Synonymy

Nathan Salmon

Abstract Alonzo Church famously provided three principal competing criteria for "strict synonymy," i.e., sameness of semantic content. These are his Alternatives (0), (1), and (2)—numbered in order of increasing course-grainedness of content. On Alternative (2), expressions are deemed strictly synonymous iff they are logically equivalent. This criterion seems hopeless as an account of the objects of propositional attitude. On Alternative (1), expressions are deemed synonymous iff they are λ-convertible. Alternative (1) also evidently conflicts with discourse about the attitudes. On Alternative (0), expressions are deemed strictly synonymous iff they are "synonymously isomorphic" in Church's sense. On Alternative (0), semantic content is so fine-grained that not even "Romeo loves Juliet" and "Juliet is loved by Romeo" qualify as strictly synonymous. A fourth alternative, here called "Alternative (3)," deems expressions strictly synonymous iff they are co-intensional, i.e., they have the same semantic extension with respect to the same possible worlds. This criterion's notion of content is even more course grained than that of Alternative (2). Several objections to Alternative (3) are considered.

Keywords Synonymy criteria · Propositional attitudes · De re belief · Semantic content · Intensional entities

Philosophers of language have undertaken extensive investigations into both the semantics and the pragmatics of ascriptions of propositional attitudes, with special

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N. Salmon (✉)
Department of Philosophy, University of California, Santa Barbara, Santa Barbara, CA, USA
e-mail: nsalmon@ucsb.edu

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attention to belief of so-called singular propositions and the concomitant notion of *de re* belief (belief *of* an object that it is such-and-such). Belief of singular propositions raises thorny difficulties involving the pragmatics of belief attributions.

Bertrand Russell presented an antinomy about propositions, now known as "the Russell–Myhill paradox," in appendix B of his brilliant book *Principles of Mathematics* (Russell, 1903). He there also suggested but dismissed a resolution, arguably by means of what is now known as "the ramified theory of types," which invokes a stratification of propositions, of propositional functions, and of related intensional entities. Later in their monumental masterpiece *Principia Mathematica*, Whitehead and Russell would develop the ramified-type theoretic resolution. Later still, the great logician Alfred Tarski employed stratification to resolve semantic antinomies, like those of the Liar sentence and Kurt Grelling and Leonard Nelson's deeply puzzling adjective "heterological."

No one has shed more light on the theory of propositions, as the semantic contents of declarative sentences and the cognitive contents of belief and various other attitudes than the great logician and philosopher, Alonzo Church. In his *Logic of Sense and Denotation* (*LSD*), Church developed rigorously a theory of propositions and of concepts more generally.[1] See (Church, 1946; 1951; 1973; 1974; 1993). (Myhill, 1951 and 1958) demonstrated that Church's initial formulation was open to Russell's *Principles* appendix B antinomy (which Myhill discovered independently). Church modified his initial formulation using stratification to obtain a consistent formulation of *LSD*. In addition, (Myhill, 1979) debunked the popular misconception that Whitehead and Russell's version of ramified-type theory, which incorporates axioms of reducibility, reinstates the very antinomies, like Russell–Myhill, that the theory was designed to resolve.

In his *LSD*, Church proposed three principal rival criteria for *strict synonymy*, i.e., for sameness of semantic content: Alternatives (0), (1), and (2), numbered in order of decreasing strictness, or decreasing fine-grainedness of concepts and propositions—and not coincidentally, in order of decreasing plausibility.[2] According to Alternative (2), expressions are strictly synonymous iff they have the same free variables and are logically equivalent. In the special case of sentences and the propositions they express, according to Alternative (2), $p = q$ iff $\vDash(p \leftrightarrow q)$.[3] Alternative (2) effectively identifies the proposition expressed by a sentence ϕ with the class of logically possible worlds with respect to which ϕ is true. According to Alternative (1),

[1] Nearly all of Church's works cited in the present paper are reprinted in (Church, 2019).

[2] Church's alternatives concern "strict synonymy" in the sense of *sameness of semantic content*, as distinct from sameness of meaning in a sense of 'meaning' on which the same semantic content may be expressed in different contexts by expressions that differ in meaning (e.g., 'I' and 'he'). For some illuminating work on Church's *LSD*, see (Kaplan, 1964); and (Anderson, 2001), at pp. 421–22. There is a valuable discussion of *LSD* and Church's three alternative criteria for strict synonymy in (Anderson, 1998). I thank Anderson for bibliographical references.

[3] Here the variables 'p' and 'q' range over propositions, $\leftrightarrow$ is the relation of material equivalence between propositions, and $\vDash$ is a logical property of contents (rather than of their expressions). Cf. (Frege, 1980), at pp. 70–71. See also (Frege, 1979), pp. 143, 197.

expressions are strictly synonymous iff they have the same free variables and one is obtainable from the other by a sequence of applications of λ-conversion and replacement of a component by a strict synonym of the same type. Although an improvement over (2), like (2) this criterion deems the conjunction "F*a* & G*a*" to be strictly synonymous with the subject-predicate sentence "(λ*x*[F*x* & G*x*])*a*." If proper names are Millian designators or "logically proper names" (and it is all but certain that they are), then according to Alternative (1) "Hesperus is brighter than Phosphorus," which might capture the content of someone's rational belief, is strictly synonymous with "Venus is a thing brighter than itself." See (Salmón, 2010).

According to the strictest of the three competing criteria, Alternative (0), expressions are strictly synonymous iff they are "synonymously isomorphic." A pair of expressions are *synonymously isomorphic* if they have the same free variables and one is obtainable from the other by a sequence of applications of: (*1*) alphabetic change of bound variable; (*2*) replacements of a component expression of a given type (e.g., a predicate) by a strictly synonymous simple (non-compound) constant of that same type; and (*3*) replacements of a component simple constant of a given type by a strictly synonymous expression (simple or compound) of that same type. According to Alternative (0), even sentences as close in meaning as "Romeo loves Juliet" and "Juliet is loved by Romeo" are not strictly synonymous. See (Church, 1954).

A number of prominent philosophers—including Jaakko Hintikka, Frank Jackson, David Lewis, Richard Montague, Robert Stalnaker, and more recently, Timothy Williamson—conceive of a proposition quite differently: as a class of metaphysically possible worlds. On this conception, $p = q$ iff $\Box(p \leftrightarrow q)$. (See footnote 3.) This yields a fourth criterion for strict synonymy, according to which there is nothing more to the semantic content of an expression than its semantic intension, i.e., its associated function from metaphysically possible worlds to semantic extensions (e.g., to truth-values). Setting aside, as irrelevant for the present purpose, subtle issues about the logic of indexicals, the fourth criterion is even more lax than Alternative (2). Using Church's system of classification in order of increasing laxness, the conception of content as mere semantic intension may be labelled "Alternative (3)." On Alternative (3), expressions are strictly synonymous iff they are semantically *co-intensional*, i.e., iff they have the same free variables and under any assignment of values to variables they have the same semantic intension. Alternative (3) effectively replaces Alternative (2)'s classes of logically possible worlds with their subclasses of metaphysically possible worlds.[4]

[4]Church would have regarded Alternative (3) as a minor variant of (2). There are even more lax criteria. According to one, which may be called 'Alternative (4)', expressions are strictly synonymous iff they are co-designative, so that designation is all there is to semantic content. This was Russell's conception of "meaning" from "On Denoting" onward. Church considered an extreme criterion, which may be called 'Alternative (5)', on which expressions are strictly synonymous iff they are co-extensional. On this criterion, materially equivalent sentences are strictly synonymous. I know of no one who has endorsed this, although given Williamson's arguments supporting Alternative (3) it is unclear why he does not instead endorse (5).

(Williamson, 2021a) dismisses the time-honored conception of propositions as composite entities structured in something like the manner of an ordered *n*-tuple of proposition-components, as in (Salmón, 1986). Williamson asserts that Alternative (3) "is the simpler and stronger framework" (pp. 314–317). Williamson presents his views further in (Williamson, 2021b); (Williamson, 2021c); and (Williamson, 2021d). He says,

> The best view [of propositions] is the very coarse-grained one that propositions are simply sets of metaphysically possible worlds . . . All the other views introduce massive complications for very meagre rewards. . . . Russellian theories project the syntactic structure of sentences onto the language-independent entities they are supposed to express. As for a full-bloodedly fine-grained approach to individuating propositions, it turns out to be inconsistent, by what is known as the Russell-Myhill paradox. The best strategy is to work with simple, coarse-grained contents but, when merely cognitive differences matter, to deal with them openly, by explicitly referring to the vehicles of content, such as sentences, or sentences in contexts.[5]

Where one's concerns are restricted to metaphysical modality, a concept's modal intension—its associated function from metaphysically possible worlds to extensions—is the only aspect of the concept that matters. (A proposition's modal intension is the characteristic, or indicator, function of the class of possible worlds in which the proposition is true.) However, the nature of propositions and concepts is in no way exhausted by their metaphysically modal characteristics. Propositions are the semantic contents of sentences. They are the objects we assert, deny, declare, announce, suggest, proclaim, insist upon, and the like. They are also objects to which we bear an array of attitudes: belief, disbelief, confidence, doubt, hope, fear, disgust, surprise, delight, resentment, wishing, and much more. (Some of these attitudes can also be directed non-propositionally.) Propositions are thus central to our mental life. They are conceptual and cognitive in nature. They are not fundamentally metaphysically modal, though they also have modal attributes.

Williamson says he prefers to avoid massive complications if the rewards are meager, and that strategically it is best to deal with "merely cognitive" differences (Williamson presumably means *non-modal differences*) among co-intensional sentences by explicitly referring to the sentences themselves instead of their semantic contents. It must be noted in response that the facts about some things are complicated, sometimes very complicated. Seeking the truth and sorting through complications is laborious, to be sure, but for those who value truth, the reward of having gotten things right is never meager, and the benefits of having avoided complexity, when they come at the cost of falsity, are ill-gotten gains. However, the resources needed to resolve the Russell–Myhill antinomy are fairly meager, and quite plausible. An appropriate restriction on naïve concept comprehension in a suitable free higher order intensional logic suffices without stratification to throw out

[5](Williamson, 2021b). The important work of Church and others on *LSD* belies Williamson's claim that "the intensionalist approach [Alternative (3)] has been far more systematically and fully developed than hyperintensionalist accounts [Alternatives (0)–(2)]."

the bath water without the baby. This much is already strongly suggested (although not required) by Russell's avoidance of impredicatively defined propositional functions. Williamson objects that any restriction on naïve comprehension is *ad hoc*.[6] On the contrary, it is well-known that unrestricted comprehension principles are generally inconsistent with the facts. As the case of ZF set theory illustrates, jettisoning naïve comprehension is not *ad hoc*; it is acknowledgment of reality.[7] Moreover, stratification is neither inelegant nor massively complicated, nor even counterintuitive. Furthermore, Church's arguments invoking the famous Church–Langford translation test demonstrate that the cognitive properties of propositions cannot be relegated in any straightforward manner to relations borne to the sentences that semantically express those propositions. In particular, 'Jones believes that water is an element' is not correctly analyzable or replaceable by 'Jones $accepts_L$ 'Water is an element'', for any of an extremely wide range of interpretations of '$accepts_L$'. In particular, it cannot be recast as 'Jones takes 'Water is an element' to be $true_L$' nor even as 'Jones believes the proposition expressed in *L* by 'Water is an element'' .

The philosophical drawbacks of Alternative (3) by comparison with any of the more discriminating alternatives are genuinely massive, on the order of a supermassive black hole. Since Alternative (3) identifies distinct concepts that share exactly the same metaphysically modal characteristics, it should come as no surprise that the criterion has a goodly number of unpalatable consequences in connection with non-modal aspects of expressions and propositions.[8] On that criterion's conception of semantic content, any co-intensional expressions are *ipso facto* strictly synonymous. This includes expressions as unalike in meaning as 'theorem of first-order logic' and 'valid formula of first-order logic'. In fact, on Alternative (3) proving theorems like Gödel's celebrated completeness and incompleteness theorems would degenerate, without exception, into an exercise in merely demonstrating utter trivialities. On (3), there is only one necessary truth, so that the proposition that water is a chemical compound is the same thing as the proposition that Peano arithmetic is not complete. According to this criterion, the sentence

[6](Williamson, 2021a), p. 315. Williamson gives a cardinality argument, misidentified with the Russell-Myhill antinomy, against (in effect) Alternative (0) with naïve property comprehension. Church's mature version of Alternative (0) is vulnerable neither to Russell–Myhill nor to Williamson's cardinality variant. Williamson's argument invokes the false premise that "on such a plenitudinous theory of properties, there are more properties of propositions than propositions, for Cantorian reasons." Like the universe of all sets, the universe of all propositions is not a set. (Even on Alternative (3), the one necessary "proposition" is not a set.) The Russell-Myhill paradox is not fully resolved by denying that there is a set of all propositions. There are variants of Russell-Myhill that do not require the purported universal set of propositions, at least not directly. Cf. Salmón 2021 and 2024. The principal paradox discussed there invokes a purported property defined by impredicative abstraction over propositions but does not straightforwardly invoke classes of propositions.

[7]Cf. (Salmon, 2021 and forthcoming).

[8]While some of the untoward consequences to be noted below are commonly known, most have not been noted before.

'Water is a compound' expresses in English both that gold is an element and that $e^{\pi i} = -1$.

The sentence 'Water covers most of the Earth's surface' clearly differs in semantic content from the significantly stronger conjunction 'Water covers most of the Earth's surface and water is a chemical compound'. The differences in content are many: The latter is conjunctive in content; its left-hand conjunct alone is not. The latter specifies the chemical nature of water, the former does not; and much more. Perhaps most telling, the latter entails that most of Earth's surface is covered by a chemical compound; the former does not have this consequence. Yet each sentence is true with respect to the very same class of metaphysically possible worlds. Alternative (3) consequently deems the two sentences strictly synonymous, and thereby flies in the face of semantic reality. Even Alternative (2) respects the dictates of common sense on this score far better than Alternative (3) does.

Alternative (2) is entirely inadequate as a criterion for identity of propositions. (Church, 1977) observed that Alternative (2) makes nonsense of the notion of logical proof, which is supposed to bring about and justify belief of q by demonstrating that it is a logical consequence of one's prior beliefs p. For on Alternative (2), if $p \Box q$, then $p = (p \ \& \ q) = (q \ \& \ p)$. Assuming that belief is closed under classical conjunction elimination—so that as a general principle, if a believes $(q \ \& \ p)$ then a believes q—it follows that on Alternative (2) belief is already closed under logical consequence.[9] As (Soames, 1985) showed independently, assuming closure of belief under conjunction elimination, and assuming further that proper names are Millian designators, it follows on Alternative (3) that one who believes that 'Hesperus' designates Hesperus (in English) and that 'Phosphorus' designates Phosphorus thereby believes that 'Hesperus' and 'Phosphorus' co-designate. Even without assuming Millianism, Church's devastating objection to Alternative (2) generalizes into a fatal collapse of Alternative (3). On Alternative (3), if $\Box(p \rightarrow q)$ then $p = (p \ \& \ q)$, so that if a believes p, then a believes $(p \ \& \ q)$. Assuming that belief is closed under conjunction elimination, according to (3) the beliefs of each of us are also closed under metaphysical-modal entailment, i.e., if $\Box(p \rightarrow q)$ and a believes p, then a believes q. It follows that on (3), one who believes any contingent proposition thereby also believes every necessary truth (that water is a compound, that $e^{\pi i} = -1$, that arithmetic is incomplete, etc.). Furthermore, on (3) there is also only one impossible falsehood, so that the propositions that water is an element and that arithmetic is complete are one and the same. According to (3), one who believes anything impossible—that water is an element, or that $1 + e^{\pi i} \geq 1$, or that arithmetic is complete, or that London and *Londres* are different cities, etc.—thereby believes *every proposition whatsoever*, whether necessary, contingent, or impossible. The steadfast advocate of Alternative (3) ultimately must deny the principle, which seems fundamental to the nature of belief, that if a believes $(p \ \& \ q)$ then a believes p.

It should be acknowledged that there are attitude-like relations toward classes of metaphysically possible worlds. For example, there is a notion of *intension-belief*,

[9] Cf. (Anderson, 1998), at pp. 157–158.

whereby *a intension-believes* a class *K* iff *a* believes some proposition *p* whose intension is *K*. This in turn yields an attenuated belief-like relation toward genuine propositions: *a (3)-believes p* iff *a* intension-believes the intension of *p*, i.e., iff *a* believes some proposition co-intensional with *p*. One who believes any necessary truth thereby (3)-believes every necessary truth; one who believes any impossible falsehood thereby (3)-believes every necessary falsehood. Unlike genuine belief, (3)-belief is not closed under conjunction elimination.

It is natural to suspect that the Alternative (3) theorist confuses the propositional attitudes with one or another of their attenuated counterparts.[10] More to the point, rejection of closure under conjunction elimination is tantamount to raising the price on proven defective goods. Alternative (3) entails that if $\Box\,(p \leftrightarrow q)$ then one who believes *p* thereby believes *q*. It follows on (3) that Kripke's Pierre (who believes that *Londres* is pretty but London is not) believes that pigs fly and London is not *Londres*. It also follows that one who believes that water is an element thereby believes that water is an element that runs uphill. It also follows that that one who believes that water runs downhill thereby believes that water is a compound that runs downhill. Even in advance of invoking closure, these consequences are already quite bad enough. Rather than making things better, adding in rejection of closure compounds the error and to that extent makes matters worse.

Taking account of its various consequences, Alternative (3) is scarcely more credible than the obviously false claim that there are just two propositions—the Great Fact and the Great Falsehood—so that $p = q$ iff $(p \leftrightarrow q)$. On (3), we all believe every proposition. Philosophical common sense demands a more reasonable conception of what we say and of how we process the world.

References

Anderson, C. A. (2001). Alternative (I*): A criterion of identity for Intensional entities. In C. Anthony Anderson & M. Zelëny (Eds.), *Logic, meaning and computation: Essays in memory of Alonzo Church* (pp. 395–427). Kluwer.

Anderson, C. A. (1998). Alonzo Church's contributions to philosophy and Intensional logic. *The Bulletin of Symbolic Logic, 4*(2), 129–171.

Church, A. (1946). Abstract of 'a formulation of the logic of sense and denotation. *Journal of Symbolic Logic, 21*(1), 31.

Church, A. (1951). A formulation of the logic of sense and denotation. In P. Henle, H. Kallen, & S. Langer (Eds.), *Structure, method and meaning: Essays in honor of Henry M* (pp. 3–24). Liberal Arts Press.

Church, A. (1954). Intensional isomorphism and identity of belief. *Philosophical Studies, 5*, 65–73.

[10] There are likewise notions of *(0)-belief*, *(1)-belief*, etc. toward propositions. See note 4 above. We define ⌜ (*n*)-believe ⌝ so that if ⌜ α believes that ϕ ⌝ is true and ϕ is deemed synonymous with ψ on Alternative (*n*), then ⌜ α (*n*)-believes that ψ ⌝ is true. In particular, one (5)-believes a proposition *p* by believing some proposition *q* that is materially equivalent to *p*. Everyone who is fallible (5)-believes every proposition.

Church, A. (1973). “Outline of a revised formulation of the logic of sense and denotation” Part 1. *Noûs, 7*(1), 24–33.
Church, A. (1974). “Outline of a revised formulation of the logic of sense and denotation” Part 2. *Noûs, 8*(2), 135–156.
Church, A. (1977). Lectures on open problems in intensional logic, UCLA, winter quarter (January–March) 1977, unpublished, recorded by Nathan Salmón.
Church, A. (1993). A revised formulation of the logic of sense and denotation, alternative (1). *Noûs, 27*(2), 141–157.
Church, A. (2019). In T. Burge & H. Enderton (Eds.), *The collected works of Alonzo Church*. MIT Press.
Frege, G. (1979). In H. Hermes, F. Kambartel, & F. Kaulbach (Eds.), *Posthumous writings*. University of Chicago Press.
Frege, G. (1980). Letter of December 9, 1906 to Edmund Husserl. In G. Gabriel, H. Hermes, F. Kambartel, C. Thiel, & A. Veraart (Eds.), *Frege's philosophical and mathematical correspondence* (pp. 70–71). University of Chicago Press.
Kaplan, D. (1964). *Foundations of intensional logic*., doctoral dissertation,. University of California at Los Angeles.
Myhill, J. (1951). Report on some investigations concerning the consistency of the axiom of reducibility. *The Journal of Symbolic Logic, 16*(1), 35–42.
Myhill, J. (1958). Problems arising in the formalization of Intensional logic. *Logique et analyse, Nouvelle Serie, 1*(2), 74–83.
Myhill, J. (1979). A refutation of an unjustified attack on the axiom of reducibility. In G. W. Roberts (Ed.), *Bertrand Russell Memorial Volume* (pp. 81–90). Humanities Press.
Russell, B. (1903), Appendix B). *The principles of mathematics* (pp. 523–528). Cambridge University Press.
Salmón, N. (1986). *Frege's puzzle*. Ridgeview.
Salmon, N. (2010). Lambda in sentences with designators: An ode to complex predication. *The Journal of Philosophy, 107*(9), 445–468.
Salmón, N. (2021). A paradox about sets of properties. *Synthese, 199*, 12777–12793. https://doi.org/10.1007/s11229-021-03353-8
Salmón, N. (2024). Russell's Law: An Ode to a Logical Theorem. Forthcoming 2024 in J. Almog and J. Pepp, eds, festschfift for David Kaplan. (Written in 2009, this item has been forthcoming since 2014.)
Soames, S. (1985). Innocence lost. *Linguistics and Philosophy, 8*(1), 59–71.
Williamson, T. (2021a). The epistemological consequences of Frege puzzles. *Philosophical Topics, 49*(2), 287–320.
Williamson, T. (2021b). Interview by Hasen Khudairi , https://www.academia.edu/48971322/Interview_with_Timothy_Williamson.
Williamson, T. (2021c). “Metametaphysics and Semantics,” talk presented to *Università della Svizzera italiana*. https://fb.watch/63BNlcGiSO/.
Williamson, T. (2021d). “Is Hyperintensionalism good science?” abstract of a talk presented to the university of St. Andrews workshop on hyperintensional metaphysics. https://www.st-andrews.ac.uk/arche/event/7288/.