation*. Oxford: Oxford University Press.

Yablo, Stephen 2012: ‘Explanation, Extrapolation, and Existence’. *Mind,* 121, pp. 1007–29.

Ylikoski, Petri 2001: *Understanding interests and causal explanation*. Phd thesis, University of Helsinki.

Ylikoski, Petri and Jaakko Kuorikoski 2010: ‘Dissecting Explanatory Power’. *Philosophical Studies,* 148, pp. 201–19.