International Journal of Quantum Foundations **4** (2018) 173 - 198

Original Paper

An Analogy for the Relativistic Quantum Mechanics through a Model of De Broglie Wave-covariant Ether

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Received: 30 November 2017 / Accepted: 15 March 2018 / Published: 28 March 2018

Abstract: Based on de Broglie's wave hypothesis and the covariant ether, the Three Wave Hypothesis (TWH) has been proposed and developed in the last century. In 2007, the author found that the TWH may be attributed to a kinematical classical system of two perpendicular rolling circles. In 2012, the author showed that the position vector of a point in a model of two rolling circles in plane can be transformed to a complex vector under a proposed effect of partial observation. In the present project, this concept of transformation is developed to be a lab observation concept. Under this transformation of the lab observer, it is found that velocity equation of the motion of the point is transformed to an equation analogising the relativistic quantum mechanics equation (Dirac equation). Many other analogies has been found, and are listed in a comparison table. The analogy tries to explain the entanglement within the scope of the transformation. These analogies may suggest that both quantum mechanics and special relativity are emergent, both of them are unified, and of the same origin. The similarities suggest analogies and propose questions of interpretation for the standard quantum theory, without any possible causal claims.

Keywords: Dirac equation; Complex vector; Emergent quantum mechanics; Emergent space time; Quantum mechanics and special relativity unification; Ether; Quantum mechanics underpinning; Quantum foundations

1. Introduction

At the end of the 1970's and during the 1980's, the Three Wave Hypothesis (TWH) was developed by Kostro (three-wave model) [1], Horodecki (Three Wave Hypothesis) [2, 3, 4, 5], and mentioned by Vigier [6, 7]. This hypothesis is based on two concepts [8]:

1. The Paris school interpretation of quantum mechanics, which is related to de Broglie's particle-wave duality [9-12], Vigier's and others' works, and
2. Einstein's special relativity considered as a limitary case of Einstein's general relativity, in which the existence of a covariant ether is assumed [13-16].

Horodecki presented TWH through a series of articles [2, 3, 4, 5]. His TWH implies that a massive particle is an intrinsically, spatially as well as temporally extended non-linear wave phenomenon [2]. This version is based on the assumption that, in a Lorentz frame, where the particle is at rest, it can be associated with an intrinsic non-dispersive Compton wave. When

the particle moves with velocity *v* (relative to the lab frame), it will be associated with the three waves: the superluminal de Broglie wave (of wavelength λ_B), a subluminal dual wave (of wavelength λ_D), and a transformed Compton wave (of wavelength λ_C) [2, 5]. In this hypothesis, there are two dispersion relations, the de Broglie wave and a proposed dual wave dispersion relation. This version of TWH suffers from a lack of experimental evidence.
The TWH is based on both covariant ether and de Broglie's wave concepts. Thus, it is a *de Broglie wave- covariant ether* model. However, for explaining the de Broglie wave-particle duality, a sub-medium (ether) has been postulated [17, 18, 19]. In this, the question arises: can the ether be an underpinning of quantum mechanics?

Considering this system of waves (Horodecki's TWH) in angular form and in a single representation of waves (instead of two dispersion relations of the de Broglie wave and dual wave) exhibits similarities with a system of two perpendicular circles [20, 21]. Thus, the combination of the three waves may form a classical kinematical bevel gear model, similar to that in Figure 1.

In spite of this system (Figure 1) being of quantum and relativistic origin, this kinematic model looks as if it has no relationship with them. This rolling circles system is totally removed from quantum mechanics, and has no experimental observation, akin to the TWH. Thus, it is either no more than a creature of a logical theoretical work, or might be of a hidden nature (unobservable). The wave function is an abstract model in quantum mechanics, and is unphysical, so as to be unobservable.

Here one may ask, does this real kinematical model, which is related originally to the de Broglie wave- covariant ether, stand behind the concept of complex form in quantum mechanics?; or is there any relationship between the system of the two rolling circles with the complex function?

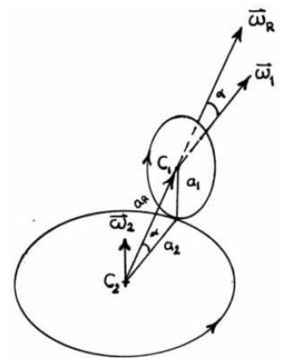


Figure 1 The bevel gear (de Broglie wave- covariant ether) [20].

Based on the above concept of two rolling circles, in a non-quantum mechanics project, a mathematical relationship between the position vector of a point in two rolling circles system and a complex vector has been explained through a proposed transformation under *partial observation* effect [22]. The model was of two rolling circles in a real plane, as in Figure 2.

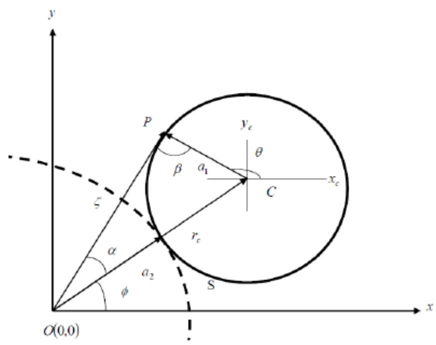


Figure 2 The rotation of point P in two rolling circles system in plane geometry [22].

The position vector ($\boldsymbol{\zeta_{1,2}}$) of a point P in the system (Figure2) is

$$\boldsymbol{\zeta_{1,2}} = \boldsymbol{r_c}\left\{\cos\,(\theta-\beta-\phi) \pm \sqrt{-sin^2(\theta-\beta-\phi)+(\frac{a_1}{r_c})^2}\right\}, \quad (1)$$

The $\boldsymbol{r_c}$, θ, β, ϕ, and a_1 are shown in Figure 2. This work [22] dealt with the scalar form of Eq. (1).

The concept of partial observation has been proposed to achieve a transformation to complex form. The partial observation is based on the inability of distinguishing the complete system due to resolution limit:

$$a_1 \ll d_\lambda \ll a_2\ , \quad (2)$$

where d_λ is the spatial resolution. This concept is based on the Rayleigh criterion (spatial resolution). Examination of the system under partial observation ($a_1 = 0$) shows a transformation to complex function:

$$\left(\boldsymbol{\zeta_{1,2}}\right)_{a_1=0} = \boldsymbol{r_c}\left\{\cos\,(\theta-\beta-\phi) \pm \sqrt{-sin^2(\theta-\beta-\phi)}\right\},$$

or

$$\left(\boldsymbol{\zeta_{1,2}}\right)_{a_1=0} = \boldsymbol{r_c}\exp \pm i\,(\theta-\beta-\phi). \quad (3)$$

Eq. (3) shows a complex phase factor (Euler form) in analogy to the wave function of quantum mechanics.

The general concept of the rolling circles model is related to the TWH, and can be transformed to an abstract model under the partial observation. This model is based originally on de Broglie wave- covariant ether. It is worth mentioning that Dirac has introduced an ether model based on a stochastic covariant distribution of subquantum motions [23]. So, is it possible for the model of two rolling circles in the real plane, and under the partial observation to analogise relativistic quantum mechanics?

However, relative to quantum physics, the above model (Figure 2) has two strange features, a real kinematics nature (rotations), and its classical nature.

1.1 Hestenes's kinematic system

The above rolling circles system exhibits a classical kinematical model, and the kinematics feature may remain (somehow) appeared after the transformation (Eq. (3)) in the complex phase factor. The concept of the kinematical model that is related to the complex phase factor has been considered by Hestenes during the 1990's. Within his geometric algebra, Hestenes proposed many concepts, like [24, 25, 26]:

- The imaginary *i* can be interpreted as a representation of the electron spin.
- Dirac's theory describes a kinematics of electron motion. It is not necessary for the kinematical rotation to be related to the pair of positive and negative energy states.
- The complex phase factor literally represents a *physical* rotation, the *zitterbewegung* rotation.
- The complex phase factor is the main feature which the Dirac wave function shares with its non-relativistic limit. Schrödinger wave function inherits the relativistic nature.

The serious concept in Hestenes proposal is the kinematic origin of the complex phase factor and the physical rotation (Zitterbewegung), but there is no experimental evidence to support these ideas yet.

1.2 Classical underpinning

Related to the classical feature of the rolling circles system, the introduction of classical physics with the foundation of quantum mechanics is not new as well. The concept of the classical underpinning of quantum mechanics has attracted many researchers. Emergent quantum mechanics tries to find in classical underpinnings many approaches to reconstruct a quantum mechanical theory. It is worth mentioning that the works of classical underpinning are based on or influenced by the quantum mechanics postulates (fully or partially) with some classical concepts [27, 28, 29, 30, 31, 32, etc.]. In these attempts, the classical bases were imposed logically within the frame of quantum postulates. These theories look like hybrid theoretical works. Whereas in the above attempt the classical model of rolling circles did not impose or postulated, it is a result of reconsideration of TWH which is within the frame of relativistic quantum mechanics.

1.3 Quantum mechanics postulates

There is no unanimous agreement on the set of the quantum mechanics postulates. Nottale and Célérier consider the quantum postulates to be in two groups, main and secondary. The main postulates are five, while the number of secondary postulates varies from one author to another; they can be derived from the main postulates [33]. However, the main postulates are (Postulates of non-relativistic quantum mechanics) [33]:

1. *Complex state function (ψ).* Each physical system is described by a state function, which determines all that can be known about the system.
2. *Schrödinger equation.* The time evolution of the wave function of a non-relativistic physical system is indicated by the time-dependent Schrödinger equation, Eq.1.

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi \,. \tag{4}$$

3. *Correspondence principle.* To every dynamic variable of classical mechanics, there corresponds in quantum mechanics a linear, Hermitian operator, which, when operating upon the wave function associated with a definite value of that observable (the eigenstate associated with a definite eigenvalue), yields this value times the wave function.
4. *Von Neumann's postulate.* If a measurement of the observable A yields some value a_i, the wave function of the system just after the measurement is the corresponding eigenstate ψ_i.
5. *Born's postulate:* probabilistic interpretation of the wave function.

For postulates of relativistic quantum mechanics, the above main postulates are exactly the same. The change is simply the free particle Hamiltonian (Dirac Hamiltonian) [34], and that is related to the second postulate above (Eq. (4)).

However, in considering the underpinning, the first two postulates are quite serious. They reflect the main features of quantum mechanics, the complexity and the mechanics, whereas the other three may look like mathematical techniques to deal with the physical problem of the complex function (wave function), and the physical interpretations.

1.4 The aim of the work

Above, we have asked these two questions:

- Does this real kinematical model which is related originally to the de Broglie wave-covariant ether explain the wave (of complex form) concept as in quantum mechanics?
- Is it possible for the model of two rolling circles in a real plane to analogise the relativistic quantum mechanics?

After Born's interpretation, the wave function has been interpreted as a probability amplitude. Our form (Eq. (3) can work as a probability amplitude as well [22] but is not a solution of the Schrödinger equation. It is not the conventional wave function. Eq. (3) exhibits an analogy for the wave function. If so, can this model to show an analogy for the relativistic quantum mechanics?

The present work is an attempt to answer these questions. Accordingly, the above-mentioned model (Figure 2) is considered. The concept of partial observation will be developed, and we try to derive analogies for the first and second postulates of the relativistic quantum mechanics (mentioned above), by using position vector with the concept of partial observation.

At the end, we will try to demonstrate a compression table, to show the similarity of the relativistic quantum mechanics equations and the special relativity equations with obtained equations. In addition to that, we will try to exhibit analogies for some of the quantum mechanics phenomena.

2. Mathematical model

The position vector of a point on a circle is a solution of the quadratic general equation of a circle in a real polar coordinate system, like:

$$a_1^2 = b^2 + r^2 - 2br\cos(\vartheta - \alpha)\,, \tag{5}$$

where a_1 , r ,b and $(\vartheta - \alpha)$ are the radius of the circle, the norm of the position vector of a point (P) on the perimeter of the circle relative to the origin (0, 0), the norm of the position vector of the circle centre (C_1) , and the angle between r and b , respectively, (see Figure 3). The polar coordinates of the centre of the circle are $(\boldsymbol{b}\,, \vartheta)$, and the coordinates of a point (P) on the perimeter of the circle are $(\boldsymbol{r}\,, \alpha)$ (Figure 3). Figure 3 is another form of Figure 2.

This circle is guided by another circle. The second circle is of radius

$$a_2 = b - a_1\,, \tag{6}$$

and its centre coordinates are (0,0). Then, the system is of two rolling circles (Figure 3).

Considering the polar vector of point $P(\boldsymbol{r}, \alpha)$, the solution of the quadratic Eq. (5) for $\boldsymbol{r}$ is

$$\boldsymbol{r} = \boldsymbol{b}\left\{\cos(\vartheta - \phi + \beta) \pm \sqrt{-\sin^2(\vartheta - \phi + \beta) + (\frac{a_1}{b})^2}\right\}. \tag{7}$$

where $\boldsymbol{r}$ is the position vector of point P. This equation is Eq. (1). The appeared ratio in Eq.(7) is

$$X \equiv \left(\frac{|\boldsymbol{a_1}|}{|\boldsymbol{\ell}|}\right)^2 = \left(\frac{a_1}{\ell}\right)^2 . \tag{8}$$

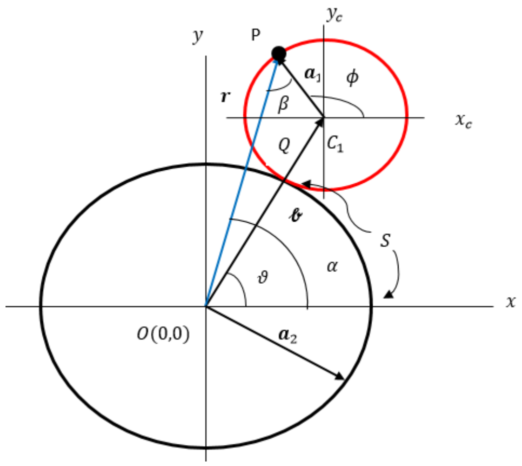


Figure 3 The real model. Rolling circles model.

The variation of the two angles ($\vartheta$and α) with time means in addition to the motion of point P around C_1 the centre of the circle (C_1) is variable (rotate around the coordinates origin (0,0)). The circle of interest is that of radius a_1 and the second circle is the guiding circle. Point Q is the point of contact between the two circles. For generality, let $a_1 < a_2$. Owing to the rolling of the circles, the motion of a point P traces out an epicycloid (hypercycloid) curve trajectory. The ratio of the system is:

$$\frac{a_2}{a_1} = \frac{\phi}{\vartheta} = \frac{\omega_1}{\omega_2} = \mu > 1 . \tag{9}$$

Eq. (9) looks like a gear ratio.

The above mentioned kinematical model (Figure 3) can be used to reformulate Eq. (7) in terms of circular motion. Let us consider these representations:

$$\frac{d\phi}{dt} = \omega_1 , \tag{10a}$$

and

$$\omega_\beta = \frac{d\beta}{dt} . \tag{10b}$$

With the aid of Figure 3:

$$\frac{d\alpha}{dt} = \omega_\alpha , \tag{10c}$$

and

$$\omega_\alpha = \omega_1 - \omega_\beta . \tag{10d}$$

The angle

$$\vartheta = \frac{\boldsymbol{s}}{\boldsymbol{a}_2} = \boldsymbol{k}_2 \cdot \boldsymbol{s} \,. \tag{10e}$$

In terms of, Eq. (6) can be rewritten as:

$$\boldsymbol{b} = \boldsymbol{a}_2 + \boldsymbol{b}\sqrt{X} \,. \tag{10f}$$

Then, in terms of kinematic parameters and X, Eq. (5) becomes,

$$\boldsymbol{r} = (\boldsymbol{a}_2 + \boldsymbol{b}\sqrt{X})\left\{\cos\left(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\right) \pm \sqrt{-sin^2\left(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\right) + X}\right\}. \tag{11}$$

where $\boldsymbol{s}$, ω_1, and ω_β represent the arc length made by point Q, the angular velocity of point P, and $\omega_\beta = d\beta/dt$, respectively. Eq. (11) gives a full description of the location of the point due to movement.

The time differentiation of Eq. (11) or the velocity equation of point P is:

$$\begin{aligned}\frac{\partial \boldsymbol{r}(r,t,X)}{\partial t} &= \frac{\partial(\boldsymbol{a}_2 + \boldsymbol{b}\sqrt{X})}{\partial t}\left\{cos\left(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\right) \pm \sqrt{-sin^2\left(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\right) + X}\right\} + (\boldsymbol{a}_2 \\ &+ \boldsymbol{b}\sqrt{X})\left\{(\omega_1 - \omega_\beta)\, sin\left(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\right) \pm \frac{(\omega_1 - \omega_\beta)\, sin(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t) cos\left(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\right) + \frac{\partial X}{\partial t}}{\sqrt{-sin^2\left(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t\right) + X}}\right\}\end{aligned} \tag{12}$$

The acceleration can be derived as well.

2.1 Unit vectors

There are two angular rotations ω_1 and ω_2, and the unit vectors of these rotations are $\hat{\boldsymbol{e}}_\phi$ and $\hat{\boldsymbol{e}}_\vartheta$ respectively. The signs $\pm$ in Eqs. (7,11, and 12) are related to the two possibilities of rotation of the point; in other words, it is related to $\hat{\boldsymbol{e}}_\phi$. The signs show the direction of angular motion (ϕ) and the unit vector $\hat{\boldsymbol{e}}_\phi$ is the axis-angle vector.

The dot product of $\hat{\boldsymbol{e}}_\vartheta$ with any perpendicular unit vector let $\hat{\boldsymbol{e}}_\varphi$ is

$$\hat{\boldsymbol{e}}_\vartheta \cdot \hat{\boldsymbol{e}}_\varphi + \hat{\boldsymbol{e}}_\vartheta \cdot \hat{\boldsymbol{e}}_\varphi = 0 \,. \tag{13}$$

The same for:

$$\hat{\boldsymbol{e}}_\vartheta \cdot \hat{\boldsymbol{e}}_\vartheta + \hat{\boldsymbol{e}}_\varphi \cdot \hat{\boldsymbol{e}}_\varphi = 2. \tag{14}$$

The square of the unit vectors

$$\hat{\boldsymbol{e}}_\vartheta \cdot \hat{\boldsymbol{e}}_\vartheta = \hat{\boldsymbol{e}}_\phi \cdot \hat{\boldsymbol{e}}_\phi = 1 \,. \tag{15}$$

The $\hat{\boldsymbol{e}}_\phi$ and $\hat{\boldsymbol{e}}_\vartheta$ are non-commutative

$$\hat{\boldsymbol{e}}_\vartheta \times \hat{\boldsymbol{e}}_\phi + \hat{\boldsymbol{e}}_\phi \times \hat{\boldsymbol{e}}_\vartheta = 0 . \qquad (16)$$

3. Physical model (Empiricist approach)

To deal with the system as a physical object, testing (measuring) in a lab is an essential approach to prove its physical existence (positivism). It is the normal case for lab observer to deal with observables. Thus, this case does not take into account classical and quantum physics. In classical physics, the observable distinguishability is related to the optical resolution (Rayleigh criterion) [35]. The spatial resolution (d_λ) is the minimum linear distance between two distinguishable points [36], and the same for the angular frequency (ω_λ). The presented concept depends on using a *monochromatic* light (λ, f). d_λ is related to the wavelength so that ω_λ is related to the light frequency (f). The system (Figure3) is fully observed when:

$$d_\lambda \ll a_1 \ll a_2 \,, \qquad (17)$$

and according to the ratio (9):

$$\omega_1 \gg \omega_2 \gg \omega_\lambda. \qquad (18)$$

Within these conditions, the lab observer (in frame Σ as shown in Figure 4) recognises a classical physical system (fully determined), and all the quantities of the system are said to be physical and can be measured.

On the other hand, the system cannot be observed when

$$a_1 \ll a_2 \ll d_\lambda, \qquad (19)$$

and

$$\omega_\lambda \gg \omega_1 \gg \omega_2 \,. \qquad (20)$$

Then the system cannot be the subject of physical study.

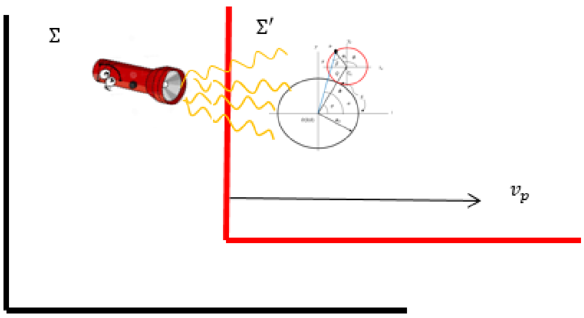


Figure 4 The lab observer frame of reference (Σ), and the system frame of reference (Σ').

3.1 Partial observation

The two cases of observation mentioned above are extreme cases. A concept of partial observation has been proposed in Ref. [22]. This case may be for a system of many parts, and some of these parts cannot be distinguished, whereas the other can be. Based on the resolution limit, the system (Figure 4) is partially resolved under the conditions:

$$a_1 \ll d_\lambda \ll a_2 \,, \tag{21a}$$

and:

$$\omega_1 \gg \omega_\lambda \gg \omega_2 \;. \tag{21b}$$

According to the ratio (9), that implies $\mu \gg 1$.
Thus for the lab observer, the partial observation may lead to the conclusion that $a_1 = \omega_2 = 0$ and the zero quantity is a practical approximation.

Inequalities (21a, 21b) describe the inability to resolve a_1 and ω_2 (missing data), whereas a_2 and ω_1 can be resolved. Partial resolution refers to the inability to completely resolve the kinematical system. Then, the quantities measured by the lab observer are:

$$\boldsymbol{a}_1 \neq \boldsymbol{a}_{1m} = 0 \text{ , and } X_m = 0 \,, \tag{22}$$

and

$$\boldsymbol{\mathscr{b}} \to \boldsymbol{a}_2 = \boldsymbol{a}_{2m}. \tag{23}$$

The subscript m indicates resolved (measured) values. The ratio μ (Eq. (9)) is a big number ($\mu \gg 1$).

When a_1 cannot be detected, then the angle β can not be detected as well, then:

$$\omega_{\beta m} = \frac{\partial \beta}{\partial t} = 0 \,. \tag{24}$$

Since $\alpha = \phi - \beta$, and Eq. (10d) then:

$$\omega_{\phi m} = \omega_{\alpha m} = \omega_{1m} \,. \tag{25}$$

Where the existence of angle β is related to the recognition of the small circle of a_1, the frequency of the angular motion can be detected indirectly (not related to rotation, which is unobservable), and the angular frequency can be measured ($\omega = 2\pi f$).

Substitution of Eqs. (22, 23 and 24) by Eq. (11) yields:

$$\boldsymbol{\mathcal{Z}}\,(\boldsymbol{s}, t, 0) = \boldsymbol{a}_{2m} \left\{ \cos\,(\boldsymbol{k}_{2m} \cdot \boldsymbol{s} - \omega_{1m} t) \pm \sqrt{-sin^2(\boldsymbol{k}_{2m} \cdot \boldsymbol{s} - \omega_{1m} t)} \right\}. \tag{26}$$

The real $\boldsymbol{r}$ transforms into complex $\boldsymbol{\mathcal{Z}}$. This form of Eq. (26) can be rewritten as (in Eulerian form):

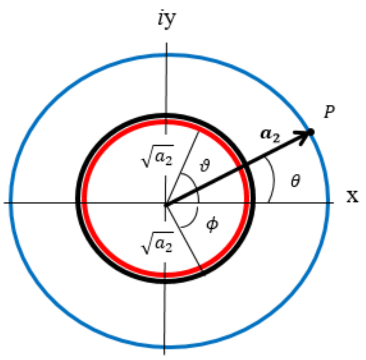
$$\boldsymbol{\mathcal{Z}}(\boldsymbol{s}, t, 0) = \boldsymbol{a}_{2m} \exp \pm i\;(\boldsymbol{k}_{2m} \cdot \boldsymbol{s} {-} \omega_{1m} t). \tag{27}$$

The equations (26, 27) are for a complex vector (Zed complex vector). Eq. (27) shows a combination of the real vector ($\boldsymbol{a}_{2m}$) and a *complex phase factor* (exp $\pm i$ ($\boldsymbol{k}_{2m} \cdot \boldsymbol{s} {-} \omega_{1m} t$)) to form the z-complex vector.

3.2 The system as seen by the lab observer

What the lab observer can get according to Eqs. (26, 27) is no more than a*bstract forms*. This z-complex vector has no physical meaning and has no macroscopic analogy. It is of an abstract nature. On the other hand, this vector can be interpreted mathematically by the lab observer in two forms, either as a position vector of a moving point as in Figure 5A, or as a sinusoidal wave (field) form as in Figure 5B in a complex plane. These two forms are *equivalents* and *indivisible* (there is no point without a sinusoidal wave and there is no sinusoidal wave without a point in the complex plane).

The partial observation acts as a filter (see Figure 6). This filter makes a separation between two different worlds, the real full deterministic mathematical world (the real position vector) and the lab observer world. The lab observer faces *difficulties* of an abstract nature. Thus, the lab observer needs to deal with this complex mathematical function to obtain physical information.



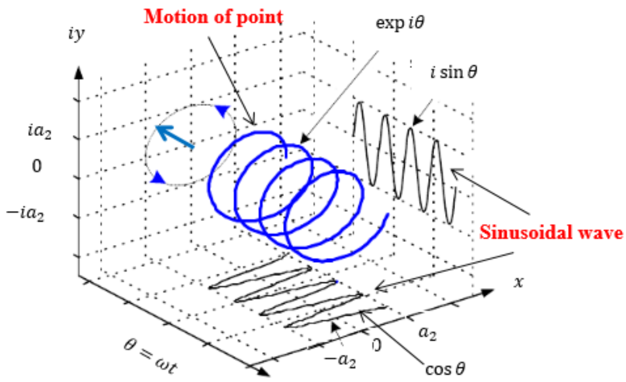


Figure 5 The z-complex vector representations. A- The point *P* location representation (polar coordinate), and B- the sinusoidal wave projection, and the point location representations ([37], with modifications).

Due to the cases being of abstract nature and the two possible representations of the z-complex vector, the lab observer may face the same situation as that of quantum mechanics in dealing with the wave function (configuration space). Accordingly, the lab observer may use some of the quantum mechanics techniques (postulates) to achieve physical information (see Figure 6).

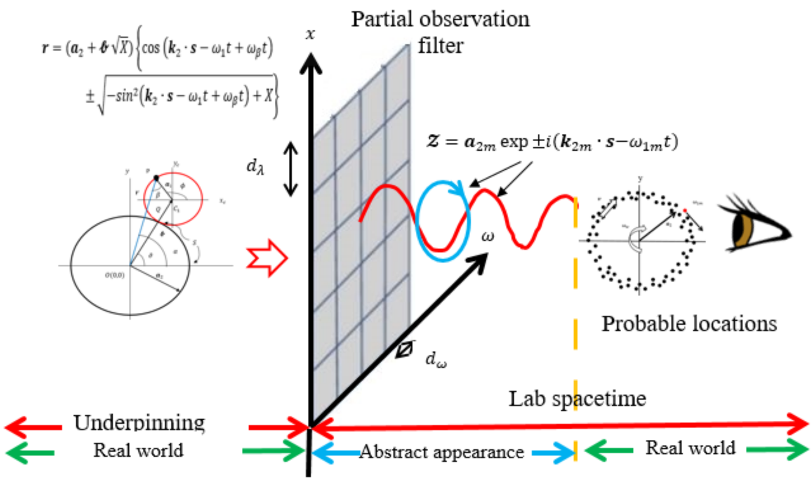


Figure 6 The system and the lab observation.

These techniques are the other postulates of quantum mechanics, like those mentioned above:

1- *Born's postulate (probabilistic interpretation of the wave function)*
2- *Correspondence principle (operators technique)*
3- *Von Neumann's postulate (wave function collapse).*

and others, like:

1- *Expectation value.*
2- *Expansion in eigenfunctions.*
3- *Eigenvalues and eigenfunctions.*

The quantum mechanics techniques may lead the lab observer to consider the complex vector in two ways:

1- The complex vector may refer to a *position vector field* (Eq. (27)). This expression leads to a concept of a *field* and its *guided point* together.
2- Regard a complex function (η) with complex phase (scaler) as:

$$\eta = A\frac{\boldsymbol{z}}{\boldsymbol{a}_{2m}} = A \exp \pm i\,(\boldsymbol{k}_{2m}\cdot \boldsymbol{s} - \omega_{1m}t), \tag{28}$$

where A is a real amplitude. This form may lead to the use of Born's rule ($\int \eta\,\eta^*\,ds = 1$), and shows a probabilistic location of a free point ($A = 1/\sqrt{2\pi\, a_{2m}}$). The probability shows an expectation value, and that means that this point is restricted (guided) in *probable* locations ($\langle r\rangle = a_{2m}$), as in Figure 7. Here too there is a point and its guide. The lab observer can recognise the observable system parameters (a_{2m}, ω_{1m}).

It looks as if the system depicted in Figure 3 is reduced to a *guided point* (represented by ω_1) of statistical wave location (a guide is represented by a_2), and both features (the point and its guide) are *indivisible* and of an equal footing (*equivalents*).

The probable locations of the point are distributed in a circular form of an average radius a_{2m} as shown in Figure 7. The ω_{1m} is attributed to the point, or is a characteristic quantity of the point (there is no recognition for an angular motion of ω_{1m}). The observable velocity of the guided point is v_p as shown in Figure 7.

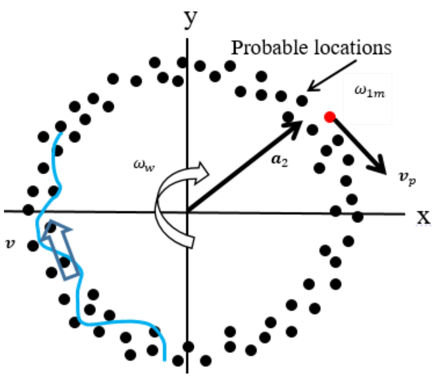


Figure 7 The observable system. The probable locations of the guided point (in real space).

3.2.1 The kinematic equations

For the lab observer who is assumed to be under the conditions of partial observation (Eqs. (22, 23, and 24)), the equation of velocity Eq. (12) transforms to (Appendix I):

$$i\frac{\partial \boldsymbol{\mathcal{Z}}}{\partial t} = (-i\, v\boldsymbol{A}\cdot\boldsymbol{\nabla} + B\omega_{1m})\boldsymbol{\mathcal{Z}}\,, \tag{29}$$

where $\boldsymbol{A}$ and B are coefficients related to the rotation of the system (Appendix I). The lab observer cannot recognise the rotations of the system, and mathematically, $\boldsymbol{A}$ and B are not the normal unit vector. Thus, the properties of the rotation unit vectors mentioned above will not be considered by the observer as rotation vectors. In reality, $\boldsymbol{A}$ and B are related to the rotations of the system, but are not unit vectors. Using the $\boldsymbol{A}$ and B instead of the $\hat{\boldsymbol{e}}_\vartheta$ and $\hat{\boldsymbol{e}}_\phi$ (in the above Eqs. (13, 14, 15, and 16)), one can find that:

$$(\pm i\hat{\boldsymbol{e}}_\vartheta)\cdot\left(\pm i\hat{\boldsymbol{e}}_\varphi\right) + (\pm i\hat{\boldsymbol{e}}_\vartheta)\cdot\left(\pm i\hat{\boldsymbol{e}}_\varphi\right) = 0\,. \tag{30}$$

The same for

$$(\pm i\hat{\boldsymbol{e}}_\vartheta)\cdot(\pm i\hat{\boldsymbol{e}}_\vartheta) + \left(\pm i\hat{\boldsymbol{e}}_\varphi\right)\cdot\left(\pm i\hat{\boldsymbol{e}}_\varphi\right) = 2. \tag{31}$$

The square of B is

$$B^2 = 1\,, \tag{32}$$

and for $\boldsymbol{A}$

$$\boldsymbol{A}^2 = \boldsymbol{A}\cdot\boldsymbol{A} = \|\boldsymbol{A}\|\|\boldsymbol{A}\| = 1\,. \tag{33}$$

Accordingly,

$$\boldsymbol{A}^2 = B^2 = 1\,. \tag{34}$$

Finally (since B is not a vector, then in dealing with the $\boldsymbol{A}$ and B in matrix form),

$$\boldsymbol{A}\times B + B\times\boldsymbol{A} = 0, \tag{35}$$

But there is a problem, where $\boldsymbol{A}$ and B are not normal unit vectors. This mathematical problem can be solved by using Dirac coefficient techniques. So these coefficients are explained by the observer in the same way as that of relativistic quantum mechanics. Accordingly, for the lab observer, the definitions of $\boldsymbol{A}$ and B may be changed to $\boldsymbol{A}'$ and B', and Equation (29) can be rewritten as:

$$i\frac{\partial \boldsymbol{\mathcal{Z}}}{\partial t} = (-i\, v\boldsymbol{A}'\cdot\boldsymbol{\nabla} + B'\omega_{1m})\boldsymbol{\mathcal{Z}}\,, \tag{36}$$

With the aid of the coefficients properties (as in relativistic quantum mechanics), the second time differentiation of Eq. (36) is

$$\frac{\partial^2 \boldsymbol{\mathcal{Z}}}{\partial t^2} = (-v^2\,\nabla^2 + \omega_{1m}^2)\boldsymbol{\mathcal{Z}}\,. \tag{37}$$

Eq. (37) looks like a complex acceleration equation for the complex velocity equation. We have to mention here that Eq. (29) can be derived from the system with the aid of partial observation, whereas Eq.(37) cannot, and only from Eq. (36).

3.3 Lab observer system

As mentioned above, owing to the two possible interpretations of Eqs. (26, 27) that there are two equivalent and indivisible pictures for the partially observed system, viz., the guided point and the probability wave.

The velocity of the probability wave (v) for the lab observer is (Figure 7), note Eq. (3-A):

$$v = \frac{\partial a_{2m'}}{\partial t} = a_{2m'}\, \omega_{w'}\,. \tag{38}$$

The quantities with the prime symbol are the quantities observed by the lab observer due to the moving frame of reference. This velocity appears in Eqs. (36and 37).The guided point parameters are v_p (the velocity of the observable guide point) and $\omega_{1m'}$, whereas the parameters of the probability wave are $a_{2m'}$ (Eq. (38)) and $\omega_{w'}$. Then, one can say that the lab observer deals with the following velocities:

1. The complex velocity $\partial \boldsymbol{Z}/\partial t$, that of Eq. (36).
2. The velocity v which appeared in Eqs. (36, and 37), and Eq. (3-A), which is related to the change of the touch point (Q) location (Figure 3). For the lab observer, a_{2m} is attributed to the probability wave (Figure 7).
3. The velocity of the guided point v_p.

The observable parameters are shown in Table 1.

Table 1 The observable parameters.

Guided point	v_p	$\omega_{1m'}$	
Probability wave	v		$a_{2m'}$

The equivalent and indivisible pictures of the the probability wave and the point (guided point), can be formulated mathematically with aid of the guided point's parameters (v_p & ω_{1m}) and the probability wave's parameters (v & $a_{2m'}$) (shown in Table 1) as:

$$v_p\, \omega_{1m'} \equiv \frac{v^2}{a_{2m'}}\,. \tag{39}$$

In this equation, the left-hand side is related to the guided point form and the right side to the probability wave.

With aid of Eq. (38), Eq. (39) can be reformulated as a ratio:

$$\frac{v}{v_p} = \frac{a_{2m'}\, \omega_{w'}}{v_p} = \frac{\omega_{1m'}}{\omega_{w'}} = \mu_L\,. \tag{40}$$

Eq. (40) may serve as a lab system (point-wave system) ratio.

From Eq. (39) (20) and Eq. (38) one can say that the velocity of the point (point velocity) is

$$v_p = \frac{v^2}{\omega_{1m'}\, a_{2m'}}\,, \tag{41}$$

or the guiding distance (Figure 7) of the guided point is

$$a_{2m'} = \frac{v^2}{\omega_{1m'}\, v_p}\,, \tag{42}$$

and one can find as well

$$v^2 = v_p \omega_{1m'} a_{2m'} = v_p v_{ph} \quad , \tag{43}$$

where v_{ph} is phase velocity and is related to the phase of Eq. (27). We have to mention here that the phase velocity can be calculated, but does not refer to any real wave.

3.3.1 Lab transformations

It has been mentioned above that the quantities with the prime symbol are the quantities observed by the lab observer due to the moving frame of reference. Now, we try to find the relationship between the lab observer's measurements and the origin system quantities.

The time variation of the function $\boldsymbol{\mathcal{Z}}$ (Eq. (29), with aid of operator postulate, is:

$$i \frac{\partial \boldsymbol{\mathcal{Z}}}{\partial t} = \omega \boldsymbol{\mathcal{Z}} , \tag{44}$$

then,

$$\omega = -i\, v\boldsymbol{A} \cdot \boldsymbol{\nabla} + B\omega_{1m} . \tag{45}$$

Same can be done for Eq. (37):

$$\omega^2 = -v^2\, \nabla^2 + \omega_{1m}^2 = \omega_{1m'}^2 . \tag{46}$$

$\omega \equiv \omega_{1m'}$ is because the lab observer cannot see only one angular frequency that is related to the system.

For the lab observer, the point velocity is v_p , and the measured angular velocity is $\omega_{1m'}$. The relationship between the ω_{1m} and $\omega_{1m'}$ can be obtained from Eq. (46) with the aide of Eqs. (43, 45, 42 and 6-A):

$$\frac{\omega_{1m}}{\sqrt{1 - \frac{{v_p}^2}{v^2}}} = \omega_{1m'} . \tag{47}$$

Eq. (47) can be formulated as

$$\omega_{1m'}{}^2 v^2 = \omega_{1m'}{}^2 {v_p}^2 + \omega_{1m}{}^2 v^2 \quad . \tag{48}$$

Substituting Eq. (47) in Eq. (42), yields

$$a_{2m} \sqrt{1 - \frac{{v_p}^2}{v^2}} = a_{2m'} \quad , \tag{49}$$

where

$$a_{2m} \equiv \frac{v^2}{\omega_{1m}\; v_p} \quad . \tag{50}$$

Substituting Eq. (47) in Eq. (39) shows that

$$\frac{\omega_w}{\sqrt{1 - \frac{{v_p}^2}{v^2}}} = \omega_{w'} \quad , \tag{51}$$

where

$$\omega_w \equiv \frac{\omega_{1m}\; v_p}{v} \quad . \tag{52}$$

According to Eqs. (38), (51) and (53)

$$v = a_{2m'}\,\omega_{w'} = a_{2m}\sqrt{1-\frac{{v_p}^2}{v^2}}\frac{\omega_w}{\sqrt{1-\frac{{v_p}^2}{v^2}}} = a_{2m}\omega_w\,. \tag{53}$$

Thus, the velocity v is invariant. From Eq. (47) the $\omega_{1m'}$ is a real quantity, then $v_p < v < v_{ph}$.

3.3.2 The system, and observable system ratios

The system ratio as in Eq. (9) is:

$$\frac{a_2}{a_1} = \frac{\phi}{\vartheta} = \frac{\omega_1}{\omega_2} = \mu > 1\,. \tag{9}$$

The lab system ratio as in Eqs. (40) and (53):

$$\frac{v}{v_p} = \frac{a_{2m'}\,\omega_{w'}}{v_p} = \frac{\omega_{1m'}}{\omega_{w'}} = \frac{\omega_{1m}}{\omega_w} = \mu_L\,. \tag{54}$$

To propose a relationship between μ and μ_L , let us *assume* that

$$v_p = a_1\,\omega_{1m'}. \tag{55}$$

Then from Eqs. (9), (54) and (55) one can find that:

$$\mu = \mu_L^2. \tag{56}$$

4. System of two guided circles

The system shown in Figure 3 is for one guided circle. A system like the one in Figure 7 has two identical guided circles A and B.

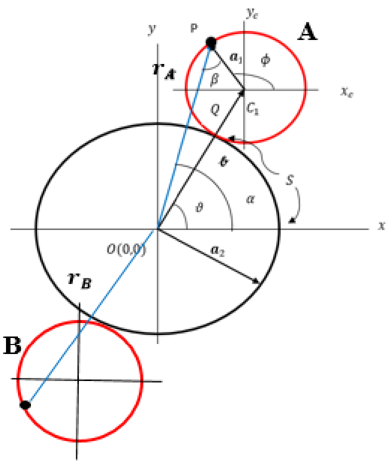


Figure 8 System of two identical guided circles.

The resultant of the position vectors of the two points:

$$\boldsymbol{r}^2 = \boldsymbol{r}_A^2 + \boldsymbol{r}_B^2 + 2\,\boldsymbol{r}_A\,\boldsymbol{r}_B \cos\theta_c\,. \tag{57}$$

The lab observer recognises two correlated points, where he/she cannot recognise each point separately. The lab observer's expression leads to:

1- $X = 0$

$$\boldsymbol{\mathcal{Z}}^2 = \boldsymbol{a}^2\left(exp^{\pm 2i\phi_A} + exp^{\pm 2i\phi_B} + 2exp^{\pm i\phi_A}\, exp^{\pm i\phi_B} \cos\theta_c\right). \tag{58}$$

We can rewrite this equation in terms of probability density (ϱ):

$$\varrho = \varrho_A + \varrho_B + 2\,\varrho_{A,B} \cos\theta_c\ . \tag{59}$$

2- The lab observer does not recognise separated parts of the system then:

$$exp^{\pm i\phi_A} = 0 \text{ and } exp^{\pm i\phi_B} = 0\,. \tag{60}$$

But two correlated points:

$$\boldsymbol{\mathcal{Z}}^2 = \boldsymbol{a}^2\left(2exp^{\pm i\phi_A}\, exp^{\pm i\phi_B} \cos\theta_c\right), \tag{61}$$

or

$$\boldsymbol{\mathcal{Z}}^2 = \boldsymbol{a}^2 2exp^{\pm i(\phi_A+\phi_B)} \cos\theta_c\,. \tag{62}$$

The complex function is

$$\mathcal{Z} = a\sqrt{2}exp^{\pm\frac{1}{2}i(\phi_A+\phi_B)}\sqrt{\cos\theta_c}\,, \tag{63}$$

or

$$\mathcal{Z}(r_A, r_B, \vartheta_A, \vartheta_B, t) = a\sqrt{2}exp^{\pm\frac{1}{2}i(\boldsymbol{k}_{2\mathrm{A}}\cdot \boldsymbol{s}_A - \omega_{1\mathrm{A}} t_A + \boldsymbol{k}_{2\mathrm{B}}\cdot \boldsymbol{s}_B - \omega_{1\mathrm{B}} t_B)}\sqrt{\cos\theta_c}\,. \tag{64}$$

It is obvious that the total dimensions of the system is 2N (N is the number of the guided circles in the plane, and 2 is related to 2 dimensions). The lab observer notes two separated points, then the dimension for the observer is 2N.

5. Remarks and Comparisons

- Comparisons

The above sections (3 and 4) demonstrate the lab observer's equations for the observed system. Table 2 compares these results with the conventional relativistic quantum mechanics forms in two sets of equations. The quantum mechanics equations are presented without $\hbar$ to show the kinematical forms.

Table 2. Comparisons of the equations of conventional quantum mechanics and special relativity with the analogical model forms.

Conventional definition	Conventional equations of quantum mechanics and special relativity	Analogical model forms	Analogical definition
Dirac wave function	$\psi_D = u_D \exp i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)$	$\boldsymbol{Z} = \boldsymbol{a}_{2m} \exp \pm i(\boldsymbol{k}_{2m}\cdot\boldsymbol{s} - \omega_{1m} t)$	*Z-complex vector*
Dirac equation	$i\frac{\partial\psi}{\partial t} = (-ic\boldsymbol{\alpha}\cdot\nabla + \beta\omega)\psi$	$i\frac{\partial\boldsymbol{Z}}{\partial t} = (-iv\boldsymbol{A}\cdot\nabla + B\omega_{1m})\boldsymbol{Z}$	*Complex velocity equation*
The coefficients	$\boldsymbol{\alpha}$ and β	$\boldsymbol{A}$ and B	*Coefficients*
Property	$\boldsymbol{\alpha}_i\boldsymbol{\alpha}_j + \boldsymbol{\alpha}_j\boldsymbol{\alpha}_i = 0$	$\boldsymbol{A}_\theta\cdot\boldsymbol{A}_\varphi + \boldsymbol{A}_\varphi\cdot\boldsymbol{A}_\theta = 0$	*Property*
Property	$\boldsymbol{\alpha}_i\boldsymbol{\alpha}_i + \boldsymbol{\alpha}_i\boldsymbol{\alpha}_i = 2$	$\boldsymbol{A}_\theta\cdot\boldsymbol{A}_\theta + \boldsymbol{A}_\theta\cdot\boldsymbol{A}_\theta = 2$	*Property*
Property	$\boldsymbol{\alpha}_i^2 = \beta^2 = 1$	$\boldsymbol{A}^2 = B^2 = 1$	*Property*
Property	$\boldsymbol{\alpha}_i\beta + \beta\boldsymbol{\alpha}_i = 0$	$\boldsymbol{A}B + B\boldsymbol{A} = 0$	*Property*
Klein-Gordon equation	$\frac{\partial^2\psi}{\partial t^2} = [-c^2\nabla^2 + \omega^2]\psi$	$\frac{\partial^2\boldsymbol{Z}}{\partial t^2} = [-v^2\nabla^2 + {\omega_{1m}}^2]\boldsymbol{Z}$	*Complex acceleration equation*
Light speed	$c < v$ c is constant	$v < v_p$ v is constant	*Wave speed*
Lorentz factor	$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$	$\frac{1}{\sqrt{1-\frac{{v_p}^2}{v^2}}}$	*Lab transformation*
Relativistic mass (angular frequency) $\times\frac{\hbar}{c^2}$	$\omega = \frac{\omega_\circ}{\sqrt{1-\frac{v^2}{c^2}}}$	$\omega_{1m'} = \frac{\omega_{1m}}{\sqrt{1-\frac{{v_p}^2}{v^2}}}$	*Lab transformation*
Length contraction	$\Delta L = \Delta L_\circ\sqrt{1-\frac{v^2}{c^2}}$	$a_{2m'} = a_{2m}\sqrt{1-\frac{{v_p}^2}{v^2}}$	*Lab transformation*
Four-vector $\times\left(\frac{\hbar}{c^2}\right)^2$	$\omega^2c^2 = \omega^2v^2 + {\omega_\circ}^2c^2$	${\omega_{1m'}}^2v^2 = {\omega_{1m'}}^2{v_p}^2 + {\omega_{1m}}^2v^2$	*Lab space*
Phase and group velocities	$c^2 = vv_{ph}$	$v^2 = v_p\omega_{1m}a_{2m}$ $v_{ph} = \omega_{1m}a_{2m}$	*Lab system velocities*
de Broglie equation	$\lambda = \frac{\hbar}{m\,v} = \frac{c^2}{\omega v}$	$a_{2m'} = \frac{v^2}{\omega_{1m'}v_p}$,	*Guiding point distance*

- The rest state of the point

Equation (47) shows the observable angular frequency of the point ($\omega_{1m'}$) and its relationship with the velocity of the point, relative to the lab observer (v_p). Let us express this frequency as

$$\omega_{1m\prime} = \omega_1 + \omega_x \,. \tag{65}$$

where ω_x is the additional quantity owing to the relative velocity (v_p). From Eq. (47) , Eq(43),and Eq. (65), we can get the ratio (in terms of the phase velocity v_{ph}):

$$\frac{\omega_1}{\omega_x} = \frac{1}{\frac{1}{\sqrt{1 - \frac{v_p}{v_{ph}}}} - 1} \,. \tag{66}$$

Figure 9 shows this relationship.

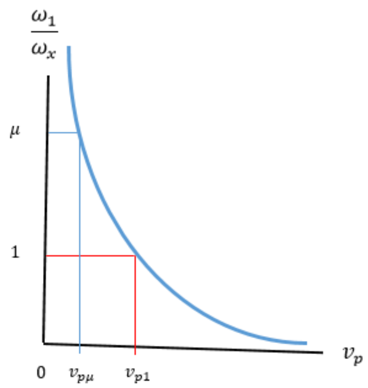


Figure 9 The relationship between the ratio ω_1/ω_x and the observable velocity of the point.

From Eq. (66), we can find the observable velocity in terms of the ratio μ

$$v_{p\mu} = \left[1 - \left(\frac{\mu}{1+\mu}\right)^2\right] v_{ph} \,. \tag{67}$$

For the ratio $\omega_1/\omega_2 = \mu$, and due to the partial observation ($\mu \gg 1$), then

$$v_{p\mu} \approx \left[1 - \left(\frac{\mu}{\mu}\right)^2\right] v_{ph} = 0 \,. \tag{68}$$

This case may represent the rest state for the lab observer. The velocity v_p is related to the guided point in lab observation.

- The point trajectory

The system may appear as in rest state, as shown above (Eq. (68)). If the system (Figure 3) is in motion, and for simplicity, let us say in linear motion. In other words, the centre O (0,0) moves in the x direction ($\boldsymbol{x}$), then the trajectory ($\boldsymbol{\mathfrak{T}}$) of the point becomes:

$$\boldsymbol{\mathfrak{T}} = \boldsymbol{x} + \boldsymbol{r} \,, \tag{69}$$

$\boldsymbol{r}$ is described by Eqs. (7) or (11). The velocity of the point is

$$\boldsymbol{v}_p = \frac{d\,\boldsymbol{\mathfrak{T}}}{dt} = \boldsymbol{v}_{p_x} + \frac{d\,\boldsymbol{r}}{d\,t} \,, \tag{70}$$

For the lab observer (when $\gg d_\lambda$, or observable) the trajectory ($\boldsymbol{Q}$) is

$$\boldsymbol{Q} = \boldsymbol{v}_{p_x} t + \boldsymbol{\mathcal{Z}} \,, \tag{71}$$

where $\boldsymbol{v}_{p_x}$ is the velocity of the centre O(0,0) in the x direction. The trajectory $\boldsymbol{Q}$ is a complex function ($\mathfrak{T} \to \boldsymbol{Q}$). The velocity of the point ($\boldsymbol{v}_{pQ}$) is:

$$\boldsymbol{v}_{pQ} = \frac{d\,\boldsymbol{Q}}{dt} = \boldsymbol{v}_{p_x} + \frac{d\,\boldsymbol{\mathcal{Z}}}{d\,t}\,, \qquad (72)$$

The complex velocity ($d\boldsymbol{\mathcal{Z}}/dt$) is described in Eq. (36). $\boldsymbol{v}_{pQ}$ is a complex velocity.

- Is the system a fine structure?

If our terminology of lab observer corresponds to the relativistic observer, then any unobservable is meant to be out of the scope of special relativity. The classical electron radius (r_e) is out of the scope of relativity, and our a_1 is out of the scope of lab observation. Then we can *assume* that

$$r_e = 2.81794092 \times 10^{-15}\, m \equiv a_1 \qquad (73)$$

For the ground state of the election in the hydrogen atom, Bohr radius (r_B) is within the scope of relativity, and our a_2 is observable for the lab observer. So, we can *assume* as well that:

$$r_B = 0.529177249\ \times 10^{-10} m \equiv a_2 \qquad (74)$$

Then, the system ratio (Eq, (7)) may be equal to:

$$\frac{a_1}{a_2} = \frac{r_e}{r_B} = 5.325136191 \times 10^{-5} = (0.00729735075)^2 = \alpha^2 \qquad (75)$$

where α is the fine structure constant. and accordingly:

$$\frac{1}{\alpha^2} = \mu = 18778.87441 \qquad (76)$$

This result may agree with the approximation of partial observation.

Electron velocity in ground state of the hydrogen atom (v) may correspond to our v_p

$$v_p \equiv 2.1876961417\ \times 10^6\ m/s \qquad (77)$$

The light speed (c) may correspond to our v as shown above. Then the lab ratio (Eq. (40)) is:

$$\frac{v_p}{v} = \frac{2.1876961417\ \times 10^6}{2.99792458 \times 10^8} = 0.007297368838 \qquad (78)$$

and

$$\frac{v_p}{v} = \frac{1}{\mu_L} \equiv \alpha \qquad (79)$$

Then $\mu_L \equiv 137.0356936$, which is equivalent to coupling constants of the electromagnetic force.

- The analogy of the wave function

As shown in Table 2, the complex vector is an analogy for the Dirac wave function. The complex vector is related to the position vector of a point in two rolling circles, but the lab observer cannot recognise that. The usage of the quantum statistical technique by the lab observer does not mean that this complex vector is related to a medium of statistical nature or may be of an epistemic nature due to the abstract feature. This problem is similar to that of the interpretation of wave function in quantum mechanics.

- Is there an explanation for the entanglement?

Equation (64) shows a single complex function of many dimensions (2N). For the lab observer, this form may be analogous to the entanglement case. This case looks like a system of instantaneous action between two separated points.

In the same way, one may suggest a case of coupled guiding circles with a guided circle as shown in Figure 9.

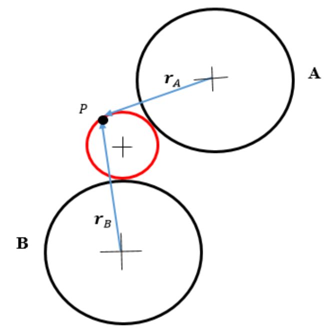


Figure 10 System of two guiding circles.

$$\mathcal{Z}_A = \boldsymbol{a}_A exp^{\pm i\phi_A} \ . \tag{80}$$

And

$$\mathcal{Z}_B = \boldsymbol{a}_B exp^{\pm i+\phi_B} \ . \tag{81}$$

The guided point is defined by two complex vectors ($\mathcal{Z}_A$ and $\mathcal{Z}_B$). So the guided point has two identical wavelengths. For the lab observer, the guided point is trapped in a region and oscillates without energy (like an object located between two springs and oscillating due to the forces of the springs). Here, the probability of finding the guided point is in two regions, as shown in Figure 11.

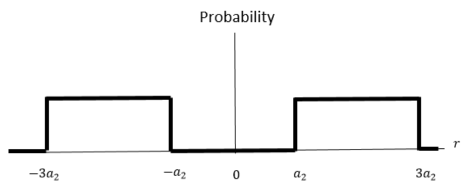


Figure 11 The probability of finding the guided point in two regions.

- Is the mass a property of the system?

Using the analogies mentioned above, one may say that

$$E \equiv \hbar\, \omega_{1m} \text{ , and } \equiv m\, v^2 \; . \tag{82}$$

Then

$$\frac{m}{\hbar} \equiv \frac{\omega_{1m}}{v^2} = \frac{\mu}{a_{2m} v} \tag{83}$$

This form shows that the mass is related to the system ratio μ. The mass looks like a property of the system.

- The principle of complementarity

Guiding and guided circles comprise a complete and *indivisible* system. For the lab observer, there is a reduced system of a guided point with a guide of the sinusoidal wave in the complex plane. These two features cannot be detected (observed) separately; in other words, both features must be associated. Neither of these features can exist without the other.

If we try to look at the matter from the slit experiment point of view, the lab observer detects the two features simultaneously:

1- The accumulation of points (light fringes) is related to the guided point feature (which-way information).
2- The separation of accumulations (dark fringes) is related to the guiding wave feature.

Thus, the interference experiment shows the two features simultaneously. The concept of a guided point mentioned above is different from the concept of *the particle* and *wave* mentioned in Bohr's principle of complementarity. The concept of a system of particle and wave has been imported from the macroscopic world, to be used in the microscopic world.
This concept (equivalent and indivisible pictures) may agree with Afshar's experiment [38].

- Partial observation and Quantum observation

Partial observation is behind the formation of the complex vector and is not related to the observation effects considered by quantum mechanics. The partial observation technique does not change the energy of the system (energy is conserved). Thus, there is no interaction with the system, and then we can regard the partial observation as an adiabatic process. This concept is closer to that of *protective measurement*, proposed within the framework of quantum mechanics by Aharonov and Vaidman [39, 40] than the *weak measurement* of small disturbances of the state [41, 42].

- Is relativistic quantum mechanics an emergent phenomenon?

The obtained $\mathcal{Z}$-complex vector, the complex velocity, and complex acceleration equations may perhaps throw light on the origin of the first and second postulates of quantum mechanics.

- Are there Zitterbewegung, and real spin?

The real rotations that are behind the complex vector is existed, as shown above in the kinematic system (Figure 3), but the lab observer cannot recognise that structure due to the partial observation problem. Both the quantum and relativity features may be based on the system deformation (due to the partial observation); thus, recognition of the rotational motions will be violated in the lab observation.

- Hidden variable or hidden system

The complex vector obtained is a result of partial observation, which causes a partially hidden system. The concept of hidden variable is based on the statistical thermodynamics frame. If the lab observer wants to establish a complete real discerption, he/she (it) should reconsider the complex vector and introduce the missing quantities without any influence of the applied statistics technique. The missing quantities are not like the hidden variables in the statistical thermodynamics.

- Is the spacetime an emergent phenomenon?

The analogy of the Lorentz factor is obtained due to the partial observation and the observer model. The similarity between the special relativity equations and the equations of the analogised model as shown in Table 2 may lead us to conclude that the spacetime is related to the same underpinning as that of quantum mechanics. Then, the spacetime is not just emergent, but is related to the same origin as quantum mechanics. The rolling circles model under the partial observation may show unification of the special relativity with quantum mechanics.

- Where is the gravity?

Within the approximation $X = 0$ of the partial observation, one can get flat spacetime formalism, which leads to a concept singularity. Ether does not exist in special relativity, or the lab observer cannot deal with the ether, as he/she (it) cannot deal with the unobservable system. According to Kostro [43] "Einstein has created two models of ether: one of the "rigid" ether [15, 16] connected with his special relativity and another of the "non-rigid" one [15] connected with his general relativity (EGR). In the two models, not any state of mechanically conceived motion (immobility included) can be attributed to the real space as such. On the other hand (especially in the EGR model), the non-atomically and non-mechanically conceived material field is never passive or quiet."

The space of underpinning structure as shown above is a curvature space; here, one may ask, is gravity related to that deep underpinning space?

- The quantum mechanics?

Gerard 't Hooft expressed his opinion about a deep level for quantum mechanics thus: "To me, it seems extremely plausible that any reasonable theory for the dynamics at the Planck scale ….. It seems quite reasonable first to try a classical, deterministic theory for the Planck domain. One might speculate then that we call quantum mechanics today, may be nothing other than an ingenious technique to handle this dynamics statistically."[44].

The present project is not in Planck scale but in a micro-scale level that cannot be detected completely. In spite of that, one may find in the present project some agreements with what has been mentioned by Gerard 't Hooft about a classical origin.

Appendix

Differentiation of the function $\boldsymbol{r}(x,t,X)$ (Eq.(11)) with respect to time is rate of change of real quantity (first order time differential equation) is:

$$\frac{\partial \boldsymbol{r}(r,t,X)}{\partial t} = \frac{\partial(\boldsymbol{a}_2+\boldsymbol{b}\sqrt{X})}{\partial t}\left\{cos\left(\boldsymbol{k}_2\cdot\boldsymbol{s}-\omega_1 t+\omega_\beta t\right) \pm\sqrt{-sin^2\left(\boldsymbol{k}_2\cdot\boldsymbol{s}-\omega_1 t+\omega_\beta t\right)+X}\right\}+(\boldsymbol{a}_2 +\boldsymbol{b}\sqrt{X})\left\{(\omega_1-\omega_\beta)\, sin\left(\boldsymbol{k}_2\cdot\boldsymbol{s}-\omega_1 t+\omega_\beta t\right) \pm\frac{(\omega_1-\omega_\beta)\, sin(\boldsymbol{k}_2\cdot\boldsymbol{s}-\omega_1 t+\omega_\beta t) cos\left(\boldsymbol{k}_2\cdot\boldsymbol{s}-\omega_1 t+\omega_\beta t\right)+\frac{\partial X}{\partial t}}{\sqrt{-sin^2\left(\boldsymbol{k}_2\cdot\boldsymbol{s}-\omega_1 t+\omega_\beta t\right)+X}}\right\}. \tag{1-A}$$

Under partial observation conditions (= 0 and $\omega_\beta t = 0$) Eq. (1-A) becomes:

$$\frac{\partial \boldsymbol{r}(r,t,0)}{\partial t} = \frac{\partial \boldsymbol{a}_{2m}}{\partial t}\left\{cos(\boldsymbol{k}_{2m}\cdot\boldsymbol{s}-\omega_{1m}t)\pm\sqrt{-sin^2(\boldsymbol{k}_{2m}\cdot\boldsymbol{s}-\omega_{1m}t)}\right\} +\boldsymbol{a}_{2m}\left\{\omega_{1m}\, sin(\boldsymbol{k}_{2m}\cdot\boldsymbol{s}-\omega_{1m}t) \pm\frac{\omega_{1m}\sin(\boldsymbol{k}_{2m}\cdot\boldsymbol{s}-\omega_{1m}t) cos(\boldsymbol{k}_{2m}\cdot\boldsymbol{s}-\omega_{1m}t)}{i\, sin(\boldsymbol{k}_{2m}\cdot\boldsymbol{s}-\omega_{1m}t)}\right\}. \tag{2-A}$$

The vector differentiation is

$$\frac{\partial \boldsymbol{a}_{2m}}{\partial t} = a_{2m}\omega\hat{\boldsymbol{e}}_\vartheta = v\,\hat{\boldsymbol{e}}_\vartheta\,. \tag{3-A}$$

Then, Eq. (2-A) becomes

$$i\frac{\partial \boldsymbol{r}(r,t,0)}{\partial t} = (i\, v\hat{\boldsymbol{e}}_\vartheta\;\cdot\boldsymbol{k}_{2m})\,\boldsymbol{a}_{2om}\{\, cos(\boldsymbol{k}_{2m}\cdot\boldsymbol{s}-\omega_{1m}t) \pm\; i\, sin(\boldsymbol{k}_{2m}\cdot\boldsymbol{s}-\omega_{1m}t)\} +\boldsymbol{a}_{2m}\omega_{1m}\{i\;\, sin(\boldsymbol{k}_{2m}\cdot\boldsymbol{s}-\omega_{1m}t)\pm cos(\boldsymbol{k}_{2m}\cdot\boldsymbol{s}-\omega_{1m}t)\}, \tag{4-A}$$

In exponential form, Eq. (4-A) becomes:

$$i\frac{\partial \boldsymbol{r}(r,t,0)}{\partial t} = (i\, v\hat{\boldsymbol{e}}_\vartheta\;\cdot\boldsymbol{k_{2m}})\,\boldsymbol{a}_{2m}\exp i\pm(\boldsymbol{k}_{2m}\cdot\boldsymbol{s}-\omega_{1m}t) + B\omega_{1m}\boldsymbol{a}_{2m}\exp i\pm(\boldsymbol{k}_{2m}\cdot\boldsymbol{s}-\omega_{1m}t) \tag{5-A}$$

Regarding that

$$\pm i\,\boldsymbol{\nabla} = \frac{1}{\boldsymbol{a_{2m}}} = \boldsymbol{k_{2m}}\,,\qquad \text{(operator)} \qquad B=\pm 1,\ \text{and}\ \boldsymbol{A}=\mp\, i\,\hat{\boldsymbol{e}}_\vartheta\ , \tag{6-A}$$

and with the aid of Eq. (6-A) then Eq. (5-A) becomes

$$i\frac{\partial \boldsymbol{\mathcal{Z}}}{\partial t} = (-i\, v\boldsymbol{A}\cdot\boldsymbol{\nabla}+B\omega_{1m})\boldsymbol{\mathcal{Z}}\,. \tag{7-A}$$

Acknowledgments: I am indebted to an anonymous referee for his remarks and report, and to Prof. Gao, the Managing Editor, for his notes. This project passed through many long stages, and at each stage, I received many comments, stimulating questions, useful criticism, and encouragements from generous scientists. Thus, I would like to thank Prof. R. Horodecki of University of Gdańsk, Prof. Claude Elbaz of Academie Europeenne Interdisciplinaire de Science, Prof. Il-Tong Cheon of Yonsei University in Korea, Prof. Goldstein of the State University of New Jersey, Dr Horn of the bbw University of Applied Sciences, Prof. Ron Johnson of the University of Surrey, Prof. Jim Al-Khalili of the University of Surrey, and Dr. Cesare Tronci of the University of Surrey.

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