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Comparing three numbers: The effect of number of digits, range, and leading zeros

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The literature is abundant with results on the cognitive processes involved in determining the larger of two numbers. In the present experiment, range, number of digits, and leading zeros were varied to determine whether some of the major results for comparing two numbers generalize to judgments of the largest of three numbers. There were consistencies as well as inconsistencies between our results and previous two-number comparison data. For example, the distance effect (Moyer & Landauer, 1967) held for three-digit numbers but was not replicated for single-digit numbers. A two-stage process is suggested, with an encoding stage and a comparison stage. At the comparison stage, strategies may vary, depending on the nature of the comparison that is to be made.

Considerable research has been aimed at determining the cognitive processes involved in selecting the larger of two numbers. Experiments on number comparison have been conducted both with the subjects selecting the larger of two numbers and with the subjects comparing a number to a standard (e.g., 55). Such experiments have included single-digit numbers (Banks, Fujii, & Kayra-Stuart, 1976; Moyer & Landauer, 1967), two-digit numbers (Dehaene, Dupoux, & Mehler, 1990; Hinrichs, Yurko, & Hu, 1981), and multidigit numbers (Hinrichs, Berie, & Mosell, 1982; Poltrock & Schwartz, 1984). One of our aims in the experiment reported here was to determine whether the processes involved in comparing several numbers are similar to those for comparing two numbers.

A result, first noted by Moyer and Landauer (1967) with respect to single-digit numbers, that consistently shows up in number comparison has been labeled the *distance effect*—namely, that the time required to select the larger of two numbers decreases as the numerical distance between them increases. They proposed an analogue model of the number-comparison process, in which the numbers are converted to an internal magnitude representation and the magnitudes are then compared. Banks et al. (1976) proposed a semantic representation as an alternative explanation. Each number is coded as large or small, and this code is used to determine the larger. If both numbers are encoded into the same category, a recoding takes place, and the response time is longer. For comparisons of numbers of more than one digit, two major competing theories have emerged: lexicographic (or place-value) models, and holistic (magnitude-comparison) models. In place-value models, the numbers are compared digit by digit, whereas holistic models propose that comparison occurs not at the digit level but after the numbers have been converted into an internal magnitude representation. Hinrichs et al. (1981) tested place-value effects with two-digit numbers and suggested several processes to account for the effects within and between decades. The numbers may be compared digit by digit without computation of magnitudes, and the rightmost digits are only utilized when the leftmost digits are identical. There may also be interference from the units digit when the units digit is larger than the target number. Hinrichs et al. proposed a mixed model in which overall magnitude of the whole number is assessed first and digit-by-digit comparison occurs only if the numbers are within the same decade. Dehaene et al. (1990) found no interference when the units digit was presented before the decades digit and concluded that a holistic model with a coding stage followed by a comparison stage best accounted for the data. The effect of number of digits and the role of zeros was studied by Hinrichs et al. (1982). The number of digits in a number is an obvious cue to its magnitude, and it influenced reaction time (RT) when the number of digits was not the same. When the number of digits in the two numbers was the same, trailing zeros led to decreased RT, and leading zeros led to increased RT.

Selecting the largest of three numbers is more complex, in that there is also the question of whether the comparison process proceeds pairwise from left to right or whether all three numbers are encoded and compared holistically. With a left-to-right pairwise comparison, reaction times

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should be faster when the target occurs farther to the left. With a holistic strategy, distance effects should be similar to those found for two-number comparisons, and one would expect longer RTs if both of the other numbers were close to the target than if only one was.

In this experiment, we used a three-number comparison task with one-, two-, or three-digit numbers. For each digit condition, the range of the numbers was varied across three ranges—wide, medium, and narrow. The presence or absence of leading zeros was tested as a between-subjects variable. The group with leading zeros always saw three digits, so for this group, the digit manipulation refers to the number of significant digits.

The range of the numbers to be compared can affect processing in several possible ways. If the distance effect is due to a holistic magnitude comparison, the overall effect of narrowing the range of numbers to be compared should be to increase the average response time. With narrower ranges, one would expect there to be more close comparisons, and average RTs should be longer for comparing numbers from a narrow as opposed to a wide range. For the two- and three-digit numbers, the narrow range was selected in such a way that at least two of the numbers had to begin with the same digit and there was a greater probability of the duplicated digits' occurring in the two larger numbers, thus forcing a comparison based on the second digit. Thus a place-value model would also predict a greater increase in RT for the narrow range and very little increase for the medium range.

The addition of leading zeros should not interact with the number of digits in the holistic model, but it would in the place-value model. In the place-value model, the one- and two-digit numbers with leading zeros should be slower than the wide-range three-digit numbers in which the initial digits differ more. In a two-stage process, with separate encoding and comparison processes, it might take longer to encode numbers with more digits. The absolute magnitude differences are greater for three-digit numbers than for two-digit numbers, and greater for two-digit numbers than for one-digit numbers, and a strict holistic magnitude interpretation might suggest that the comparison stage should be faster for the numbers with more digits. The condition with leading zeros, in which all of the comparisons involved three digits but with redundant leading zeros for the one- and two-digit numbers, should yield a different pattern of results than should the condition without leading zeros. If a magnitude comparison is used but it takes time to process more digits, then wide-range two-digit numbers with leading zeros should be similar to narrow-range three-digit numbers.

METHOD

General Procedure

The subjects' task was to select the largest of three numbers. The experiment, which was conducted in one session, consisted of a test of spatial ability followed by the experimental task. (The spatial ability test will not be discussed further here, for it was unrelated to performance on the experimental task.) Subjects were tested individually, and all parts of the experiment were controlled by a Macintosh IIci computer equipped with a portrait monitor and an extended keyboard. The subjects were seated approximately 20 in. from the screen and used the right hand to select the answers with the auxiliary numeric keypad. Instructions, emphasizing speed and accuracy, were presented on the computer screen, and practice and data collection were programmed into the tasks.

Numbers Task

For each problem, subjects saw three numbers evenly spaced along a horizontal row centered in a window in the middle of the screen. The numbers were in Size 18 Courier font and were presented as sets of dark digits on a light background. The subjects were instructed to press the "7," "8," or "9" key on the right-hand number pad to indicate the location of the largest number—left, center, or right. Several kinds of feedback were given to encourage speed and accuracy. Above the three numbers were displayed the words "Proficiency Level =" followed by a percentage score and, to the right, the word "Score =" followed by a numeric score. "Proficiency" was based on the most recent 10 problems and was initialized to 100%. The subject began each trial with a score of 200, from which 10 points were subtracted for each wrong response and 10 points were added for each correct response. In addition, prerecorded voice messages stored in the computer were used as warnings if performance fell below certain levels. If subjects did not respond within approximately 3,300 msec, they heard "time is running out," spoken in a male voice. If proficiency fell below 80%, they heard "too many errors," in a male voice, and if both time and proficiency were out of bounds, they heard "situation critical, respond *now*," in a female voice. The program recorded RT with tick-count accuracy (16.625 msec) and also collected error and correct response data.

Design and Procedure

The effect of leading zeros was a between-subjects variable, with one group of subjects getting leading zeros and the other no leading zeros. Number of digits and range were within-subjects variables. In the leading zeros condition, there were exactly three digits in all of the numbers, so that one-digit numbers were preceded by two zeros (e.g., 006), two-digit numbers were preceded by one zero (e.g., 043), and three-digit numbers had no zeros, exactly as in the no zeros condition. For each digit condition, the range of possible numbers could cover a wide, medium, or narrow range, as can be seen in Table 1. Within a given range, the three numbers to be presented on each trial were selected randomly without replacement. The subjects in the no zeros condition saw the same sets of one-, two-, and three-digit numbers presented in the zeros condition, right-justified without the leading zeros.

The experiment consisted of practice followed by 18 test trials. Practice consisted of three problems from each of the nine experimental conditions. Each set of three practice problems was separated by a screen that instructed the subjects to continue when ready by pressing a letter key on the keyboard. Practice was specific to the assigned test condition—zeros or no zeros. Each test trial consisted of 20 three-number problems. The three levels of each of the two within-subjects variables—number of digits and range—were blocked within trials. The nine experimental conditions were presented in a pseudorandom order and partially counterbalanced across the three groups of subjects in such a way that one-, two-, and three-digit numbers and wide, medium, and narrow ranges all occurred on early, middle, and late trials. To further control for practice and fatigue effects, each condition was presented once in the first 9 trials, and the same conditions were presented in the reverse order on the last 9 trials. The subjects were assigned to the groups in the order of their arrival, and males and females were rotated separately through the experimental groups.

Table 1
Number Ranges for the Three Difficulty Levels for Each of the Different Numbers of Digits

No. Digits	Range: Wide	Medium	Narrow
1	0–9	2–7	3–6
2	10–99	30–69	45–59
3	100–999	300–699	450–599

Subjects

Sixty-three students from the University of Maryland undergraduate psychology department subject pool volunteered to participate for extra course credit. Complete data were collected from 62 of the subjects—37 females and 25 males.

RESULTS

The effect of number of digits and leading zeros on RT and on errors is shown in Figure 1 for the wide, medium, and narrow ranges. Separate analyses of variance were carried out for RT and for errors. The data for the first and second nine trials were combined for all analyses after an analysis by presentations showed a small improvement from the first to the second presentation and otherwise an almost identical pattern of results for the two presentations. An analysis of variance was conducted with zeros as a between-groups factor and number of digits and range as within-groups factors. Multiple comparisons were made with the Tukey *HSD* test (Winer, 1971), with $p < .01$, and this level applies to all comparisons reported below as being significant.

The presence or absence of leading zeros did not lead to a significant difference in overall RT between the two groups [$F(1,60) = 1.87, p < .20$], but there was a significant interaction of number of digits and leading zeros [$F(2,120) = 28.52, p < .0001$]. For the one- and two-digit numbers, RTs were significantly slower for the zeros group than for the no zeros group. The three-digit numbers had no leading zeros for either group, and RTs did not differ significantly.

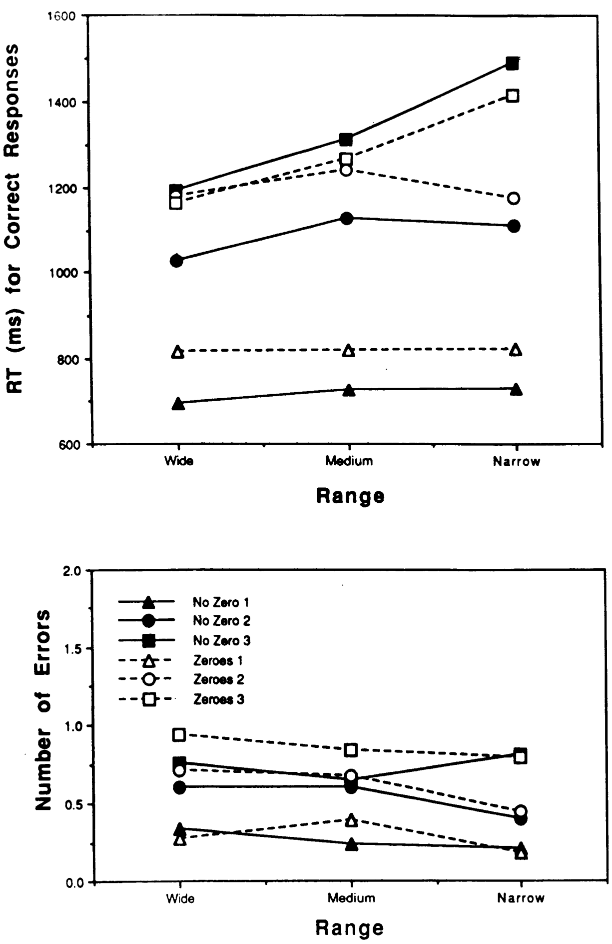

Figure 1. The effect of number of digits and leading zeros on reaction time (RT) and errors for the wide, medium, and narrow ranges.

Table 2
Correlations of RTs with the Magnitude of the Differences between the Target and Each of the Distractors

	No Zeros Group		Zeros Group	
No. Digits	Smaller Difference	Larger Difference	Smaller Difference	Larger Difference
One	−0.238	−0.193	−0.230	−0.180
Two	−0.280	−0.240	−0.254	−0.156
Three	−0.355	−0.329	−0.340	−0.307

For both groups, RTs increased as the number of digits increased, and there was a significant main effect of number of digits [$F(2,120) = 1,059.34, p < .0001$]. The overall effect of range was significant [$F(2,120) = 134.47, p < .0001$], and there was also a significant interaction of number of digits and range [$F(4,240) = 57.24, p < .0001$]. For the three-digit numbers, RTs were significantly slower for the medium as opposed to the wide range and for the narrow as opposed to the medium range. For the two-digit numbers, RTs were significantly slower for the medium as opposed to the wide range, but RTs were not slower for the narrow range than they were for the medium range. For single-digit numbers, RTs did not differ significantly across the wide, medium, and narrow ranges.

In general, there were very few errors. The number of digits had a significant effect on errors, with errors increasing as the number of digits increased [$F(2,120) = 44.11, p < .0001$]. No other effects or interactions were significant for errors.

As a further test of the distance effect, Pearson correlations of RT with the magnitude of the difference between the target and each of the distractors were calculated for the wide-range conditions.[1] These are shown in Table 2. Consistent but modest negative correlations were obtained, indicating that RTs were longer the less difference there was between the target and the distractors. The correlations were slightly larger when there were more digits and therefore bigger numerical differences. Finally, an analysis of the position of the target (left, middle, or right) revealed no significant differences due to target position.

DISCUSSION

The effect of leading zeros did not support a strictly sequential digit-by-digit comparison process. RTs with one- and two-digit numbers with leading zeros were not slower than RTs with wide-range three-digit numbers. There was also no increase in RT from the medium to the narrow range for two-digit numbers, even though the narrow range always had some repetition of leftmost digits and a high probability that the repetition would include the target.

Our results agree with those of Hinrichs et al. (1981), who also found that RTs were shorter for close comparisons with a narrow range as opposed to a wide range of two-digit numbers. Poltrock and Schwartz (1984) argue strongly for a sequential place-value comparison process. They found that when multidigit numbers were compared, RT increased linearly with the number of digits preceding the leftmost differing digit. The exception was that the decrease was less when only the rightmost digits differed, and this effect disappeared when a letter was appended to the end of the numbers. It seems reasonable to suppose that differences at the end of a string stand out more and are easier to process than differences in the middle of a string. However, the fact that for our results, RT was faster for two-digit numbers with leading zeros than for narrow-range three-digit numbers and faster for single-digit numbers with two leading zeros than for narrow-range two-digit numbers argues against a simple sequential place-value comparison. Furthermore, there was no support for a left-to-right comparison of the three numbers.

Our results likewise showed only weak agreement with holistic magnitude-comparison models. Decreasing the range yielded consistent increases in RT only for the three-digit numbers, with no significant effect of range for the single-digit numbers, and the two-digit numbers somewhere in between. Within the wide ranges, there were only modest correlations between RT and the magnitude of the distance between the target and the distractors. The distance effect was greater for the distractor that was closer to the target than it was for the more distant distractor, suggesting that closer competitors do require more processing than far ones do. Still, even the more distant distractor showed consistent small negative correlations, indicating that comparison did indeed take longer when both distractors were close to the target. A possible explanation for the lack of an increase in RT for medium- and narrow-range single-digit numbers and for narrow-range two-digit numbers is suggested by the semantic model of Banks et al. (1976). As the range narrows, the concept of what is "large" varies less from problem to problem, and the criterion for selecting the largest number does not have to be constantly readjusted.

The effect of leading zeros supports the concept of a separate encoding stage. The RT increment due to leading zeros did not differ for one-digit and two-digit numbers or across ranges. RT increased as number of digits increased, and numbers with leading zeros took longer to process than did numbers without zeros. This suggests that numbers with more digits take longer to encode, and it agrees with the function of zeros as a placeholder in our number system. Although zeros cannot be ignored in the encoding of numbers, they seem to be processed differently from nonzero leading digits. Unlike zeros, nonzero digits differed with range, and the effect of adding nonzero digits was greater than that of adding zeros, even when the first digit was more or less redundant, as was the case with the narrow-range two- and three-digit numbers.

Our results suggest a two-stage process involving an encoding stage and a comparison stage. At the comparison stage, our results show points of agreement and of disagreement with both the digit-by-digit comparison models and with the holistic magnitude-comparison models. It seems that a contextualist approach (Jenkins, 1974) best accounts for the results of this experiment. That is, the context of the type of comparisons in each block would determine the most effective strategy for that particular block. Sometimes it may be effective to look only at the rightmost digit (e.g., single-digit numbers with leading zeros), and at other times it may be effective to use a holistic search for "large" numbers (e.g., wide-range three-digit numbers).

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NOTE

1. Correlations were not obtained for the medium and narrow ranges, because the range of possible differences between target and distractors was intentionally limited for these conditions.

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