Indeterminacy and Triviality

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Penultimate version. Final version forthcoming in the *Australasian Journal of Philosophy*

Abstract. Suppose you're certain that a claim, say "Frida is tall", does not have a determinate truth value. What attitude should you take towards it? This is the question of the cognitive role of indeterminacy. This paper presents a puzzle for theories of cognitive role. Many of these theories vindicate a seemingly plausible principle: if you are fully certain that A, you are rationally required to be fully certain that A is determinate. Call this principle "Certainty". We show that Certainty, in combination with some minimal side premises, entails a very implausible claim: whenever you're certain that it's indeterminate whether A, it is rationally required that you reject A. This is a surprising result, which requires abandoning at least some intuitive views about indeterminacy and cognitive role.

1 Introduction: indeterminacy and cognitive role

Suppose that you're certain that (1) lacks a determinate truth value. I.e., you are fully confident that it is not determinately true and not determinately false.

(1) Frida is tall.

What attitude should you take towards (1)? Reject it? Suspend judgement? Adopt middling confidence? Answering this question is taking a stance on the *cognitive role of indeterminacy*.[1] It is the first step in building a theory of rational belief appropriate to sentences and propositions that lack determinate truth values.[2] We'll talk in this paper as if there were a single all-embracing notion of indeterminacy, generating a single cognitive role question. Nothing hangs on this: if there are many species of indeterminacy, then there will be just as many cognitive role questions.

[1] For explicit attempts to answer this question, see Field 2000, 2004, 2008; Schiffer 2003; Dorr 2003; Smith 2008; Williams 2014a,b; Bacon 2018

[2] For attempts to talk about vague desire and rational belief, desire and decision in the context of vagueness, see for example Edgington 1997; Williams 2016.

One class of answers to the cognitive role question recommend something that is rationally incompatible with belief. We call all these answers *exclusionary*. Middling confidence, suspension of judgement, rejection, a special mode of uncertainty—all these answers are exclusionary in the relevant sense. Despite their differences, exclusionary answers agree on the following: less than full confidence that A is determinate rationally requires less than full confidence that A. The claim that we take our starting point is the contrapositive of this:

> CERTAINTY (preliminary statement): If a subject is certain of A, they are rationally required to be certain that A is determinate.

CERTAINTY is not uncontroversial. But it's plausible enough to be taken seriously. Our paper shows that CERTAINTY, in combination with minimal side premises, gives rise to a puzzle.

We provide a full argument for CERTAINTY in the body of the paper, but here is a quick consideration to motivate it. (For now, we idealize away from the possibility of higher-order indeterminacy.) Suppose that an agent is less than fully confident that it's determinately true that Frida is tall. Assuming the usual link between credence and dispositions to bet, if this agent is also fully confident that Frida is tall, they should be willing to take bets at any odds (or at least, at extremely unfavorable odds) on the proposition that Frida is tall. But this combination seems strange. In other words, it seems strange to take bets at any odds on the proposition that Frida is tall, while having some credence that it's not determinately true that Frida is tall. If you agree, you should be sympathetic to CERTAINTY.

Among the various exclusionary positions, perhaps the least plausible is the following: when you're certain that it's indeterminate whether Frida is tall, it is rationally required that you utterly reject Frida being tall. This is *rejectionism*. It has defenders but it is a minority position.[3] Most find it surprising as a thesis about indeterminacy associated with borderline cases of paradigmatically vague properties. As we point out below, rejectionism is intolerable for at least some applications of indeterminacy—for example, for thesis that the openness of the future consists in indeterminacy of future contingents.[4] It is also intolerable on many leading accounts of the semantics of indeterminacy—for example, the thesis that indeterminate sentences or propositions have linearly ordered intermediate degrees of truth, which is most naturally paired with the rival exclusionary thesis that in the circumstances mentioned, one have an intermediate degree of belief in Frida being tall.[5]

[3]On our reading of him, the leading defender is Field 2004. As discussed below, there are some delicate exegetical issues.

[4]Discussion of the indeterminate future dates back to Aristotle. For a clear recent articulation of the view on which future contingents are indeterminate, see Barnes and Cameron 2009.

[5]See Smith 2008 for a discussion of the degrees of truth approach to indeterminacy, and Smith 2009 for an explicit articulation and defence of this anti-rejectionist position. Truth value gap semantics for indeterminacy by contrast form a neat theoretical package with indeterminacy, but in conversation, truth value gap theorists who we have asked about the issue typically bridle at the

The puzzle raised by this paper is that CERTAINTY, together with minimal side premises, entails rejectionism. This is a surprising result, which requires abandoning at least some of our intuitive views about indeterminacy and cognitive role. As we point out in the final sections, there is at least a formal analogy between our argument and various triviality arguments that have been presented in the literature on conditionals and modals.[6] Perhaps this points towards a general solution. One tempting moral, which would apply both to our puzzle and to standard triviality arguments, is that standard Bayesian assumptions about learning need to be revised when dealing with modal, conditional, and determinacy operators. But what we say leaves open the idea that our puzzle requires a different solution.

Before we start, let us make three clarifications.

First, nothing in our argument directly presupposes any particular treatment of the semantics of indeterminacy. In particular, we think of indeterminate claims as claims that are not determinately true nor determinately false. That is close to a platitude, and in particular does not commit us to the the gap theorist's distinctive claim that they are neither true nor false.

Second, we don't need the assumption that there is a unitary phenomenon deserving the label 'indeterminacy'. What we say is fully compatible with a bold pluralism about indeterminacy, on which 'indeterminacy' picks out different phenomena in different cases. What matters to us is that the puzzle arises for any kind of indeterminacy. If pluralism about indeterminacy is true, we have a plurality of puzzles rather than one.

Third, we assume that determinacy and indeterminacy are primarily properties of propositions.For current purposes, we understand propositions as structured entities that have truth conditions, but are not identical to them. Some theorists might be skeptical about the claim that propositions are indeterminate. To sidestep this concern, the argument can be reframed by using sentences. This could be done in a variety of ways. One option is to use Fodor-style sentences of mentalese. Another option is to rephrase our talk of a subject having credence n in a proposition p as being simply shorthand for an attitude that is appropriately described as "having credence n that Frida is tall". On this option, the argument could be stated without appealing to any reified notion of a proposition.

We proceed as follows. In §2, we formalize the main claims. In §3, we show how rejectionism follows from CERTAINTY. In §4, we survey the possible reactions to the argument. §§5-8 give a more in-depth discussion of the problem.

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suggestion that they should embrace rejectionism. Rejectionism is not popular even among those who one might expect to be its friends.

[6]The literature on triviality results was started by Lewis 1976; see Hájek and Hall 1994 for an overview of early triviality results. For more recent results, see Bradley 2000, 2007; see also Charlow 2016; Russell and Hawthorne 2016; Goldstein forthcoming for extensions of triviality results beyond conditionals.

2 Formalizing the main claims

Let ‘DET’ stand for "it is determinate that". We work with a space of degrees of belief, but make only weak assumptions about it. There could be three degrees of belief (belief, agnosticism, full disbelief), or there could be infinitely many degrees of belief, modelled by the real numbers between $[0,1]$, or by intervals drawn from $[0,1]$. All we assume is that these degrees of belief are at least partially ordered by comparative strength ($\geq$), and that there is a strongest degree of belief (represented by 1, which we'll label certainty) and a weakest degree of belief (represented by 0, which we'll label rejection).[7] We use ‘$\mathcal{C}$’ to denote the set of all rational belief states. Also, we assume that degrees of belief are defined over propositions, and we use Roman sans-serif capitals (‘A’, ‘B’, ‘C’, ...) as metavariables ranging over propositions.

We assume that agents have both categorical degrees of belief in propositions, and conditional degrees of belief in one proposition given another. If Cr picks out such a belief state we use $Cr(\mathsf{A})$ to pick out a degree of categorical belief in A and $Cr(\mathsf{B} \mid \mathsf{A})$ the degree of conditional belief in B given A. We understand conditional degree of belief in terms of update: $Cr(\mathsf{B} \mid \mathsf{A})$ denotes the posterior degree of belief in B had by a rational agent with prior credence function Cr, upon learning A (with certainty, as total information).[8] On standard Bayesian accounts, the conditional credence $Cr(\mathsf{B} \mid \mathsf{A})$ is also set, by definition, to be equal to the ratio of the unconditional credences $Cr(\mathsf{A} \wedge \mathsf{B})$ and $Cr(\mathsf{A})$. But, on the current proposal, this is a substantial claim—indeed, one of the routes to block the argument will consist precisely in denying the ratio formula.

Let us now formalize our main claims. Our starting principle can be expressed as follows:

$$\forall Cr \in \mathcal{C} : Cr(\mathsf{A}) = 1 \Rightarrow Cr(\text{DET } \mathsf{A}) = 1 \qquad (\text{CERTAINTY})$$

[7] In Hartry Field's work on the cognitive role question for indeterminacy, he moves between point valued and interval valued formalisms. In the point valued formalism, his view is that a rational agent certain that A is indeterminate should have credence 0 in A. In the interval valued setting, his thesis is that a rational agent certain that A is indeterminate should take interval-valued attitude [0,1], toward A. Starting from the point-valued representation, is natural to read him as a rejectionist, with that one adopt a state of minimal confidence to known-indeterminate claims (an interpretation that's enforced, we submit, by his discussion of norms of logic and the liar paradox). Redescribed in the interval valued formalism, however, the corresponding "strength of belief" will be $[0,1]$. This is perfectly consistent, so long as one adopts a reading of interval-valued strengths of belief where intervals $[0,x]$ are all states of minimal confidence. There is a rival (and more common) reading of interval valued formalism on which $[0,1]$ represents an agnostic state, strictly more confident than $[0,0]$ but less confident than $[1,1]$ and incomparable to e.g. $[0.5,0.5]$ (more generally, $[a,b] < [c,d]$ iff $b < c$). If one forced this reading onto Field's theory, then he would not count as a rejectionist by our lights, and the argument to follow, if sound, would be a reductio of the position, rather than an argument for it.

[8] An alternative that works equally well for our purposes is to understand conditional degree of belief in terms of supposition: $Cr(\mathsf{B} \mid \mathsf{A})$ denotes an agent's degree of belief in B, on the supposition that A. This alternative might allay some worries raised by the update-based construal of conditional probabilities. (For example, one might worry that update involves learning a proposition with certainty only in rare occasions.) Thanks to an anonymous referee here.

The rejectionist thesis is:

$$\forall Cr \in \mathcal{C} : Cr(\neg\textsc{det}\ \mathsf{A} \wedge \neg\textsc{det}\neg\mathsf{A}) = 1 \Rightarrow Cr(\mathsf{A}) = 0 \tag{rejectionism}$$

To prove REJECTIONISM, we prove a claim that entails it, namely the inequality:

$$\forall Cr \in \mathcal{C} : Cr(\textsc{det}\ \mathsf{A}) \geq Cr(\mathsf{A}) \tag{equiv1}$$

The entailment from EQUIV1 to REJECTIONISM goes via two principles: that A being indeterminate and it being determinate that A are inconsistent; and that if one is certain of one of a pair of inconsistent propositions, one is rationally required to reject the other. The antecedent of REJECTIONISM tells us that we are certain of A being indeterminate, so we must reject DET A which is inconsistent with it. Then by EQUIV1 we must reject A, i.e. the consequent of REJECTIONISM.[9]

Let us also observe that EQUIV2 is pretty clearly true, and so we can strengthen our conclusion to the very informative identity EQUIVALENCE:[10]

$$\forall Cr \in \mathcal{C} : Cr(\textsc{det}\ \mathsf{A}) \leq Cr(\mathsf{A}) \tag{equiv2}$$

$$\forall Cr \in \mathcal{C} : Cr(\textsc{det}\ \mathsf{A}) = Cr(\mathsf{A}). \tag{equivalence}$$

3 The proof

We are to prove EQUIV1 from CERTAINTY. Note that when $Cr(\mathsf{A}) = 0$, EQUIV1 holds, so we may assume $Cr(\mathsf{A}) \neq 0$.

We assume three side premises. The first two are constraints on rational degrees of belief:

$$Cr(\mathsf{A} \mid \mathsf{A}) = 1 \tag{identity}$$

$$Cr(\mathsf{B} \mid \mathsf{A}) = 1 \Rightarrow Cr(\mathsf{B}) \geq Cr(\mathsf{A}) \tag{bound}$$

The third side premise is a closure principle, stating that that the result of updating a rational credence on proposition C (itself of non-zero credence) is a rational credence function:

$$\forall \mathsf{C} : Cr(\bullet) \in \mathcal{C} \wedge Cr(\mathsf{C}) \neq 0 \Rightarrow Cr(\bullet \mid \mathsf{C}) \in \mathcal{C} \tag{closure}$$

[9] In fact, via this route we get a stronger principles than REJECTIONISM, i.e. a principle with a weaker antecedent:

$$\forall Cr \in \mathcal{C} : Cr(\neg\textsc{det}\ \mathsf{A}) = 1 \Rightarrow Cr(\mathsf{A}) = 0 \tag{rejectionism*}$$

In our discussion, we stick with REJECTIONISM because it seems to be the philosophically more interesting principle.

[10] We can also argue for it given a few more principles: (a) rational degree of belief doesn't drop over logical consequence; (b) determinacy is factive: DET $\mathsf{A} \models \mathsf{A}$.

We say more in defense of these side premises below. For the moment, let us flag that all of them are entailed by standard Bayesian tenets about credence—though one doesn't need to be a Bayesian to endorse them.

With these assumptions in place, it's simple to state the proof. Start with an arbitrary rational belief state Cr. By an instance of closure, $Cr(\bullet \mid \mathsf{A})$ is a rational belief state. We argue:

1. $Cr(\mathsf{A} \mid \mathsf{A}) = 1$ (from IDENTITY, applied to $Cr \in \mathcal{C}$)
2. If $Cr(\mathsf{A} \mid \mathsf{A}) = 1$, $Cr(\textsc{det}\ \mathsf{A} \mid \mathsf{A}) = 1$ (CERTAINTY, applied to $Cr(\bullet \mid \mathsf{A}) \in \mathcal{C}$)
3. $Cr(\textsc{det}\ \mathsf{A} \mid \mathsf{A}) = 1$ (from 1 and 2)
4. $Cr(\textsc{det}\ \mathsf{A}) \geq Cr(\mathsf{A})$ (from 2 and BOUND, applied to $Cr \in \mathcal{C}$)

The last line is the relevant instance of EQUIV1, as required.

4 Reactions to the argument

The argument is valid, so there are just five things one can do in response:

i. Accept EQUIV1 and so accept REJECTIONISM

ii. Reject IDENTITY

iii. Reject BOUND

iv. Reject CLOSURE

v. Reject CERTAINTY

On the face of it, none of these options seems particularly plausible. Yet at least one of them has to be right. So the proof in §3 poses a puzzle.

Notice that the puzzle holds for every single proposition. This means that we cannot respond to the puzzle simply by denying that one of the principles holds in general. E.g., suppose you think CLOSURE fails as a general thesis, because some propositions are not rationally learnable. That gets you out of one instance of the argument, but does nothing to help you get out of other instances of the argument involving propositions that are rationally learnable.

So we should distinguish two ways of reacting to the puzzle. On the one hand, we might defend an across-the-board solution. For example, we might endorse REJECTIONISM for every single proposition; or one may resist the argument by denying that IDENTITY ever holds. On the other, we might defend a piecemeal solution. For example, we might hold that CERTAINTY fails for certain propositions, and REJECTIONISM is true for others.

As we noted, some philosophers believe that 'indeterminacy' fails to pick out a unitary phenomenon. On this view, the indeterminacy of the open future is one thing, the indeterminacy of borderline vague adjectives is another, and so on. We do not take any stance on that issue. Take any species of indeterminacy that you like: we can run the argument above for an instance of that specific kind, and the choice between (i)-(v) is forced for that specific instance. A pluralist about indeterminacy could think that different cases call for different solutions. The awkward fact, however, is that for many species of indeterminacy, none of the available solutions seems attractive.

In the next sections, we discuss options (i)–(v). We won't try to settle definitively which of them is right, but we will steer the debate in directions that seem plausible to us.

4.1 Accepting REJECTIONISM

As we said, some cognitive role theorists endorse REJECTIONISM. So one might think that we just provided an argument for an across-the-board endorsement of this position. We want to resist this conclusion, which we find particularly implausible.

Why is REJECTIONISM so implausible? As just mentioned, indeterminacy shows up in many different cases. There are indeterminate occurrences of paradigm gradable adjectives (*tall*, *bald*, *red*), but there are also indeterminate occurrences of the relation *being the same person as*. Believers in the open future hold that future contingents are indeterminate. The conditional *if I roll a fair die, it will land even* is classified as indeterminate on many theories. In many of these cases, rejectionism straightforwardly conflicts with common sense. One obvious case is that of future contingents. My attitude to the indeterminate future contingent *I will catch the train this afternoon* is uncertainty, not utter disbelief. More in general, our processes of deliberation about the future seem to presuppose that propositions about the future should receive positive credence, even though it is indeterminate whether they are true. So endorsing REJECTIONISM about future contingents would be disruptive both for our ordinary deliberations and for our philosophical theorizing about them.[11] Similar considerations apply to other kinds of indeterminacy. For example, assigning positive credence to indeterminate claims about personal identity is arguably central to understanding moral and self-interested concern for the future.[12]

Perhaps these considerations can be overridden via decisive theoretical arguments.[13] But, absent these, we think that REJECTIONISM should not be in-

[11] An anonymous referee raises a worry: what if we say that it's irrational to believe that the future is indeterminate? In that case, also the rejectionist can grant that subjects have rational positive credence towards propositions about the future. We grant the referee's point, though we hasten to point out that the view that they describe is very strong.

[12] For an argument for an exclusionary answer to the cognitive role question on this sort of basis, see Williams 2014b.

[13] A referee suggests we might be being overly charitable to rejectionism, and that it is not even a prima facie option for the cognitive role of (an instance of) indeterminacy. If the referee is right, that would only intensify the puzzle this paper is articulating, but we do not endorse anything so strong. The formal articulation of rejectionism described in Williams 2016 should help interested readers see rejectionism could at least be made coherent, whether or not it is plausible.

voked as an across-the-board solution.

4.2 Rejecting IDENTITY

Rejecting IDENTITY (repeated below), whether across the board or for some cases, also seems implausible.

$$Cr(\mathsf{A} \mid \mathsf{A}) = 1 \qquad \text{(IDENTITY)}$$

All we need to derive IDENTITY are two principles that seem very safe. The first is simply the logical validity of iteration. The second is a principle saying that, if A entails B, the conditional probability of the latter given the former is 1.

$$\mathsf{A} \vDash \mathsf{A} \qquad \text{(ITERATION)}$$

$$\text{If } \mathsf{A} \vDash \mathsf{B}, \text{ then } Cr(\mathsf{B} \mid \mathsf{A}) = 1 \qquad \text{(ENTAILMENT)}$$

At the very least, it's extraordinarily implausible that we will be able to appeal to the rejection of IDENTITY as an across-the-board response to the puzzle.[14]

4.3 Rejecting BOUND

Consider now BOUND:

$$Cr(\mathsf{B} \mid \mathsf{A}) = 1 \Rightarrow Cr(\mathsf{B}) \geq Cr(\mathsf{A}) \qquad \text{(BOUND)}$$

Let us observe first that BOUND is entailed by the classical construal of conditional probability, spelled out in RATIO, together with the principle that one's credence in a conjunct is an upper bound in one's credence in a conjunction.[15]

$$Cr(\mathsf{A} \wedge \mathsf{B}) = Cr(\mathsf{B} \mid \mathsf{A}) \times Cr(\mathsf{A}) \qquad \text{(RATIO)}$$

$$Cr(\mathsf{A}) \geq Cr(\mathsf{A} \wedge \mathsf{B}) \qquad \text{(CONJUNCTION)}$$

CONJUNCTION seems extremely plausible. Of course, we could find a weird enough logic for conjunction that invalidates it. But for current purposes we won't question it. Assuming that CONJUNCTION is safe, then, rejecting BOUND entails rejecting RATIO.

Classical Bayesians take RATIO to be definitional of conditional probability. But recall that in §2 we explicitly disavowed this construal. Rather, we defined conditional credence in terms of update, and left it as an issue to be adjudicated whether RATIO holds. So one option is to deny that the notion of conditional probability that captures update can be defined in the usual way. We come back to this option in §7.

[14]Though see the discussion of degrees of determinacy and conditional probability in Williams 2016 for a precedent.

[15]Proof: assume $Cr(\mathsf{B} \mid \mathsf{A}) = 1$. Then, via RATIO, $Cr(\mathsf{A} \wedge \mathsf{B}) = Cr(\mathsf{A})$. Via CONJUNCTION, $Cr(\mathsf{B}) \geq Cr(\mathsf{A} \wedge \mathsf{B})$; by replacing $Cr(\mathsf{A} \wedge \mathsf{B})$ with $Cr(\mathsf{A})$ in the inequality it follows that $Cr(\mathsf{B}) \geq Cr(\mathsf{A})$.

Incidentally, notice that RATIO assumes that multiplication is well-defined on degrees of belief. So, in order to claim that BOUND follows from RATIO, we need more substantial assumptions about degrees of belief than the ones we have taken up in §2.

4.4 Rejecting CLOSURE

CLOSURE says that the result of updating a rational credence function on proposition C (itself having non-zero credence) is also a rational credence function.

In §2, we defined conditional probabilities just as the probabilities that are reached by a rational agent via update. So we are guaranteed that, for any proposition C that captures an agent's total evidence, conditonalizing on C has to lead to a rational credal state. This leaves room for one way in which CLOSURE could fail. It might be that some propositions cannot serve as a rational agent's total evidence. In particular, we might claim that one can only learn perfectly determinate propositions: learning A always entails also learning DET A. We will return also to this claim later on.

4.5 Rejecting CERTAINTY

CERTAINTY (repeated below) is the principle we have introduced in this paper.

$$\forall Cr \in \mathcal{C} : Cr(\mathsf{A}) = 1 \Rightarrow Cr(\textsc{det}\ \mathsf{A}) = 1 \qquad (\textsc{certainty})$$

We have already defended it informally in the introduction, and we think it is extremely compelling. It follows from extant non-rejectionist accounts of the cognitive role of indeterminacy, such as Smith 2009, so at least some in the firing line certainly can't escape in this way. This section pinpoints the theoretical damage incurred by anyone denying CERTAINTY, by presenting an explicit arguments for it from general premises.

The argument starts from a weak version of the exclusionary idea, which we can formalize as follows:

$$\forall Cr \in \mathcal{C} : Cr(\neg\textsc{det}\ \mathsf{A}) = 1 \Rightarrow Cr(\mathsf{A}) < 1 \qquad (\textsc{weak exclusionism})$$

In addition to WEAK EXCLUSIONISM, we will use the following assumption:

> ASSUMPTION. If S is fully confident that it is determinate whether A obtains, and S has less than full confidence in A, then there is some world w that is doxastically possible for S, such that S has full confidence in $\neg$A, conditional on w.

If we set aside indeterminacy entirely, it is very plausible that less than full confidence in a proposition entails that there is some doxastic possibility relative to which we have full confidence in the negation of the proposition. Considerations of indeterminacy may mean we may not want to endorse it in full generality, but we should continue to endorse the version stated, since it includes the *caveat* that from the point of view of the subject, there's no relevant indeterminacy involved. The subsidiary assumption needs no such *caveat*. It is simply that when you are fully confident in a proposition, you are fully confident in it conditional on the obtaining of any world that is doxastically possible for us.

What we give now is an argument for CERTAINTY, for the special case that the subject in question is fully confident that there is no higher order indeterminacy in the proposition being targeted. We argue that under that extra assumption, the contrapositive of CERTAINTY obtains. So suppose that $Cr(\text{DET A}) < 1$. By the main theoretical assumption, there must be some world w given non-zero credence such that $Cr(\neg\text{DET A} \mid w) = 1$. But then, by the exclusionary role for indeterminacy (and strictly appealing to the relevant instance of CLOSURE) we have that $Cr(\mathsf{A} \mid w) < 1$. The subsidiary premise now kicks in: since w was doxastically possible for us, it cannot be the case that $Cr(\mathsf{A}) = 1$. Contraposing, when $Cr(\mathsf{A}) = 1$, it cannot be that $Cr(\text{DET A}) < 1$, i.e. it must be that $Cr(\text{DET A}) = 1$.

The argument won't run as stated when we have higher order indeterminacy in the picture, and we'll soon be considering that issue systematically. It may be natural to think that there is higher-order indeterminacy in some cases, e.g. paradigmatic borderline cases of vague predicates.[16] But thinking that there is higher-order indeterminacy is very unnatural in others, e.g. future contingents or indeterminacy in personal identity. For these applications, the argument in this section suggests that CERTAINTY is viable and one should look elsewhere to solve the puzzle.

5 Roadmap

The rest of the paper expands the discussion in three directions. In §6 we show how the argument can be generalized in various ways, to sidestep different kinds of resistance maneuvers. In §7, we discuss in detail the possibility of blocking the argument by rejecting closure, in the light of a result developed by *[name omitted]*. In §8, we explore the analogy with triviality arguments for modals and conditionals.

6 Generalizations

This section generalizes the argument. Some theorists might try to block the argument by rejecting CERTAINTY, appealing to various motivations. We show that these attempts still lead to results that are unacceptable to non-rejectionists.

6.1 Hedging CERTAINTY

Here is one reason to think that CERTAINTY is too strong. Suppose you are walking through a rose garden, looking down a line of roses that incrementally change from clear red to clear orange. You might think that you can remain certain that a rose is red in cases where we have at least some doubt whether

[16]A clarification: here and in what follows we assume that borderline cases are cases of indeterminacy. (Some theorists, notably epistemicists, do not subscribe to this use of 'borderline', since they take all borderline sentences to have determinate truth values.)

the rose is determinately red.[17] That is: while confidence in a rose being determinately red cannot be dramatically lower than your confidence in it being red, one might think that it is rationally permissible for there to be a slight drop between the latter and the former.

If we take this suggestion on board, we will replace CERTAINTY with the following modified principle:

$$Cr(\mathsf{A}) = 1 \Rightarrow Cr(\text{DET } \mathsf{A}) \approx 1 \qquad \text{(HEDGED CERTAINTY)}$$

Here $x \approx y$ presupposes a new relation among degrees of belief: that of being near one another. If degrees of belief are modelled by real numbers in $[0,1]$, $x \approx y$ may be read as $|x - y| < \varepsilon$ for some small ε.

The following variant of BOUND is just as plausible as the original:[18]

$$Cr(\mathsf{B} \mid \mathsf{A}) \approx 1 \Rightarrow Cr(\mathsf{B}) \gtrsim Cr(\mathsf{A}) \qquad \text{(HEDGED BOUND)}$$

But now, a result follows that is similar to our original one. Putting HEDGED CERTAINTY and HEDGED BOUND together with IDENTITY and CLOSURE, the argument proceeds as before, with the conclusion:

$$Cr(\text{DET } \mathsf{A}) \gtrsim Cr(\mathsf{A}) \qquad \text{(HEDGED EQUIV1)}$$

And this establishes a hedged version of rejectionism:

$$Cr(\neg\text{DET } \mathsf{A} \wedge \neg\text{DET}\neg\mathsf{A}) = 1 \Rightarrow Cr(\mathsf{A}) \approx 0 \qquad \text{(HEDGED REJECTIONISM)}$$

Dialectically, this is just as bad for anti-rejectionists as the original result. Their thesis was that the appropriate response to indeterminacy was some state of uncertainty—middling credence, agnosticism, or whatever—that is incompatible with hedged rejection.

6.2 Higher Order Indeterminacy

Our interlocutor may at this point withdraw even from HEDGED CERTAINTY, and reframe her worry. Perhaps the real worry with our almost-borderline red rose was higher order vagueness. If there are higher order borderline cases—borderline cases between it being indeterminate whether a rose is red and it being determinate whether a rose is red—then, she reasons, it must be possible to have a determinately red rose that is not determinately determinately red. Certainty that such a rose is red may be appropriate (since the rose *is* determinately red). But *ex hypothesi*, the rose is not determinately determinately red. Hence the appropriate attitude towards the proposition *the rose is determinately red* could be the very kind of uncertainty appropriate to propositions that are indeterminate. (Note too that, suspiciously, we idealized away from higher order indeterminacy in motivating CERTAINTY).

[17] Thanks to *[name omitted]* for this kind of case.

[18] Here we're understanding $\gtrsim$ as follows: $x \gtrsim y \iff x \geq y \vee x \approx y$.

We might question our interlocutor's case, since it is not clear that we can (with rational certainty) identify a case as one of determinate but not determinate-determinate redness, as she supposes. We think she should not have given up on HEDGED CERTAINTY so quickly. But rather than push this point, we develop another route to something tantamount to our original conclusion. One of the reasons for interest in the variant presented below is that—like the original, but unlike the hedged version just given—it is very neutral on the quantity and structure of degrees of belief, not requiring notions like "near certainty" in its formulation.

This variant of our argument uses a weakened determinacy operator, $\textsc{det}_w$. We drop CERTAINTY and instead start from:

$$Cr(\mathsf{A}) = 1 \Rightarrow Cr(\textsc{det}_w\mathsf{A}) = 1 \qquad \text{(WEAK CERTAINTY)}$$

What is weak determinacy? If determinacy means: has degree of truth 1, then weak determinacy may be: having degree of truth at least 0.75. If determinacy requires that a proposition be true on every sharpening, then weak determinacy may be: being true on three quarters of the sharpenings.

We run the argument exactly as before, substituting $\textsc{det}_w$ for DET throughout and using only the original side premises IDENTITY, BOUND, and CLOSURE. We obtain:

$$Cr(\textsc{det}_w\mathsf{A}) \geq Cr(\mathsf{A}) \qquad \text{(WEAK EQUIV1)}$$

And this establishes another variant of rejectionism:

$$Cr(\neg\textsc{det}_w\mathsf{A} \wedge \neg\textsc{det}_w\neg\mathsf{A}) = 1 \Rightarrow Cr(\mathsf{A}) = 0 \qquad \text{(WEAK REJECTIONISM)}$$

$\textsc{det}_w$ is entailed by DET, but does not entail it, so it is easier to be confident that $\textsc{det}_w$ applies to a proposition than that DET applies to it. WEAKENED CERTAINTY is indeed a weaker claim that CERTAINTY. Dually, the notion of indeterminacy that is defined out of weak determinacy is stronger than straight indeterminacy: only when you are certain that A is a 'central case' of indeterminacy should you be certain that neither it nor its negation is even weakly determinate. But of course, rivals to rejectionism who think that uncertainty is called for when you are certain that something is indeterminate will a fortiori think that uncertainty (rather than rejection) is called for in these central cases of indeterminacy. So WEAK REJECTIONISM is not something they can live with.[19]

6.3 Regularity

We consider one final attempt at sidestepping the argument. Some Bayesians say that it is irrational to ever become certain of any proposition that is not a

[19] Higher order weak determinacy is little explored, but of obvious relevance here. For example, on Williamson's fixed-width margin of error models for higher order vagueness (1992; 1994) nothing is higher-order weakly determinate at all orders. This could form the basis for an independent objection to WEAK CERTAINTY. However, our initial investigations show that there are natural variants of these models that avoid this feature. An objection from this quarter would have to dig into the plausibility of the various detailed modelling assumptions in play.

logical truth: rationality requires that we always remain open to the possibility of error. This is a controversial general thesis about rationality, known as 'regularity'.In the context of our argument, regularity is relevant because it clashes badly with CLOSURE.[20] CLOSURE states that, if a credence function Cr counts as rational, the credence function $Cr(\bullet \mid \mathsf{A})$ that we obtain by conditonalizing on A also counts as rational. Regularity states that conditionalizing on a proposition A is rational only if A is a logical truth.

To address these concerns, we make use of notions of approximate certainty again. Assume that we cannot rationally learn contingent propositions with certainty. Plausibly, though, we are able to become *nearly* certain of them: we write $Cr(\bullet \uparrow \mathsf{C})$ for the result of updating on C in the sense of becoming almost certain of it. In the typical Bayesian framework where degrees of belief are modelled by the unit interval $[0,1]$, $Cr(\bullet \uparrow \mathsf{C})$ can be characterized as the result of Jeffrey-conditionalizing on a partition that includes C, and where C's coefficient is $1-\varepsilon$, where ε is the very constant used to characterize $\approx$ earlier.

We can now run a variant of our argument with the following premises, built around a notion of APPROXIMATE CLOSURE specifically designed to appeal to fans of the regularity constraint. For this, we need not just notions of approximate equality $\approx$, but approximate approximate equality $\approx\approx$, approximate approximate approximate equality $\approx\approx\approx$ etc.[21] The argument runs:

$$Cr(\mathsf{A}) \approx 1 \Rightarrow Cr(\textsc{det}\ \mathsf{A}) \approx 1 \qquad \text{(APPROXIMATE CERTAINTY)}$$

$$Cr(\mathsf{A} \uparrow \mathsf{A}) \approx 1 \qquad \text{(APPROXIMATE IDENTITY)}$$

$$Cr(\mathsf{B} \uparrow \mathsf{A}) \approx 1 \Rightarrow Cr(\mathsf{B}) \gtrapprox Cr(\mathsf{A}) \qquad \text{(APPROXIMATE BOUND)}$$

$$\forall \mathsf{C} : Cr(\bullet) \in \mathcal{C} \wedge Cr(\mathsf{C}) \neq 0 \Rightarrow Cr(\bullet \uparrow \mathsf{C}) \in \mathcal{C} \qquad \text{(APPROXIMATE CLOSURE)}$$

The argument then proceeds exactly as before, with the conclusion:

$$Cr(\textsc{det}\ \mathsf{A}) \gtrapprox Cr(\mathsf{A}) \qquad \text{(APPROX EQUIV1)}$$

And this establishes an approximate version of rejectionism:

$$Cr(\neg\textsc{det}\ \mathsf{A} \wedge \neg\textsc{det}\neg\mathsf{A}) \approx 1 \Longrightarrow Cr(\mathsf{A}) \approx\approx\approx 0 \qquad \text{(APPROX REJECTIONISM)}$$

This is no better for rivals to rejectionism than was the original conclusion. If you think that agnosticism or middling credence is the right response to indeterminacy, then you shouldn't think that if we're nearly certain that something is indeterminate, we're forced to be within a small distance of 1 (approximately approximately approximately equal to 1)—but that is what approximate rejectionism tells us.

Even for those who do not insist on regularity, the above form of our argument holds interest. One reaction to the argument that we discuss in §7 below

[20] This response was first put to us by *[name omitted for anonymous review]* Compare Lewis 1986.

[21] If credences are real numbers, we have $x \approx y$ is true iff $|x-y| \leq \varepsilon$, and analogously, $x \approx\approx y$ iff $|x-y| \leq 2\varepsilon$, $x \approx\approx\approx y$ $|x-y| \leq 3\varepsilon$, etc. $x \gtrapprox y := x \geq y \vee x \approx\approx y$.

holds that some propositions could be rationally learned with certainty (*pace* regularity), but that they have to be perfectly determinate. But the discussion in this section shows that, to resist all versions of the argument, one must hold that possibly vague propositions are unlearnable in a much stronger sense: we cannot even learn them in the Jeffrey-conditionalization sense.

Before moving on, let us we point out that the resources that we have deployed throughout this section can be brought together. Approximate and hedged versions of our argument can be combined; we discuss the resulting principles in a footnote.[22]

7 Denying closure or ratio: restricting principles to perfectly determinate propositions

In §4, we saw that two of the most promising strategies for resisting the argument were linked to changing our understanding of conditional probability and update. In this section, we investigate these routes in further detail.

Let us first consider denying CLOSURE (repeated below).[23]

$$\forall \mathsf{C} : Cr(\bullet) \in \mathcal{C} \wedge Cr(\mathsf{C}) \neq 0 \Rightarrow Cr(\bullet \mid \mathsf{C}) \in \mathcal{C} \qquad (\textsc{closure})$$

Each instance of CLOSURE follows from two claims. First: a rational agent with prior belief state Cr, who learns C as total information with certainty, has posterior (categorical) beliefs given by $Cr(\bullet \mid \mathsf{C})$. Second: the particular C involved in the instance of closure is learnable: it is possible to learn it, with certainty, as total information.

Resistance on the first point is ruled out, given the way we are understanding conditional probability in the present context. In §2, we have simply stipulated that $Cr(\mathsf{B} \mid \mathsf{A})$ denotes the posterior degree of belief in B had by a rational agent with prior credence function Cr, upon learning A with certainty as total information. So the only route to deny CLOSURE is to target the second condition: we might deny that some propositions can be learned as one's total information.

[22]Someone might have the concern that a belief in A being within ϵ of 1 doesn't guarantee that our belief in DET A is within ϵ of 1. But this interlocutor may endorse a suitably hedged variant of the principle: that the consequent follows if A meets some tighter bound–within some δ of 1, where $\delta < \varepsilon$. Writing $\simeq$ for this tighter approximation, we can combine approximate and hedged versions of our argument via the following premises:

$$Cr(\mathsf{A}) \simeq 1 \Rightarrow Cr(\textsc{det}_w \mathsf{A}) \approx 1 \qquad (\textsc{hedged weak approx certainty})$$

$$Cr(\mathsf{A} \uparrow \mathsf{A}) \simeq 1 \qquad (\textsc{hedged approx identity})$$

$$Cr(\mathsf{B} \uparrow \mathsf{A}) \approx 1 \Rightarrow Cr(\mathsf{B}) \gtrapprox Cr(\mathsf{A}) \qquad (\textsc{approximate bound})$$

$$\forall \mathsf{C} : Cr(\bullet) \in \mathcal{C} \wedge Cr(\mathsf{C}) \neq 0 \Rightarrow Cr(\bullet \uparrow \mathsf{C}) \in \mathcal{C} \qquad (\textsc{approximate closure})$$

There are contexts where this variant of our argument—strengthened in several dimensions—is required.

[23]For discussion of this material, we are indebted to *names omitted for blind review* .

Even though CLOSURE involves universal quantification over propositions, all we need to run an instance of our argument is a particular instance of CLOSURE. So, if we want to pursue an across-the-board solution to the problem via this route, we need to deny all instances of CLOSURE that involve those propositions for which the conclusion is unacceptable, by categorizing the proposition in question as not rationally learnable. That raises the question: what propositions are left as learnable? Here is a partial result: [*name omitted*] has shown in correspondence, *modulo* standard classical Bayesian assumptions CERTAINTY is simply equivalent to the claim that all learnable propositions are perfectly determinate.[24] In a classical context, then, CLOSURE fails because $Cr(\bullet \mid \mathsf{C})$ only picks out a rational credence function when C is perfectly determinate. (Strictly speaking, $Cr(\bullet \mid \mathsf{C})$ is simply undefined for other C, since $Cr(\bullet \mid \mathsf{C})$ represents the result of *rationally* updating Cr on C.)

This is not the place to adjudicate the suggestion that every learnable proposition is perfectly determinate. Let us just notice that this claim is highly controversial, and that it has been forcefully denied recently. For example, Andrew Bacon (2018) argues that the totality of what we learn through perception, reflection and testimony is inexact and potentially vague information.

Now, let us turn to the other option: denying RATIO (repeated below), with the goal of invalidating BOUND.

$$Cr(\mathsf{A} \wedge \mathsf{B}) = Cr(\mathsf{B} \mid \mathsf{A}) \times Cr(\mathsf{A}) \qquad \text{(RATIO)}$$

Building on our discussion of the failure of CLOSURE, there is a natural way to motivate the failure of RATIO. This time we grant that subjects may rationally update on propositions that are not perfectly determinate, and hence that $Cr(\bullet \mid \mathsf{C})$ is well-defined for all C with positive credence. But we may now claim that RATIO holds if and only if the proposition that is updated on is perfectly determinate, and *[name's]* result shows us that this restricted claim is tenable in a classical Bayesian setting, minus the usual definition of conditional probability.

Choosing this route might be a plausible option for those who want to explore a solution similar to the denial of closure, but want to allow that we may learn not perfectly determinate propositions. So far as we can see, the main hurdle for this route is to develop a plausible philosophical justification for the restriction of RATIO. We leave this task to future work.

8 Analogies with modal triviality

Our proof has close relatives in the literature on conditionals and modality. A number of theorists (Stalnaker 1970, Adams 1975, Edgington 1995) have

[24] More precisely: *[name]* introduces the notion of a *determinacy fixed-point*, defined as follows:

A is a determinacy fixed-point just in case $\text{DET}\ \mathsf{A} = \mathsf{A}$.

[name] shows that CERTAINTY is equivalent to the claim that evidential propositions are determinacy fixed-points.

pointed out an intuitive constraint on credences in conditionals: a subject's credences in a conditional should line up with their conditional credences in the consequent, given the antecedent.

Stalnaker's Thesis.
For all A, B, and for all $Cr \in \mathcal{C}$: $Cr(\mathsf{A} > \mathsf{B}) = Cr(\mathsf{B} \mid \mathsf{A})$

The unrestricted endorsement of Stalnaker's Thesis is notoriously problematic. Appealing to Stalnaker's Thesis and to standard Bayesian principles, Lewis (1976) shows that we can prove that the probability of a conditional $\mathsf{A} > \mathsf{B}$ has to be identical to the probability of its consequent—an unacceptable result.

Recent literature on triviality has pointed out that similarly unacceptable consequences can be reached via assumptions that are strictly weaker and no less intuitive. Also, it has been pointed out that triviality is not confined to conditionals, but rather generalizes to modalized statements of various sort.[25] *[reference omitted]* lays out a template for generating triviality results of this kind. This template starts from a constraint of the following form, for specific A and B:

$$\forall Cr \in \mathcal{C} : Cr(\mathsf{A}) = 1 \Rightarrow Cr(\mathsf{B}) = 1 \qquad \text{(TRIVIALITY SCHEMA)}$$

From TRIVIALITY SCHEMA, using standard Bayesian principles, we can prove that $Pr(\mathsf{A}) \leq Pr(\mathsf{B})$. Our CERTAINTY is, of course, a particular instance of TRIVIALITY SCHEMA, and EQUIV1 is the local instance of the schematic consequence mentioned. The proof we gave in section 2 can be run schematically, and refines the premises needed for this schematic connection.

For concreteness, let us consider the following way of instantiating TRIVIALITY SCHEMA: we replace MUST A (with MUST understood epistemically) for B.

$$\forall Cr \in \mathcal{C} : Cr(\mathsf{A}) = 1 \Rightarrow Cr(\text{MUST } \mathsf{A}) = 1 \qquad \text{(MUST CONSTRAINT)}$$

The consequence of this (assuming the side-premises we use above) is that credence in A is a lower bound on the credence of MUST A. But of course, whenever you're uncertain whether A is true or not, your credence in A should be higher than your credence in MUST A, since the latter should be zero or near-zero.

Once we see the analogy between the indeterminacy and the modal cases, it is tempting to seek a unified solution to the two puzzles. Different theorists will have different inclinations on this issue.

On the one hand, a uniform solution seems *prima facie* desirable. Once we see TRIVIALITY SCHEMA, the puzzle appears to be generated by some abstract, shared features of the logic of determinacy and epistemic modality. On the other, it might be that the explanatory resources we need to appeal to are different from case to case. For example, it seems plausible to us that for the case of modal and conditionals the solution will involve denying CLOSURE or RATIO. In fact, this response follows from a natural idea: rational learning about

[25] The first point is due to Richard Bradley (see e.g. 2000; 2007); for examples of triviality arguments applied to epistemic modals, see e.g. Russell and Hawthorne 2016, Goldstein forthcoming.

the world is invariably accompanied by learning about our own epistemic response: hence e.g. rationally learning A is invariably accompanied by learning MUST A. But, as we saw in the previous section, the corresponding claim for the case of determinacy is controversial. So it's unclear that a uniform response is desirable.

9 Conclusion

We have given an argument that starts from a plausible principle about determinacy and credence, i.e. CERTAINTY, and, via three plausible side-premises, leads to a controversial claim about cognitive role, i.e. REJECTIONISM.

Seeing this outcome, one might start questioning CERTAINTY. But, as we have argued, CERTAINTY is plausible. We have seen that there are routes to denying BOUND (via denying RATIO) and CLOSURE, but this strategy leads into controversial territory. Other solutions, like switching to weaker variants of CERTAINTY or endorsing regularity, also won't defeat all versions of the argument. We conclude that our puzzle raises a substantial challenge, which is not easily addressed by any extant account of belief and indeterminacy.[26]

[26]Thanks to Thomas Brouwer, Mike Caie, Branden Fitelson, and Jason Turner for extensive help and discussion. Thanks also to audiences at the Josh Parsons memorial conference at St Andrews, the Pacific APA 2019, the Australian National University, and Glasgow University.

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