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A kinematic model for a partially resolved dynamical system in a Euclidean plane

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Abstract
The work is an attempt to transfer a structure from Euclidean plane (pure geometrical) under the physical observation limit (resolving power) to a physical space (observable space). The transformation from the mathematical space to physical space passes through the observation condition. The mathematical modelling is adopted. The project is based on two stapes: (1) Looking for a simple mathematical model satisfies the definition of Euclidian plane; (2)That model is examined against three observation resolution conditions (resolved, unresolved and partially resolved). The simplest mechanical model satisfies the definition of Euclidian plane is a planetary gear. The interesting examination of the mechanical model is that is under partial resolution. That examination shows analogous equation for Euler's formula. The derived complex formula contains the resolved (observable) quantities of the mechanical system and satisfies the linear wave equation. The interpretation of this complex formula is: it is a function related to the position vector of a point in the small wheel of the partially resolved planetary gear system. The function is in terms of the observable quantities only. The work shows the possibility of transformation from real to complex space. The work is purely classical but the result of the partial resolution shows a function similar to the Quantum mechanics wave function.

Keywords: *Circular motion, Resolution power, Complex vector.*

1 Introduction

The Euler's formula **(**or Euler identity) is a pure mathematical expression, and composed of real and imaginary parts. Owing to the periodical nature, this formula has many wave applications in classical physics like wave analysis in electromagnetic field and optics. In this type of analysis, the physical quantity is represented by complex amplitude. The problem of this complex formulation is its imaginary component, which has no real meaning in physics. In the classical applications only the real part is considered. Thus, complex amplitude is not more than a technical wave representation tool, and do not reflect the real nature of the waves.

Not like classical physics, the complex formulation is genuine in quantum mechanics. The complex wave function (ψ) is the solution of Schrödinger wave equation. In this case the physical meaning is necessary. Physics could not get explanation for this complex wave function only through a real quantity (probability density,$\psi\psi^*$). This interpretation is the Copenhagen Interpretation of quantum mechanics. In spite of this statistical approach complex wave function has no physical (real) meaning. Euler's formula is a real analogous to the complex wave function.

Classical physics approaches have nothing to do with this wave function, and there is on classical model can be represented by Euler's formula.

It is possible to observe a segment of a line, object or a time interval, but the observation depends on the detection or observation tool and the size of the object or the limit of the interval. Imaging devices are limited by their resolution power.

The resolution power may be defined as the shortest distance between two points on the object that can still be distinguished by the observer as separate entities. The resolving power of an optical system depends on the wavelength (λ) of the light, the refractive index of the medium between the lens and the specimen (n), the aperture of the observing system, and the geometrical arrangement. As an example, the linear resolving power of a microscope is (Khare, 2009).

$$d_\lambda = 1.22\frac{\lambda}{2n\sin\theta} \quad (1)$$

where d_λ is the minimum linear distance between two distinguishable points and θ is half of the aperture angle of the objective. In the well resolved case, a distance (Δx) is quite clear, and the intensity peaks of the imaging device are well separated. For example, $\Delta x\rangle\rangle\rangle d_\lambda$. It is similar for the time interval when the minimum time interval (t_λ) is smaller than the measured interval (Δt) or $\Delta t\rangle\rangle\rangle t_\lambda$. The minimum time interval (t_λ) is related to the light frequency (f).

To recognise a dynamical event, both the space segment and time interval must be measured. Many researchers have reported the time and space resolving power (Miyake et al, 1998; Alvisi et al, 1999 and Yaroshenko, 2000). High resolution power is needed to measure the space segment and time interval. In this project, the focus is on the time-like interval or subluminal motion ($\beta = v/c\langle 1$). The particle velocity (v) is

$$v = \frac{\Delta x}{\Delta t} \quad (2)$$

And, for high resolution power for both space and time, it is

$$\Delta x\rangle\rangle d_\lambda \text{ and } \Delta t\rangle\rangle t_\lambda \quad (3)$$

However, because

$$v = \frac{\Delta x}{\Delta t}\langle\langle\frac{d_\lambda}{t_\lambda} \approx c \quad (4)$$

the condition of high time resolution is justified for any space resolution power.

Experimentally, there are two distinct cases, either resolved with a different level of clarity or the unresolved case. Thus, there will be either a segment of a line or a point. The same is true for the time interval.

The subluminal motion ($v\langle c$) of a point can be recognised accurately on the macroscopic scale ($\Delta x\rangle\rangle\rangle d_\lambda$ and $\Delta t\rangle\rangle\rangle t_\lambda$), where both measurements of space and time are accurate. In the case of subluminal motion on the micro scale ($\Delta x\langle\langle\langle d_\lambda$), the space segment is unresolved and cannot be recognised. However, because the motion is subluminal, it is still possible for the time interval (Δt) to be resolved or measured using the same light of observation. In this case, the motion cannot be observed, but the time can be measured through an associated phenomenon with motion similar to that of the radiation of a charged particle. This radiation can lead to a speculation of an unresolved space segment or motion. This case may be defined as partially resolved. The partial resolution refers to the possibility of observation of part of a system of two dimensions ($\Delta z, \Delta y$) when $\Delta z\rangle\rangle\rangle d_\lambda$ and the other part cannot be resolved $\Delta y\langle\langle\langle d_\lambda$. This system is partially resolved relative to a certain wavelength of light and imaging device, where the observed radiation is due to unresolved motion. Some of the space segments and the time intervals are resolved, whereas the others are unresolved.

The problem of partial resolution is an interesting subject in image processing, astronomy, modern visual technology, and simulations (Boden et al, 2009; Hsing Shih et al, 2010 and Umetani et al, 1989). In a system (of more than one dimension), there may be three possible cases: the resolved, unresolved, and partially resolved system cases.

With aid of mathematical modelling technique for a classical model the present work tries to transform to Euler's formula. The work is based on two stapes:

1-Looking for a simple mechanical model satisfies the definition of Euclidian plane.

2 -The model will be examined against the classical observation resolution conditions; the resolved, unresolved, and partially resolved system cases. The unresolved dimension will be regarded as of zero length in relative to the observation tool.

2 Motion in two dimensional Euclidean space

The Euclidean plane is a two dimensional space (R^2) in the Euclidean geometry and may be represented by the inner dot product (Prasolov & Tikhomirov, 2001). The two vectors $\vec{r}$ and $\vec{\zeta}$ (Euclidean vectors) are in the plane, the angle between them is α, and the trigonometric dot product is

$$\vec{r}\cdot\vec{\zeta} = \|\vec{r}\|\|\vec{\zeta}\|\cos\alpha \tag{5}$$

where $\|\ \|$ represents the norm. This combination represents a system of three vectors. The algebraic form based on the dot product is the cosine law:

$$\|\vec{a}\|^2 = \|\vec{r}\|^2 + \|\vec{\zeta}\|^2 - 2\|\vec{r}\|\|\vec{\zeta}\|\cos\alpha \tag{6}$$

A simple system in the Euclidean plane should satisfy the following three conditions:
1-The three vectors are linearly dependent, which is shown in eq. (6).
2-The three vectors are variables. The circular motion is a simple model to demonstrate the variable vectors. A simple model, which satisfies this condition, is shown in fig(1). Point P is rotate in the Cartesian coordinate system defined by x_c, y_c and anchored to the origin (C). The radius of rotation around x_c, y_c is a_1.
The location of the point P relative to the origin point O may be defined as

$$\vec{\zeta} = \vec{r}_c + \vec{a}_1 \tag{7}$$

where $\vec{\zeta}$ and $\vec{r}_c$ represent the position vectors of points P and C relative to the x, y coordinates. The angle α is the angle opposite $\vec{a}_1$. The origin point O (0,0) is considered as a fixed reference point. Using the cosine law to formulate $\vec{\zeta}$ in terms of $\vec{a}_1, \vec{r}_c$ and α gives

$$\|\vec{a}_1\|^2 = \|\vec{r}_c\|^2 + \|\vec{\zeta}\|^2 - 2\|\vec{r}_c\|\|\vec{\zeta}\|\cos\alpha \tag{8}$$

When the angle ϕ changes, the vector $\vec{r}_c$ changes as well.

In this case, the model fits two mutually externally tangent circles or a mechanical gear system of two wheels in the plane (fig(1)). In this model, the first wheel (of radius a_1) rolls around the second wheel of radius a_2. Then, the distance r_c is

$$r_c = |\vec{r}_c| = |\vec{a}_2| + |\vec{a}_1| \tag{9}$$

The centre of the second circle or wheel is fixed to the origin point O. The rolling of the small wheel can change the form of the triangle P, C, O in the plane. This model satisfies the three variable vectors.

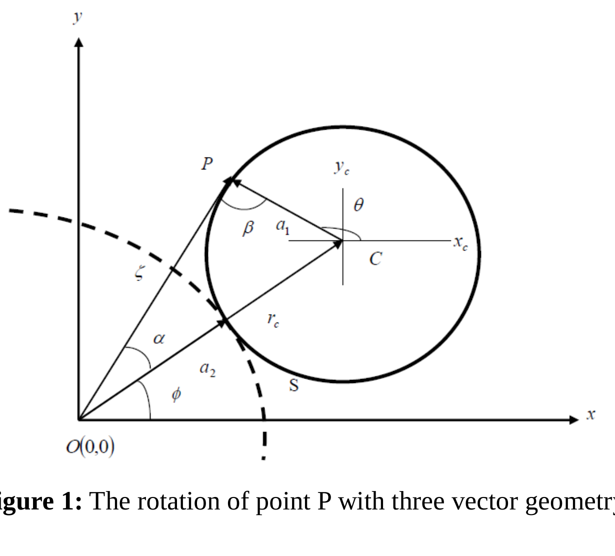

Figure 1: The rotation of point P with three vector geometry

3 - To examine the motion of point P around x_c, y_c, the observation of point P is considered relative to the position vector ($\vec{r}_c$) of the centre of rotation (x_c, y_c). In other words,

$$\zeta = f(r_c, \alpha) \tag{10}$$

Therefore, eq(8) can be rearranged as a quadratic equation:

$$\|\vec{\zeta}\|^2 - \|\vec{\zeta}\| B + C = 0 \tag{11}$$

$$B = -2r_c \cos\alpha \text{ and } C = r_c^{\ 2} - a_1^{\ 2} \tag{12}$$

The roots of the quadratic equation (eq. (8)) are

$$\|\vec{\zeta}\|_{1,2} = \zeta_{1,2} = r_c \cos\alpha \pm r_c \left\{ -\sin^2\alpha + \frac{a_1^{\ 2}}{r_c^{\ 2}} \right\}^{\frac{1}{2}} \tag{13}$$

This form for $\vec{\zeta}$ satisfies the function (10). For a simpler form, let the ratio

$$\frac{a_1}{r_c} = \Re$$

such that

$$\zeta_{1,2} = r_c \cos\alpha \pm r_c \left\{ -\sin^2\alpha + \Re^2 \right\}^{\frac{1}{2}} \tag{14}$$

These roots are real. The maximum value of the angle α is

$$\alpha_M = \sin^{-1}\Re \tag{15}$$

The relative angular displacement α can be represented in terms of the rotational angles θ and ϕ. According to fig(1), the angular displacement of P around the rotation centre C (θ) is

$$\vec{\theta} = (\vec{\alpha} + \vec{\beta}) + \vec{\phi} \tag{16}$$

Then, the relative angular displacement (α) is

$$\vec{\alpha} = \vec{\theta} - \vec{\beta} - \vec{\phi} \tag{17}$$

where $0 \le \beta \le \pi$. Then, eq.(14) becomes

$$\zeta_{1,2} = r_c \cos(\theta - \beta - \phi) \pm r_c \left\{ -\sin^2(\theta - \beta - \phi) + \Re^2 \right\}^{\frac{1}{2}} \tag{18}$$

The position of point P is controlled by the vector $\vec{a}_1, \vec{r}_c$ .eq. (18) satisfies the three conditions of the dynamical system. To examine the rotational motion, the gear ratio is

$$\frac{a_2}{a_1} = \frac{\omega}{\omega_2} \tag{19}$$

where the angular velocity of the point P is

$$\omega = \frac{d\theta}{dt} \tag{20}$$

and ω_2 is the angular velocity of the rolling motion around the origin point.

$$\alpha = \omega t - \omega_\beta t - \phi \tag{21}$$

Where, $$\omega_\beta = \frac{d\beta}{dt} \tag{22}$$

For the rolling motion on the arc s , the angle is

$$\phi = \frac{s}{a_2} \tag{23}$$

Then, eq. (18) becomes

$$\left\|\bar{\zeta}\right\| = \zeta_{1,2} = r_c \cos(\omega t - \omega_\beta t - \frac{s}{a_2}) \pm r_c \left\{ -\sin^2(\omega t - \omega_\beta t - \frac{s}{a_2}) + \Re^2 \right\}^{\frac{1}{2}} \tag{24}$$

The motion of point P has an epicycloids curve trajectory [10], as shown in Figure 4. eq(24) is the kinematical equation for the motion of point P in the Euclidean plane and relative to the position vector of the rotation of the point.

3 Observation

For a physical system, consider a trapped charged particle (of mass $m_\circ$ and charge q) in a static magnetic field (B). In this case, the electromagnetic radiation (cyclotron radiation) is related to the charge angular acceleration (a_r) [11]. The instantaneous power radiated (P) of the charge is (Larmor equation)

$$P = \frac{2}{3}\frac{q^2}{c^3}\left|\bar{a}_r\right|^2 \tag{25}$$

where c is the light velocity and ω is the particle angular frequency. This cyclotron radiation is emitted by moving charged particles trapped in a magnetic field. The cyclotron radiation frequency is

$$\omega = \frac{qB}{m_\circ} \tag{26}$$

which is the angular velocity of the trapped particle (Larmor frequency). In this case, the angular frequency of the trapped charge can be predicted by an observer regardless of the recognition of angler motion. For non relativistic velocities, the radiated field is sinusoidal, and with the aid of a Fourier transformation, a single frequency is detected.

For the model described above, the rotation of the charged particle P around x_c, y_c is responsible for cyclotron radiation. Then, the cyclotron radiation frequency is ω , which is equal to the angular frequency of the radiating particle.

3.1 Experimental observation

The experiment is as follows:

1-The dynamical system is of fixed dimensions and is represented by eqs(14 or 24).

2-In examining the system, the observation tool will be changed according to the three resolution conditions to show how the system might appear to an observer for the three possible resolutions.

3-Any resolved space segment or time interval is measurable. The resolved and unresolved parameters of the system are substituted in the modelling equation (eqs(14 or 24)).

However, observation of the system will be conducted throughout three proposed experiments according to the role of the resolution and as follows:

I - Resolved observation condition ($d_\lambda \langle\langle\langle a_1 \langle a_2$ and $t_\lambda \langle\langle\langle \Delta t$). In this case, all the dimensions of the system are larger than the minimum linear distinguishable segment (d_λ), and thus

$$d_\lambda \approx 0 \tag{27}$$

$$a_1 = a_{1m} \tag{28}$$

$$a_2 = a_{2m} 1 \tag{29}$$

$$\bar{\alpha} = \alpha_m \tag{30}$$

and, $$\omega = \omega_m \tag{31}$$

The subscript m refers to the measured value. All the parameters are measured with high acceptable accuracy. The position vector in terms of the measured values is

$$\zeta_{1,2} = r_{cm} \cos \alpha_m \pm r_{cm} \left\{ -\sin^2 \alpha_m + \Re_m{}^2 \right\}^{\frac{1}{2}} \tag{32}$$

which is a macroscopic level of observation, and the observer can recognise both vectors $\vec{a}_1$ and $\vec{a}_2$. In this case, the theoretical model fits the observed system. Fig(2) shows the variation of the magnitude of the position vector with the angle ϕ.

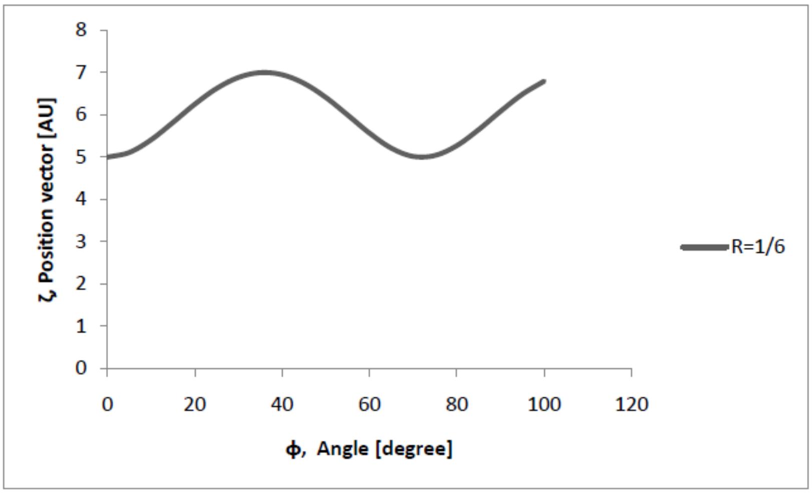

Figure 2: The variation of the position vector magnitude with the angle ϕ. The ratio $a_1 / a_2 = 1/5$.

II - Unresolved case ($d_\lambda \rangle\rangle\rangle a_2 \rangle\rangle a_1$ and $t_\lambda \langle\langle\langle \Delta t$). In this case, the system cannot be resolved and appears as an emitting particle localised at the origin point only. The emission may be attributed to the unresolved angular motion or any speculative radiation mechanism. This picture does not fit the theoretical model above. The third possibility is considered in the next section.

4 Partially resolved system

The partially resolved case is the third possibility ($a_1 \langle\langle\langle d_\lambda \langle\langle a_2$ and $t_\lambda \langle\langle\langle \Delta t$); the line segment a_2 is well recognised as in the first case, whereas a_1 has insufficient resolution, as in the second case. a_1 may be considered as an undistinguishable segment of the line. In other words, it may appear as a point. Thus, for the observer, the measured quantities are (see Table (I) for partial resolution quantities)

$$a_{1m} = 0 \tag{33}$$

Then,

$$r_{cm} = a_{2m} \tag{34}$$

$$\bar{\alpha}_m = 0 \tag{35}$$

Because the cyclotron radiation frequency can be measured and is equal to the charge particle angular frequency, ω can be estimated, or

$$\omega = \omega_m \tag{36}$$

Thus, the parameters of eq(14) are either observables (a_2, ω) or unobservable (a_1, α). Then, the theoretical model above is reduced to

$$(\zeta_{1,2})_{\Re=0} = a_{2m}\cos\alpha_m \pm a_{2m}\left\{-\sin^2\alpha_m + \Re_m^{\ 2}\right\}^{\frac{1}{2}} = a_{2m} \tag{37}$$

This result shows the position vector of an emitting particle rotating around the origin point. The result is acceptable because it is a real quantity and is related to a real physical macroscopic description of the system. fig.(3) shows the constancy of the position vector (circular motion) due to the partial resolution case. The difference between fig.(3) and fig(2) is quite clear.

Table I: The partial resolved system.

Resolved quantities $d_\lambda \langle\langle\langle a_2$ and $t_\lambda \langle\langle\langle \Delta t$	Unresolved quantities $a_1 \langle\langle\langle d_\lambda$ and $t_\lambda \langle\langle\langle \Delta t$
r_c	a_1
ω (cyclotron radiation detection)	$\Re$
ϕ	ω_β
a_2 , k_2	α

However, the radiation is not related to the rotation around the original point. Thus, the description of eq.(37) is not adequate.

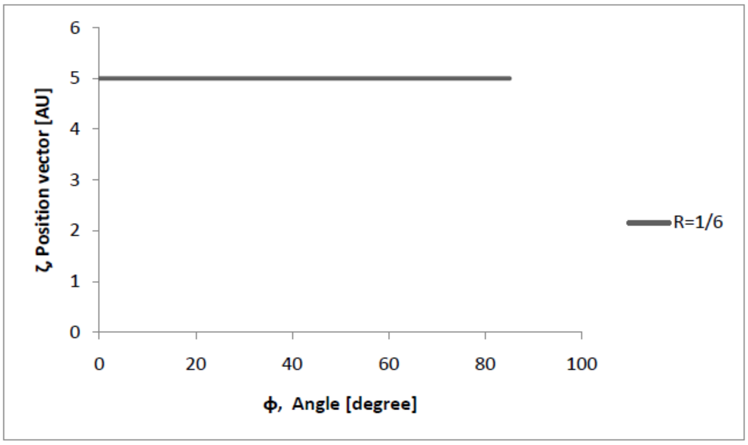

Figure 3: The position vector in the case of a partially resolved structure, where $a_{2m} = 5unit$. The ratio $a_1/a_2 = 1/5$.

In the proposed model, the observed radiation is due to unresolved angular motion around x_c, y_c. Thus, there are two points that should be clarified:

1- The description of eq(37) cannot offer any form that may lead to an explanation of the observed cyclotron radiation frequency (ω).

2- There is no relationship between the radiation frequency and the angular motion around the origin point.

3- As in the case of the unresolved system, the radiation may be attributed to unresolved motion. However, this scenario does not fit the above theoretical model well.

The condition of partial resolution ($a_1 \langle\langle\langle d_\lambda \langle\langle a_2$) neglects some useful information in eq(24), whereas the radiation measurement provides more information that is not used.

The present task is an attempt to use all the information provided and substitute it into eq(24) without violating the results of eq(37).
It was assumed above that the observer knows the model of radiation (eq(37)) already. Below is an attempt to use all of the measurements of the well resolved dimensions. The new approach is as follows:

1- Because $a_1 \langle\langle\langle d_\lambda \langle\langle a_2$, the parameters of eq(24) are either observables (a_2, ω, ϕ) or unobservable ($a_1, \alpha, \omega_\beta$). Then, eq(37) can be rewritten according to eq(24) as

$$(\zeta_{1,2})_{\Re=0} = a_{2m}\cos(\omega_m t - \omega_{\beta m} t - \frac{s}{a_{2m}}) \pm a_{2m}\left\{-\sin^2(\omega_m t - \omega_{\beta m} t - \frac{s}{a_{2m}}) + \Re_m{}^2\right\}^{\frac{1}{2}} = a_{2m} \quad \text{(38-a).}$$

Because the ratio $\Re_m = 0$,

$$(\zeta_{1,2})_{\Re=0} = a_{2m}\cos(\omega_m t - \omega_{\beta m} t - \frac{s}{a_{2m}}) \pm a_{2m}\left\{-\sin^2(\omega_m t - \omega_{\beta m} t - \frac{s}{a_{2m}})\right\}^{\frac{1}{2}} = a_{2m} \quad \text{(38-b)}$$

2- Eq(38-b) is of a complex form and can be formulated in Euler’s formula form:

$$\left(\zeta_{1,2}\right)_{\Re=0} = a_{2m}\exp \pm i(\omega_m t - \omega_{\beta m} t - \frac{s}{a_{2m}}) = a_{2m} \quad (39)$$

Accordingly, the phase equals zero and contains the unresolved quantity.

3- Eq(39) can be rearranged as

$$\left(\zeta_{1,2}\right)_{\Re=0} = a_2 \frac{\exp \pm i\left(\omega_m t - \frac{s}{a_{2m}}\right)}{\exp \pm i\omega_{\beta m} t} = a_{2m} \quad (40)$$

or

$$\frac{\exp \pm i\left(\omega_m t - \frac{s}{a_{2m}}\right)}{\exp \pm i\omega_{\beta m} t} = 1 \quad (41)$$

A new definition (η, η_β) for the complex functions is

$$\frac{\exp \pm i\left(\omega_m t - \frac{s}{a_{2m}}\right)}{\exp \pm i\omega_{\beta m} t} = \frac{\eta}{\eta_\beta} \quad (42)$$

The unity arises from the macroscopic observations. Eq(41) is a separated form of resolved and unresolved quantities. The denominator ($\omega_\beta t = \beta$) is related to the rotation around (x_c, y_c), which belongs to the unresolved structure, whereas the nominator relates to the resolved quantities.

4- This approach cannot deal with the unresolved rotational structure. Thus, the denominator of eq(42) has no meaning and will not be considered in this work.
In eq(42), η is the new function that is related to both rotational motions and contains the resolved

quantities only. It is

$$\eta_{1,2}(r,\theta,t) = \exp \pm i(\omega_m t - \frac{s}{a_{2m}}) \tag{43}$$

The subscripts 1 and 2 refer to the positive and negative phases. Because this form has wave features, it possible to state that

$$\frac{s}{a_{2m}} = \vec{k}_{2m} \cdot \vec{s} \tag{44}$$

where k_{2m} is the wave number. Then,

$$\eta_{1,2}(r,\theta,t) = \exp \pm i(\omega_m t - \vec{k}_{2m} \cdot \vec{s}) \tag{45}$$

The phase is a variable quantity, and the function (η) is continuous. This complex wave cannot reflect any physical meaning. Within the present approach, this complex form (eq(43)) is related to the gear model, but the complex form has no physical meaning.

Next, possible information is extracted. In physics, such a case can be found in quantum mechanics, where the physical meaning arises from the statistical explanation. The function depends on space and time. The separation of the temporal-spatial dependence is

$$\eta_{1,2}(r,\theta,t) = \exp \mp i\vec{k}_{2m} \cdot \vec{s} \exp \pm i\omega_m t \tag{46}$$

$$\eta_{1,2}(r,\theta,t) = \eta(r,\theta) \exp \pm i\omega_m t \tag{47}$$

$\eta(r,\theta)$ is stationary function, and it depends only on the spatial coordinate.

$$\eta_{1,,2}(r,\theta) = \exp \mp i\vec{k}_{2m} \cdot \vec{s} \tag{48}$$

To normalise the constant, the general form of this function is

$$\eta_{1,2}(r,\theta) = A \exp \mp i\vec{k}_{2m} \cdot \vec{s} \tag{49}$$

A is an arbitrary constant. The functions to be considered are

$$\eta_1(r,\theta) = A \exp i\vec{k}_{2m} \cdot \vec{s} = \eta(r,\theta) \tag{50}$$

and

$$\eta_2(r,\theta) = A \exp - i\vec{k}_{2m} \cdot \vec{s} = \eta^*(r,\theta) \tag{51}$$

According to the partial resolution results, the particle is restricted to $r = \alpha_{2m}$. However, statistically speaking, there should be a range, $0 \le s \le 2\pi a_{2m}$, such that

$$A^2 \int_{-\infty}^{\infty} \eta(r,\theta)\eta^*(r,\theta) ds = 1 \tag{52}$$

Then,

$$A = \frac{1}{\sqrt{2\pi a_{2m}}} \tag{53}$$

The normalised function becomes

$$\eta(r,\theta) = \frac{1}{\sqrt{2\pi a_{2m}}} \exp \mp i\vec{k}_{2m} \cdot \vec{s} \tag{54}$$

and the function becomes

$$\eta(r,\theta) = \frac{1}{\sqrt{2\pi a_{2m}}} \exp \mp i\vec{k}_{2m} \cdot \vec{s} \quad \text{at} \quad 0 \le s \le 2\pi a_{2m} \tag{55}$$

$$\text{and} \qquad \eta(r,\theta) = 0 \ \text{ elsewhere} \tag{56}$$

Using the normalised function, the expectation value of the particle position is

$$\int_{-\infty}^{\infty} \eta(r,\theta) s \eta^*(r,\theta) ds = \langle 2\pi a \rangle = 2\pi a_{2m} \tag{57}$$

The variance of s is ($\sigma_s{}^2$)

$$\sigma_s{}^2 = \int_{-\infty}^{\infty} (s - 2\pi a_{2m})^2 |\eta|^2 ds = \langle s^2 \rangle - (2\pi a_{2m})^2 \tag{58}$$

The standard deviation (uncertainty of position) is

$$\Delta s = \sigma_s = \sqrt{\langle s^2 \rangle - (2\pi a_{2m})^2} \tag{59}$$

Then, the uncertainty is

$$\Delta s \geq 0 \tag{60}$$

For the temporal function,

$$\eta_{T1,2}(t) = \exp \pm i\omega_m t \tag{61}$$

the normalised function is

$$\eta_{T,1,2}(t) = \sqrt{\frac{\omega_m}{2\pi}} \exp \pm i\omega_m t \tag{62}$$

and the expected time of finding the particle is

$$\int_{-\infty}^{\infty} \eta_T(t) t \eta_T{}^*(t) dt = \left\langle \frac{2\pi}{\omega_m} \right\rangle = T_m \tag{63}$$

This new result (eq(57)) represents the expected position of the particle for the observer, and it is different than that of eq(37). Thus, the expected time for the particle to be at that position is represented in eq(63).
The new function η ($\eta \in C$) describes the system better than the real function ζ ($\zeta \in R$) in the case of partial resolution. The new picture is of a probabilistic nature.

This new function does not represent a real physical quantity, and it is not like the macroscopic case. However, within this new consideration, the position of the point is of a probabilistic nature, and a_2 may represent the expected position or the most probable position of the emitting particle.
This complexity may be justified as a result of the partially resolved system.

5 Conclusions

The aid of mathematical modelling technique for a system in Euclidean, demonstrates the possibility of the transformation from real space to complex space. The technique demonstrates the feature for the structure before and after transformation. In the three transformation cases the results are easy to comprehend. Thus, the technique is an efficient educational approach for this complicated problem.
The main conclusions for the examination of the mechanical model under partial resolution are:

1-The complex features are due to two groups of reasons:

A. The nature of the system

- The system is of two space dimensions.
- The three vectors ($\vec{a}_1$, $\vec{\zeta}$,and $\vec{r}_c$) are linearly dependent.
- The observation of point P is considered relative to the position vector of the rotation centre.
- The three vectors are variables.

- The three variable vector model fits two mutually externally tangent circles or a mechanical gear system of two wheels in the plane.
- The vectors are not equal ($a_1/a_2 \langle\langle\langle 1$).
- The dynamical system is classical (subluminal velocity $c\rangle\rangle v$).

B. The observation conditions

- Time measurement is more accurate ($t \geq t_\lambda$) due to the case of subluminal velocity $c\rangle\rangle v$.
- Partial resolution of the system ($a_1\langle\langle\langle d_\lambda \langle\langle a_2$).
- There is a phenomenon related to the unresolved structure (the electromagnetic radiation in the above treatment).

2- The new picture depends on the resolved parameters of the system only.
3- Within the frame of partial resolution, there is no other alternative for a more accurate description of the system.
4- The continuous function obtained does not relied on any continuous medium beyond it.
5- The adopted statistical technique does not relied on any stochastic nature or a huge number of microscopic entities as in thermodynamics.
6- The particle linear velocity relative to the centre O is

$$v = \omega_{2m} a_{2m} \tag{64}$$

whereas the complex function shows a phase velocity of

$$v_{ph} = \frac{\omega_m}{k_2} = \omega_m a_{2m} \tag{65}$$

This velocity has no real existence where no part of the system moves with this velocity. Because $\omega_m \rangle \omega_{2m}$,

$$v \langle v_{ph} \tag{66}$$

Using wave terminology, the particle velocity corresponds to the wave group velocity, and the system appears to have normal dispersion.

7- The complex function satisfies a linear wave equation.

The work throws light on the concept of the wave function.

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