Measuring Ontological Simplicity

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Standard approaches to ontological simplicity focus either on the number of things or types a theory posits or on the number of fundamental things or types a theory posits. In this paper, I suggest a ground-theoretic approach that focuses on the number of something else. After getting clear on what this approach amounts to, I motivate it, defend it, and complete it.

What should we be counting when it comes to ontological simplicity? The dominant approach has us counting the things or types of things a theory quantifies over.[1] An increasingly popular one has us counting the fundamental things or types of things a theory quantifies over.[2] In this paper, I offer a novel ground-theoretic approach to ontological simplicity that has us counting something else. The first part states this approach, the second motivates it, the third defends it, and the fourth completes it.

We should contrast ontological simplicity with elegance (Baker 2016; Bennett 2017, 227-8) and ideological simplicity (Sober 2001, 14; Cowling 2013). Elegance is concerned with the neatness or gracefulness of the laws, inferences, and explanations posited by a theory. Ideological simplicity with the notions a theory uses in describing the world. Neither will be discussed in this paper.

It is standard to distinguish between *qualitative* simplicity and *quantitative* simplicity. For some, all that matters is qualitative (Lewis 1973, 87). For others, quantitative also matters (Nolan 1997; Sober 2009, fn. 7). In this paper, a neutral stance is adopted. Where the reader is free to opt for either disjunct, we can express this neutrality with the following disjunction: what matters when it comes to simplicity is the number of F things (quantitative) or F types of things (qualitative), where 'F' specifies the kinds of things I claim we should be focusing on when it comes to simplicity.

Telling us what to measure when it comes to simplicity is one thing. Telling us that we should, all else being equal, prefer simpler theories is another. So in arguing for what we should be counting, I am not *ipso facto* arguing for the following command

> Do not multiply what counts against simplicity without necessity!

Indeed, invoking this command only makes sense when we are trying to decide between two or more *competing* theories. But since comparisons of simplicity can be

[1] For a recent defense of this approach, see Baron & Tallant (2018), Da Vee (2020), and Thunder (2021).

[2] See Schaffer (2007, 189; 2009, 361; 2015), Cameron (2010, 250), Sider (2013, 240), and Bennett (2017, 220-9). For an overview of reasons to favor this view, see Saenz (2020).

made between non-competing theories (a theory which quantifies only over my left shoe is seemingly simpler than one which quantifies only over the real numbers), we need to separate accepting an approach to simplicity from accepting the above command. Since my concern is with the former more foundational issue, anything I say about the latter I say only as it relates to the approach to simplicity on offer.

Since grounding is integral to this approach to simplicity, some words about it are in order. As I am understanding it, grounding is *metaphysical dependence*. Because of this, it is able to relate ontologically diverse things: there is no in principle bar to facts, individuals, and properties being dependent things. This kind of neutrality is apropos. A theory of simplicity should work just as well for those who think that grounding relates individuals and properties (or entities) as it does for those who think it relates only facts.

In keeping with orthodoxy, I treat grounding as irreflexive, transitive, and asymmetric. Taking grounding to be primitive, fundamentality and partial grounding are defined in the standard ways: x is fundamental $\leftrightarrow_{df.}$ x is not grounded; $x_1, \ldots, x_n$ partially ground $y \leftrightarrow_{df.} x_1, \ldots, x_n$ ground y or $\exists z_1, \ldots, z_n(x_1, \ldots, x_n, z_1, \ldots, z_n$ ground $y)$.[3]

1. The Approach

1.1 The Relation

The relation needed for this paper's approach to simplicity is that of something *being independent of* some things. Here is how to understand it:

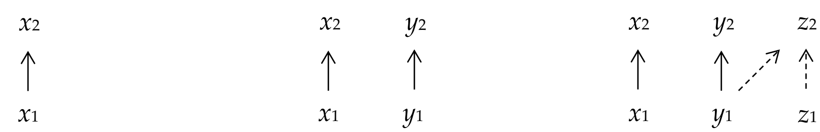
> **Independence.** x is independent of some things $\leftrightarrow_{df.}$ x is none of them, none of their partial grounds, not partially grounded in any of them, and not partially grounded in any of their partial grounds.

Think of independence as a way of capturing what it means for something to be wholly unconnected, free, and distinct from some things. Of course, there are other ways of capturing these notions (modal, mereological, and spatiotemporal). But here, grounding is given pride of place.[4]

[3]As the reader can see, on pain of entailing that non-existent things are fundamental, negation takes narrow scope in 'x is not grounded'. This makes it equivalent to x exists and there is nothing that grounds x. Thanks to a referee for pointing this out.

[4]For a modal way, see Armstrong (1989, x). For a mereological way, see Gendler & Hawthorne (2002, 21). And for a spatiotemporal way, see Lewis (1986, 88).

Let me illustrate independence. Where the solid arrow represents grounding and the dashed partial grounding, consider the following three grounding structures (the last of which has y_1, z_1 collectively grounding z_2):



first structure second structure third structure

Now in the first structure, nothing is independent of anything. In the second, each of the xs is independent of the ys (and vice-versa). And in the third, although each of the xs is independent of the zs (and vice-versa), this is not true of each of the ys. Since y_1 partially grounds z_2 and y_2 is grounded in a partial ground of z_2, each of the ys is not independent of z_2. But then each of the ys is not independent of the zs.

1.2 Some Formal Features

In order to get a better handle on independence, let's look at some of its formal features. Where in what follows, 'I' is our variably polyadic predicate for it, '$Ixy_1, \ldots, y_n$' means that x is independent of $y_1, \ldots, y_n$. Now we should accept

Irreflexivity. $\sim Ixx$.

Since independence implies non-identity, nothing is independent of itself.

We should also accept

Symmetry. $Ixy \rightarrow Iyx$.

To see why, assume that y is not independent of x. So y is either identical to x, one of x's partial grounds, partially grounded in x, or partially grounded in some of x's partial grounds. But then x is either identical to y, partially grounded in y, partially grounds y, or partially grounded in some of y's partial grounds. So x is not independent of y. So the contrapositive of Symmetry is true and so Symmetry is true.

We should not accept

Transitivity. $(Ixy \ \& \ Iyz) \rightarrow Ixz$.

Symmetry and Transitivity have it that if x is independent of y, then x is independent of itself. Since this contradicts Irreflexivity, Transitivity is false.

We should accept

Distribution. $Ixy_1, \ldots, y_n \rightarrow (Ixy_1 \,\&\ldots\&\, Ixy_n)$.

To see why, assume that x is not independent of y_1. So x is either identical to y_1, one of y_1's partial grounds, partially grounded in y_1, or partially grounded in some of y_1's partial grounds. So x is not independent of $y_1, \ldots, y_n$. Since this reasoning generalizes to any of $y_2, \ldots, y_n$, the contrapositive of Distribution is true and so Distribution is true.

Where 'X_1', ..., 'X_n' range over pluralities, we should accept

Collection. $(IxX_1 \,\&\ldots\&\, IxX_n) \rightarrow IxX_1, \ldots, X_n$.

To see why, assume that x is not independent of $X_1, \ldots, X_n$. So x is either one of the things among $X_1, \ldots, X_n$, a partial ground of one of these things, partially grounded in one of these things, or partially grounded in some partial ground of one of these things. But on any of these, it is not true that x is independent of X_1 and ... and X_n. So the contrapositive of Collection is true and so Collection is true. (And from Collection and Distribution we get: $(Ixy_1 \,\&\ldots\&\, Ixy_n) \leftrightarrow Ixy_1, \ldots, y_n$.)

Where $y_1, \ldots, y_m$ is a proper sub-plurality of $y_1, \ldots, y_n$, we should accept

Contraction. $Ixy_1, \ldots, y_n \rightarrow Ixy_1, \ldots, y_m$.[5]

To see why, assume that x is independent of $y_1, \ldots, y_n$. By Distribution, x is independent of y_1 and ... and y_n and so independent of y_1 and ... and y_m. But then by Collection, x is independent of $y_1, \ldots, y_m$. So Contraction is true.

We should not, however, accept the converse of Contraction

Expansion. $Ixy_1, \ldots, y_m \rightarrow Ixy_1, \ldots, y_n$.

To see why, assume that x is independent of $y_1, \ldots, y_m$. So by Expansion, x is independent of $y_1, \ldots, y_m, x$. But then by Distribution, x is independent of itself. Since this contradicts Irreflexivity, Expansion is false.

These features of independence help give us a grasp on the logic of independence and so on independence. But they also help drive home some interesting results and point to differences between independence and related notions. For some of these results and differences, see the appendix.

[5] $y_1, \ldots, y_m$ is a proper sub-plurality of $y_1, \ldots, y_n \leftrightarrow_{df.}$ each thing among $y_1, \ldots, y_m$ is among $y_1, \ldots, y_n$ but not vice-versa.

1.3 The Independence Approach

I have defined independence, illustrated it, and listed a number of its formal features. I now want to state this paper's approach to simplicity using it. Consider then a plurality which meets the following condition: for any x among this plurality, x is independent of any proper sub-plurality of this plurality which does not have x among it. Since any such plurality is a plurality of things each of which is independent of the others, let us say of such a plurality that it is a plurality of independent things.[6]

For each theory under consideration, consider those largest pluralities of independent things that are also maximal: for any such plurality, there can be nothing in the theory that is independent of it.[7] Now according to the independence approach, all that matters when it comes to making comparisons of simplicity are the sizes of these pluralities. Quantifying over any such plurality in theory T with the variable 'X_{T}' and in theory T* with the variable 'X_{T^*}', here is this paper's approach to ontological simplicity:

> **The Independence Approach.** T is simpler than T* $\leftrightarrow_{df.}$ X_{T} is smaller than X_{T^*}.[8]

Now in order to be an informative approach, we need to know what makes it that one plurality is smaller than another. For now, assume that the largest number of independent things a theory posits is finite. (In §4, I drop this finitist assumption and show what happens when we permit pluralities of independent things that are infinite in number.) Given this, we can state our approach as follows:

> T is simpler than T* $\leftrightarrow_{df.}$ the number of things in X_{T} is less than the number of things in X_{T^*}.[9]

So T is simpler than T* just in case the largest number of independent things T posits is less than the largest number T* posits.[10]

[6] Note that pluralities of one are pluralities of independent things. For any x among such a plurality, any proper sub-plurality of this plurality is such that x is independent of it on account of pluralities of one having no proper sub-pluralities. So vacuously, a plurality of one is a plurality of things each of which is independent of the others.

[7] What does largeness amount to here? Though I like to talk in terms of pluralities rather than sets, it will perhaps do here to put it set-theoretically: largest in the sense of having the greatest cardinality.

[8] And so, for co-simplicity, T and T* are co-simple $\leftrightarrow_{df.}$ X_{T} is the same size as X_{T^*}.

[9] This is neutral over whether it is quantitative or qualitative simplicity that is at issue. For example, if one wants to focus on qualitative simplicity, then the number of things in X_{T} amounts to the number of types in X_{T}, where one is free to understand types as they see fit: properties, predicates, sets or pluralities of things, or what have you (notice though that if types are pluralities, then the independence approach requires that one makes sense of pluralities of pluralities, and so of super-pluralities).

[10] There is a brief snag. Since the approach requires quantifying over pluralities of independent things,

Applying the approach, let us compare the simplicity of three theories, the first of which has the first grounding structure depicted in §1.1, the second the second structure, and the third the third structure. In the first theory, the largest number of independent things is one: x_1 and x_2 are these largest pluralities (recall that pluralities of one are pluralities of independent things). In the second, the largest number is two: x_1, y_1 and x_1, y_2 and x_2, y_1, and x_2, y_2 are these largest pluralities. And in the third, the largest number is three: x_1, y_1, z_1 and x_1, y_2, z_1 and x_2, y_1, z_1 and x_2, y_2, z_1 are these largest pluralities. So the first theory is simpler than the second, which is simpler than the third, which seems right given their grounding structures.

The independence approach allows us to give sense to the notion of the *width* of a theory. Width is measured in terms of the size of the largest pluralities of independent things a theory has. The larger the size, the wider the theory. Contrast this with the *height* of a theory, which is measured in terms of the size of the largest pluralities that form a grounding chain.[11] The larger the size, the taller the theory. The grounding structures displayed in §1.1 help illustrate this distinction nicely. In having no plurality that forms a grounding chain which exceeds two, any theory which has one of these grounding structures has the same height as any theory which has one of the others. But as seen in the previous paragraph, they do not have the same width.

There are four important features of the present approach. First, given that simplicity is a theoretical virtue, we get the following command

> **The Shaver.** Do not multiply independent things without necessity!

(so called because it tells us to shave, if we can, things each of which is independent of the others). Second, this approach does not require a fundamental level in order

it would seem that we cannot infer from it that theories according to which there is nothing are simpler than ones according to which there is something. There are a few ways to respond. The one I prefer quantifies over the degenerate 'empty plurality' and has it that the largest plurality of independent things in a theory that posits nothing is this plurality (thanks to Jonathan Schaffer for this suggestion). Now taken at face value, this involves quantifying over zero things and so involves a *zero or more* plural quantifier. Though there are plural logics that involve such quantifiers (Burgess 2004), some doubt their intelligibility. Fortunately, there is a trick that allows us to define a zero or more plural quantifier into a standard one or more one (Boolos 1984, 444). Let us translate '$\exists XFX$' as 'Either there are some X that are F, or F*', where F* is the result of replacing each x among X that are F with $x \neq x$. So '$\exists XFX$' means that there are some things that are F or everything that is among some things that are F is not self-identical. Since the right-hand disjunct is trivially satisfied in a theory according to which there is nothing, so is $\exists XFX$.

[11] $x_1, \ldots, x_n$ form a grounding chain $\leftrightarrow_{df.}$ partial grounding is connected over $x_1, \ldots, x_n$. And R is connected over $x_1, \ldots, x_n \leftrightarrow_{df.}$ for any x_i and x_j among $x_1, \ldots, x_n$, either Rx_ix_j, Rx_jx_i, or $x_i = x_j$. Notice that this permits degenerate grounding chains since it entails that for anything, it forms a grounding chain (since for anything, it is self-identical). This is desirable since it allows us to assign a height to theories that posit only fundamental things.

for claims of relative simplicity to hold. So it is consistent with grounding never bottoming out and our thinking that it never bottoms out (more on this later). Third, the approach is consistent with *grounding nihilism*. Take a theory which eschews grounding.[12] For such a theory, nothing metaphysically owes its existence and nature to the nature and existence of something else (for if something did, then it would be grounded). But then everything is independent of everything else. Here then, the independence approach is, in practice, the same as an approach which counts everything (for such a theory, The Shaver and Ockham's Razor are equivalent). And so, even if it requires that we make *sense* of grounding, the independence approach does not require that we *posit* grounding when it comes to simplicity. Grounding is not foisted on anyone. Fourth, this approach understands simplicity relationally. This counts in its favor — or so it seems to me. In order for something to count against the simplicity of a theory, how it relates to the rest of the things in that theory matters. In particular, what grounding relations (or lack thereof) stand between it and everything else matters. And this is precisely what the independence approach says.

There is more that can be said. For example, we can define a notion of partial independence that, interestingly enough, bears on the epistemology of simplicity given the present approach. And it would be an oversight if something were not said about how this approach to simplicity relates to the notorious "nothing-over-and-above" relation. Because of this, and because discussing such issues now would interrupt the flow of the paper, I have reserved doing so for the appendix.

2. Motivating the Approach

2.1 An Argument

Schaffer (2015), Bennett (2017, 220-1), and Fiddaman & Rodriguez-Pereyra (2018, 3-4) have argued that when it comes to simplicity, the number of things a theory posits is not all that matters. One way of showing this is to compare theories that differ, not in the number of things they posit, but in the number of fundamental things they posit. For example, consider the difference in the grounding structure between a monist and a dualist theory:

[12] For an outline of such a view, see van Inwagen (2014).

$$\begin{tikzcd} y \\ x \arrow[u] \end{tikzcd}$$

monist structure

$$\begin{tikzcd} x & y \end{tikzcd}$$

dualist structure

In the monist structure, y is grounded and x is fundamental. In the dualist structure, both are fundamental. Now, in line with Schaffer, Bennett, and Fiddaman & Rodriguez-Pereyra, theories with these grounding structures are not on par: in fully accounting for y by means of x, such monist theories are simpler theories. But then we should not look to the number of things when it comes to simplicity.

What, though, should we look to? Some say the fundamental since monist theories have less fundamental things than dualist theories (Schaffer 2015; Bennett 2017, 220-9). But this is too quick. For consider any bottomless monist theory which has the following monist structure, and any bottomless dualist theory which has the following dualist structure (the ellipses tell us that it is grounds all the way down):

$$\begin{tikzcd} y_2 \\ x_2 \arrow[u] \\ y_1 \arrow[u] \\ x_1 \arrow[u] \\ \vdots \arrow[u] \end{tikzcd}$$

bottomless monist structure

$$\begin{tikzcd} x_2 & y_2 \\ x_1 \arrow[u] & y_1 \arrow[u] \\ \vdots \arrow[u] & \vdots \arrow[u] \end{tikzcd}$$

bottomless dualist structure

Here, the salient facts are the same: in both the non-bottomless and the bottomless cases, we are able to get monist structure from dualist structure by having the *x*s ground, and so account for, the *y*s to their "right". And so, just as it was with the first pair of theories, so it is with the second: bottomless monist theories are simpler than bottomless dualist theories. But then we should not look to the number of fundamental things when it comes to simplicity.[13]

Again though, what should we look to? Independent things! In dualist theories, the largest number of independent things is two. In monist theories, it is one. So according to the independence approach, monisms (bottomless or not) are simpler than dualisms (bottomless or not). This gets the seeming facts about simplicity right. In all this, we have reason to accept the independence approach.

2.2 Unity

The independence approach identifies simplicity with a kind of unity; a unity amongst the things, taken collectively, a theory posits. Call this 'ontological unity'.[14] Given it, a simpler theory is a more unified theory (because it has less independent things) and a more unified theory is a simpler theory (again, because it has less independent things).

Ontological unity is a function of the relations that things stand in. Consider Oppenheim and Putnam's (1958) well-known paper on the unity of science. There, they give an almost entirely ontological account of this unity by appealing to micro-reduction. The 'micro' in 'micro-reduction' has to do with parthood. They say "the reduction of B2 to B1 is a micro-reduction: B2 is reduced to B1; and the objects in the universe of discourse of B2 are wholes which possess a decomposition into proper parts all of which belong to the universe of discourse of B1" (6). And so the behavior of individual cells is to be explained in terms of their biochemical constitution (given the levels Oppenheim and Putnam employ, from the cellular to the molecular level) and the behavior of molecules are to be explained in terms of atomic physics (from the molecular to the atomic level). The picture then is one where the various

[13] Appealing to bottomless cases allows us to avoid the following response to monist and dualist cases: what makes monisms preferable to dualisms is not that the former are simpler than the latter, but that the former leave fewer things ungrounded (Da Vee 2020, 3681). This, however, is not so in the bottomless case. Bottomless theories leave nothing ungrounded and yet bottomless monisms still seem simpler than bottomless dualisms. This helps us to see that the seemings we are having in these monist and dualist cases are seemings about the relative simplicity of these theories (as opposed to seemings about something else). Here at least, the content of these seemings are not so opaque, as some seem to suggest.

[14] This notion can and should be contrasted with other notions of unity. Prominent here is epistemological/pragmatic unity, which often has to do with definability, derivability, and explanation, each of which are frequently understood in terms of semantic/logical notions. For a classic account of this kind of unity in science, see Nagel (1961). For an influential response, see Fodor (1974). For a nice introduction to these matters, see Tahko (2021).

non-fundamental levels in science (social groups, multicellular living things, cells, molecules, atoms) micro-reduce to one (elementary particles). Whatever its problems, it is clear why this is an account of the *unity* of science. The various branches of science, for them, will micro-reduce to a single branch. Here, the "height" of science, how many levels of science there are, matters not.[15] It is the "width" that matters, where width is measured in terms of the number of branches of science that are not micro-reduced (or are not micro-reduced to some same branch). That is, what matters is the number of branches that are independent of each other, independence being understood mereologically and not ground-theoretically.[16]

Here, we see the same kind of unity in the independence approach to simplicity. Recall the distinction between the width of a theory and the height, where the former is measured in terms of the size of the largest pluralities of independent things a theory posits. Like simplicity, what matters is width when it comes to unity. The grounding structures depicted in §1.1 illustrate this nicely. The first structure is more unified than the second which is more unified than the third. And the most natural and straightforward explanation of this has everything to do with their width. This is also clear in monist and dualist theories. Monisms are more unified than dualisms precisely because they have less independent things (they are, after all, monisms).

That the independence approach identifies simplicity with ontological unity yields two nice things. First, it explains why focusing on just the number of fundamental things will not do. Since there can be ontological unity *sans* fundamentality, the unity of a theory is not a function of the number of fundamental things. Second, it lowers the number of potentially distinct theoretical virtues. If ontological simplicity were a

[15]They only require that there must be several levels and that the number of levels must be finite. With respect to the *unity* of science, the first requirement seems odd. If there is only one level, then we have an extremely unified picture. But with respect to the unity of *science*, the first requirement is mandatory. The branches of science make up a hierarchy. Turning to the second requirement, it too seems odd with respect to the *unity* of science. How would the unity of science be jeopardized if there were an infinite number of levels (turtles all the way down, or up, as it were)? Of course, if it were turtles all the way down, then the non-fundamental levels would not micro-reduce to one. But reducing to one is only necessary for unity if we first assume that it is not turtles all the way down. And if it were turtles all the way up (but not down), then the non-fundamental levels *would* reduce to one! At best then, and like the first requirement, this requirement only seems plausible when it comes to the unity of *science* (the levels of science seem to be finite in number).

[16]But of course, we can understand it grounding-theoretically. Especially if wholes are grounded in their parts. Indeed, the independence approach to simplicity applies nicely to Oppenheim and Putnam's picture. The branches of science are unified *and* yield a simple ontology because the objects in the universe of discourse of any branch of science are not independent of the objects in the universe of discourse of any other branch. And they are not independent because, given that wholes are grounded in their parts, the objects in the universe of discourse for any one branch are either partial grounds of, partially grounded in, or partially grounded in some of the partial grounds of the objects in the universe of discourse for any other branch.

matter of the number of things posited, then ontological unity and simplicity would and could come apart. The same holds if ontological simplicity were a matter of the number of fundamental things posited (since, as seen in the above bottomless theories, unity is not a function of fundamentality).

2.3 A Flexible Approach

As seen above, bottomless monist and dualist grounding structures tell against thinking that when it comes to simplicity, fundamental things are what we should be counting. I want to continue to push this line by providing further cases that the independence approach can, but a fundamentality approach cannot, make sense of.

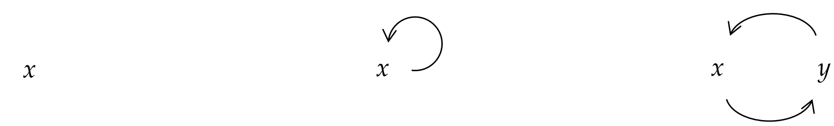
Let us begin by comparing a foundationalist structure which posits one and only one fundamental thing with a mixed structure which posits a fundamental thing and something which has no fundamental ground. Where the ellipsis tells us that it is grounds all the way down, we have

a foundationalist structure a mixed structure

In positing y, the mixed structure posits something over and above x. But then it posits something over and above everything in the foundationalist structure. So any theory with this foundationalist structure is simpler than any theory with this mixed structure. And the independence approach can make sense of this. Since the largest number of independent things in the mixed structure (two or more) is greater than the largest number in the foundationalist structure (one), it follows from this approach that any theory with the latter structure is simpler than any theory with the former structure. And since this cannot be captured by an approach to simplicity that counts only fundamental things (both structures posit the same number of fundamental things), the independence approach accommodates a greater range of data.

There are other ways of showing what we just did. For example, suppose we get rid of x in both of the above structures. Then we have a nihilist structure (which is to say that we have no structure) on one side and an infinitist structure (which is to say that we have some things but no fundamental things) on the other. Now any measure of simplicity should have any theory with the nihilist structure coming out as simpler than any theory with the infinitist structure. And the independence approach does. The largest number of independent things in the infinitist structure (one or more) is greater than the largest number in the nihilist structure (zero). But an approach to simplicity that counts only fundamental things does not (both structures have no fundamental things).

There are other ways of denying the existence of a fundamental level. Consider



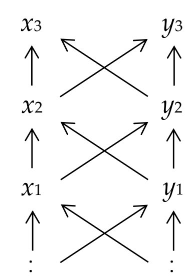
a foundationalist structure a reflexive structure a symmetric structure

If to be fundamental is to be ungrounded, then every theory with reflexive or symmetric structure lacks fundamental things. So an approach to simplicity that counts only fundamental things has it that any theory with either of these structures is simpler than a theory with the above foundationalist structure and as simple as a theory with a nihilist one! This is not so for the independence approach. The largest number of independent things in the reflexive structure is one (pluralities of one are, vacuously, pluralities of independent things). The same holds for the symmetric structure since x and y are not independent of each other. And so theories with these reflexive or symmetric structures are just as simple as ones with the above foundationalist structure and less simple than theories with a nihilist one.

Perhaps a revision in our notion of fundamentality is called for. Let us say that to be fundamental is to be not grounded or if grounded, then grounded only in itself. But this helps little: given this notion of fundamentality, any theory with the above symmetric structure still has no fundamental things. But then any theory with this structure is still simpler than a theory with the above foundationalist structure and as simple as a theory with a nihilist one. So let us revise this notion further by saying that for something to be fundamental is for it to be ungrounded or if grounded, then grounded only in something that it grounds. This will make x and y in the symmetric structure fundamental. Notice though that an approach to simplicity that counts only fundamental things will have it that each of x and y in the symmetric structure costs something that the other does not since, given the revised notion of fundamentality, each is fundamental. But this gets the facts wrong. Since each of x and y grounds, and so accounts for, the other, counting both is to double count. So this last notion of fundamentality does not help. An approach to simplicity that counts only fundamental things has a hard time making sense of the data.

Here is a revealing comparison. Where the ellipses tell us that the grounding structure is preserved all the way down, consider the following two structures:

a linear infinitist structure



a criss-crossed infinitist structure

Now, any theory with this linear structure seems simpler than any theory with this criss-crossed structure. After all, for any level L in the criss-crossed structure, it has no less than two things whereas for any level L in the linear structure, it has no more than one. But here, we must tread carefully. Notice that in the criss-crossed structure, each of the ys is nothing over and above some of the xs since each of the ys is grounded in some of the xs. So, once we have the xs, the ys come for free. The same holds in reverse: each of the xs is nothing over and above some of the ys since each of the xs is grounded in some of the ys. So, once we have the ys, the xs come for free. But then, that "linear" theories are simpler than "criss-crossed" theories is no longer so clear. What initially seemed to be the case now looks doubtful.

Let me motivate this a bit differently. Notice that for the criss-crossed structure, each thing is so bound up with everything else that to get rid of some is to get rid of all. For example, removing x_3 removes everything below it (grounds necessitate what they ground).[17] So x_2, y_2, x_1, y_1, ... would go. But in no longer having a ground, y_3 would also go. So everything would go! Or to go lower down the hierarchy, removing x_2 removes everything below it. But then in no longer having a ground, y_2 would go. But then in longer having a ground, x_3 and y_3 would go. Again, everything would go! In the criss-crossed structure then, nothing stands apart from anything else. All is bound to all. In this respect, the criss-crossed structure and the linear structure are the same: both are highly unified. (Indeed, this unity claim holds for any criss-crossed structure and so holds for an infinitely extended version of the above structure where the grounding structure is not only "bottomless" but also "sideless".)

Continuing, ignore everything that occurs below x_1 and y_1 in the criss-crossed structure and assume that, for all practical purposes, x_1 and y_1 are fundamental. Given this ignoring, we should no longer think that the criss-crossed structure is as unified as the linear structure. In disregarding what occurs below x_1, and so in treating x_1 as fundamental, we have no reason to think that removing it would result in any other thing being removed (the same holds for y_1). But then in disregarding what occurs below x_1, we have no reason to think that each of the things that we are not disregarding (x_1, y_1, and everything above them) is bound up with every other. We see then that for any level L in the criss-crossed structure, focusing on the things in L and ignoring what occurs below has implications when it comes to assessing the simplicity of a theory with this structure. And that this is so explains why a theory with this structure seems less simple than one with the linear structure. It seems less simple because we tend to do what was just done: ignore what occurs below. More carefully, in assessing the simplicity of a theory with this criss-crossed structure, we tend to focus *only* on the number of things within some level or other and so pay no attention to the way in which these things are grounded in their grounds. And the point here is that we should not do this. We should not measure simplicity in this way. Since what occurs below is relevant to how bound up or unified things are above, we should not ignore or disregard any of the lower parts of a theory when it comes to simplicity.

In light of all this, that any theory with the above linear structure is simpler than one with the above criss-crossed structure is no longer so clear.[18] And that it is not less

[17] Or at least, grounds *plus* appropriate background conditions (enablers) necessitate what they ground. For reasons to think that grounds do not always necessitate what they ground, see Leuenberger (2014), Skiles (2015), and Richardson (2019). For reasons to think that grounds do, see Trogdon (2013).

[18] Of course, any such "criss-crossed" theory has a complexity that any such "linear" theory does not. But this complexity is found in its grounding structure taken as a whole. It is not found in its ontology.

simple is, unsurprisingly, what the independence approach says. Given their grounding structure, there is nothing in such "criss-crossed" theories that is independent of any other thing. Since this is also true of "linear" theories, the thing to say is not that the latter theories are simpler than the former, but that they are co-simple. (An approach to simplicity that counts only fundamental things also entails this. But as should now be clear, it entails this for the wrong reason.)

There is something else we can glean from all this. In response to worries infinitist structures pose for a fundamentality approach to simplicity, Schaffer (2015, 663-4) suggests the following

> T is simpler than T* iff there is a level L such that, if L were fundamental, then T* would have more fundamental things than T where for every level $L\sim$ lower than L, if $L\sim$ were fundamental, then T* would have more fundamental things than T.

Suppose that the non-fundamental level that x_1 appears on in both the linear and criss-crossed structure is L. Since if L were fundamental, a theory with the criss-crossed structure would have more fundamental things than one with the linear structure, and since for every level $L\sim$ lower than L, if $L\sim$ were fundamental, a theory with the criss-crossed structure would have more fundamental things than one with the linear structure, it follows from Schaffer's suggestion that the latter theory is simpler than the former.

But this is the wrong result. And the above bi-conditional gives us this result because it does what it should not. In going to counterfactual scenarios where L is fundamental, this bi-conditional is making the simplicity of a theory a function of how simple it would be were some non-fundamental level fundamental. But then in going to counterfactual scenarios where L is fundamental, it is overlooking how bound up the things in L are in the actual scenario by disregarding the ways in which these things are grounded in their grounds. In short, in going to these scenarios, it ignores what is happening at levels lower than L in the actual scenario (the same holds when we go to counterfactual scenarios where $L\sim$ is fundamental). But for reasons already given, no approach to simplicity should do this.

There are other structures and so other comparisons we can make.[19] But here,

As the above pictures make clear, criss-crossed structures are not as neat and graceful as linear structures (lines are neater than criss-crosses). And so "linear" theories are more elegant than "criss-crossed" ones. That's the sense, if any, in which a theory which has the above criss-crossed structure is less simple than a theory which has the linear one.

[19]Some of these are, like the criss-crossed structure, revealing. For one such structure, see §3.1.

we have seen enough to see the power of the independence approach. It gets the facts right in cases involving theories with infinitist, nihilist, reflexive, and symmetric structures. And it gets the facts right *for the right reasons* when comparing theories with linear and criss-crossed infinitist structures. This is not so for an approach that focuses only on fundamental things. In all this then, the independence approach proves superior.

2.4 Independence and Fundamentality

In spite of fundamentality being the wrong thing to focus on when it comes to simplicity, fundamentality and simplicity are related. To see why, assume that every non-fundamental thing is fully grounded in some fundamental things.[20] From this, we can prove the following

> **Equivalence.** n is the number of fundamental things in T $\leftrightarrow$ n is the largest number of independent things in T.

Proof: since, if some things are fundamental, then each is independent of the others, it cannot be that the number of fundamental things in T is greater than the largest number of independent things in T (from here on out, 'in T' will be dropped).

So suppose that the largest number of independent things is greater than the number of fundamental things. Now these independent things cannot all be grounded. For if they were, then since we are supposing that there are more of them than there are fundamental things, some of them would share a partial ground (if there are more grounded things than fundamental things, then it must be that at least two grounded things share a partial ground). But then each of these independent things would not be independent of the others. Since we are supposing that they are, they cannot all be grounded.

Suppose then that they are not all grounded. So some are fundamental and some are grounded.[21] Now let us say that m of them are fundamental and that n of them are grounded. So the largest number of independent things is $m + n$. And since these n grounded things are independent of these m fundamental things, it cannot be that the former are partially grounded in any of the latter. So these n grounded things must be grounded in some other fundamental things. But then in order to avoid these

[20] This is what Dixon (2016, 446) and Rabin and Rabern (2016, 63) say that foundationalism about grounding should amount to.

[21] They cannot all be fundamental since we are assuming that the number of them is greater than the number of fundamental things.

n grounded things sharing a partial ground, the number of these other fundamental things had better be at least n. And if so, then the number of fundamental things is at least $m + n$. But then the largest number of independent things is not greater than the number of fundamental things. Since this contradicts our supposition that it is greater, it cannot be that these independent things are not all grounded.

Now since these independent things are either all grounded or not all grounded, and since both disjuncts lead to a contradiction on the assumption that the largest number of independent things is greater than the number of fundamental things, this assumption must not be true. And from this and that the number of fundamental things cannot be greater than the largest number of independent things, it follows that the number of fundamental things is the same as and the largest number of independent things. Thus, Equivalence.

From Equivalence (and recall, we only get Equivalence by assuming that every non-fundamental thing is fully grounded in some fundamental things), it follows that T has less fundamental things than T* if and only if the largest number of independent things T posits is less than the largest number T* posits. So from the independence approach, T has less fundamental things than T* if and only if T is simpler than T*. So fundamental things *are* relevant to simplicity. But what makes them relevant is not that they are fundamental, but that each is independent of the others. That is, adding fundamental things to a theory does not result in a less simple theory in virtue of the fundamentality of the things added, but in virtue of their independence of the fundamental things already there. And this difference, which is a difference in what "makes" for comparative simplicity, makes *all* the difference. It is the difference that allows the independence approach to get the facts right in cases where there are no fundamental things. But then it is the difference that makes the independence approach an especially attractive approach.

3. Defending the Approach

3.1 Egality

Where the dashed arrows represent partial grounding, consider these two grounding structures

$$\begin{tikzcd} y & & z \\ & x \arrow[ul] \arrow[ur] & \end{tikzcd}$$

a hierarchical structure

$$\begin{tikzcd} y \arrow[rr, dashed, leftrightarrow] \arrow[dr, dashed, leftrightarrow] & & z \arrow[dl, dashed, leftrightarrow] \\ & x & \end{tikzcd}$$

an egalitarian structure

A theory with this hierarchical structure is simple. Everything boils down to a single thing. This is not so for a theory with the above egalitarian structure. Given it, everything is grounded in no less than everything *taken collectively*. (This differs from a theory with the symmetric structure considered in §2.3. There, everything is grounded in everything *taken individually*.) However, since the largest number of independent things in each theory is one, then neither is simpler than the other given the independence approach. But the theory with the hierarchical structure *is* simpler. So the independence approach is not the right approach.[22]

It is helpful to state this reason for thinking that one theory is simpler than the other in terms of the notion of a complete minimal basis. Say that $x_1, \ldots, x_n$ form a complete basis $\leftrightarrow_{df.}$ each of the grounded things are grounded in $x_1, \ldots, x_n$ or some proper plurality of $x_1, \ldots, x_n$. Then say that $x_1, \ldots, x_n$ form a complete minimal basis $\leftrightarrow_{df.}$ $x_1, \ldots, x_n$ form a complete basis and no proper plurality of $x_1, \ldots, x_n$ forms a complete basis. So, why think that a theory with hierarchical structure is simpler? Because the complete minimal basis in it (x) is smaller than the complete minimal basis in a theory with the above egalitarian structure (x, y, z).

Now for hierarchically structured theories that have complete minimal bases (and as we have seen, not all do), this reason for thinking that one theory is simpler than another seems right. But this is not always so when it comes to non-hierarchical theories. Here is why.

In a theory with the above egalitarian structure, the ontological demands that x makes are no different than the ones made by y (they are x, y, z). Grounding and being grounded in the same things, neither requires more or less than the other and so neither is something over and above the other. So x costs no more than y and y

[22] I thank Jonathan Schaffer for raising this objection.

costs no more than x. But then x does not count against the simplicity of this theory any more than y does and vice-versa. Since all of this holds for z as well, nothing in such an egalitarian structured theory counts against its simplicity any more than anything else. Because of this, it is a mistake to count everything when measuring such a theory's simplicity. If the cost of x is no different than that of y's, then counting both is to double count. Saying otherwise has it that x's ontological demands are distinct from y's. But that is false.[23]

What has happened here? How is it that the complete minimal basis in the egalitarian structure is x, y, z and yet neither x, y, nor z costs any more than any other? The answer is that the things that form a complete minimal basis collectively ground each other. And so each *merely* partially grounds each other (if even one fully grounded the rest, they would not form a complete minimal basis). In egalitarian structures then, the rules have changed. The size of a structure's complete minimal base is no indication of the simplicity of a theory with that structure.

There are two things we can take away from this. First, given that nothing counts against the simplicity of a theory with egalitarian structure any more than anything else, such a theory is no less simple than a theory with the above hierarchical structure. And this is what the independence approach says. Far then from being a problem for such an approach, in the end, this objection from egality serves to confirm it.

Second, notice that no matter how large we increase the complete minimal basis in an egalitarian structured theory (four, five, six, ... aleph null, ...), nothing in such a basis would count against the simplicity of this theory any more than anything else. So increasing the size of this basis does not result in a less simple theory. This is important since it shows us where the problem really lies. The problem is not with the independence approach. It is not with *whether* one theory is simpler than another. It is with *how* simplicity in egalitarian structured theories is achieved. The proponent of such a theory can claim that it is a virtue of her theory that it can postulate a whole host of things at no extra cost. But this "advantage" has all the marks of theft over honest toil. After all, the total number of things that form a complete minimal basis can be increased *ad infinitum* without a corresponding decrease in simplicity. This should not be possible. But it is in an egalitarian framework. So much the worse then not for the facts which make for simplicity, but for the egalitarian framework which

[23] The unity test we employed earlier with respect to criss-crossed structure can be applied here. Once x is removed in the above egalitarian structure, so is what grounds it. But what grounds it grounds everything else. So once x is removed, everything is removed. *Mutatis mutandis* for y and z. Here then, nothing "stands apart" from anything else. In this respect, both the hierarchical and egalitarian structures are the same.

exploits these simplicity-making facts in a most unattractive way.

3.2 Profligacy

Suppose that theory T posits ten fundamental and no grounded things and that theory T* posits nine fundamental and 1,000 grounded things. Now, if this is the only difference between them, then according to Fiddaman & Rodriguez-Pereyra (2018, 344), "[T] is the better theory, since [T*] is unnecessarily profligate".[24] Since this contradicts the independence approach, then if they are right, this approach gets things wrong.

Fiddaman & Rodriguez-Pereyra think that T* is unnecessarily profligate on account of positing more things than T without a corresponding advantage. But this is not a good reason for thinking that T* is *objectionably* profligate. Notice another way in which T* can be said to be profligate. Grounded things exist and do the work they do because their grounds exist and do the work they do. For example, baseballs exist and do the work they do — break windows, bruise mitts, and dent bats — because their parts arranged baseball-wise exist and do the work they do — break windows, bruise mitts, and dent bats. But then the 1,000 grounded things exist and do the work they do because the nine fundamental things exist and do the work they do. Here then, T* is profligate: any theory that posits the nine fundamental things that T* posits but no grounded things will be, with respect to the work the things in it do, just as adequate as T*. We see then that T* is profligate on account of its positing *superfluous* things: things that do no more work than some of the other things the theory posits.[25]

Now, that T* should be rejected on account of its positing grounded, and so superfluous, things is an extreme claim since it amounts to a ban on grounded things: according to this claim, any theory with grounded things should be rejected in favor of a theory just like it *sans* these grounded things. But it is also a false claim. As Marcus (2001, 75) says, "As overdetermination is ordinarily conceived . . . overdetermining causes are thought of as both independent and sufficient for their effects". But since grounded things are not independent of their grounds, grounded things do not overdetermine (or problematically overdetermine) the work their grounds do. So grounded things are not superfluous in a problematic kind of way. But then even if T* is profligate on account of positing grounded, and so superfluous, things, it is not

[24]For a similar verdict, see Baron & Tallant (2018, 600).

[25]There are two ways for something to be superfluous: the superfluous can be superfluous in virtue of failing to do any work (so they are idle) or in virtue of doing work, but not doing new work (so they overdetermine). In T*, the 1,000 grounded things are superfluous not because they fail to do work, but because the work they do is not new. For an excellent paper on this and related matters, see Barnes (2000).

objectionable because of this.

What bearing does this have on Fiddaman & Rodriguez-Pereyra's insisting that T is simpler than T*? As just seen, that T* posits more things than a theory that posits just its fundamental things is no mark against it. But then, where T** is gotten from T* by eliminating the latter's grounded things, we should accept

T* and T** are co-simple.

Now since T posits ten fundamental things, T** nine, and since neither posits grounded things, it is uncontroversial that

T** is simpler than T.

And from this and that T* and T** are co-simple, it follows that

T* is simpler than T,

contradicting Fiddaman & Rodriguez-Pereyra's judgement. Since that T** is simpler than T is uncontroversial, if they want to maintain their claim that T* is objectionably profligate, they need to show that T* and T** are not co-simple. In short, they need to take the extreme route and argue that theories with grounded things should be rejected in favor of theories without them.

3.3 Likelihoods

Though this paper's concern is not over whether preference should be given to simpler theories, there is a way of justifying such a preference that tells against the independence approach.

Here is an attractive idea: simpler theories are more likely to be true because they are better supported by the data. Huemer (2009, 221) elaborates on this when he says that "a simple theory can accommodate fewer possible sets of observations than a complex theory can . . . [so the] realization of its predictions is consequently more impressive than the realization of the relatively weak predictions of the complex theory". Where T is a theory and E is our evidence, we can see this at work in Bayes's Theorem:

$$P(T|E) = [(P(E|T) \times P(T)] \ / \ P(E).$$

Consider a complex theory T_c, a simple theory T_s, and some evidence E. Now the likelihood of any theory T given E is P(E|T). And the claim here is that T_s typically has the higher likelihood. Huemer (2009, 223) says

> if $[T_s]$ is compatible with and neutral between possible items of evidence E_1 and E_2, while $[T_c]$ is compatible with and neutral among E_1, E_2, E_3 and E_4 (where the E_i are mutually exclusive), then $P(E_1|[T_s]) = \frac{1}{2}$, whereas $P(E_1|[T_c]) = \frac{1}{4}$. $[T_s]$ takes a greater risk, since it would be refuted by E_3 or E_4, but if E_1 or E_2 is observed, $[T_s]$ is supported twice as strongly as T_c.

Assuming then that the prior probabilities of T_s and T_c are the same, if $P(E|T_s) > P(E|T_c)$, it follows from Bayes's Theorem that $P(T_s|E) > P(T_c|E)$.

This seems all well and good. But Baron & Tallant (2018, 610) yield it in a way that tells against the independence approach.[26] They start by considering a simple case. Suppose that T posits one fundamental thing A and three grounded things C, D, and E and that T* posits two fundamental things A and B and one grounded thing C. Now an experiment is performed and the result is that derivative C exists. In light of this, which theory is more probable? Assuming that the priors are the same, Baron & Tallant have it that the theory with more independent things is. They say "the probability of performing the experiment and it showing us that C exists given [T] is $\frac{1}{3}$ and the probability of performing the experiment and it showing us that C exists given [T*] is 1". (610) After this, they claim that a theory with "more entities or entity types will always be less probable than a theory with less in relation to a given piece of evidence, regardless of what those entities or entity types are". (610) Here then, that simplicity should be measured in terms of independence is not supported by an intuitive account of what makes simpler theories preferable. But since any adequate approach to simplicity should, the independence approach is not the right approach.

What should we think of this argument? Put to the side the controversial claim that a preference for simpler theories can be justified.[27] Notice instead that a harmless change in the evidence has it that T is the more probable theory. For suppose that the result of the experiment is that fundamental A exists. Assuming that both theories have the same priors, T comes out as more probable: given it, the probability of performing the experiment and it showing us that A exists is 1. Given T*, the probability of performing the experiment and it showing us that A exists is $\frac{1}{2}$. Here then, the likelihoods favor the theory that is said to be simpler by the independence approach. And so the likelihoods do not *always* favor the theory with less entities or entity types,

[26]Their target is not this approach. It is Schaffer's (2015) fundamentality approach. Still, what they say in their paper tells just as much against this paper's approach as it does Schaffer's.

[27]Swinburne (1997, 51) and Sider (2013, 239) think that it cannot (and think this all while believing that simpler theories are, all else being equal, more likely to be true). And French (2014, 57) says that "it is more or less accepted that there is no argument that demonstrates that simplicity tracks the truth in the scientific case". But if there is no argument in the scientific case, it is doubtful that there is any at all.

contra Baron & Tallant.[28]

Here is another worry. In order for us to infer that one theory is more probable than another on the basis of their likelihoods, we have to assume that their priors are the same. But why make such an assumption in the present context? Baron & Tallant answer

> when we are at the point of choosing between theories using theoretical virtues, ... we already know that the theories at issue do not come apart in any of the normal ways, and so something extra is needed to select between them. If our priors were not equal between the theories, then the theories would probably come apart in a standard way, and so considerations of parsimony would be less likely to weigh in. (609)

On the contrary, one would have thought that when it comes to choosing between theories on the basis of the theoretical virtues, such theories predict, and predict equally well, the evidence; the 'all else being equal' clause seems to rule out a difference in the likelihoods. As Sober says (2009, 130), the command to choose the simpler theory all else being equal is "meant to apply when the likelihoods "fail to discriminate" between "X exists" and "X does not exist"".[29] But then if simplicity is to have Bayesian import, it must be reflected in the priors and not the likelihoods. Far from thinking that if our priors were not equal between theories, matters involving simplicity would be less germane, it is precisely with respect to the priors that such matters seem to have import.[30]

The debate between nominalism and platonism provides us with a nice example.[31] According to the former, there are no numbers. According to the latter, there are num-

[28]In fact, the likelihoods can be used to show that a theory's positing more things than another can favor accepting it if in so doing, it says less about what does not exist. Suppose that there are only three possible things A, B, and C. Further suppose that according to T**, only A and B exist and that according to T***, only A exists. Now an experiment is performed and the result is that C does not exist. Assuming that both theories have the same priors, the theory with more things comes out as more probable: the probability of performing the experiment and it showing us that C does not exist is 1 given T** but $\frac{1}{2}$ given T***.

[29]See Sider (2013, 241). See also Brenner (2015, 335), who says that "Simplicity considerations are generally brought in to decide between competing theories which are equally capable (or very nearly equally capable) of explaining our evidence".

[30]See Jeffreys (1931, 38-9), Howson (1988, 81-2), Swinburne (1997, 56), and Huemer (2009, 219-220), each of which use simplicity to assign probabilities to priors that are not based on empirical evidence (first priors). For some who use simplicity to assign probabilities to priors that are based on empirical evidence (non-first priors), see Sober (1990, 79-84) and Jansson & Tallant (2017).

[31]Here, I assume that appeals to simplicity in metaphysics are appropriate. For some who think they are not, see Huemer (2009), Kriegel (2013, 17-19), and Willard (2014). For a defense of the claim that they are, see Tallant (2013), Brenner (2017), and Bradley (2018).

bers and they are independent of the physical world. Now suppose that our evidence involved the truth of various mathematical *sentences*, S_m.[32] Further suppose that both the platonist and the nominalist could tell an equally plausible story that yielded that S_m are true. So $P(S_m \text{ are true}|\text{Platonism}) = 1$ and that $P(S_m \text{ are true}|\text{Nominalism}) = 1$. Here, the likelihoods are the same. So if simplicity is to have Bayesian import, it must be reflected in the priors. This comports well with philosophical methodology: if the likelihoods are the same, the nominalist would declare victory (or a significant advantage) and the platonist defeat (or a significant disadvantage) on grounds of simplicity. But then, at least for those nominalists and platonists who are Bayesians, simplicity is reflected in the priors and not the likelihoods.[33]

4. Completing the Approach

I have so far assumed that the largest number of independent things a theory posits is finite. Given this, comparisons of simplicity can proceed based on the largest number of independent things theories have. But what happens when the theories being compared each posit an infinity of independent things? *If the infinities involved differ in size*, then comparisons can proceed based on the largest number of independent things theories have. But they cannot proceed in this way when the infinities involved are the same. For suppose that the number of things in theory T numbers the natural numbers and that each of these is independent of the others. Further suppose that this is true of theory T* and that the things in T are a proper sub-plurality of the things in T* (this is possible for infinities). In spite of the number of things in T and T* being equal, T is the simpler theory.

4.1 The Basic Idea

Notice what this calls for: an account of what makes it that one plurality is smaller than another that works for all theories, and so works for theories that posit an infinite number of independent things. For convenience sake, let us, for now, restrict

[32] The appeal to sentences is important given certain brands of nominalism. If one is a Quinean about ontological commitment, then it can be that '$2 \times 3 = 6$' is true so long as the proposition it expresses is one that does not involve quantifying over numbers.

[33] Sober (2015, 272-6) is skeptical that we can assign a higher prior to either nominalism or platonism. He also thinks that the mere fact that nominalism is simpler than platonism is no mark in its favor. I suspect that this last belief of his stems from his assumption that simplicity is not a fundamental epistemic goal. Of course, not everyone agrees with him on this (Swinburne 1997; Sider 2013, 239). Number me with these sober dissenters.

ourselves to theories each of whose things is independent of the others. And let us assume that the number of things in these theories is of the same infinite size. Given this, distinguish between a pair of theories each of which has an infinite number of independent things that the other does not and a pair of theories where this is false. That is, distinguish between a pair of theories each of which *unshares* an infinite number of independent things with the other and a pair of theories where this is false.

An Infinity Unshared. Assume that theory T_a posits an infinite number of abstracta and theory T_c an infinite number of concreta. So, concreta and abstracta being mutually exclusive, each theory unshares an infinity of independent things with the other. But which theory is simpler? Or are they co-simple? Neither. They are instead simplicity *incommensurable*. Here is a "small-addition argument" for this that mimics the small-improvement argument found in the literature on value incommensurability (Chang 1997). Intuitively, T_a is neither more nor less simple than T_c: since each has an infinite number of independent things that the other does not, there is no basis on which one can be simpler than the other. Now, take T_a and add to it something that is independent of the things in it. Call the theory that results from this addition 'T_{a+}'. Now T_a is simpler than T_{a+} since everything in T_a is among everything in T_{a+} but not vice versa. But T_c is not simpler than T_{a+} (the reason for thinking this is the same as the reason for thinking that T_a is neither more nor less simple than T_c). And from this, it follows that T_a and T_c are not co-simple. Here is why. Assume for *reductio* that

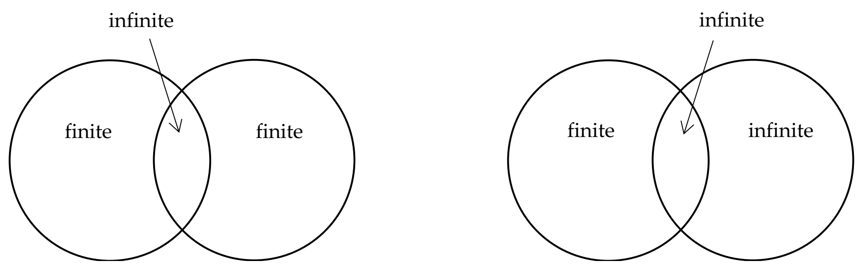
T_a and T_c are co-simple.

Since T_a is simpler than T_{a+}, it follows from T_a and T_c being co-simple that

T_c is simpler than T_{a+}.

But as we have just seen, it is not. So T_a and T_c are not co-simple. And since neither is more nor less simple than the other, it must be that they are simplicity incommensurable. So, when it comes to theories that unshare an infinity of independent things, such theories are simplicity incommensurable.

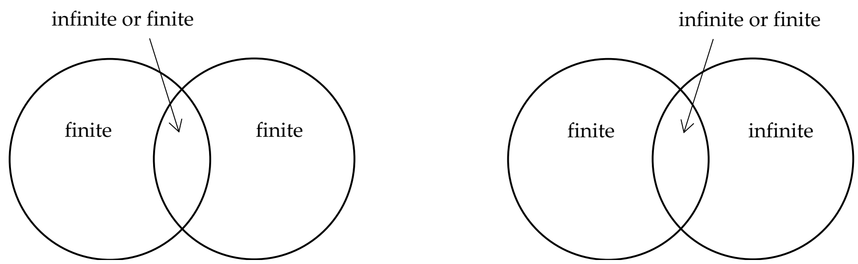
A Finitude Unshared. Let us turn to pairs of theories where it is false that each unshares an infinity of independent things with the other. So, either each theory has a mere finite (possibly zero) number of things that the other does not or only one does. (Since we are dealing with theories that have an infinity of independent things, it must be that these theories share an infinity of such things.) Let us represent these ways by means of the following Venn diagrams.



Now in order to make comparisons of simplicity, ignore those things that these theories share and focus only on the unshared things. Looking at the left-hand diagram, suppose that the number of these things in one theory is m and that the number of these things in the other is n. Then if $n > m$, the first theory will be simpler, if $m > n$, the second theory will be simpler, and if $m = n$, they will be co-simple. And of course, in the right-hand diagram, the theory that has a mere finite number of such things is simpler than the one that has an infinity.

Here then, when it comes to theories that have an infinite number of independent things, a basis involving finite numbers has been established on which to make judgments of simplicity. In order to know which theory is simpler, all we have to do is look at the number of their *unshared* things. If both theories unshare a finite number of things, or one unshares a finite number of things and the other an infinite, then matters involving finitude suffice to generate comparisons of simplicity (in the second case, since one unshares an infinite number of things, it also unshares a finite number of things that is greater than the number of things unshared by the other). This basis also predicts why in cases where each theory unshares an infinite number of things, no such comparisons can be made. Since no basis involving finite numbers can be had, no such comparisons can be made.

Because a basis has been established on which judgments of simplicity can be made for theories which posit an infinity of independent things, we can start to give a general account of what makes it that one plurality of independent things is smaller than another. Let us begin by no longer assuming that both theories have an infinity of independent things. This yields the following Venn diagrams (note that these diagrams are consistent with both theories having, and only having, things that the other does not).



Making comparisons of simplicity here proceeds in the same manner as before. Again, ignore the shared things, focus on the unshared things, and make comparisons of simplicity on the basis of the number of these unshared things.

4.2 Expanding the Basic Idea

We have so far restricted ourselves to theories each of whose things is independent of the others. Doing so made it easy to see the basic idea, which is to ignore the shared things, focus on the unshared things, and make comparisons of simplicity on the basis of the number of these unshared things. But we need to expand on this idea by looking at scenarios where this restriction is not in place.[34]

Where the ellipses tell us that there are an infinite number of *a*'s and an infinite number of *b*'s, and where the grounding structure (or lack thereof) is preserved, consider the following two theories, each of which agree on the number and identity of things at, and only at, the fundamental level:

$$\begin{array}{cccc} b_1 & b_2 & b_3 & \dots \\ \uparrow & \uparrow & \uparrow & \\ a_1 & a_2 & a_3 & \dots \end{array} \qquad\qquad \begin{array}{cccc} & & & \\ & & & \\ a_1 & a_2 & a_3 & \dots \end{array}$$

theory T_1 theory T_2

Now, in comparing the simplicity of T_1 and T_2, we want to see what ontological costs,

[34] I thank a referee for showing me the need to say more.

if any, each makes that the other does not. That is, we want to see what unshared things, in each theory, we should be looking at and count these things. But how should we go about doing this? Here is a way we should not:

> Choose *any* largest, maximal, plurality of independent things in T_1 and *any* largest, maximal, plurality of independent things in T_2, ignore the things these pluralities share, focus on the unshared things, and make a comparison of simplicity on the basis of the number of these unshared things.

Why is this a way we should not? Because it yields inconsistent results. For example, $b_1, b_2, b_3, \ldots$ in T_1 unshares an infinite number of things with $a_1, a_2, a_3, \ldots$ in T_2 and vice-versa whereas $a_1, a_2, a_3, \ldots$ in T_1 and $a_1, a_2, a_3, \ldots$ in T_2 unshare nothing. So, given the first pair of pluralities, T_1 and T_2 are simplicity incommensurable but, given the second pair, they are not (being instead co-simple).

How then should we go about comparing the simplicity of T_1 and T_2? As follows. First, take any of those largest, maximal, pluralities of independent things in T_1 that overlap the *most* with some largest, maximal, plurality of independent things in T_2.[35] Now, since $a_1, a_2, a_3, \ldots$ is one of these pluralities, and since it overlaps the most with the largest, maximal, plurality of independent things in T_2 ($a_1, a_2, a_3 \ldots$), then it is the plurality we should take.[36] Second, ignore the things these pluralities share. What remains in $a_1, a_2, a_3, \ldots$ in T_1 (nothing) are the relevant unshared things.

Do the same thing with T_2. Take any of those largest, maximal, pluralities of independent things in T_2 that overlap the most with some largest, maximal, plurality of independent things in T_1. Since $a_1, a_2, a_3, \ldots$ is the only largest maximal plurality of independent things in T_2, and since it overlaps the most with $a_1, a_2, a_3, \ldots$, which is one of the largest, maximal, pluralities of independent things in T_1, then it is the plurality we should take. Next, ignore the things these pluralities share. What remains in $a_1, a_2, a_3, \ldots$ in T_2 (nothing) are the relevant unshared things.

[35] What does overlapping the most amount to? Suppose we have the natural numbers, the natural numbers minus the number one, and the natural numbers minus the number one and the number two. Each of these pluralities share the same number of things (they each share an infinite number of numbers). But the first plurality overlaps more with the second plurality than it does with the third. We can say then that, where X, Y, W, and Z are pluralities of the same size, X overlaps more with Y than W does with $Z \leftrightarrow_{df.}$ X shares a greater number of things with Y than W does with Z or X unshares a smaller number of things with Y than W does with Z.

[36] There are an infinite number of pluralities that are among those largest, maximal, pluralities of independent things in T_1. In addition to $a_1, a_2, a_3, \ldots$ and $b_1, b_2, b_3, \ldots$, we have $b_1, a_2, a_3, \ldots$ and $a_1, b_2, a_3, \ldots$ and $a_1, a_2, b_3, \ldots$ etc. But, with the exception of the first plurality, none of these overlaps the most with the largest, maximal, plurality of independent things in T_2 ($a_1, a_2, a_3, \ldots$) since each unshares at least one thing with such a plurality.

Now, what matters when it comes to making comparisons of simplicity is the number of things that remain, and so the number of relevant unshared things. Since nothing remains in $a_1, a_2, a_3, \ldots$ in T_1 and nothing remains in $a_1, a_2, a_3, \ldots$ in T_2, then T_1 and T_2 are co-simple, which is the result we want (or so it seems to me).

Let us consider a slightly more complicated case. Consider the following two theories, neither of which share anything at the fundamental level (the first has the odd-numbered *a*s whereas the second has the even-numbered *a*s) but where, at the second level, T_3 includes something (b_1) that T_4 does not but not vice-versa:

$$\begin{array}{cccc} b_1 & b_2 & b_3 & \ldots \\ \uparrow & \uparrow & \uparrow & \\ a_1 & a_3 & a_5 & \ldots \end{array} \qquad\qquad \begin{array}{cccc} b_2 & b_3 & b_4 & \ldots \\ \uparrow & \uparrow & \uparrow & \\ a_2 & a_4 & a_6 & \ldots \end{array}$$

theory T_3 theory T_4

Following the example set by our last case, take any of those largest, maximal, pluralities of independent things in T_3 that overlap the most with some largest, maximal, plurality of independent things in T_4. Since $b_1, b_2, b_3, \ldots$ is one of these pluralities, and since it overlaps the most with $b_2, b_3, b_4, \ldots$ in T_4, then it is a plurality we are free to take.[37] Second, ignore the things these pluralities share. What remains in $b_1, b_2, b_3, \ldots$ in T_3 (b_1) are the relevant unshared things.

Do the same thing with T_4. Take any of those largest, maximal, pluralities of independent things in T_4 that overlap the most with some largest, maximal, plurality of independent things in T_1. Since $b_2, b_3, b_4, \ldots$ is one of these pluralities, and since it overlaps the most with $b_1, b_2, b_3, \ldots$ in T_3, then it is a plurality we are free to take. Next, ignore the things these pluralities share. What remains in $b_2, b_3, b_4, \ldots$ in T_4 (nothing) are the relevant unshared things.

Looking at the number of things that remain, and so at the number of relevant unshared things, since b_1 is what remains in $b_1, b_2, b_3, \ldots$ in T_3 and nothing remains in $b_2, b_3, b_4, \ldots$ in T_4, then it is T_4 that is the simpler theory, demanding less of the world than T_3.

[37] $a_1, b_2, b_3, \ldots$ is another one of these pluralities, overlapping just as much with $b_1, b_2, b_3, \ldots$ in T_4.

4.3 The Expression

We brought out the basic idea by working with theories each of whose things is independent of the others. We have expanded on this idea by applying it to theories where some things are not independent of others. It is now time to turn all of this into an expression of the independence approach. Take then any of those largest, maximal, pluralities of independent things in T, X_{T}, that overlap the most with some largest, maximal, plurality of independent things in T* and ignore the things that are shared between these pluralities.[38] What remains in X_{T}, if anything, are the relevant unshared things in T. Do the same thing for T*, taking any of those largest, maximal, pluralities of independent things in T*, X_{T^*}, that overlap the most with some largest, maximal, plurality of independent things in T and ignore the things that are shared between these pluralities. What remains in X_{T^*}, if anything, are the relevant unshared things in T*. Focusing then on these pluralities, if the number of unshared things in X_{T} is m and the number of unshared things in X_{T^*} is at least n, then if $n > m$, X_{T} is smaller than X_{T^*}. We can now give a fully general and perspicuous expression of the independence approach:

> **The (Completed) Independence Approach.** T is simpler than T* $\leftrightarrow_{df.}$ the number of unshared things in X_{T} is less than the number of unshared things in X_{T^*}.

Notice that, given this expression, if the number of unshared things in X_{T} is less than the number of unshared things in X_{T^*}, then it must be that either both numbers are finite, one is finite and the other is infinite, or one is a smaller infinity than the other.

Notice also that if the above tells us what it is for T to be simpler than T*, then in order for T and T* to be co-simple, it must be that the number of unshared things in X_{T} and the number of unshared things in X_{T^*} is finite. This condition on co-simplicity should not come as a surprise. We proved earlier, when working with theories each of whose things is independent of the others, that if the number of unshared things in X_{T} is the same as the number of unshared things in X_{T^*}, then if this number is infinite, T and T* are simplicity incommensurable. And it follows from this that if T and T* are co-simple, and so not simplicity incommensurable, then the number of unshared things in X_{T} and the number of unshared things in X_{T^*} is finite.

[38] This assumes that for T, there is some largest, maximal, plurality of independent things (thanks to a referee for pointing this out). But this seems right. Put differently, for any theory, there is always a theory with a larger number of independent things. But then for any theory, and so for T, there is some largest, maximal, plurality of independent things. Or if there is not, this needs to be shown.

Before closing, I want to show that this completed expression of the independence approach is equivalent to our initial, finitist, expression when we assume that the largest number of independent things a theory has is finite. That is, given this assumption, we can prove the following:

> **Equivalence*.** The number of unshared things in X_{T} is less than the number of unshared things in $X_{\mathrm{T}^*} \leftrightarrow$ the number of things in X_{T} is less than the number of things in X_{T^*}.

Proof: assume that the number of unshared things in X_{T} is m and that the number of unshared things in X_{T^*} is n, where $n > m$. (Recall that X_{T} is among those largest, maximal, pluralities of independent things in T that overlaps the most with some largest, maximal, plurality of independent things in T*. *Mutatis mutandis* for X_{T^*}.) Now, it cannot be that the number of shared things in X_{T} is greater than the number of shared things in X_{T^*}. For if it were, then there would be some largest, maximal, plurality of independent things in T* that overlaps more with some largest, maximal, plurality of independent things in T than does X_{T^*}. But by assumption, there is not. By identical reasoning, it cannot be that the number of shared things in X_{T^*} is greater than the number of shared things in X_{T}. So the number of shared things in X_{T} is the number of shared things in X_{T^*}. But then, since the number of unshared things in X_{T} is less than the number of unshared things in X_{T^*}, the number of things in X_{T} is less than the number of things in X_{T^*}.

Going in the other direction, assume that the number of things in X_{T} is less than the number of things in X_{T^*}. Since, as just seen, the number of shared things in X_{T} is the number of shared things in X_{T^*}, then if the number of things in X_{T} is less than the number of things in X_{T^*}, the number of unshared things in X_{T} is less than the number of unshared things in X_{T^*}. Thus, Equivalence*.

So, given our completed expression, in cases where the largest number of independent things is finite, the simplicity of a theory "boils down" to the largest number of independent things a theory has. And this, of course, is the result we want.

5. Closing

The independence approach to simplicity is an attractive approach. In appealing only to grounding, it is cheap. In making simplicity a matter of unity, it is conservative. In getting the facts right in various grounding scenarios, it is flexible. And in yielding surprising results in non-standard grounding structures (criss-crossed and egalitarian ones), it is illuminating.

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Appendix

In this paper's approach to simplicity, independence takes center stage. But *partial independence* also deserves our attention. Here is one way to understand it:

> **Partial.** x is partially independent of some things $\leftrightarrow_{df.}$ x is none of them, none of their partial grounds, not grounded in any of them, and not grounded in any of their partial grounds.[39]

Note the difference between independence and partial independence. Unlike the former's *definiens*, the latter's appeals to grounding in its third and fourth conjuncts. Because of this, independence is stronger than partial independence. So if something is independent of some things, it is partially independent of those things (this is so because if something is not partially grounded in some things, then it is not grounded in those things). But if something is partially independent of some things, it does not follow that it is independent of those things (this is so because if something is not grounded in some things, it does not follow that it is not partially grounded in those things). The third structure in §1.1 demonstrates this last claim: z_2 is partially independent, but not independent, of z_1.[40]

[39] There are other notions of partial independence, some of which are stronger than others. The weakest says that x is partially independent of some things just in case it is none of them, none of their grounds, not grounded in any of them, and not grounded in any of their grounds. Given that we have more than one notion of partial independence, why focus on the one described in the main text? Because it is the weakest such notion that bears an important relationship to independence and so to simplicity (see below).

[40] There are also formal differences. In the third structure, z_2 is partially independent of y_2 but not vice-versa and is partially independent of y_1, z_1 when taken individually but not when taken collectively. So partial independence is not symmetric and does not satisfy its version of Collection.

From Part to Full. Still, even if that x is partially independent of some things does not entail that it is independent of those things, it plausibly entails that something is. That is, the following seems true:

> **Part to Full.** x is partially independent of $y_1, \ldots, y_n \rightarrow \exists z$($z$ is independent of $y_1, \ldots, y_n$).[41]

In being none of $y_1, \ldots, y_n$, none of their partial grounds, not grounded in any of them, and not grounded in any of their partial grounds, x requires the existence of something wholly unconnected to $y_1, \ldots, y_n$. As an example, my body is partially independent of my legs (it is neither of them, neither of their partial grounds, not grounded in them, and not grounded in any of their partial grounds). Given this, something must be independent of my legs. And something is! My torso, arms, neck, and head are each independent of my legs.[42]

Part to Full has an important consequence. Suppose that each of $y_1, \ldots, y_n$ is independent of the others and that x is partially independent of them. Then by Part to Full, some z is independent of $y_1, \ldots, y_n$. From this, it follows by Contraction that z is independent of any proper sub-plurality of $y_1, \ldots, y_n$. It also follows that each of $y_1, \ldots, y_n$ is independent of any plurality involving only the others and z.[43] And these jointly entail that each of $z, y_1, \ldots, y_n$ is independent of the others.

This result is important. It shows us that adding something that is partially independent of some independent things results in a larger plurality of independent things. So adding to a theory something that is partially independent of the things in that theory results in a larger plurality of independent things. Given this paper's approach to simplicity, it follows that adding partially independent things to a theory is tantamount to decreasing the simplicity of that theory.

This has epistemological import. Knowing that something is partially independent of some things requires knowing less than knowing that something is indepen-

[41]Given Part to Full and that independence is stronger than partial independence, we get: $\exists x$(x is partially independent of $y_1, \ldots, y_n$) $\leftrightarrow \exists z$($z$ is independent of $y_1, \ldots, y_n$).

[42]There are models of grounding where Part to Full fails. Suppose that x is merely partially grounded in everything else. Or suppose that we have a partially pedestalled chain (Dixon 2016, 454-8) and so a chain where every non-fundamental thing is merely partially grounded in the fundamental things. Now on account of finding these models objectionable *qua* models of grounding, they should not tell against Part to Full. But even if they were not objectionable, this would do little practically speaking. The majority of cases of grounding we concern ourselves with are not instances of such bizarre models. So in the majority of cases, Part to Full holds. At worst then, we can add a 'normally' operator and say that normally, if x is partially independent of $y_1, \ldots, y_n$, then $\exists z$(z is independent of $y_1, \ldots, y_n$).

[43]Proof: if z is independent of $y_1, \ldots, y_n$, then by Distribution z is independent of each of $y_1, \ldots, y_n$. By Symmetry, each of $y_1, \ldots, y_n$ is independent of z. Since each of $y_1, \ldots, y_n$ is independent of the others, by Collection each of $y_1, \ldots, y_n$ is independent of any plurality involving only the others and z.

dent of those things (this is because knowing that something is not grounded in some things requires knowing less than knowing that something is not partially grounded in those things). All else being equal then, knowledge of partial independence is easier to have than knowledge of independence. So, given Part to Full, we can know the harder by means of the easier. Here then, the notion of partial independence proves useful when it comes to the epistemology of simplicity.

Nothing Over and Above. Partial independence allows us to make sense of the well-known (though not always well-understood) notion of being nothing over and above some things.[44] Because of this, it allows us to relate this familiar notion to this paper's approach to simplicity. Here is the thought:

> x is nothing over and above some things $\leftrightarrow_{df.}$ x is not partially independent of them.[45]

This is to give a "broad" account of being nothing over and above. It is not just that grounded things are nothing over and above their grounds (Schaffer 2009, 353; 2015, 647-8; Bennett 2017, 221-2). Things are also nothing over and above those things that they are among, those things that they partially ground, and those things whose partial grounds ground them. The first should be uncontroversial, but the last two might seem false. The parts of an apple collectively ground it. "But", it will be claimed "the stem of the apple is something over and above the apple". Why though? Is it because this stem can exist in the absence of the apple (just pluck the stem and eat the apple)? This is a bad reason. In spite of being nothing over and above whatever grounds it, the apple can exist in the absence of these grounds (apples can survive the destruction of some of their parts). Is it because the apple can exist in the absence of the stem? But this is, at best, a reason to think that the stem is something over and above the apple *when the stem does not partially ground the apple* (when plucked from the apple, say). It is not a reason to think that the stem is something over and above the apple *when it partially grounds it*. Here is why. Given that the apple is grounded in its parts, the ontological cost of the apple (if I may put it this way) just is the ontological cost of its parts. Given that it is nothing over and above its parts, its cost cannot be more than the cost of its parts. But it also seems false to say that it is less. If the parts ground the

[44] For some prominent appeals to this notion, see Lewis (1991, 81), Armstrong (1997, 12), Schaffer (2009, 353), Sider (2015), and Bennett (2017, 221-2).

[45] And so x is something over and above some things $\leftrightarrow_{df.}$ x is partially independent of them.

apple, in what way does committing to it commit you to less than them?[46] Given that it is grounded in them, to commit to it *is* to commit to them. And so, given that the apple is grounded in its parts, the cost of it *is* the cost of its parts. But from this and that nothing among a plurality is something over and above that plurality, nothing among the parts of the apple is something over and above the apple (being nothing over and above is transitive). So the stem, which is among these parts, is nothing over and above the apple. And this is what the above account of being nothing over and above says.[47]

When it comes to simplicity, this account of being nothing over and above yields the right results. Given that the stem partially grounds the apple, it is nothing over and above the apple. And so it counts no more against the simplicity of a theory than the apple. And this is right given the independence approach to simplicity. Given that the stem partially grounds the apple, the independent things required by the stem is at most a proper plurality of the independent things required by the apple. But then from the independence approach to simplicity, the stem counts no more against the simplicity of a theory than does the apple. The opposite does not hold. The apple is not nothing over and above the stem. It is very much over and above it. And so it should be that the apple is partially independent of the stem. And it is! It is not the stem, does not partially ground the stem, is not grounded in the stem, and is not grounded in any of the stem's partial grounds. Given this and Part to Full, it follows that the apple requires a larger plurality of independent things than does the stem (which, intuitively, it does). But then from the independence approach to simplicity, the apple counts more against the simplicity of a theory than does the stem. This is exactly as it should be.

Turning now to independence, here is the claim

> x is strongly something over and above some things $\leftrightarrow_{df.}$ x is independent of them.

This captures an intuitive notion. As seen above, the apple is something over and above the stem. It is not the stem, does not partially ground the stem, is not grounded

[46]Of course, something can be nothing over and above some things and cost less than them. Take the stem, skin, and core of the apple. Taken collectively, the stem is nothing over and above these things. But it does cost less than them. Why? Because the skin and core are something over and above it.

[47]But what about the claim that things are nothing over and above those things whose partial grounds ground them? Given that things are nothing over and above their grounds and that partial grounds are nothing over and above what they partially ground (as has just been argued), it follows from the transitivity of being nothing over and above that things are nothing over and above those things whose partial grounds ground them.

in the stem, and is not grounded in any of the stem's partial grounds. But it is partially grounded in the stem. And so, in spite of being something over and above the stem, it is not *strongly* something over and above the stem. And it is not strongly something over and above the stem because it is not independent of the stem; it is not wholly unconnected, free, and distinct from the stem.

Given our distinction between independence and partial independence, we have three key notions: being nothing over and above some things, being something over and above some things, and being strongly something over and above some things. Of course, with respect to some things, whatever stands in the first relation to these things cannot stand in the second and third. Whatever stands in the second relation to these things need not stand in the third. But whatever stands in the third relation to these things can and must stand in the second.

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