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NEUTROSOPHIC FILTERS

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ABSTRACT

In this paper we introduce the notion of filters on neutrosophic set which is considered as a generalization of fuzzy filters studies in [6], the important neutrosophic filters has been given. Several relations between different neutrosophic filters and neutrosophic topologies are also studied here. Possible applications to computer sciences are touched upon.

KEYWORDS: Fuzzy Filters, Neutrosophic Sets, Neutrosophic Filters, Neutrosophic Topology, Neutrosophic Ultrafilters

INTRODUCTION

The fuzzy set was introduced by Zadeh [11] in 1965, where each element had a degree of membership. The intuitionstic fuzzy set (Ifs for short) on a universe X was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. After the introduction of the neutrosophic set concept [8, 9, 10].

The fundamental concepts of neutrosophic set, introduced by Smarandache in 2002 [7, 8] and Salama in 2012[10], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts[1, 2, 3, 4, 5, 7, 11], such as a neutrosophic set theory

PRELIMINARIES

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [8, 9], Atanassov in [1, 2, 3], Salama [10] and Kul Hur at el [6]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^-0, 1^+[$ is nonstandard unit interval.

Definition 2.1. [10]

Let T, I, F be real standard or nonstandard subsets of $[^-0, 1^+]$, with

Sup_T=t_sup, inf_T=t_inf

Sup_I=i_sup, inf_I=i_inf

Sup_F=f_sup, inf_F=f_inf

n-sup=t_sup+i_sup+f_sup

n-inf=t_inf+i_inf+f_inf,

T, I, F are called neutrosophic components

Definition 2.2. [10]

Let X be a non-empty fixed set. A neutrosophic set (NS for short or ($(A \in N^X)$) A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ Where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A .

Definition 2.3 [10] The NSs 0_N and 1_N in X as follows:

0_N may be defined as:

(0_1) $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$

(0_2) $0_N = \{\langle x, 0, 1, 1\rangle : x \in X\}$

(0_3) $0_N = \{\langle x, 0, 1, 0\rangle : x \in X\}$

(0_4) $0_N = \{\langle x, 0, 0, 0\rangle : x \in X\}$

1_N may be defined as:

(1_1) $1_N = \{\langle x, 1, 0, 0\rangle : x \in X\}$

(1_2) $1_N = \{\langle x, 1, 0, 1\rangle : x \in X\}$

(1_3) $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$

(1_4) $1_N = \{\langle x, 1, 1, 1\rangle : x \in X\}$

BASIC PROPERTIES OF NEUTROSOPHIC FILTERS

Definition 3.1. Let N be a neutrosophic subsets in a set X. Then N is called a neutrosophic filter on X, if it satisfies the following conditions:

(N_1) Every neutrosophic set in X containing a member of N belongs to N .

(N_2) Every finite intersection of members of N belongs to N .

(N_3) O_N not in N .

In this case, the pair (X, N) is called neutrosophic filtered by N .

It follows from (N_2) and (N_3) that every finite intersection of members of N is not O_N . Furthermore, there is no neutrosophic set. We obtain the following results.

Proposition 3.1. The condition (N_2) is equivalent to the following two conditions

(N_{2a}) The intersection of two members of N belongs to N .

$(N_{2b})\ 1_N$ belongs to N

Proposition 3.2. Le N be a non-empty neutrosophic subsets in X satisfying (N_1).Then,

- $1_N \in N$ iff $N \neq O_N$
- $O_N \notin N$ iff $N \neq$ all neutrosophic subsets of X.

From the above Propositions (3.1) and (3.2), we can characterize the concept of neutrosophic filter:

Theorem 3.1. Let N be a neutrosophic subsets in a set X. Then N is neutrosophic filter on X, if and only if it is satisfies the following conditions

- Every neutrosophic set in X containing a member of N belongs to N.
- If $A, B \in N$, then $A \cap B \in N$.
- $N^X \neq N \neq O_N$.

Proof: It's clear.

Theorem 3.2. Let $X \neq \phi$. Then the set $\{1_N\}$ is a neutrosophic filter on X. Moreover if A is a non-empty neutrosophic set in X, then $\{B \in N^X : A \subseteq B\}$ is a neutrosophic filter on X

Proof: Let. $N = \{B \in N^X : A \subseteq B\}$. Since $1_N \in N$ and $O_N \notin N$, $O_N \neq N \neq N^X$. Suppose $U, V \in N$, then $A \subseteq U, A \subseteq V$. Thus $\mu_A(x) \le \min(\mu_U(x), \mu_V(x))$, $\sigma_A(x) \le \min(\sigma_U(x), \sigma_V(x))$ or $\sigma_A(x) \le \max(\sigma_U(x), \sigma_V(x))$ and $\gamma_A(x) \le \max(\gamma_U(x), \gamma_V(x))$ for all $x \in X$. So $A \subseteq U \cap V$ and hence $U \cap V \in N$.

COMPARISON OF NEUTROSOPHIC FILTERS

Definition 4.1. Let N_1 and N_2 be two neutrosophic filters on a set X. Then N_2 is said to be finer than N_1 or N_1 coarser than N_2 if $N_1 \subset N_2$

If also $N_1 \neq N_2$, then N_2 is said to be strictly finer than N_1 or N_1 is strictly coarser than N_2.

Two neutrosophic filters are said to be comparable, if one is finer than the other. The set of all neutrosophic filters on X is ordered by the relation N_1 is coarser than N_2, this relation is induced the inclusion relation in N^X.

Proposition 4.1. Let $(N_j)_{j \in J}$ be any non-empty family of neutrosophic filters on X. Then $N = \cap_{j \in J} N_j$ is a neutrosophic filter on X. In fact N is the greatest lower bound of the neutrosophic set $(N_j)_{j \in J}$ in the ordered set of all neutrosophic filters on X.

Remark 4.1.The neutrosophic filter by the single neutrosophic set 1_N is the smallest element of the ordered set of all neutrosophic filters on X.

Theorem 4.1. Let A be a neutrosophic sets in X. Then there exists a nutrosophic filter $N(A)$ on X containing A iff for any finite subset $\{S_1, S_2, ..., S_n\}$ of A, $\cap_{i=1} S_i \neq O_N$. In fact $N(A)$ is the coarest neutrosophic filter containing A.

Proof ($\Rightarrow$) Suppose there exists a nutrosophic filter $N(A)$ on X containing A. Let B be the set of all the finite intersections of members of A. Then by (N_2), $B \subset N(A)$. By (N_3), $O_N \notin N(A)$. Thus for each member B of B, Hence the necessary condition holds

($\Leftarrow$) Suppose the necessary condition holds. Let $N(A) = \{A \in N^X : A \text{ contains a member of B}\}$. Where B is the family of all the finite intersections of members of A. Then we can easily check that $N(A)$ satisfies the conditions in Definition3.1

The neutrosophic filter $N(A)$ defined above is said to be generated by A and A is called a sub - base of $N(A)$.

Corollary 4.1. Let N be a neutrosophic filter in a set X and A neutrosophic set. Then there is a neutrosophic filter $N^{/}$ which is finer than N and such that $A \in N^{/}$ iff and A neutrosophic set. Then there is a neutrosophic filter $N^{/}$ which is finer than N and such that $A \in N^{/}$ iff $A \cap U \neq O_N$ for each $U \in N$.

Corollary 4.2 A set φ of a neutrosophic filter on a non-empty set X, has a least upper bound in the set of all neutrosophic filters on X iff for all finite sequence $(N_j)_{j \in J}, 0 \leq j \leq n$ of elements of φ and all $A_j \in N_j (1 \leq j \leq n)$, $\cap_{j=1} A_j \neq O_N$

Corollary 4.3. The ordered set of all neutrosophic filters on a non-empty set X inductive.

If Λ is a sub base of a neutrosophic filter N on X, then N is not in general the set of neutrosophic sets in X containing an element of Λ; for Λ to have this property it is necessary and sufficient that every finite intersection of members of Λ should contain an element of Λ. Hence we have the following result:

Theorem 4.2. Let β is a set of neutrosophic sets on a set X. Then the set of neutrosophic sets in X containing an element of β is a neutrosophic filter on X iff β has the following two conditions

(β_1) The intersection of two members of β contain a member of β.

(β_2) $\beta \neq O_N$ and $O_N \notin \beta$.

Definition 4.2. Let Λ and β are neutrosophic sets on X satisfying conditions (β_1) and (β_2) is called a base of neutrosophic filter it generates. Two neutrosophic bases are said to be equivalent, if they generate the same neutrosophic filter.

Remark 4.2. Let Λ be a subbase of neutrosophic filter N. Then the set β of finite intersections of members of Λ is a base of filter N.

Proposition 4.2. A subset β of a neutrosophic filter N on X is a base of N iff every member of N contains a member of β.

Proof ($\Rightarrow$) Suppose β is a base of N. Then clearly, every member of N contains an element of β. ($\Leftarrow$) Suppose the necessary condition holds. Then the set of neutrosophic sets in X containing a member of β coincides with N by reason of $(N_j)_{j \in J}$.

Proposition 4.3. On a set X, a neutrosophic filter $N^{/}$ with base $\beta^{/}$ is finer than a neutrosophic filter N with base β iff every member of β contains a member of $\beta^{/}$.

Proof This is an immediate consequence of Definitions 4.2 and 4.4.

Proposition 4.4. Two neutrosophic filters bases β and $\beta^{/}$ on a set X are equivalent iff every member of β contains a member of $\beta^{/}$ and every member of $\beta^{/}$ and every member of $\beta^{/}$ contains a member of β.

NEUTROSOPHIC ULTRAFILTERS

Definition 5.1. A neutrosophic ultrafilter on a set X is a neutrosophic filter N such that there is no neutrosophic filter on X which is strictly finer than N (in other words, a maximal element in the ordered set of all neutrosophic filters on X).

Since the ordered set of all the neutrosophic filters on X inductive, Zorn's lemma shows that

Theorem 5.1. If N be any neutrosophic ultrafilter on a set X, then there is a neutrosophic ultrafilter than N .

Propostition 5.1. Let N be a neutrosophic ultrafilter on a set X. If A and B are two neutrosophic subsets such that $A \cup B \in N$, then $A \in N$ or $B \in N$.

Proof: Suppose not. Then there exist neutrosophic sets A and B in X such that $A \notin N, B \notin N$ and $A \cup B \in N$. Let $\Lambda = \{M \in N^X : A \cup M \in N\}$. It is straightforward to check that Λ is a neutrosophic filter on X, and Λ is strictly finer than N , since $B \in \Lambda$ .This contradiction the hypothesis that N is a neutrosophic ultrafilter.

Corollary 5.1. Let N be a neutrosophic ultrafilter on a set X and let $(N_j)_{1 \le j \le n}$ be a finite sequence of neutrosophic sets in X. If $\underset{j=1}{\cup} N_j \in N$, then at least one of the N_j belongs to N .

Definition 5.2. Let A be a neutrosophic set in a set X. If U is any neutrosophic set in X, then the neutrosophic set $A \cap U$ is called trace of U an A and denoted by U_A . For all neutrosophic sets U and V in X, we have $(U \cap V)_A = U_A \cap V_A$.

Definition 5.3. Let A be a neutrosophic set in a set X. Then the set Λ_A of traces an $A \in N^X$ of member of Λ is called the trace of Λ an A .

Propsition 5.2. Let N be a neutrosophic filter on a set X and $A \in N^X$. Then the trace of N_A of N an A is a neutrosophic filter iff each member of N meets A .

Proof. From the result in Definition5.3, we see that N_A satisfies (N_2).If $M \cap A \subset P \subset A$, then $P = (M \cup P) \cap A$. Thus N_A satisfies (N_1). Hence N_A is a neutrosophic filter iff it satisfies (N_3). i.e. iff each member of N meets A .

Definition 5.4. Let N be a neutrosophic filter on a set X and $A \in N^X$. If the trace N_A of N an A , then N_A is said to be induced by N an A .

Propsition 5.3. Let N be a neutrosophic filter on a set X induced a neutrosophic filter N_A on $A \in N^X$. Then trace β_A on A of a base β of N is a base of N_A .

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