GALILEO VS. ARISTOTLE ON FREE FALLING BODIES

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ABSTRACT

This essay attempts to demonstrate that it is doubtful if Galileo's famous thought experiment concerning falling bodies in his 'Dialogues Concerning Two New Sciences' (Galileo 1954: 61-64) actually does succeed in proving that Aristotle was wrong in claiming that "bodies of different weight [...] move [...] with different speeds which stand to one another in the same ratio as their weights," (Galileo 1954: 61). (Part I); and further that it is likewise doubtful that that argument *does* or even *can* establish Galileo's own famous 'Law of Falling Bodies,' *viz.,* that regardless of their weight all bodies fall with the same speed. (Part II)

ZUSAMMENFASSUNG

Widerlegt Galileo Aristoteles' Theorie des freien Falls?

Aristoteles vertrat bekanntlich die These, "Körper von verschiedenem Gewicht fallen mit Geschwindigkeiten, die im selben Verhältnis zueinander stehen wie ihre Gewichte". Mit einem berühmten Gedankenexperiment (aus den 'Dialogues Concerning Two New Siences') will Galileo diese These widerlegen und zugleich sein eigenes Fallgesetz beweisen, nach dem alle Körper sich im freien Fall gleich schnell bewegen, wie schwer sie auch sein mögen. Im ersten Teil des Aufsatzes wird dargelegt, dass Galileos Argumente gegen Aristoteles aber durchaus zweifelhaft sind; der zweite Teil stellt darüber hinaus in Frage, ob das Gedankenexperiment wirklich den Schluss auf Galileos Fallgesetz erlaubt, bzw. ob eine solche apriorische Herleitung überhaupt denkbar wäre..

INTRODUCTION

In the chapter entitled 'First Day,' of 'Dialogues Concerning Two New Sciences' (1638), Galileo orchestrates a discussion about Aristotle's natural philosophy between a fictitious master, Salviati (who famously stands for Galileo himself), and his two students, Sagredo and Simplicio (the latter often serving as Aristotle's advocate). They focus especially on Aristotle's assumption that

> bodies of different weight […] move in one and the same medium with different speeds which stand to one another in the same ratio as the weights; so that, for example, a body which is ten times as heavy as another will move ten times as rapidly as the other. (Galileo 1954: 61)[1]

Salviati's (i.e., Galileo's) stance is that "we may deny [Aristotle's] assumptions." (Galileo 1954: 61).

As presented, the first reason for doing so, for denying Aristotle's assumptions, is voiced by the young Sagredo, who claims to have *made the empirical test* and to have *observed* that Aristotle is wrong (cf. Galileo 1954: 62).[2] Having thus given founding expression to the modern scientific method, Galileo allows the master, Salviati to further add that "even *without further experiment*, it is possible to prove clearly, by means of a short and conclusive argument, that a heavier body does not move more rapidly than a lighter one […]." (Galileo 1954: 62; my emphasis)

Before examining his argument in detail, I note that it is indeed possible to show "*without further experiment*" that an allegedly true law statement is incorrect and does not therefore state a law of nature. Granted, at first glance this fact may seem odd, especially so as the spirit of modern science is to progress via experimental confirmation or refutation. Nonetheless, there is a way to show "*without further experiment,*" that a law hypothesis is wrong: that is, by showing that it is self-contradictory or that it entails contradictory statements. In what follows I will argue that Galileo's

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[1] Note that when I speak about Aristotle I entirely rely on what Galileo reports Aristotle to have said.

[2] More important, in the chapter "Third Day" (cf. esp. Galileo 1954: 175) we find Galileo's meticulous descriptions of his inclined plane experiments. At this place he proves the correct law of free falling bodies, namely that "the free motion of a heavy falling body is continuously accelerated […] The distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity." (Galileo 1954: 153)

argument succeeds in demonstrating only that Aristotle's law *strongly suggests* a self-contradiction but not that it *strictly entails* one and that defensive strategies against his argument are therefore possible. Acting as Aristotle’s advocate I will show how his assumptions might be defended (Part I).

At the end of his proof, Galileo claims something even stronger. He concludes: "we infer therefore that large and small bodies [heavy and light bodies; MAS] move with the same speed," (Galileo 1954: 64)[3]—a formulation of his own Law of Falling Bodies. Yet if I am right and Galileo does not first succeed in refuting Aristotle's law, then *a fortiori*, he cannot establish his own law. Further again, I will demonstrate, that even if we were to assume the success of his first task Galileo’s second claim, his law, still does not follow from that refutation alone but rather has need of further assumptions.[4]

As an aside it is noted that success on Galileo's part would amount to an *a priori* proof of a law hypothesis with empirical content. Whilst I do not wish to involve myself here in any argument as to whether such a thing is at all possible, I note that it would strike the empirically minded as very doubtful indeed.

When considering Galileo’s argument some philosophers, like James Robert Brown (Brown 2000) or Simon Blackburn (Blackburn 1996), have accepted Galileo's proof (or parts of it). In his article on *Thought Experiments* in Blackwell's Companion to the Philosophy of Science Brown writes: "Galileo showed that all bodies fall at the same speed with a brilliant thought experiment that started by destroying the then reigning Aristotelian account." (Brown 2000: 529) After a reconstruction of Galileo's argument Brown, I believe wrongly, concludes

> That's the end of Aristotle's theory; but there is a bonus, since the right account is now obvious: they all fall at the same speed […] This is said to be a priori (though still fallible) knowledge of nature, since no new data are involved, nor is the conclusion derived from old data, nor is it some sort of logical truth. (Brown 2000: 529)

Blackburn is more cautious. First of all he only thinks that Galileo refutes the Aristotelian view "that a heavy body falls faster than a lighter one" without claiming that Galileo also proves his own law and, secondly, Blackburn presents Galileo's proof in a slightly altered form from the

[3] The quote continues: "provided they are of the same specific gravity" where Galileo means density by "gravity" (compare (Galileo 1954: 68ff)). Since we are concerned with *free* falling bodies the *gravity* of bodies can be disregarded.
[4] Which are, I have to concede, very likely to be accepted by many people.

original and in so doing weakens the defensive strategy that I propose. I will outline his reformulation at a later point.[5]

PART I

Let me return to Salviati, Sagredo, and Simplicio. Before he starts his proof, Salviati adds to Aristotle's main claim[6] that its speed is a fixed value for each body: "each falling body acquires a definite speed fixed by nature, a velocity which cannot be increased or decreased or diminished except by the use of force or resistance," (Galileo 1954: 62-3). Then Salviati's main argument begins. He illustrates that Aristotle's claim apparently leads to two mutually contradictory consequences so that it can be rejected by *reductio*. Here is the first part:

> (GALILEO_1) If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will be partly retarded by the slower, and the slower will be somewhat hastened by the swifter. (Galileo 1954: 63)

He adds a quantitative reformulation:

> If a large stone moves with a speed of, say, eight while a smaller moves with a speed of four, then when they are united, the system will move with a speed less than eight… (Galileo 1954: 63)

And here is the second, contradicting part:

> (GALILEO_2) … but the two stones when tied together make a stone larger than that which before moved with the speed of eight. Hence the heavier body [i.e., the new which consists of the two former ones; MAS] moves with less speed than the lighter [i.e., the former heavier one; MAS]; an effect which is contrary to your supposition. (Galileo 1954: 63)[7]

Reductio complete:

> Thus you see how, from your assumption [directed to Simplicio;

[5] I must mention papers by Gendler (1998) and Norton (1996) who make, in a larger context and thus previously unknown to me, points similar to those I am going to present.

[6] "Bodies of different weight […] move in one and the same medium with different speeds which stand to one another in the same ratio as the weights"

[7] Actually, next to speed and weight Galileo uses the concepts large and small. I identify large with heavy and small with light to simplify the argument. I believe that Galileo's proof is thereby not altered. Also, I will use the terms 'mass' and 'weight' interchangeably. Their difference—a weight is the gravitational force a mass experiences in a gravitational field—has no direct bearing on the argumentation.

> MAS] that the heavier body moves more rapidly than the lighter one, I infer that the heavier body moves more slowly. (Galileo 1954: 63)

In due course I will show not only how Aristotle could answer this challenge, but also how a counterargument proposed by Galileo—in the guise of Simplicio—unwittingly almost succeeds in refuting the argument by Salviati.

As an appetiser for the serious argumentation against Galileo's proof consider the following story:

> Little Aristotle argues: "A bigger portion of ice cream is better than a smaller one (within certain limits)." "That's not correct,", his younger cousin Galileo answers, "they all have the same quality!". His argument is this: "The small portion together with the big portion (fill both portions from their old bowls into a new one) is better than the former big one, because it is even bigger. But, at the same time, it is less good since we have to take the average of the big and the small one's quality, i.e., the low quality of the small will reduce the high quality of the former big one. *Reductio* complete, bigger ice cream portions are—contrary to what you think, Aristotle—not better than smaller ones. They all have the same quality." Speaks thus. He no doubt reaches for the bigger portion and tries to leave Aristotle with the small one.

I now turn to the serious argument. To achieve my aim it is helpful to translate each of Aristotle's claims into a semi-formal language.

Aristotle's and Galileo's starting points are these:

0	ARISTOTLE$_0$: "Each falling body [weight, MAS] acquires a definite speed fixed by nature." (Galileo 1954: 62-3) (Theorem 0 is presupposed by Theorem 1 but does not enter the argument itself explicitly.)	$\forall w \exists s(S(w)=s)$ with s being a speed and w being the weight of a certain body.
1	ARISTOTLE$_1$: "Bodies of different weight […] move in one and the same medium with different speeds which stand to one another in the same ratio as the weights so that, for example, a body which is ten times as heavy as another will move ten times as rapidly as the other. (Galileo 1954: 61)	$\forall w \forall w^* (w/w^* = S(w)/S(w^*))$; with w and w* being weights of two different bodies, S(w) and S(w*) their speeds.
1*	ARISTOTLE$_{1*}$: A consequence of 1 which goes into the argument (instead of 1 itself)	$\forall w \forall w^* (w>w^* \supset S(w)>S(w^*))$
2	GALILEO$_1$: "If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will be partly retarded by the slower, and the slower will be somewhat hastened by the swifter." (Galileo 1954:	$\forall w \forall w^* (S(w)>S(w^*) \supset S(w)>S(w+w^*)>S(w^*))$

	63)	
3	GALILEO$_2$: "The two stones when tied together make a stone larger than [the biggest of the two; MAS]." (Galileo 1954: 63)[8]	$\forall w \forall w^* (w>w^* \supset w+w^* > w)$

Now, Galileo's proof reads like this:

4	Premise	$w_1 > w_2$
5	From 4, 1*	$S(w_1) > S(w_2)$
6	From 5, 2	$S(w_1) > S(w_1 + w_2) > S(w_2)$
7	From 3, 4	$w_1 + w_2 > w_1$
8	From 7, 1*: contradiction to 6	$S(w_1 + w_2) > S(w_1)$

A main element of this *reductio* is that speeds combine to average (cf. **2**), where weights add (cf. **3**). This would be impossible however if speed were directly proportional to weight, (cf. **1***).

I said, when I announced my counterargument, that Galileo's proof only *suggests* the failure of Aristotle's assumption but that the failure does not strictly follow. That it *prima facie* leads to success is undeniable. Here is why it does, nonetheless, ultimately fail:

What we can conclude from the formal proof is only that at least one of the premises **1*** (ARISTOTLE$_{1*}$), **2** (GALILEO$_1$), **3** (GALILEO$_2$), or **4** must be rejected or amended. **1*** is, of course, the premise Galileo chooses to reject, since **2** and **3** are just tailored for this purpose. But this is not cogent.

We can also argue against **2** (GALILEO$_1$): if we combine two bodies, a heavier w_1 and a lighter one w_2, to one bigger body w_1+w_2, then, if this is properly done and really *one body* is the result, this bigger body $W=w_1+w_2$ has its own natural speed which is, according to Aristotle's claim, higher than the speed of the former heavier body w_1 (and *a fortiori* higher than the speed of the former lighter body w_2). Hence, contrary to **2** (GALILEO$_1$) and **6**: $S(W) > S(w_1) > S(w_2)$.

If, on the other hand, the two bodies are still two bodies after having tied them together—that is, the unification has not been done properly—then the slower one will retard the quicker and *vice versa*. In this case, the masses do not add to a bigger sum, i.e., there is no entity which has the weight $W=w_1+w_2$, and, hence, a higher speed. Contrary to **3** (GALILEO$_2$) and **7**: the sum w_1+w_2 does just not exist.

[8] Needless to say that $\forall i\ (m_i > 0)$.

Against **4** we can't really argue. **4** is an exemplification of our common belief that there are at least two different bodies with two different masses.

Arguing against **2** and **3** in the above way, we take the sting out of Galileo's argument. We are not confronted with a *reduction* anymore, but rather with the question whether the resulting speed is to be calculated by

$$s_3 = S(w_1 \text{ plus } w_2) = S(w_1 + w_2) = S(W)$$

or by

$$s_3^* = S(w_1) \text{ plus } S(w_2) = (S(w_1) + S(w_2))/2 = (s_1 + s_2)/2$$

where $s_3 > s_3^*$. The answer depends on whether the two bodies have been properly united to one single body or not.

The problem is then to spell out exactly what would constitute a *proper unification* of bodies. It is precisely our lack of clarity on this issue that lends such credibility to Galileo's proof on first inspection. Indeed, Blackburn's reformulation of Galileo's proof serves to further underline that difficulty. In his Dictionary of Philosophy under the entry 'Thought Experiments' Blackburn writes

> [Galileo] asks us to imagine a heavy body made into the shape of a dumbbell and the connecting rod gradually made thinner, until it is finally severed. The thing is one heavy body until the last moment, and then two light ones, but it is incredible that this final snip alters the velocity dramatically." (Blackburn 1996: 377)[9]

I concede that our immediate intuition here is that once the two sides of the dumbbell (this one body) are separated, we have two bodies.

In summary then, I have so far shown that although Galileo's proof strongly suggests the failure of Aristotle's law of free falling bodies it is not logically decisive. For his proof to be conclusively dismissed however, we need a solid criterion by which to establish when two or more bodies are properly united.

Yet, it is Galileo himself who gives us an idea how to achieve that aim. He lets Simplicio put forward his doubts against Salviati and provides thereby (and unbeknownst to himself) the needed criterion.

Simplicio challenges GALILEO_1:

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[9] We could, as another defence strategy, state the nearly unbelievable, namely, that, in fact, the resulting two bodies will slow down instantaneously after the final snip (note that Blackburn himself says that it is "incredible" to believe in a dramatic velocity change; not that it is absolutely inconceivable.) That this is not sustainable is very clear, but for empirical reasons. Galileo's proof would have to resort to empirical data at this point and would no longer be a proof "without further experiments".

> It appears to me that the smaller stone when added to the larger increases its weight and by adding weight I do not see how it can fail to increase its speed or, at least, not to diminish it. (Galileo 1954: 63)

Weight, so I read in Simplicio's words, is the ultimate unit that has to be determined first; speed comes second. And this is close to saying that when really one body is the result, there aren't any lower speeds to be added (that is averaged) anymore.

Salviati's answer to this objection is that tying a small body on top of a large body only adds weight if the stones are at rest:

> A large stone placed in a balance not only acquires additional weight by having another stone placed upon it, but even by the addition of a handful of hemp its weight is augmented. (Galileo 1954: 63)

In motion, however,

> do you think that the hemp will press down upon the stone and thus accelerate its motion or do you think the motion will be retarded by a partial upward pressure? […] You must therefore conclude that, during free and natural fall, the small stone does not press upon the larger and consequently does not increase its weight as it does when at rest. (Galileo 1954: 64)

So here is Galileo spelling out one possible criterion for when a proper weight addition (a body unification) is achieved and when it is not: *viz.,* you cannot unite two bodies / weights (or cut one into two) while they are falling; if you want to unite two bodies to one single one the two bodies have to be at rest. Now, there is no contradiction between GALILEO_1 and GALILEO_2 anymore: either you put the two stones together while still resting, then they—together as one body—acquire a higher speed when you let them fall (contra GALILEO_1); or you put them together while falling and GALILEO_2 turns out to be false.

I want to underline that this criterion for united bodies does not of course immunise Aristotle's theory against all attack. One will find in an experiment that two stones, which are united while at rest, fall nearly precisely like two stones that are united while falling. This is, however, an empirical finding and does not count anymore as "argument without further experiment".

Finally, I concede that our intuition about Blackburn's dumbbell are clearly contrary to this artificial criterion. Galileo's argument is very suggestive. It is, however, not logically compelling.[10]

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[10] It is, of course, possible to invent other criteria for proper body fusions and fissions—at full moon, 6am; when glued together with superglue; chained; etc.—which would decide over the question whether their weights can be added and create a new natural speed or not.

PART II

I now turn to my final section. Galileo concludes the whole quarrel not by saying that Aristotle is wrong and hence by declaring that it is not true that "bodies of different weight […] move in one and the same medium with different speeds which stand to one another in the same ratio as the weights", but with the even stronger claim that "we infer therefore that large and small bodies [heavy and light bodies; MAS] move with the same speed." (Galileo 1954: 64) This is Galileo's own law of gravitation. It was not the initial goal of Salviati, Sagredo, and Simplicio's discussion to prove it. My claims in Part II are: (A) that he *does not* succeed to prove his own law and (B) that it would count against a still appreciated empiricist dogma if he *could* do it in this way.

Starting with (A), let us suppose, contrary to what I have shown in Part I, that Galileo successfully proved Aristotle wrong. There would then of course be innumerable options left as to what bodies might do when falling. Perhaps, for example, bodies of different weight might move with different speeds that stand to one another in the opposite ratio as the weights, i.e. lighter bodies would fall faster than heavier bodies. Or worse again, perhaps different bodies could do different things. The negation of Aristotle's 'bodies of different weight […] move […] with different speeds' states that 'not all bodies of different weight move with different speeds'. In short then, the most that Galileo succeeds in demonstrating is that there are some bodies that do not accord to Aristotle's law.[11]

Hence, the first of my doubts is confirmed: Galileo's proof *does not*, as it stands, establish his famous 'Law of Falling Bodies' (apart from the fact that my Part I has already shown that Galileo did not refute Aristotle and therefore did—*a fortiori*—not establish his own law).

Before considering (B), the 'cannot' part of my goal, I want to consider an amendment of Galileo's proof. Lets—contrary to the facts—take for granted that Galileo did succeed in proving Aristotle wrong.

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[11] I suspect that one could run Galileo's argument again with Aristotle's law in existentially quantified form: '*there are* two bodies, one heavier, one lighter, such that the heavier falls faster than the lighter'. The result would be the negation, i.e., 'there aren't any such bodies' (provided, of course, we buy the general argument structure).

Then we could get his own law in two further steps. (i) By running through the argument again exchanging all instances of '*faster*' by '*slower*' and *vice versa* (which would reflect an anti-Aristotelian law). Thereby we would establish that not all bodies fall with speeds which stand in opposite relation to the weights. If we finally (ii) suppose that nature is uniform and the speed of all bodies is fixed in the very same way (whatever that way is) we have to conclude that they all—no matter what their weight is—fall with the same speeds: Galileo's own law.[12]

Turning now finally to (B), to my claim that Galileo not only *does not*, but also *cannot* prove his own law. What if Galileo did succeed with his proof? Then he would have shown a law with empirical content "without further experiment". Yet, even if some law hypotheses are formulated on the basis of pure thought we feel that we either need empirical tests to confirm them or empirical data have been presupposed tacitly. This is why I believe Galileo could not achieve his goal with such a kind of proof[13] and Galileo might even agree. It was he after all who taught us an empiricist lesson against the Aristotelian creed. Indeed, before he comes up with his proof Galileo (alias Sagredo) assures us that he has shown the validity of his law already empirically:

> I […] who *have made the test* can assure you that a cannon ball weighing one or two hundred pounds, or even more, will not reach the

[12] Here's the formal proof against anti-Aristotle:

1*'	$\forall w \forall w^* (w>w^* \supset S(w)<S(w^*))$	New 1*
2	$\forall w \forall w^* (S(w)>S(w^*) \supset S(w)>S(w+w^*)>S(w^*))$	Unchanged 2
3	$\forall w \forall w^* (w+w^*>w^*)$	Unchanged 3
4	$w1 > w2$	Premise
5	$S(w1) < S(w2)$	4, 1*'
6	$S(w1) < S(w1 + w2) < S(w2)$	5, 2
7	$w1 + w2 > w1$	3, 4
8	$S(w1 + w2) < S(w1)$	7, 1*': Contradiction to 6

From the original argument we (supposedly) get that

$\neg \forall w \forall w^*(w>w^* \supset S(w)>S(w^*))$; i.e.,

$\exists w \exists w^*(w>w^* \wedge S(w) \leq S(w^*))$

from the altered argument that

$\neg \forall w \forall w^*(w>w^* \supset S(w)<S(w^*))$;

$\exists w \exists w^*(w>w^* \wedge S(w) \geq S(w^*))$

and therefore—with the uniformity assumption—that

$\forall w \forall w^* (S(w)=S(w^*))$:

Galileo's own law.

[13] That is, unless there is some sort of empirical input tacitly included in his arguments.

> ground by as much as a span ahead of a musket ball weighing only half a pound, provided both are dropped from a height of 200 cubits." (Galileo 1954: 62; my emphasis)

I let Galileo have the last word on this issue:

> Among the safe ways to pursue truth is the putting of experience before any reasoning, we being sure that any fallacy will be contained in the latter, at least covertly, it not being possible that a sensible experience is contrary to truth. And this also is a precept much esteemed by Aristotle and placed [by him] far in front of the value and force of the authority of everybody in the world. (From a letter to Fortunio Liceti, September 1640, here quoted from (Drake 1978: 409))

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