# Analyticity and Apriority (1993)

The logical positivists invoked various notions of analyticity, or 'truth by convention', to explain the special modal and epistemological character of logic and mathematics, as well as of other nonempirically based assertions. The central idea of at least one version of the argument is that the postulates of, for example, arithmetic, do not describe independently existing fact, and instead constitute linguistic conventions, which represent decisions to use expressions in a certain way, with such-and-such meaning. Such decisions are 'conventions' in the sense that alternatives were available and furthermore the choices made among the alternatives do not require epistemic justification. Rather, they are to be justified on pragmatic grounds.<sup>1</sup>

On reflection, the fundamental claim that the postulates of arithmetic (or of any other subject) require no epistemic justification is really quite puzzling. In fact, conventionalism is based on serious philosophical errors. A linguistic convention which is supposed to be justified pragmatically rather than epistemically is not strictly a piece of cognitive information at all. It is not a truth or a fact; it is a decision, a commitment, a resolve. It is precisely because one's stipulations are in this way prescriptive rather than descriptive, that the justification for their adoption is pragmatic rather than epistemic. This feature already poses a significant challenge for the conventionalist account of the arithmetic postulates and of other *a priori* statements (statements whose contents are knowable independently of experience). The famous Peano Postulates, for example, describe paradigmatic facts concerning natural numbers; it is generally presumed that they state necessary facts that are knowable *a priori*. Linguistic conventions, while they are not themselves facts, do of course create, or give rise to, facts. That a particular expression, 'successor' for example, has the meaning it does—even when that meaning was secured by explicit stipulation—is every bit a knowable fact. But the linguistic convention per se, the resolve to use the expression with that meaning, is not the right sort of thing to be a piece of knowledge, properly speaking. Furthermore, the facts to which conventions give rise are, by the very nature of their source, contingent rather than necessary, and knowledge of those facts

The present chapter was presented as commentary on a paper by Richard Creath (cited in note 1 below) to the UCLA Carnap and Reichenbach Centennial Symposium, October 1991. An argument related to (though also significantly different from) one to be given below was presented by James Cain in "Are Analytic Statements Necessarily *A Priori?*" *Australasian Journal of Philosophy*, 69, 3 (September, 1991), pp. 334–337. (I discovered Cain's article only after the present article had gone to press.)

<sup>&</sup>lt;sup>1</sup> Richard Creath provides a sympathetic exposition of this version of the argument in 'Carnap's Conventionalism,' delivered to the UCLA Carnap and Reichenbach Centennial Symposium.

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is generally *a posteriori* (epistemically justified only by way of experience) rather than *a priori*. This poses a further, serious difficulty for the conventionalist's attempt to accommodate the necessary apriority of mathematics. I think it ultimately impossible that these pressing challenges to conventionalism can be satisfactorily met.<sup>2</sup>

There is a related problem with the conventionalist idea that the postulates of arithmetic, or of some other subject, are conventions for which a pragmatic rather than epistemic justification is appropriate, and with the related notion that, as David O. Brink put it, 'convention is the mother of necessity',<sup>3</sup> i.e. that the necessity of mathematics has its source in convention. I strongly suspect that these conventionalist ideas involve a conceptual confusion, one that remains widespread in contemporary analytic philosophy. Essentially, it is the failure to distinguish between the semantic cognitive content of a declarative sentence *S* and the logically independent, metatheoretic proposition that *S* itself is true.<sup>4</sup>

To take an example of a widely discussed linguistic convention, consider the sentence

(M) The Standard Bar (assuming it exists) is exactly one meter long at time t,

in the context of someone's having introduced the expression 'meter' as a word for a unit of length which is exactly the length at time t of a particular stick, the Standard Bar. We assume the Standard has a particular length at t. Let l be that length. The decision to use the word 'meter' as a name for l, together with the semantic facts created by this decision, must be sharply distinguished from the independent, preexisting fact about the Standard Bar that it has the very length l at t. As we have seen, it is arguable-and indeed it is part of at least one version of the conventionalist account—that the decision to use 'meter' in this way is not a piece of knowledge, since it is not a natural, extralinguistic fact but a man-made convention, a resolve, and that therefore a pragmatic rather than an epistemic justification is appropriate. The stipulation creates or gives rise to the fact that the phrase 'one meter' designates the length l of the Standard at t, and hence also the fact that the sentence (M) is true. Nevertheless, the fact that the Standard has the particular length l at t is in no way a result of linguistic stipulation or decision. That fact, unlike the semantic facts concerning 'one meter' and (M), would have obtained regardless of whatever linguistic conventions one might have chosen to adopt. As Saul Kripke observed in opposition to Wittgenstein's cryptic remarks concerning this example, the fact that the Standard

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<sup>&</sup>lt;sup>2</sup> Perhaps the conventionalist is prepared to eschew the mathematical facts themselves in favor of the metamathematical facts created by linguistic convention. Creath, who sees this approach to the problem as far preferable to an epistemology that invokes Russellian-type nonempirical acquaintance with the subject matter of mathematics, endorsed the approach in response to my objection at the UCLA Carnap-Reichenbach symposium. But what becomes of the conventionally established meta-theoretic facts when the object-theoretic facts are discarded? If the best, or only, way of getting rid of the bath water involves throwing out the baby, one should probably give serious consideration to finding a use for grey water.

<sup>&</sup>lt;sup>3</sup> See Alan Sidelle, *Necessity, Essence, and Individuation* (Cornell University Press, 1989), at p. vi.

<sup>&</sup>lt;sup>4</sup> In my book *Frege's Puzzle* (Atascadero, Ca.: Ridgeview, 1986, 1991), I highlight this distinction as a special case of the more general distinction between information *semantically contained* in a sentence and information merely *pragmatically imparted* by utterances of the sentence.

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is one meter long at *t* is surely a statable piece of knowledge, and one that obtains only contingently.<sup>5</sup>

When philosophical questions concerning the epistemological status of a particular sentence are under investigation—whether it is a sentence from theoretical science, from mathematics, or from everyday life—our concern is not one of providing an historical or causal explanation of how the sentence came to be true (or perhaps I should say *assertable*), but one of providing a philosophical account of how one might come to know the proposition that is the cognitive content of the sentence. In particular, even if it is taken as settled that the decisions or conventions that resulted in the truth of (M) require only pragmatic justification, and even if it is taken as settled that the resulting fact that (M) is true is thereby knowable somehow *a priori*, we must consider anew the justification for the fact *semantically described* or *encoded* by (M).

How is knowledge of the fact semantically described or encoded by (M) to be justified? Sentence (M) and others like it have been offered by Kripke and David Kaplan, and discussed by many others, as nontrivial counterexamples to the thesis—which was the dominant view among the logical positivists—that any proposition that is knowable *a priori* is true by necessity.<sup>6</sup> The following similar, and in some respects purer, example of what is alleged to be the same phenomenon is due to Kaplan. If one introduces the expression 'Newman-1' as a name for the first child to be born in the twenty-second century, then the sentence

(N) If anyone will be the first child born in the twenty-second century, it will be Newman-1

is supposed to describe a fact that might have been otherwise yet is knowable *a priori* by the speaker who adopts this convention.<sup>7</sup> If Kaplan and Kripke are correct, one might try to make a case, along the conventionalist's lines, for the claim that (M) and (N) are justified pragmatically rather than epistemically. (I ignore for present purposes the significant fact mentioned above that a decision or convention that is justified pragmatically rather than epistemically termed '*a priori*', since it is not strictly a knowable fact at all.) However, Keith Donnellan and a few others, citing the distinction mentioned earlier between the semantic content of a sentence S and

<sup>7</sup> The 'Newman-1' example first appeared in Kaplan's "Quantifying In," in D. Davidson and J. Hintikka, eds., *Words and Objections: Essays on the Work of W. V. Quine* (Dordrecht: D. Reidel, 1969), pp. 206–242, at 228–229. There, however, Kaplan's intuitions were aligned much more closely with common sense. (Cf. also pp. 220–221 of that work.)

For some reason, in Kaplan's more recent writings (cited in the previous note) the description has become 'the first child to be born in the 21st Century'. Since the controversy concerning this sort of example is not likely to be settled during the current decade, we should do well to return to the original example, as I am doing here.

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<sup>&</sup>lt;sup>5</sup> Kripke, *Naming and Necessity* (Harvard University Press, 1972, 1980), at p. 54.

<sup>&</sup>lt;sup>6</sup> Kripke, Naming and Necessity, pp. 54–56, 63, 79n; Kaplan, "Demonstratives" and the "Afterthoughts" thereto, in J. Almog, J. Perry, and H. Wettstein, eds., *Themes from Kaplan* (Oxford University Press, 1988), pp. 481–614, at 536–539, 550, 560, 597, 604–607. Cf. Kaplan, "Dthat," in P. French, T. Uehling, and H. Wettstein, eds., *Contemporary Perspectives in the Philosophy of Language* (Minneapolis: University of Minnesota Press, 1979), pp. 383–400, at 397. Kripke has modified his view of the epistemological status of sentences like (*M*) and (*N*) (below) since the appearance of Naming and Necessity.

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the metatheoretic proposition that S is true, criticized Kaplan's and Kripke's account of the epistemological status of sentences like (M) and (N).<sup>8</sup> Exposing a fallacy in Kripke's treatment of the matter, Donnellan argued persuasively that knowledge of the facts described by (M) and (N) are knowable only *a posteriori* (i.e. by means of experience), requiring a straightforwardly empirical justification.

I believe, with Donnellan and company, that (M) and (N) fail as examples of the contingent *a priori.*<sup>9</sup> The fact described by (M) is a nonlinguistic fact concerning the length of a particular object. That the Standard has length *l* is paradigmatically *a posteriori*. In fact, I propose to turn Kaplan and Kripke on their heads by taking these same examples a step further. It is my contention that these very same examples may be seen as demonstrating the falsity of an even more cherished thesis, virtually unchallenged in analytic philosophy: that all analytic sentences—or, if one prefers, all sentences that are true by convention—state facts that are knowable *a priori*.

Whether (M) and (N) qualify as genuinely analytic, or true by convention, depends in large measure on precisely what is meant in calling a sentence 'analytic' or 'true by convention'. A number of definitions or explications of analyticity have been proposed. My favorite is a proposal by Hilary Putnam. In an exposition of W. V. Quine's famous (if little understood) attack on the analytic–synthetic distinction, Putnam suggests that a sentence may be termed 'analytic' if it is deducible from the sentences in a finite list at the top of which someone who bears the ancestral of the graduate-student relation to Carnap has printed the words 'Meaning Postulate'.<sup>10</sup> This definition not only acknowledges the central importance of Carnap's contribution to the role of the analytic–synthetic distinction in analytic philosophy, but it has the additional virtue that it accords to those few among us who bear this special relationship to Carnap an authority that strikes me as only fitting. Unfortunately, there are those who fail to appreciate the virtues of Putnam's definition. For them I should like to propose a variation on Carnap's own explication of analyticity.

In his Introduction to Semantics, Carnap distinguished between what he called *pure* semantics and descriptive semantics.<sup>11</sup> Descriptive semantics was concerned with the

<sup>8</sup> Donnellan, "The Contingent *A Priori* and Rigid Designators," in P. French, T. Uehling, and H. Wettstein, eds., *Contemporary Perspectives in the Philosophy of Language* (Minneapolis: University of Minnesota Press, 1979), pp. 49–60; Michael Levin, "Kripke's Argument Against the Identity Thesis," *The Journal of Philosophy*, 72, 6 (March 27, 1975), pp. 149–167, at 152*n*2; Alvin Plantinga, *The Nature of Necessity* (Oxford University Press, 1974), at p. 8–9*n*1.

<sup>9</sup> I assume that the cognitive content of (or the fact semantically described by) (M) is the proposition (fact) that the Standard has the particular length l at t. For an account like Donnellan's, based on the theory of Russellian singular propositions as the semantic contents of sentences involving names and similar devices, but differing from Donnellan's in significant respects, compare *Frege's Puzzle*, pp. 140–142, and my "How to Measure the Standard Metre," *Proceedings of the Aristotelian Society* (New Series), 88 (1987/1988), pp. 193–217. (I do not deny that there are examples of contingent *a priori* truths.)

<sup>10</sup> Putnam, "The Meaning of 'Meaning'," in K. Gunderson, ed., *Minnesota Studies in the Philosophy of Science, VII: Language, Mind, and Knowledge* (Minneapolis: University of Minnesota Press), pp. 131–193, at 174.

<sup>11</sup> Carnap, *Introduction to Semantics and Formalization of Logic* (Harvard University Press, 1942, 1943), volume I, section 5, at pp. 11–13.

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semantical features of a natural language, with all its diachronic vicissitudes, while pure semantics was concerned exclusively with artificial languages ('semantical systems') whose semantics is stipulated. The former was an empirical science, whereas the latter consisted entirely of definitions for semantical expressions like 'designatesin-L' and 'true-in-L' and their logical consequences. Carnap's distinction between descriptive and pure semantics corresponds roughly to the distinction between a law of nature and a law passed by the legislature. Although Carnap did not explicitly propose doing so, his notion of pure semantics might have been extended to cover artificial bits of a natural language, as for example the name 'Newman-1' or, perhaps, certain legislative decrees by L'Academie francaise.

The definition of analyticity that I propose is based on a somewhat different distinction, between what I call pure semantics and applied semantics, analogous to the distinction between pure and applied mathematics. It is a purely semantic fact about English that the definite description 'the inventor of bifocals' designates (denotes, refers to) the inventor of bifocals. It is also a semantic fact about English that 'the inventor of bifocals' designates Benjamin Franklin. But the latter is a fact of applied semantics; it obtains partly in virtue of the nonlinguistic, historical fact that it was Benjamin Franklin who invented bifocals. Similarly, whereas it is a purely semantic fact about English that 'Snow is white' is true if and only if snow is white, it is an applied semantic fact that 'Snow is white' is true. As with Carnap's notion, pure semantics, in my sense, consists of appropriate recursive definitions for semantic expressions like 'true-in-L' and 'designates-in-L' and their logical consequences. For Carnap, however, any semantical matter concerning a natural language—including its pure semantics, in my sense-was ipso facto a matter of descriptive semantics. With my notion of pure semantics, the language L whose semantics is under consideration may be 'historically given', the product of natural evolution rather than of legislation. On the other side of the coin, the 'appropriateness' of the semantic definitions is crucial for my notion. A definition for truth-in-English that has the consequence that 'Snow is white' is true if and only if grass is green, while it may not involve any falsehood, is inappropriate. It has smuggled in some applied semantics.<sup>12</sup>

Certain sentences are special in that their truth value is settled entirely by pure semantics. It is a purely semantic fact about English for example that 'All married men are married' is true. For this fact is a logical consequence of the purely semantic fact that 'All married men are married' is true if and only if all married men are married. My proposal, finally, is that we call a true sentence 'analytic' if its truth is in this way a fact of pure rather than applied semantics.<sup>13</sup> This notion is related to Carnap's

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<sup>&</sup>lt;sup>12</sup> Thus while my distinction between pure and applied semantics is free of some of the controversial philosophical underpinnings of Carnap's distinction between pure and descriptive semantics, it is not free of all such. The same thing is true, for essentially the same reason, of Donald Davidson's program of providing a theory of meaning for a language by supplying a theory of truth.

<sup>&</sup>lt;sup>13</sup> This notion of analyticity differs from that given in *Frege's Puzzle*, pp. 133–135. The latter is roughly the notion of a sentence whose proposition content is a logical truth. I argued there that sentences like 'Cicero is Tully' are analytic in the latter sense. In "Relative and Absolute Apriority" (Philosophical Studies, forthcoming), I argue that 'Cicero is Tully' is also analytic in the sense proposed here.

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notion of '*L*-truth', which he proposed as constituting an explication equally of analyticity and of necessity—although *L*-truth corresponds more closely to (and indeed is an important precursor to) the contemporary notion of *logical truth* as truth in all models for the language.<sup>14</sup>

The proposed definition includes certain sentences in addition to those that have the form of a logical validity. A sentence like 'All husbands are married', assuming 'husband' is synonymous with 'married man' also qualifies as analytic under the definition. For it is a purely semantic fact about English that the adjective 'married' (correctly) applies to all married individuals, and it is also a purely semantic fact that the noun 'husband' applies only to married men. In fact, assuming 'husband' and 'married man' are synonymous, the purely semantic fact that 'husband' applies only to husbands is identical with the fact that 'husband' applies only to married men. It is a truth of logic that if 'married' applies to all married individuals and 'husband' applies only to married men, then 'married' applies to any individual to which 'husband' applies. Given the further purely semantic fact that the English construction 'All Ns are A' is true if and only if the adjective A applies to anything to which the NP N applies, it follows that 'All husbands are married' is true. Alternatively, it is a fact of pure semantics for English that 'All husbands are married' is true if all husbands are married. That all husbands are married is nothing more than the logical truth that all married men are married. The truth of 'All husbands are married' is thus logically settled by pure rather than applied semantics.

Let us return to sentence (N). Given the manner in which the designation of 'Newman-1' is fixed, the fact that 'Newman-1' designates the first child to be born in the 22nd Century, and hence also the resulting fact that (N) is true, are facts of pure rather than applied semantics. One may also say, therefore, that (N) is analytic; it is, in a straightforward sense, true by convention. Similarly for (M).<sup>15</sup> Indeed, the truth of either sentence is settled by 'pure semantics' in both Carnap's sense (as extended above to incorporate stipulated bits of natural language) and my own.

The notion of a sentence's truth being a logical consequence of pure rather than applied semantics is, roughly, a notion of 'truth solely by virtue of meaning'.<sup>16</sup> The

Interestingly, the further fact that 'Newman-1' designates Newman-1 is a fact of applied rather than pure semantics, since it obtains only by virtue of the nonlinguistic fact that Newman-1 (i.e. that very future person) will be the first child to be born in the twenty-second Century. Cf. "How to Measure the Standard Metre," at pp. 200-201n10.

<sup>16</sup> I do not mean the phrase (or its cognates, e.g. 'true in virtue of meaning alone') in the traditional sense, which rules out that the sentence in question describes an extralinguistic fact and is in that sense true partly by virtue of a feature of the world. Nor (as will shortly become even more evident) do I wish to be associated with the philosophical thesis, which has traditionally gone hand in hand with the analytic-synthetic distinction, that sentences like 'All husbands are married' are devoid of extralinguistic, factual content. (Cf. note 12 above.) Indeed, I think it is obvious that even logical validities like 'All married men are married', since they are contentful and true,

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<sup>&</sup>lt;sup>14</sup> Introduction to Semantics and Formalization of Logic, volume I, pp. 60–61, 79–80, 134–137; Meaning and Necessity (University of Chicago Press, 1947, 1956), section 2, at pp. 9–10.

<sup>&</sup>lt;sup>15</sup> This is, of course, a purely terminological matter of decision. Those sentences are counted synthetic under the alternative notion of analyticity given in *Frege's Puzzle*. (See note 13 above.) They are also deemed synthetic on Kripke's alternative definition of analyticity—in *Naming and Necessity*, p. 39—though for a different reason. See especially pp. 56*n*, 122–123*n* of that work. Interestingly, the further fact that 'Newman-1' designates Newman-1 is a fact of applied rather

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epistemologically charged term '*a priori*' is less appropriate for this notion than the more semantic epithet 'analytic'. Nevertheless, I have often felt that this form of analyticity may be what is meant by particular uses of '*a priori*'.<sup>17</sup> The notion of truth-asa-consequence-of-semantics-alone does have an epistemological dimension: for any sentence whose truth value is a logical consequence of pure semantics, anyone competent in the language is *ipso facto* in possession of sufficient information to determine that truth value by logic—never mind that knowledge of pure semantics for a natural language, and hence competence in the language, is gained only by means of experience. This might explain the Kaplan–Kripke stance with respect to (*M*) and (*N*). What originally prompted the claim that those sentences are *a priori* was the recognition that they belong, in some sense, with sentences for which knowledge of the meaning—however empirical that knowledge may be—is sufficient to establish their truth.<sup>18</sup>

Even if (M) and (N) are declared analytic, it is widely recognized nowadays that it does not follow that their contents are necessary truths. Still, it is usually assumed that the content of any sentence that is true solely by virtue of meaning is *a priori*. I maintain that (M) and (N), though analytic in the suggested sense, are both contingent and *a posteriori*; their contents are not only contingent but also knowable only by means of experience. Whereas the philosophical significance of the existence of propositions that are both contingent and *a priori* is apparent, the philosophical significance of the fact that such conventionally true sentences as (M) and (N) express contingencies even though their truth is a matter of pure semantics is less so. One consequence (noted by Kaplan, in 'Demonstratives', p. 540) is that Quine was wrong to see the 'second grade of modal involvement' as recasting analyticity, which is a meta-theoretic notion, as the object-language notion of necessity. Carnap was equally wrong to identify necessity with truth by pure semantics.

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describe facts — typically extralinguistic (albeit particularly unexciting) facts that are both necessary and knowable *a priori*. There is a natural and straightforward sense in which such a sentence is, like any contentful and true sentence, true "in virtue of' both its meaning and the extralinguistic fact that it describes. A better phrase for the notion of analyticity that I am embracing here is 'true as a consequence of meaning alone'. An analytic sentence, in the sense in which I am using the term, is a contentful sentence which is true (and hence true in virtue of both its meaning and some fact about the world), and for which the very fact that it is true is itself a logical consequence entirely of purely semantic facts about the sentence.

<sup>&</sup>lt;sup>17</sup> Cf. my "How Not to Become a Millian Heir," *Philosophical Studies*, 62, 2 (May 1991), pp. 165–177, at p. 172. <sup>18</sup> Kripke says: "What...is the *epistemological* status of the statement [(M)], for someone who

<sup>&</sup>lt;sup>18</sup> Kripke says: "What.. is the *epistemological* status of the statement [(M)], for someone who has fixed the metric system by reference to [the Standard Bar]? It would seem that he knows it *a priori*. For if he used [the Standard Bar] to fix the reference of the term 'one meter', then as a result of this kind of 'definition' (which is not an abbreviative or synonymous definition), he knows automatically, without further investigation, that [the Standard Bar] is one meter long" (*Naming and Necessity*, p. 56). But what the reference-fixer knows automatically as a result of his reference-fixing definition is that (*M*) is true (in his own idiolect); he knows automatically without investigating the Standard Bar that however long it is, that length is designated by the phrase 'one meter'. He does not automatically know of that length that the bar is exactly (or even roughly) that long. For extended discussion see the articles by Donnellan and me cited in notes 8, 9, and 13 above.

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If I am correct, another consequence is that analyticity, in this sense, is no more a guarantee of apriority (knowability independently of experience) than it is of necessity. In order to explain the special modal and epistemological status of necessary *a priori* sentences, it is not sufficient to assert (whether rightly or wrongly) that they are analytic, or true by convention.

Consider the following mathematical postulate:

(P)  $\pi$  is the ratio of the circumference of a circle to its diameter (if there is a fixed such ratio).

It is very plausible that the term ' $\pi$ ' is, in some sense, *defined* by (*P*). This is in fact significantly more plausible than the prospect that the expressions 'natural number', '0', and 'successor' are somehow implicitly but simultaneously defined by the Peano Postulates.<sup>19</sup> For (*P*) at least determines the extension of ' $\pi$ '. Indeed, the truth of (*P*) is analogous in many ways to the truth of (*M*) and (*N*). To use Kripke's phrase, the definite description 'the ratio of the circumference of a circle to its diameter' *fixes the reference* of ' $\pi$ ', without thereby turning ' $\pi$ ' into a synonym for the description.<sup>20</sup> One point of disanalogy with the case of (*M*) and (*N*) is that the reference-fixing definite description involved here is a rigid designator; (*P*) contains a necessary truth. The various analogies with (*M*) and (*N*), however, amply demonstrate that the analyticity, or conventional truth, of (*P*) does not account for its necessity—otherwise (*M*) and (*N*) should be necessary as well. An alternative account is required.

A second striking disanalogy with the case of (M) and (N) is that it is not at all plausible that (P) is *a posteriori*. The epistemic justification of purely mathematical knowledge is very different from that concerning the lengths of bars and the birthdates of persons. On the other hand, the central point of analogy remains: the epistemic justification for the mathematical fact described by (P) is independent of the justification for the metamathematical fact that (P) is true. In order to know that (P)is true, one need only know how ' $\pi$ ' is defined. That is pure semantics. It is also *a posteriori*. To say that (P) is not *a posteriori*, however, is not yet to say that it is *a priori*. For it is arguable that the content of (P) is not knowable at all. Exactly what is involved in coming to know of the number,  $\pi$ , that it is the ratio of the circumference of a circle to its diameter (assuming there is such a ratio)—and even the question of whether it is possible for us to gain this purely mathematical, nonsemantic knowledge—are vexing matters that raise delicate issues in the philosophy of mathematics and epistemology generally.<sup>21</sup> The analyticity of (P) is of no help here.

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<sup>&</sup>lt;sup>19</sup> This prospect may have been first proposed by Richard Dedekind in "*Was Sind und Was Sollen die Zahlen?*" (section 10), one year before Peano proposed taking Dedekind's conditions as postulates for arithmetic. See William and Martha Kneale, *The Development of Logic* (Oxford University Press, 1962, 1986), at pp. 469–473. Cf. the position defended in Paul Benacerraf, "What Numbers Could not Be," *The Philosophical Review*, 74 (1965), pp. 47–73.

<sup>&</sup>lt;sup>20</sup> Cf. Naming and Necessity, p. 60.

<sup>&</sup>lt;sup>21</sup> Cf. 'How to Measure the Standard Metre,' pp. 211–212.