Identity

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Identity. From very early days of quantum theory it was recognized that quanta were statistically strange (see →Bose-Einstein statistics). Suspicion fell on the *identity* of quanta, of how they are to be counted [1], [2]. It was not until Dirac's [1902-1984] work of 1926 (and his discovery of →Fermi-Dirac statistics [3]) that the nature of the novelty was clear: the quantum state of exactly similar particles of the same mass, charge, and spin must be *symmetrized*, yielding states either symmetric or antisymmetric under permutations. This is the *symmetry postulate* (SP).

The SP further implies that expectation values of particle observables are invariant under permutations. The latter looks temptingly like the sort of principle on which one might hope to found the theory of quantum identity. It is called the *indistinguishability postulate* (IP) – see →indistinguishabiity. But it turns out to be weaker than the SP, the principle we are interested in.

The question we shall pose is this: what does the SP tell us about quantum ontology? By a large margin, the consensus today is that the founding fathers were on to something, and that the SP implies or otherwise reflects a *failure of particle identity* in quantum mechanics, whether identity over time, or identity at a time (or identity *simpliciter*, without regard to time). For quantum mechanics itself, even for exactly similar particles, does not require the SP; such particles can perfectly well be described by unsymmetrized states and their superpositions.

Identity over time. It is common to most →interpretations of quantum mechanics that the underlying ontology need not be localized – that particles have no trajectories. In which case, there may be no good criterion of particle identity over time.

Of course that cannot be the whole story: unsymmetrized quantum mechanical systems also lack trajectories, but obey Maxwell-Boltzmann statistics [4]. In fact, it is already over-simplistic: the existence or otherwise of trajectories is not an all or nothing affair. It is true that no continuous sequence of $1-$particle states defines a curve in configuration space (or momentum space or any other sub-manifold of the classical phase space), but there are certainly evolutions under which symmetric and antisymmetric states define smooth curves ('orbits') of 1-particle states in *quantum* state space (Hilbert space) – see →indistinguishability. In terms of these the SP appears to have only a humble role, as ruling out any further fact as to which particle is attached to which orbit. The same can be said of the analogous symmetrization postulate as applied to classical particle trajectories [5].

This point has appeared puzzling to some. Doesn't the SP imply the IP? If particles can be associated with 1-particle states, or orbits of such, why can't they be individuated accordingly, in violation of the IP? Surely in the classical

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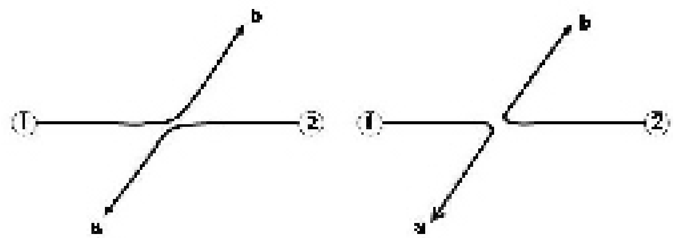

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Figure 1: Feynman diagrams for direct and exchange transition amplitudes

case we can always distinguish the particle by the trajectory, in violation of the IP? [6 p.7-8]. But this is to confuse the question of which particle is in which state, or sequence of states, or trajectory, which cannot be determined by any observation according to the IP, with the question of what distinguishes the states, or sequences of states or trajectories from each other, which in principle is perfectly observable [7]. The atoms ($1-$particle states) in the bottle of helium by the door are distinguishable from those ($1-$particle states) in the laser trap in the corner.

The SP then, blocks the question of which particle is in which state, or sequence of states. Classically, by mean of the trajectories, one can still say of two particles at two different times if they are the same or different $-$ whether or not they lie on the same trajectory. In quantum mechanics, where orbits of $1-$particle states may not be defined at all, there can be no such guarantee (this independent of symmetrization). This and the SP now lead to something new. For the SP implies that given two exactly similar particles with momenta in directions a and b, the state (a, b) (to use Dirac's notation [3]) is the same as (b, a); we should read these states as unordered pairs; but now given two particles initially in the state $(1, 2)$, and finally in the state (a, b), understood as unordered pairs, there will in general be *two* ways of linking them - by a transition $1 \rightarrow a$, $2 \rightarrow b$, and the 'exchange' transition $2 \rightarrow a$, $1 \rightarrow b$. If both transition amplitudes are appreciable, they may interfere with each other, and their relative phase will make a difference to the total transition probability. The relative phase is in turn different for symmetric states than for antisymmetric ones [8].

This point was in Feynman's [1918-1988] view the key to understanding quantum statistics. The rule is:

Bosons (Amplitude direct) + (Amplitude exchanged)

Fermions (Amplitude direct) - (Amplitude exchanged).

In Feynman's notation [9], $\langle a|1\rangle = a_1$ is the amplitude for particle 1 to scatter in direction a, and similarly $\langle a|2\rangle = a_2$, etc.. The total amplitude is the sum (bosons) or difference (fermions) of the amplitudes for the two Feynman diagrams shown in Fig 1:

$$\langle a|1\rangle\langle b|2\rangle \pm \langle b|1\rangle\langle a|2\rangle = a_1 b_2 \pm b_1 a_2.$$

The probability for bosons as $a \to b$ is then $\lim_{a\to b} |a_1 b_2 + b_1 a_2|^2 = 4|b_1 b_2|^2$; for fermions it vanishes. In the case of unsymmetrized particles, one of the processes $\langle a|1\rangle\langle b|2\rangle$, $\langle b|1\rangle\langle a|2\rangle$ results, with probability $|a_1 b_2|^2$ and $|b_1 a_2|^2$ respectively; in the limit $a \to b$ one cannot tell which has occurred, and the probabilities should be summed to obtain $2|b_1 b_2|^2$, exactly half the cross-section for bosons. Bosons, relative to unsymmetrized particles, act as though they attract one another, whilst fermions repel.

The point dovetails neatly with the →Copenhagen interpretation. According to this, if the experimental set-up permits the determination of the path (trajectory, orbit), taken by the particle – as would be possible if the particles differed in their state-independent properties (but which could also be ensured by other means) – there could be no interference effects (think of the two-slit experiment). This is reflected in the formalism by rules for using the measurement postulates: whether we should first take the absolute square of the amplitudes and then add, or add the amplitudes and then take the absolute square.

One might wonder if such a close link to the problem of measurement is a virtue of Feynman's approach. On the other hand, one could say the link was obvious from the beginning, purely on the basis of →Bohmian mechanics. In that theory trajectories are introduced explicitly, but one can still derive the same transition probabilities, consistent with quantum statistics.

Identity at a time or identity simpliciter. Does the SP pose a still deeper challenge to the concept of identity? Many think it does, and point to the apparent failure in quantum mechanics of Leibniz's [1646-1716] theory of identity, in particular his *principle of identity of indiscernibles* (PII).

Yet the history to this suggestion is curious, for when the PII was first brought up in the context of the SP, by Weyl [1885-1955], the principle was supposed to be vindicated, not undermined:

> The upshot of it all is that the electrons satisfy Leibniz's principium identitatis indiscernibilium, or that the electronic gas is a 'monomial aggregate' (Fermi-Dirac statistics). In a profound and precise sense physics corroborates the Mutakallimûn: neither to the photon nor to the (positive and negative) electron can one ascribe individuality. As to the Leibniz-Pauli Exclusion Principle, it is found to hold for electrons but not for photons. [10, p.247].

Quantum mechanics, for Weyl, posed no special problem for Leibniz's philosophy, at least as goes fermions.

For those focused on the question of quantities assigned to particles on the basis of their place in the $N-$fold tensor product of $1-$particle states, these comments made no sense. They are determined as expectation values of operators of the form

$$\langle \Psi, I \otimes ... \otimes I \otimes A \otimes I \otimes ... \otimes I \Psi \rangle$$

(where A is a $1-$particle observable). Include by all means other statistical properties, and marginal probability distributions, likewise attributed to particles or particle pairs of $k-$tuples on the basis of their place in the tensor product structure; if Ψ is symmetrized, every particle (or particle pair or $k-$tuple) has exactly the same $1-$particle expectation value for A, and the same statistical properties and marginal probability distributions. It seems, then, that the PII must *comprehensively* fail in quantum mechanics, for fermions as well as bosons, as claimed by Margineau [1901-1997] [11]. Similar conclusions were reached by others in subsequent studies [12], [13].

There is, however, a rather obvious rejoinder to this argument, namely that by particles we really mean $1-$particle states and properties. Our concern is not with which particle has which state or property, but with what those states and properties are. At least in *some* circumstances, particles may be identified with $1-$particle states. Thus in $2-$particle case, for $\{\phi_i\}$ an orthonormal basis for the 1-particle space, consider states of the form:

$$\Psi^{ij}_{\pm} = \frac{1}{\sqrt{2}}(\phi_i \otimes \phi_j \pm \phi_j \otimes \phi_i),\ i \neq j. \tag{1}$$

Ψ^{ij}_{+} is symmetric; Ψ^{ij}_{-} is antisymmetric. In Dirac's notation, they are states (i, j), understood as an unordered pair. As such they *manifestly* describe two particles, one being state ϕ_i, one being state ϕ_j; one having property P_{ϕ_i}, the other property P_{ϕ_j} (where P_ϕ is the projection on the state ϕ). It was understandable for Weyl to speak of the 'Leibniz-Pauli Exclusion Principle', at least in the case of electrons, in certain circumstances – in atoms subject to sufficiently strong external fields, so as to completely remove every energy degeneracy. In that case each electron is uniquely identified by its four quantum numbers.

But these are special cases. In the case of superpositions of vectors $\Psi^{ij}_{\pm}$, more than two 1-particle states are involved; there may be no pair of distinguished properties, one for each particle. And of course even if there are definite 1-particle states or properties for each particle, in the case of bosons there could spell trouble: they may be precisely the same (as with product states $\phi_j \otimes \phi_j$). Even for a state of the form (1) there may be a difficulty, as with the spherically symmetric singlet state of spin of two spin-$\frac{1}{2}$ particles. This state can be written in many ways:

$$\Psi^0_- = \frac{1}{\sqrt{2}}(\phi^x_+\phi^x_- - \phi^x_-\phi^x_+) = \frac{1}{\sqrt{2}}(\phi^y_+\phi^y_- - \phi^y_-\phi^y_+) = \frac{1}{\sqrt{2}}(\phi^z_+\phi^z_- - \phi^z_-\phi^z_+) \tag{2}$$

where $\phi^x_\pm$ are eigenstates of the $x-$component of spin, etc.,as exploited by Bohm [1917-1992] in his formulation of the →EPR thought experiment. It seems each particle must have every component of spin, or none.

We should be clearer on what the PII actually says. It is usually stated as the principle "it is not possible for there to exist two individuals possessing all their properties (relational and non-relational) in common" [14 p.9] (where the principle is the stronger the fewer the admissible properties and relations). Traditionally, philosophical debates on this principle have centered on what is to count as admissible: surely not relations involving identity and proper names, which threaten to trivialize the PII altogether. But there has been less interest in questions of logical form, and the meaning of 'relational properties'. If indeed *properties*, then they correspond to complex monadic predicates, presumably involving relations with other things only through bound quantification. But this is not the only, or the most important way in which relations are used in predication. Restricted to these, the PII is unnecessarily stringent. Why not allow that things may be discerned by relations as well as relational properties? But take *this* step and it is not obvious that the PII fails in quantum mechanics.

For the sake of clarity, the point is worth formalizing. Let L be a first-order language with a finite primitive vocabulary. Let s and t be $L-$terms (variables or proper names). Then the principle stated in terms of relational properties has the form:

$$s = t \underset{def}{=} \bigwedge_{\text{all primitive } L\text{-predicates } F} [\forall\forall...\forall F(...s...) \leftrightarrow \forall\forall...\forall F(...t....)] \tag{3}$$

where, if F is an $n-$ary predicate, there are $n-1$ quantifiers $\forall$ (so that $\forall\forall...\forall F$ is $1-$ary). This clearly fails to capture the full generality of relational predication: on the RHS of (3) should be conjoined conditions of the form:

$$\forall\forall...\forall[F(...s...) \leftrightarrow F(...t...)] \tag{4}$$

Proceeding in this way, one arrives at a definition of identity that, unlike (3), satisfies the formal axioms of identity and is essentially unique. As such it was championed by Quine [1908-2000][15].

Given this, if s and t are exactly similar, but $s \neq t$, they need not differ in any relational property, but only if for some F (4) is false. (4) would fail, for example, if for some dyadic F, $F(st)$ is true and F is irreflexive. F may even be symmetric too, thus incorporating permutation symmetry [5], [7].

As applied to quantum mechanics, it would then be enough, to discern electrons in the singlet state of spin, that they satisfy an irreflexive relation. And so they do: in the state (2), the relation 's has opposite $x-$component of spin to t' is clearly irreflexive and clearly true. Indeed, analogous statements hold for every component of spin, as Eq.(2) shows. But this does not imply the electrons each have any definite component of spin; compare 's is one mile apart from t', which may be true, for the space-time relationist, even though neither s nor t has any particular position in space.

A similar relation of anticorrelation for any state of the form (1) is easily specified:

$$(P_{\phi_i} - P_{\phi_j}) \otimes (P_{\phi_i} - P_{\phi_j})\Psi^{ij}_{\pm} = -\Psi^{ij}_{\pm}. \tag{5}$$

The generalization to superpositions of finitely-many such states is

$$\sum_{i.j=1}^{d}(P_{\phi_i}-P_{\phi_j})\otimes(P_{\phi_i}-P_{\phi_j})\sum_{i\neq j=1}^{d}c_{ij}\Psi_{\pm}^{ij}=-\sum_{i\neq j=1}^{d}c_{ij}\Psi_{\pm}^{ij} \tag{6}$$

where $c_{ij}=c_{ji}$. Since for fermions the RHS of (6) is the most general state possible, fermions, at least in finite dimensions, are always discernible. Evidently the same cannot be said of bosons; symmetric product states, such as $\phi_j\phi_j$, can be discerned by these methods only if subject to an evolution which leaves them entangled [16].

The upshot is that violation of the PII is neither sufficient nor necessary for the SP. But it would be wrong to conclude that the two principles are completely unrelated. There is, indeed, a very simple sense in which the PII together with exact similarity implies the SP, for they imply that states of affairs that differ only by permutations of particles should be identified – in Dirac's notation, that (a,b) and (b,a) be identified. But then the same principles should apply to classical statistical mechanics as well (for classical particles may surely be exactly similar); the explaination of quantum statistics cannot be traced to these – or not in isolation from other features of quantum mechanics, whether to do with identity over time, or the discrete nature of probability measures on Hilbert space [5], in line with early suggestions by Planck [1858-1947] and Lorentz [1853-1928] [17].

Literature

[1] P. Ehrenfest: Welche Züge der Lichtquantuenhypothese spielen in der Theorie der Wärmestrahlung eine wesentliche Rolle, *Annalen der Physik*, **36**, 91-118 (1911).
[2] L. Natanson, Über die statistische Theorie der Strahlung, *Physikalische Zeitschrift* **12**, 659-666 (1911).
[3] P.A.M. Dirac: On the theory of quantum mechanics, *Proceedings of the Royal Society (London)* **A112**, 281-305 (1926)
[4] P. Ehrenfest and G. Uhlenbeck: Die wellenmechanische Interpretation der Botzmannschen Statistik neben der neueren Statistiken, *Zeitschrift für Physik*, **41**, 24-26 (1927)
[5] S. Saunders, On the explanation for quantum statistics, *Studies in the History and Philosophy of Modern Physics* **37** 192-211 (2006)
[6] A. Bach, *Indistinguishable Classical Particles*, Berlin: Springer-Verlag (1995)
[7] S. Saunders, Are quantum particles objects?, *Analysis* **66,** 52-63 (2006)
[8] R. Oppenheimer: On the quantum theory of electronic impacts, *Physical Review* **32,** 361-376 (1928)
[9] R. Feynman: *The Feynman Lectures on Physics*, Vol.III, R. Feynman, R. Leighton and M. Sands (eds.), Reading, Mass: Addison-Wesley (1965)
[10] H. Weyl, *Philosophy of Mathematics and Natural Science*, Princeton: Princeton University Press (1949).

[11] H. Margenau, The Exclusion Principle and its Philosophical Importance, *Philosophy of Science* **11,** 187-208 (1944).
[12] S. French and M. Redhead, Quantum Physics and the Identity of Indiscernibles, *British Journal for the Philosophy of Science* **39**, 233-246 (1988).
[13] D. Dieks, Quantum Statistics, Identical Particles and Correlations, *Synthese* **82**, 127-155, (1990).
[14] S. French and D. Krause, *Identity in Physics: A Historical, Philosophical and Formal Analysis*, Oxford: Clarendon Press, (2006).
[15] W. V. Quine, *Set Theory and Its Logic*, Cambridge, Masssachusetts: The Belnap Press of Harvard University Press (1969).
[16] F. Muller and S. Saunders, Discerning Fermions, *British Journal for the Philosophy of Science* **59** (2008).
[17] O. Darrigol, Statistics and cominbatorics in early quantum theory, II: Early symptoma of indistinghability and holism, *Historical Studies in the Physical and Biological Sciences* **21**, 237-298 (1991).