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The Characteristic Function of a Neutrosophic Set

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Abstract. The purpose of this paper is to introduce and study the characteristic function of a neutrosophic set. After given the fundamental definitions of neutrosophic set operations generated by the characteristic function of a neutrosophic set (Ng for short), we obtain several properties, and discussed the relationship between neutrosophic sets generated by Ng and others. Finally, we introduce the neutrosophic topological spaces generated by Ng. Possible application to GIS topology rules are touched upon.

Keywords: Neutrosophic Set; Neutrosophic Topology; Characteristic Function.

1 Introduction

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory. After the introduction of the neutrosophic set concepts in [2-13]. In this paper we introduce definitions of neutrosophic sets by characteristic function. After given the fundamental definitions of neutrosophic set operations by Ng, we obtain several properties, and discussed the relationship between neutrosophic sets and others. Added to, we introduce the neutrosophic topological spaces generated by Ng .

2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [7- 9], Hanafy, Salama et al. [2- 13] and Demirci in [1].

3 Neutrosophic Sets generated by Ng

We shall now consider some possible definitions for basic concepts of the neutrosophic sets generated by Ng and its operations.

3.1 Definition

Let X is a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form $A=\langle x,\mu_A(x),\sigma_A(x),\nu_A(x)\rangle$ where $\mu_A(x),\sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\gamma_A(x)$) respectively of each element $x\in X$ to the set A .and let $g_A : X\times[0,1]\to[0,1]=I$ be reality function, then $Ng_A(\lambda)=Ng_A(\langle x,\lambda_1,\lambda_2,\lambda_3\rangle)$is said to be the characteristic function of a neutrosophic set on X if

$$Ng_A(\lambda)=\begin{cases}1 & \text{if } \mu_A(x)=\lambda_1,\sigma_{A(x)}=\lambda_2,\nu_A(x)=\lambda_3\\ 0 & \text{otherwise}\end{cases}$$

Where $\lambda=(\langle x,\lambda_1,\lambda_2,\lambda_3\rangle)$. Then the object $G(A)=\langle x,\mu_{G(A)}(x),\sigma_{G(A)}(x),\nu_{G(A)}(x)\rangle$ is a neutrosophic set generated by Ng where

$$\mu_{G(A)}=\sup_{\lambda_1}\{Ng_A(\lambda)\wedge\lambda\}$$

$$\sigma_{G(A)}=\sup_{\lambda_2}\{Ng_A(\lambda)\wedge\lambda\}$$

$$\nu_{G(A)}=\sup_{\lambda_3}\{Ng_A(\lambda)\wedge\lambda\}$$

3.1 Proposition

1) $A\subseteq^{Ng} B\Leftrightarrow G(A)\subseteq G(B)$.

2) $A =^{Ng} B \Leftrightarrow G(A) = G(B)$

3.2 Definition

Let A be neutrosophic set of X. Then the neutrosophic complement of A generated by Ng denoted by A^{Ngc} iff $[G(A)]^c$ may be defined as the following:

(Ng^{c1}) $\langle x, \mu^c{}_A(x), \sigma^c{}_A(x), \nu^c{}_A(x)\rangle$

(Ng^{c2}) $\langle x, \nu_A(x), \sigma\ {}_A(x), \mu_A(x)\rangle$

(Ng^{c3}) $\langle x, \nu_A(x), \sigma^c{}_A(x), \mu_A(x)\rangle$

3.1 Example. Let $X = \{x\}$, $A = \langle x, 0.5, 0.7, 0.6\rangle$, $Ng_A = 1$, $Ng_A = 0$. Then $G(A) = (\langle x, 0.5, 0.7, 0.6\rangle)$
Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the $G(0_N)$ and $G(1_N)$ as follows $G(0_N)$ may be defined as:

i) $G(0_N) = \langle x, 0, 0, 1\rangle$

ii) $G(0_N) = \langle x, 0, 1, 1\rangle$

iii) $G(0_N) = \langle x, 0, 1, 0\rangle$

iv) $G(0_N) = \langle x, 0, 0, 0\rangle$

$G(1_N)$ may be defined as:

i) $G(1_N) = \langle x, 1, 0, 0\rangle$

ii) $G(1_N) = \langle x, 1, 0, 1\rangle$

iii) $G(1_N) = \langle x, 1, 1, 0\rangle$

iv) $G(1_N) = \langle x, 1, 1, 1\rangle$

We will define the following operations intersection and union for neutrosophic sets generated by Ng denoted by $\cap^{Ng}$ and $\cup^{Ng}$ respectively.

3.3 Definition. Let two neutrosophic sets $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ and

$B = \langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle$ on X, and

$G(A) = \langle x, \mu_{G(A)}(x), \sigma_{G(A)}(x), \nu_{G(A)}(x)\rangle$,

$G(B) = \langle x, \mu_{G(B)}(x), \sigma_{G(B)}(x), \nu_{G(B)}(x)\rangle$. Then $A \cap^{Ng} B$ may be defined as three types:

i) type I: $G(A \cap B) =$

$\langle \mu_{G(A)}(x) \wedge \mu_{G(B)}, \sigma_{G(A)}(x) \wedge \sigma_{G(B)}(x), \nu_{G(A)}(x) \vee \nu_{G(B)}(x)\rangle$

ii) Type II:

$G(A \cap B) =$

$\langle \mu_{G(A)}(x) \wedge \mu_{G(B)}, \sigma_{G(A)}(x) \vee \sigma_{G(B)}(x), \nu_{G(A)}(x) \vee \nu_{G(B)}(x)\rangle$.

ii) Type III:

$G(A \cap B) =$

$\langle \mu_{G(A)}(x) \times \mu_{G(B)}, \sigma_{G(A)}(x) \times \sigma_{G(B)}(x), \nu_{G(A)}(x) \times \nu_{G(B)}(x)\rangle$

$A \cup^{Ng} B$ may be defined as two types:

Type I :

$G(A \cup B) =$

$\langle \mu_{G(A)}(x) \vee \mu_{G(B)}, \sigma_{G(A)}(x) \wedge \sigma_{G(B)}(x), \nu_{G(A)}(x) \wedge \nu_{G(B)}(x)\rangle$ ii) Type II:

$G(A \cup B) =$

$\langle \mu_{G(A)}(x) \vee \mu_{G(B)}, \sigma_{G(A)}(x) \vee \sigma_{G(B)}(x), \nu_{G(A)}(x) \wedge \nu_{G(B)}(x)\rangle$

. **3.4 Definition**

Let a neutrosophic set $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ and $G(A) = \langle x, \mu_{G(A)}(x), \sigma_{G(A)}(x), \nu_{G(A)}(x)\rangle$. Then

(1) $[\ \ ]^{Ng} A = \langle x : \mu_{G(A)}(x), \sigma_{G(A)}(x), 1 - \nu_{G(A)}(x)\rangle$

(2) $<>^{Ng} A =$
$\langle x : 1 - \mu_{G(A)}(x), \sigma_{G(A)}(x), \nu_{G(A)}(x)\rangle$

3.2 Proposition

For all two neutrosophic sets A and B on X generated by Ng, then the following are true

1) $(A \cap B)^{cNg} = A^{cNg} \cup B^{cNg}$.
2) $(A \cup B)^{cNg} = A^{cNg} \cap B^{cNg}$.

We can easily generalize the operations of intersection and union in definition 3.2 to arbitrary family of neutrosophic subsets by generated by Ng as follows:

3.3 Proposition.

Let $\{A_j : j \in J\}$ be arbitrary family of neutrosophic subsets in X generated by Ng, then

a) $\cap^{Ng} A_j$ may be defined as :

1) Type I :
$G(\cap A_j) = \langle \wedge \mu_{G(A_j)}(x), \wedge \sigma_{G(A_j)}(x), \vee \nu_{G(A_j)}(x)\rangle$,

2) Type II:
$G(\cap A_j) = \langle \wedge \mu_{G(A_j)}(x), \vee \sigma_{G(A_j)}(x), \vee \nu_{G(A_j)}(x)\rangle$,

b) $\cup^{Ng} A_j$ may be defined as :

1) $G(\cup A_j) = \langle \vee \mu_{G(A_j)}(x), \wedge \sigma_{G(A_j)}(x), \wedge \nu_{G(A_j)}(x)\rangle$ or

2) $G(\cup A_j) = \left\langle \vee \mu_{G(A_j)}(x), \vee \sigma_{G(A_j)}(x), \wedge \nu_{G(A_j)}(x) \right\rangle$.

3.4 Definition

Let f. X → Y be a mapping .

(i) The image of a neutrosophic set A generated by Ng on X under f is a neutrosophic set B on Y generated by Ng , denoted by f (A) whose reality function g_B : Y x I → I=[0, 1] satisfies the property

$$\mu_{G(B)} = \sup_{\lambda_1} \{Ng_A(\lambda) \wedge \lambda\}$$

$$\sigma_{G(B)} = \sup_{\lambda_2} \{Ng_A(\lambda) \wedge \lambda\}$$

$$\nu_{G(B)} = \sup_{\lambda_3} \{Ng_A(\lambda) \wedge \lambda\}$$

(ii) The preimage of a neutrosophic set B on Y generated by Ng under f is a neutrosophic set A on X generated by Ng, denoted by f^{-1} (B) , whose reality function g_A : X x [0, 1] →[0, 1] satisfies the property G(A) = G (B) o f

3.4 Proposition

Let $\{A_j : j \in J\}$ and $\{B_j : j \in J\}$ be families of neutrosophic sets on X and Y generated by Ng , respectively. Then for a function f: X → Y, the following properties hold:

(i) If $A_j \subseteq^{Ng} A_k$;i , j∈J, then $f(A_j) \subseteq^{Ng} f(A_k)$

(ii) If $B_j \subseteq^{Ng} B_k$, for j , K ∈ J, then

$f^{-1}(B_j) \subseteq^{Ng} f^{-1}(B_K)$

(iii) $f^{-1}\left(\underset{jEJ}{\cup}^{Ng} B_j \right) =^{Ng} \underset{jEJ}{\cup}^{Ng} f^{-1}(B_j)$

3.5 Proposition

Let A and B be neutrosophic sets on X and Y generated by Ng , respectively. Then, for a mappings f : X → Y , we have :

(i) $A \subseteq^{Ng} f^{-1}(f(A))$ (if f is injective the equality holds) .

(ii) $f(f^{-1}(B)) \subseteq^{Ng} B$ (if f is surjective the equality holds) .

(iii) $[f^{-1}(B)]^{Ngc} \subseteq^{Ng} f^{-1}(B^{Ngc})$.

3.5 Definition . Let X be a nonempty set, Ψ a family of neutrosophic sets generated by Ng and let us use the notation

$$G(\Psi) = \{ G(A) : A \in \Psi \}.$$

If (X , G (Ψ)= $N\tau$) is a neutrosophic topological space on X is Salama's sense [3] , then we say that Ψ is a neutrosophic topology on X generated by Ng and the pair (X , Ψ) is said to be a neutrosophic topological space generated by Ng (ngts , for short). The elements in Ψ are called genuine neutrosophic open sets. also , we define the family

$$G(\Psi^c) = \{ 1 - G(A) : A \in \Psi \}.$$

3.6 Definition

Let (X , Ψ) be a ngts . A neutrosophic set C in X generated by Ng is said to be a neutrosophic closed set generated by Ng , if 1- G(C) ∈ G (Ψ) = $N\tau$.

3.7 Definition

Let (X , Ψ) be a ngts and A a neutrosophic set on X generated by Ng . Then the neutrosophic interior of A generated by Ng , denoted by, ngintA, is a set characterized by G(intA) = $\underset{G(\Psi)}{\mathrm{int}}$ G(A) , where $\underset{G(\Psi)}{\mathrm{int}}$ denotes the interior operation in neutrosophic topological spaces generated by Ng .Similarly, the neutrosophic closure of A generated by Ng , denoted by ngclA , is a neutrosophic set characterized by G(ngclA)= $\underset{G(\psi)}{cl}$ G(A) , where $\underset{G(\psi)}{cl}$ denotes the closure operation in neutrosophic topological spaces generated by Ng .

The neutrosophic interior gnint(A) and the genuine neutrosophic closure gnclA generated by Ng can be characterized by :

$gnintA =^{Ng} \cup^{Ng} \{ U : U \in \Psi \text{ and } U \subseteq^{Ng} A \}$

$gnclA =^{Ng} \cap^{Ng} \{ C : C$ is neutrosophic closed generated by Ng and $A \subseteq^{Ng} C \}$

Since : $G(\text{gnint } A) = \cup \{ G(U) : G(U) \in G(\Psi), G(U) \subseteq G(A) \}$

$G(\text{gncl } A) = \cap \{ G(C) : G(C) \in G(\Psi^c), G(A) \subseteq G(C) \}$.

3.6 Proposition . For any neutrosophic set A generated by Ng on a NTS (X , Ψ) , we have

(i) $\text{cl } A^{Ngc} =^{Ng} (\text{ int } A)^{Ngc}$

(ii) $\text{Int } A^{Ngc} =^{Ng} (\text{ cl } A)^{Ngc}$

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