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Neutrosophic Crisp α-Topological Spaces

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Abstract.
In this paper, a generalization of the neutrosophic topological space is presented. The basic definitions of the neutrosophic crisp α-topological space and the neutrosophic crisp α-compact space with some of their characterizations are deduced. Furthermore, we aim to construct a netrosophic crisp α-continuous function, with a study of a number its properties.

Keywords: Neutrosophic Crisp Set, Neutrosophic Crisp Topological space, Neutrosophic Crisp Open Set.

1 Introduction

In 1965, Zadeh introduced the degree of membership5 and defined the concept of fuzzy set [15]. A degree of non-membership was added by Atanassov [2], to give another dimension for Zadah's fuzzy set. Afterwards in late 1990's, Smarandache introduced a new degree of indeterminacy or neutrality as an independent third component to define the neutrosophic set as a triple structure [14]. Since then, laid the foundation for a whole family of new mathematical theories to generalize both the crisp and the fuzzy counterparts [4-10]. In this paper, we generalize the neutrosophic topological space to the concept of neutrosophic crisp α-topological space. Moreover, we present the netrosophic crisp α-continuous function as well as a study of several properties and some characterization of the neutrosophic crisp α-compact space.

2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [12,13,14], and Salama et al. [4, 5,6,7,8,9,10,11]. Smarandache introduced the neutrosophic components T, I, F which respectively represent the membership, indeterminacy, and non-membership characteristic mappings of the space X into the non-standard unit interval $\left]^-0, 1^+\right[$.

Hanafy and Salama et al. [3,10] considered some possible definitions for basic concepts of the neutrosophic crisp set and its operations. We now improve some results by the following.

2.1 Neutrosophic Crisp Sets

2.1.1 Definition

For any non-empty fixed set X, a neutrosophic crisp set A (NCS for short), is an object having the form $A=\langle A_1, A_2, A_3\rangle$ where A_1, A_2 and A_3 are subsets of X satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$.

2.1.2 Remark

Every crisp set A formed by three disjoint subsets of a non-empty set X is obviously a NCS having the form $A=\langle A_1, A_2, A_3\rangle$.

Several relations and operations between NCSs were defined in [11].

For the purpose of constructing the tools for developing neutrosophic crisp sets, different types of NCSs ϕ_N, X_N, A^c in X were introduced in [9] to be as follows:

2.1.3 Definition

ϕ_N may be defined in many ways as a NCS, as follows:

i) $\phi_N = \langle \phi, \phi, X\rangle$, or
ii) $\phi_N = \langle \phi, X, X\rangle$, or
iii) $\phi_N = \langle \phi, X, \phi\rangle$, or
iv) $\phi_N = \langle \phi, \phi, \phi\rangle$.

2.1.4 Definition

X_N may also be defined in many ways as a NCS:

i) $X_N = \langle X, \phi, \phi\rangle$, or
ii) $X_N = \langle X, X, \phi\rangle$, or
iii) $X_N = \langle X, \phi, X\rangle$, or
iv) $X_N = \langle X, X, X\rangle$.

2.1.5 Definition

Let $A=\langle A_1, A_2, A_3\rangle$ a NCS on X, then the complement of the set A, (A^c for short) may be defined in three different ways:

(C_1) $A^c=\langle A_1^c, A_2^c, A_3^c\rangle$,

(C_2) $A^c=\langle A_3, A_2, A_1\rangle$

(C_3) $A^c=\langle A_3, A_2^c, A_1\rangle$

Several relations and operations between NCSs were introduced in [9] as follows:

2.1.6 Definition

Let X be a non-empty set, and the NCSs A and B in the form $A=\langle A_1, A_2, A_3\rangle$, $B=\langle B_1, B_2, B_3\rangle$, then we may consider two possible definitions for subsets $(A\subseteq B)$

$(A\subseteq B)$ may be defined in two ways:

1) $A\subseteq B \Leftrightarrow A_1\subseteq B_1, A_2\subseteq B_2$ and $A_3\supseteq B_3$ or

2) $A\subseteq B \Leftrightarrow A_1\subseteq B_1, A_2\supseteq B_2$ and $A_3\supseteq B_3$

2.1.7 Proposition

For any neutrosophic crisp set A, and the suitable choice of ϕ_N, X_N, the following are hold:

i) $\phi_N\subseteq A$, $\phi_N\subseteq\phi_N$.

ii) $A\subseteq X_N$, $X_N\subseteq X_N$.

2.1.8 Definition

Let X is a non-empty set, and the NCSs A and B in the form $A=\langle A_1, A_2, A_3\rangle$, $B=\langle B_1, B_2, B_3\rangle$. Then:

1) $A\cap B$ may be defined in two ways:

i) $A\cap B=\langle A_1\cap B_1, A_2\cap B_2, A_3\cup B_3\rangle$ or

ii) $A\cap B=\langle A_1\cap B_1, A_2\cup B_2, A_3\cup B_3\rangle$

2) $A\cup B$ may also be defined in two ways:

i) $A\cup B=\langle A_1\cup B_1, A_2\cap B_2, A_3\cap B_3\rangle$ or

ii) $A\cup B=\langle A_1\cup B_1, A_2\cup B_2, A_3\cap B_3\rangle$

2.1.9 Proposition

For any two neutrosophic crisp sets A and B on X, then the followings are true:

1) $(A\cap B)^c=A^c\cup B^c$.

2) $(A\cup B)^c=A^c\cap B^c$.

The generalization of the operations of intersection and union given in definition 2.1.8, to arbitrary family of neutrosophic crisp subsets are as follows:

2.1.10 Proposition

Let $\{A_j : j\in J\}$ be arbitrary family of neutrosophic crisp subsets in X, then

1) $\underset{j}{\cap} A_j$ may be defined as the following types :

i) $\underset{j}{\cap} A_j=\langle\cap A_{j1}, \cap A_{j_2}, \cup A_{j_3}\rangle$, or

ii) $\underset{j}{\cap} A_j=\langle\cap A_{j1}, \cup A_{j_2}, \cup A_{j_3}\rangle$.

2) $\underset{j}{\cup} A_j$ may be defined as the following types :

i) $\underset{j}{\cup} A_j=\langle\cup A_{j1}, \cup A_{j_2}, \cap A_{j_3}\rangle$ or

ii) $\underset{j}{\cup} A_j=\langle\cup A_{j1}, \cap A_{j_2}, \cap A_{j_3}\rangle$.

2.1.11 Definition

The Cartesian product of two neutrosophic crisp sets A and B is a neutrosophic crisp set $A\times B$ given by

$A\times B=\langle A_1\times B_1, A_2\times B_2, A_3\times B_3\rangle$.

2.1.12 Definition

Let (X,Γ) be $NCTS$ and $A=\langle A_1,A_2,A_3\rangle$ be a NCS in X. Then the neutrosophic crisp closure of A ($NCcl(A)$ for short) and neutrosophic crisp interior ($NCint(A)$ for short) of A are defined by

$NCcl(A)=\cap\{K: K$ is a $NCCS$ in X and $A\subseteq K\}$

$NCint(A)=\cup\{G: G$ is a $NCOS$ in X and $G\subseteq A)$,

Where NCS is a neutrosophic crisp set and $NCOS$ is a neutrosophic crisp open set. It can be also shown that $NCcl(A)$ is a $NCCS$ (neutrosophic crisp closed set) and $NCint(A)$ is a $NCOS$ (neutrosophic crisp open set) in X.

3 Neutrosophic Crisp α-Topological Spaces

We introduce and study the concepts of neutrosophic crisp α-topological space

3.1 Definition

Let (X,Γ) be a neutrosophic crisp topological space (NCTS) and $A=\langle A_1,A_2,A_3\rangle$ be a NCS in X, then A is said to be neutrosophic crisp α-open set of X if and only if the following is true: $A\subseteq NCint(NCcl(NCint(A)))$.

3.2 Definition

A neutrosophic crisp α-topology (NCαT for short) on a non-empty set X is a family Γ^α of neutrosophic crisp subsets of X satisfying the following axioms

i) $\phi_N, X_N\in\Gamma^\alpha$.

ii) $A_1\cap A_2\in\Gamma^\alpha$ for any A_1 and $A_2\in\Gamma^\alpha$.

iii) $\underset{j}{\cup} A_j\in\Gamma^\alpha$ $\forall$ $\{A_j : j\in J\}\subseteq\Gamma^\alpha$.

In this case the pair (X,Γ^α) is called a neutrosophic crisp α-topological space ($NC\alpha TS$ for short) in X. The elements in Γ^α are called neutrosophic crisp α-open sets (NCαOSs for short) in X. A neutrosophic crisp set F is α-closed if and only if its complement F^C is an α-open neutrosophic crisp set.

3.3 Remark

Neutrosophic crisp α-topological spaces are very natural generalizations of neutrosophic crisp topological spaces, as one can prof that every open set in a NCTS is an α-open set in a NCαTS

3.4 Example

Let $X=\{a,b,c,d\}$, ϕ_N, X_N be any types of the universal and empty sets on X, and A, B are two neutrosophic crisp sets on X defined by $A=\langle\{a\},\{b,d\},\{c\}\rangle$,

$B=\langle\{a\},\{b\},\{c\}\rangle$, $\Gamma=\{\phi_N, X_N, A\}$ then the family $\Gamma^\alpha=\{\phi_N, X_N, A, B\}$ is a neutrosophic crisp α-topology on X.

3.5 Definition

Let $(X,\Gamma_1^\alpha),(X,\Gamma_2^\alpha)$ be two neutrosophic crisp α-topological spaces on X . Then Γ_1^α is said be contained in Γ_2^α (in symbols $\Gamma_1^\alpha\subseteq\Gamma_2^\alpha$) if $G\in\Gamma_2^\alpha$ for each $G\in\Gamma_1^\alpha$. In this case, we also say that Γ_1^α is coarser than Γ_2^α .

3.6 Proposition

Let $\{\ \Gamma_j^\alpha : j\in J\}$ be a family of NCαTS on X . Then $\cap\Gamma_j^\alpha$ is a neutrosophic crisp α-topology on X . Furthermore, $\cup\Gamma_j^\alpha$ is the coarsest NCαT on X containing all α-topologies.

Proof

Obvious.

Now, we can define the neutrosophic crisp α-closure and neutrosophic crisp α-interior operations on neutrosophic crisp α-topological spaces:

3.7 Definition

Let (X,Γ^α) be NCαTS and $A=\langle A_1,A_2,A_3\rangle$ be a NCS in X, then the neutrosophic crisp α- closure of A (NCαCl(A) for short) and neutrosophic crisp α-interior crisp (NCαInt (A) for short) of A are defined by

$$NC\alpha Cl(A)=\cap\{K : K \text{ is an NCS in X and } A\subseteq K\}$$
$$NC\alpha Int\,(A)=\cup\{G : G \text{ is an NCOS in X and } G\subseteq A\}$$

where NCS is a neutrosophic crisp set, and NCOS is a neutrosophic crisp open set.

It can be also shown that $NC\alpha Cl\ (A)$ is a NCαCS (neutrosophic crisp α-closed set) and $NC\alpha Int(A)$ is a NCαOS (neutrosophic crisp α-open set) in X .

a) A is a NCα-closed in X if and only if $A=NC\alpha Cl(A)$.

b) A is a NCα-open in X if and only if $A=NC\alpha Int(A)$.

3.8 Proposition

For any neutrosophic crisp α-open set A in (X,Γ^α) we have

(a) $NC\alpha Cl(A^c)=(NC\alpha Int(A))^c$,

(b) $NC\alpha Int(A^c)=(NC\alpha Cl(A))^c$.

Proof

a) Let $A=\langle A_1,A_2,A_3\rangle$ and suppose that the family of all neutrosophic crisp subsets contained in A are indexed by the family $\{A_j\}_{j\in J}=\{<A_{j_1},A_{j_2},A_{j_3}>: j\in J\}$. Then we see that we have two types defined as follows:

Type1: $NC\alpha Int(A)=<\cup A_{j_1},\cup A_{j_2},\cap A_{j_3}>$

$(NC\alpha Int(A))^c=<\cap A_{j_1}^c,\cap A_{j_2}^c,\cup A_{j3}^c>$

Hence $NC\alpha Cl(A^c)=(NC\alpha Int(A))^c$

Type 2: $NC\alpha Int(A)=<\cup A_{j_1},\cap A_{j_2},\cap A_{j_3}>$

$(NC\alpha Int(A))^c=<\cap A_{j_1}^c,\cup A_{j_2}^c,\cup A_{j3}^c>$.

Hence $NC\alpha Cl(A^c)=(NC\alpha Int(A))^c$

b) Similar to the proof of part (a).

3.9 Proposition

Let (X,Γ^α) be a NCαTS and A,B are two neutrosophic crisp α-open sets in X . Then the following properties hold:

(a) $NC\alpha Int(A)\subseteq A$,

(b) $A\subseteq NC\alpha Cl(A)$,

(c) $A\subseteq B\Rightarrow NC\alpha Int(A)\subseteq NC\alpha Int(B)$,

(d) $A\subseteq B\Rightarrow NC\alpha Cl(A)\subseteq NC\alpha Cl(B)$,

(e) $NC\alpha Int\ (A\cap B)=NC\alpha Int(A)\cap NC\alpha Int(B)$,

(f) $NC\alpha Cl(A\cup B)=NC\alpha Cl(A)\cup NC\alpha Cl(B)$,

(g) $NC\alpha Int(X_N)=X_N$,

(h) $NC\alpha Cl(\phi_N)=\phi_N$.

Proof. Obvious

4 Neutrosophic Crisp α-Continuity

In this section, we consider $f{:}X\longrightarrow Y$ to be a map between any two fixed sets X and Y.

4.1 Definition

(a) If $A=\langle A_1,A_2,A_3\rangle$ is a NCS in X, then the neutrosophic crisp image of A under f, denoted by $f(A)$, is the a NCS in Y defined by $f(A)=\langle f(A_1), f(A_2), f(A_3)\rangle$.

(b) If f is a bijective map then f^{-1}: $Y\longrightarrow X$ is a map defined such that: for any NCS $B=\langle B_1,B_2,B_3\rangle$ in Y, the neutrosophic crisp preimage of B, denoted by $f^{-1}(B)$, is a NCS in X defined by $f^{-1}(B)=\langle f^{-1}(B_1), f^{-1}(B_2), f^{-1}(B_3)\rangle$.

Here we introduce the properties of neutrosophic images and neutrosophic crisp preimages, some of which we shall frequently use in the following sections.

4.2 Corollary

Let A = $\{A_i : i\in J\}$, be NCαOSs in X, and B = $\{B_j : j\in K\}$ be NCαOSs in Y, and $f : X\to Y$ a function. Then

(a) $A_1\subseteq A_2 \Leftrightarrow f(A_1)\subseteq f(A_2)$, $B_1\subseteq B_2\Leftrightarrow f^{-1}(B_1)\subseteq f^{-1}(B_2)$,

(b) $A \subseteq f^{-1}(f(A))$ and if f is injective, then $A = f^{-1}(f(A))$.

(c) $f^{-1}(f(B)) \subseteq B$ and if f is surjective, then $f^{-1}(f(B)) = B$.

(d) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i)$, $f^{-1}(\cap B_i)) \subseteq \cap f^{-1}(B_i)$,

(e) $f(\cup A_i) = \cup f(A_i)$, $f(\cap A_i) \subseteq \cap f(A_i)$.

(f) $f^{-1}(Y_N) = X_N$, $f^{-1}(\phi_N) = \phi_N$.

(g) $f(\phi_N) = \phi_N$, $f(X_N) = Y_N$, if f is subjective.

Proof

Obvious.

4.3 Definition

Let (X, Γ_1^{α}) and (Y, Γ_2^{α}) be two NCαTSs, and let $f : X \to Y$ be a function. Then f is said to be α-continuous iff the neutrosophic crisp preimage of each NCS in Γ_2^{α} is a NCS in Γ_1^{α}.

4.4 Definition

Let (X, Γ_1^{α}) and (Y, Γ_2^{α}) be two NCαTSs and let $f : X \to Y$ be a function. Then f is said to be open iff the neutrosophic crisp image of each NCS in Γ_1^{α} is a NCS in Γ_2^{α}.

4.5 Proposition

Let (X, Γ_o^{α}) and (Y, ψ_o^{α}) be two NCαTSs.

If $f : X \to Y$ is α-continuous in the usual sense, then in this case, f is α-continuous in the sense of Definition 4.3 too.

Proof

Here we consider the NCαTs on X and Y, respectively, as follows : $\Gamma_1^{\alpha} = \{\langle G, \phi, G^c \rangle : G \in \Gamma_o^{\alpha}\}$ and $\Gamma_2^{\alpha} = \{\langle H, \phi, H^c \rangle : H \in \Psi_o^{\alpha}\}$,

In this case we have, for each $\langle H, \phi, H^c \rangle \in \Gamma_2^{\alpha}$, $H \in \Psi_o^{\alpha}$,

$$f^{-1}\langle H, \phi, H^c \rangle = \langle f^{-1}(H), f^{-1}(\phi), f^{-1}(H^c) \rangle$$
$$= \langle f^{-1}H, \phi, (f^{-1}(H))^c \rangle \in \Gamma_1^{\alpha}.$$

4.6 Proposition

Let $f : (X, \Gamma_1^{\alpha}) \to (Y, \Gamma_2^{\alpha})$.

f is continuous iff the neutrosophic crisp preimage of each CNαCS (crisp neutrosophic α-closed set) in Γ_1^{α} is a CNαCS in Γ_2^{α}.

Proof

Similar to the proof of Proposition 4.5.

4.7 Proposition

The following are equivalent to each other:

(a) $f : (X, \Gamma_1^{\alpha}) \to (Y, \Gamma_2^{\alpha})$ is continuous .

(b) $f^{-1}(CN\alpha Int(B) \subseteq CN\alpha Int(f^{-1}(B))$ for each CNS B in Y.

(c) $CN\alpha Cl(f^{-1}(B)) \subseteq f^{-1}(CN\alpha Cl(B))$ for each CNC B in Y.

4.8 Corollary

Consider (X, Γ_1^{α}) and (Y, Γ_2^{α}) to be two NCαTSs, and let $f : X \to Y$ be a function.

if $\Gamma_1^{\alpha} = \{f^{-1}(H) : H \in \Gamma_2^{\alpha}\}$. Then Γ_1^{α} will be the coarsest NCαT on X which makes the function $f : X \to Y$ α-continuous. One may call it the initial neutrosophic crisp α-topology with respect to f.

5 Neutrosophic Crisp α-Compact Space

First we present the basic concepts:

5.1 Definition

Let (X, Γ^{α}) be an NCαTS.

(a) If a family $\{\langle G_{i_1}, G_{i_2}, G_{i_3} \rangle : i \in J\}$ of NCαOSs in X satisfies the condition $\cup \{\langle G_{i_1}, G_{i_2}, G_{i_3} \rangle : i \in J\} = X_N$, then it is called an neutrosophic α-open cover of X.

(b) A finite subfamily of an α-open cover $\{\langle G_{i_1}, G_{i_2}, G_{i_3} \rangle : i \in J\}$ on X, which is also a neutrosophic α-open cover of X , is called a neutrosophic crisp finite α-open subcover.

5.2 Definition

A neutrosophic crisp set $A = \langle A_1, A_2, A_3 \rangle$ in a NCαTS (X, Γ^{α}) is called neutrosophic crisp α-compact iff every neutrosophic crisp open cover of A has a finite neutrosophic crisp open subcover.

5.3 Definition

A family $\{\langle K_{i_1}, K_{i_2}, K_{i_3} \rangle : i \in J\}$ of neutrosophic crisp α-compact sets in X satisfies the finite intersection property (FIP for short) iff every finite subfamily $\{\langle K_{i_1}, K_{i_2}, K_{i_3} \rangle : i = 1,2,...,n\}$ of the family satisfies the condition $\cap \{\langle K_{i_1}, K_{i_2}, K_{i_3} \rangle : i = 1,2,...,n\} \neq \phi_N$.

5.4 Definition

A NCαTS (X, Γ^{α}) is called neutrosophic crisp α-compact iff each neutrosophic crisp α-open cover of X has a finite α-open subcover.

5.5 Corollary

A NCαTS (X, Γ^{α}) is a neutrosophic crisp α-compact iff every family $\{\langle G_{i_1}, G_{i_2}, G_{i_3} \rangle : i \in J\}$ of neutrosophic crisp α-compact sets in X having the the finite intersection properties has nonempty intersection.

5.6 Corollary

Let (X, Γ_1^{α}), (Y, Γ_2^{α}) be NCαTSs and $f : X \to Y$ be a continuous surjection. If (X, Γ_1^{α}) is a neutrosophic crisp α-compact, then so is (Y, Γ_2^{α}).

5.7 Definition

(a) If a family $\left\{\left\langle G_{i_1}, G_{i_2}, G_{i_3}\right\rangle : i \in J\right\}$ of neutrosophic crisp α-compact sets in X satisfies the condition $A \subseteq \cup\left\{\left\langle G_{i_1}, G_{i_2}, G_{i_3}\right\rangle : i \in J\right\}$, then it is called a neutrosophic crisp open cover of A.

(b) Let's consider a finite subfamily of a neutrosophic crisp open subcover of $\left\{\left\langle G_{i_1}, G_{i_2}, G_{i_3}\right\rangle : i \in J\right\}$.

5.8 Corollary

Let $\left(X, \Gamma_1^{\alpha}\right)$, $\left(Y, \Gamma_2^{\alpha}\right)$ be NCαTSs and $f : X \to Y$ be a continuous surjection. If A is a neutrosophic crisp α-compact in $\left(X, \Gamma_1^{\alpha}\right)$, then so is $f(A)$ in $\left(Y, \Gamma_2^{\alpha}\right)$.

6. Conclusion

In this paper, we presented a generalization of the neutrosophic topological space. The basic definitions of the neutrosophic crisp α-topological space and the neutrosophic crisp α-compact space with some of their characterizations were deduced. Furthermore, we constructed a netrosophic crisp α-continuous function, with a study of a number its properties.

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