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Chapter 15

STUDYING NEUTROSOPHIC VARIABLES

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Abstract

This chapter presents the neutrosophic variables, which are a generalization of the classical random variables obtained from the application of the neutrosophic logic on classical random variables. The neutrosophic variable have change because of randomize and indeterminacy, and the values it have represent are the possible results and the possible indeterminacy. Then the neutrosophic-randomized variables are classified into two types of discrete and continuous neutrosophic random variables, and we define the expected value and variance of the neutrosophic random variable then offer some illustrative examples. (Neutrosophic logic a new non-classical logic that was founded by the American philosopher and mathematical Florentin Smarandache, which he introduced as a generalization of fuzzy logic especially the intuitionistic fuzzy logic).

Keywords: Neutrosophic logic, Neutrosophic variable, discrete, continuous, Neutrosophic expected value and variance.

1. Introduction

Random variables are used to express the results of a randomized experiment with numerical values rather than names or attributes within the framework of classical logic, so that the random variable changes due to randomize.

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We present the neutrosophic variables; it is generalization for classical variables. That generalization is going to help us to not neglecting any result when conducting an experiment.

In the neutrosophic logic, the variable has change because of randomize and indeterminacy, and call it, the neutrosophic variable, its values are the results of random experiment (including the indeterminate results), we can say that the neutrosophic variable is variable can have indeterminate result.

Florentin Smarandache in his book "Introduction to Neutrosophic statistics" [6], introduced the neutrosophic variable and mentioned the difference between randomize and indeterminate.

Many researchers have worked in this framework such as Aslam in [18,19,20,21] .

- In the neutrosophic logic, as in classical logic, the randomized variables can be classified into two types:

  1- Discrete neutrosophic random variables

  2- Continuous neutrosophic random variables.

Neutrosophic Random Variable:

Assuming that X is the sample space for a neutrosophic-randomized experiment, the neutrosophic randomized variable z is define as a function on the sample space X.

So that the sample space for a neutrosophic-randomized experiment is a space consisting of all the possible results of a randomized experiment which it contains undetermined results.

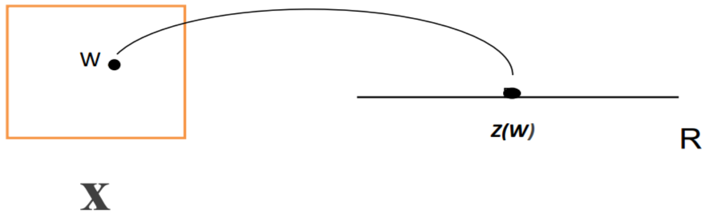
Notes:

1- The neutrosophic random variable Z gives a single real value or an indeterminate value for each element of the sample space X.

2- The neutrosophic random variable z represents either the value of the result of indetermination or application his range is a space sample X ,and its corresponding range is the set of real numbers R that is :

$$Z : X \rightarrow R$$

3- If $w \in X$ is a sample point, its image under the influence of the neutrosophic variable Z is Z(w) which is either an indeterminate or a real value:



$w \xrightarrow{z} z(w) \in R$

Or

$w \xrightarrow{z} z(w) \in I$

(Where "I" set for possible indeterminacy [4, 8, 9])

For clarification:

The image of w on straight line of the real numbers in the classical Case has a value, but assuming that part of this straight line has been cleared and a picture of w has been dropped in this Missing part, then we get an indeterminate result.

The set:

$$Z(X) = \{ z \in I \quad \text{or} \quad z \in R \; : z(w) = z \; , w \in X \; \}$$

Is the extent of application Z, called the set of possible values of the neutrosophic random variable Z, and its subset is the set of real numbers plus the set of possible indeterminacy:

$Z(X) \subseteq R + I$

Discrete Neutrosophic Random Variable:

The neutrosophic random variable Z is discrete if the set of possible values Z(X) is a discrete set (or countable).

Note:

Neutrosophic randomized variables are either:

Limited: has a finite number of possible outcomes and possible indeterminacy.

Unlimited: has an infinite number of possible outcomes or possible indeterminacy.

The unlimited neutrosophic randomized variables are either countable or non-countable.

The probability mass function for Discrete Neutrosophic Variable:

Definition: If z is a discrete neutrosophic variable, the probability mass function of z is denoted by the symbol $f_z(z)$.

Where possible values for z are finite or infinite.

Defined as follows:

$$f_z(z) = \begin{cases} NP(Z = z) & ; \quad z \in Z(X) \\ 0 & ; \quad o.w \end{cases}$$

Properties of the probability mass function:

The probability mass function $f_z(z) = NP\,(Z = z)$ Check the following:

1- $f_z(z) = NP(z) = \big(\, p(z_1), p(z_2), p(z_3)\big) \quad ; \quad 0 \le p_1 \,, p_2 \,, p_3 \le 1$

2- $\sum_{\forall\, z} f_z(z) = \sum NP(z) = \sum\big(\, p(z_1), p(z_2), p(z_3)\big) = (1,1,1) = 1_N$

Where: NP is a neutrosophic probability [10, 11, 12].

Expected Value and Variance of a Discrete Neutrosophic Random Variable:

Let X a discrete neutrosophic probability space with events $x_1, x_2, \dots \dots, x_r$, the chances of their occurrence respectively $p_1, p_2, \dots \dots, p_r$, with the indeterminacy $I_1, I_2, \dots \dots, I_S$, then the neutrosophic expected value we express it with the symbol NE given in form :[6]

$$NE = \sum_{j=1}^{r} n_j \; p_j + \sum_{k=1}^{s} m_k \; I_k$$

Where:

n_j : Possible numerical results of probabilities p_j , $\forall j$.

m_k: Possible numerical results of probability of occurrence the indeterminacy I_k , $\forall$ k .

- under the same previous hypotheses and depending on the properties of classical variance , We define the variance of the discrete neutrosophic random variable , express it with the symbol NV , given in form :

$$NV = \left(\sum_{j=1}^{r} {n_j}^2 \, p_j + \sum_{k=1}^{s} {m_k}^2 \; I_k\right) - \; (NE)^2$$

Example:

Suppose that we have a jar containing:

5 cards with the symbol A , 3 cards with the symbol B and 2 of the not selected cards (code has been cleared). The numerical results of the bet of a group of people to extract the card A is the loss of 200 $ and to extract the card B is to earn 300 $ while to extract an unspecified card is the loss of 100$.

What is the neutrosophic expected value and variance?

The neutrosophic expected value:

$$\mathrm{NE} = -2\,.\left(\frac{5}{10}\right) + 3\,.\left(\frac{3}{10}\right) - 1.\left(\frac{2}{10}\right) = -0.30\ \$$$

The variance:

$$\mathrm{NV} = \left((-2)^2\left(\frac{5}{10}\right) + (3)^2\left(\frac{3}{10}\right) + (-1)^2\left(\frac{2}{10}\right) \right) - (-0.30)^2$$

$$= 4{,}9 - 0.09 = 4.81$$

Examples of discrete neutrosophic random variables:

All examples such as the experience of throwing dice on an irregular surface. Throw a dice on a regular surface but two of the dice surface was cleared. Throw a coin on a surface have cracks. Jar containing non-specific cards as well as get an equalizer to two teams in the game of football (equalizer option not provided by classic logic).

All the examples mentioned may give us an unspecified result; we note that they represent examples of discrete neutrosophic random variables.

Example:

The Meteorological Center reports that there is a possibility of rainfall tomorrow by 0.46, but that does not mean that the probability of not rainfall is 0.54 because there are other factors of the weather may affect not reported by meteorological reports, for example, cloudy or foggy or otherwise.

If, for example, we assume that the chance for tomorrow's weather to be clear (there is no rain) is 0.45 we note that:

$$1 - 0.46 - 0.45 = 0.09$$

So, neutrosophic probability:

$$\mathrm{NP(A)} = (\ 0.46\ , 0.09\ , 0.45)$$

Where A represents the chance of rain.

- From this example, we note that the set of possible results for the weather tomorrow (rainy - cloudy - sunny ....) is a discrete set.

Continuous Neutrosophic Random Variable: Is a random variable, whose values and possible indeterminacy are a period or union of a number of periods.

For any continuous neutrosophic random variable z, there is a function symbolized by the symbol $f_N(z)$ called probability density function, through which we find the probability of accidents expressed by the neutrosophic random variable z

$NP\ (a < z < b) = \int_a^b f_N(z)\ dv = \int_a^b g(z)\ dz + \int_a^b i(z)\ dz$

Where:

$g(z)$ The specified part of function $f_N(z)$

$i(z)$ The undefined part of the function $f_N(z)$

$f_N(z) \in [\ g(z)\ \ ,\ \ g(z) + h(z)]$

Where $h(z)$:

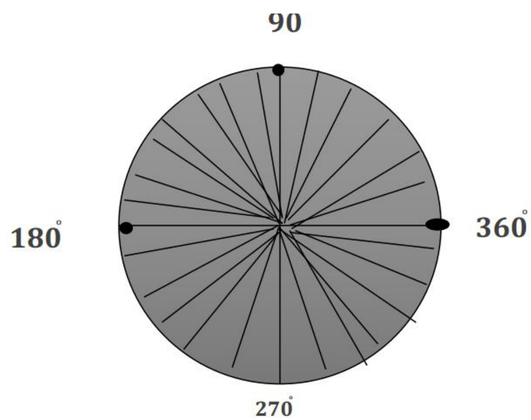
$h(z) \geq 0\ ;\ i(z) \in [\ 0\ , h(z)\ ]$

and (dv) is neutrosophic Measure [10] .

Examples:

Example (1): **[12]**

Let us have the turntable:



Continuous sample space is $X = [0\ , 360]$.

And assuming that the turntable has been cleared between 270° and 360°, if the turntable stops at any point of this area we will not be able to read the number and then we get the result of non-specific.

$NP\ (I) = \frac{1}{4}$

Where "I" represents that we get an indeterminate result.

To find a possible turntable stop between 90 and 100, note that we have a continuous neutrosophic random variable

$NP([\,90\,,100]) = \left(\,p\,[90\,,100]\,,p(I),p\,\overline{[90\,,100]}\right)$

$= \left(\frac{10}{360}\,,\frac{90}{360}\,,\frac{260}{360}\right)$

Example (2):

Let us have regular coins that have two sides H and T , It was thrown on an irregular surface and it is assumed that the chance of getting a coin stuck in a slit on the surface (getting a status indeterminate) is $p(I) = 0.02$

Because the coin is balanced, the probability of getting T or H is an equal probability

$P(H) = P(T) = \frac{1-0.02}{2} = 0.49$

Neutrosophic Probable space is: $X = \{H\,,T\,,I\,\}$

Where I represents getting indeterminate

So:

$NP(H) = NP(T) = (\,0.49\,,0.02\,,0.49\,)$

When you throw the coin three times... What is the neutrosophic probability to get HTT?

The solution:

The resulting neutrosophic space is:

$\{H\,,T\,,I\,\}\,,\{H\,,T\,,I\,\}\,,\{H\,,T\,,I\,\}$

Which equals:

={HHH , HHT ,HTH , HTT , THH , THT , TTH , TTT, IHH , IHT , ITH , ITT , HIH , HIT , TIH , TIT , HHI , HTI , THI , TTI , IIH , IIT , IHI , ITI , HII , TII , III }

We have $3^3 = 27$ element.

In which:

P(HHH)=P(HHT)=………=P(TTT)= $(0.49)^3 = 0.117469$

P(IHH)= P(IHT)=………..=P(TTI)= $(0.49)^2(0.02) = 0.004802$

P(IIH)=P(IIT)=…………..= P(TII)=(0.49)$(0.02)^2 = 0.000196$

P(III)=$(0.02)^3 = 0.000008$

The total summation for the probability of obtaining an indeterminate is:

P (total indeterminacy) = 12$(0.004802) + 6(0.000196) + (0.000008) = 0.058808$

Thus the chance of HTT occurrence is:

$P(HTT) = \;(0.49)^3 = 0.117649$

While the chance of non-occurrence of HTT is:

$$P(\overline{HTT}) = 7(0.117649) = 0.823543$$

Finally:

$$NP(HTT) = (0.117649\ , 0.058808\ , 0.823543\ )$$

In classical probability when:

P (indeterminacy) = 0

We get: $P(HTT) = (0.5)^3 = 0.125$

In neutrosophic, we write:

$$NP(HTT) = ((0.5)^3\ , 0\ , 7(0.5)^3) = (0.125\ , 0\ , 0.875)$$

- Note that the probability of throwing a coin three times in respectively and getting the HTT is Smaller in the neutrosophic probability space than the classical probability space because the opportunity is quite positive in order to obtain the indeterminacy $0.125000 > 0.117649$.

<u>Example (3):</u>

Assuming we have two football teams, team A will play against team B and team C against team D, and we have:

$$NP(Win\ \ A\ ) = (\ 0.7\ \ , 0.2\ , 0.1)$$

Which means that A has (0.7) a chance to profit, (0.2) unspecified and (0.1) to loss

$$NP(Win\ C) = (\ 0.3\ \ , 0.5\ , 0.2)$$

So what is the neutrosophic probability because both teams A and C win in the game of football?

The solution:

We have the following neutrosophic probabilistic space:

$$\{W_A, I_{A,B}, L_A\} * \{W_C, I_{C,D}, L_C\}$$

Where:

W_A represents the team's win A .

$I_{A,B}$ represents The two teams tied A , B .

L_A represents team loss A .

Similarly for $W_C, I_{C,D}, L_C$, and therefore:

$$\{W_AW_C\ , W_AI_{C,D}\ , W_AL_C\ , I_{A,B}W_C\ , I_{A,B}I_{C,D}\ , I_{A,B}L_C\ , L_AW_C\ , L_AI_{C,D}\ , L_AL_C\}$$
$$= \{0.21\ , 0.35\ , 0.14\ , 0.06\ , 0.10\ , 0.04\ , 0.03\ , 0.05\ , 0.02\ \}$$

These latter figures represent possible outcomes of the chance of events.

1- In classical probability we say that:

$$\mathrm{P(Win\ A\ and\ Win\ C)} = (0.7)(0.3) = 0.21$$

While the probability of a counter event: 1-0.21= 0.79

Note that in both football games, there are either at least tied or at least one of the teams A or C are lose.

2- In neutrosophic probability the results are more accurate:

a) $\mathrm{NP(\,Win\ A\ and\ Win\ C\,)} =$

$$\left(\begin{array}{c} \mathrm{P\,(Win\ A\ and\ Win\ C)\,,\ \ P\,(\ at\ least\ one\ A\ or\ C\ tied\ )} \\ \mathrm{,\,p\,(A\ and\ c\ \ lose\ one\ and\ the\ other\ wins\ or\ both\ lose\,)} \end{array}\right)$$

=(0.21 , 0.35+0.06+0.10+0.04+0.05 , 0.14+0.03+0.02)

= (0.21, 0.60, 0.19)

b) $\mathrm{NP(\,Win\ A\ and\ Win\ C\,)} =$

$$= \left(\ \mathrm{P\,(Win\ A\ and\ Win\ C)\,,\ \ P\,(\ at\ least\ one\ A\ or\ C\ tied\ )\ ,p\,\big((\ at\ least\ one\ A\ or\ C\ \ lose\,)}\right)$$

$$= (0{,}21\,, 0.35 + 0.06 + 0.10\,, 0.14 + 0.04 + 0.03 + 0.05 + 0.02)$$

$$= (0.21\,, 0.51\,, 0.28)\,.$$

c) Another solution using neutrosophic logic

Considering that:

$$\mathrm{P_1} = \{\,\mathrm{Win\ \ A}\} = (0.7\,, 0.2\,, 0.1\,)$$

$$\mathrm{P_2} = \{\,\mathrm{Win\ \ C}\,\} = (\,0.3\,, 0.5\,, 0.2\,)$$

Then using the neutrosophic symbol (and) in the form of $\wedge_{\mathrm{N}}$

$$\mathrm{P_1} \wedge_{\mathrm{N}} \mathrm{P_2} = (0.7 \wedge_{\mathrm{F}} 0.3 \ \ , 0.2 \ \vee_{\mathrm{F}} 0.5 \ \ , 0.1 \vee_{\mathrm{F}} 0.2)$$

- Depending on, the fuzzy logic, we can write:

$$\mathrm{P_1} \wedge_{\mathrm{N}} \mathrm{P_2} = (\min(0.7\ , 0.3\,)\ , \max(\ 0.2\ , 0.5)\ \ , \max(\ 0.1\ , 0.2\,))$$

$$= \ (\,0.3\ ,,0.5\ , 0.2)$$

CONCLUSION

We conclude that the neutrosophic logic can be applied in any scientific field where the indeterminacy finds its place.

- The quality of the study varies according to the type of indeterminacy we have, whether related to the physical space or related to the elements of this space.

- Linking the sample points for a randomized experiment with neutrosophic random variables gives us a valid basis for the study because it represents all the results of the randomized experiment, with that which it is not explicitly defined.

- The generalization of data to include non-specific cases ignored by classical logic actually affects the value of the final probability and therefore cannot be ignored and excluded from the framework of the study.

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