Multiple Generality in Scholastic Logic

Boaz Faraday Schuman

In natural language, ambiguity abounds. In particular, the presence of multiple quantificational terms has been long and widely recognized as a potential source of confusion. Here is a famous example in English, much discussed nowadays in introductory logic courses:

(1) Everyone loves someone

A proposition like (1) is ambiguous, because it is not entirely clear which of the quantificational terms ('everyone', 'someone') is the main one—that is, which one takes widest scope. In the more natural reading of (1), scope is determined by word order, so that 'everyone' is the main quantificational term, and 'someone' falls under it. If we read (1) this way, we understand it along the following lines:

(1a) For every person P, there is someone (or other) whom P loves

A proposition like (1a) will be true if everyone loves someone *or other*, and so it can be true even if there is no one specific person (say, Falstaff) whom everyone loves. But (1) also permits the following reading, which gives wide-scope to the particular term 'someone':

(1b) There is a (specific) person P whom everyone loves

Note that (1b) entails (1a), but not the other way around: if everyone loves one specific person, then everyone loves someone (or other). But even if everyone loves someone (or other), it does not follow that one

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specific person is loved by everyone. So the truth conditions for the (1a) and (1b) come apart.

Granted, it is far more natural to read (1) as (1a) than as (1b). Indeed, outside of logical contexts, we might not register the difference. But this is a pragmatic matter, not a semantic one. This fact is made plain by a famous joke from *Saturday Night Live*: "Every minute a man is mugged in New York City. We are going to interview him tonight."[1] The fact that the initial and obvious understanding of the first proposition of the joke can be cancelled by the second without contradiction shows us that the less obvious meaning was available all along. Similarly, we might add a clause to (1), as follows:

(1c) Everyone loves someone, and that special someone is Sir John.

Adding the right-hand conjunct thus rules out (1a) without falsifying (1). So the interpretation given by (1b) is genuinely available all along, even though it is far less natural than that (1a).

Nowadays, we use variable-binding quantifiers to disambiguate ambiguities like that of (1): in (1a), the particular quantifier 'some' (of 'someone') is within the scope of the universal quantifier 'every' (of 'everyone'). In (1b) the orders are reversed. Accordingly, the two would be symbolized as follows:

($1a_S$) $\forall x \exists y(\text{Loves}(x, y))$
(*For every x, there exists a y, such that x loves y*)

($1b_S$) $\exists y \forall x(\text{Loves}(x, y))$
(*There exists a y, such that for every x, x loves y*)[2]

[1] Cited by Dave Barker-Plummer, Jon Barwise, and John Etchemendy, *Language, Proof and Logic*, 2nd ed. (Stanford: CSLI Publications, 2011), 314.

[2] I am aware that this way of symbolizing these propositions will only work if our domain is restricted just to humans. If the domain is broader (if, for instance, it includes horses or axes or anything else), we will have to specify that it is *people* we are talking about, and so ($1a_s$) will look like this: $\forall x \exists y((\text{Person}(x) \wedge \text{Person}(y)) \rightarrow \text{Loves}(x,y))$, and ($1b_s$) will likewise incorporate a conditional with a conjunctive antecedent. But I have opted for the simpler way of symbolizing these, following the dictum Quine attributes to Adolf Meyer: "where it doesn't itch, don't scratch." See W. V. O. Quine, *Word and Object* (Cambridge, MA: MIT Press, 1960), 146.

Hence the modern quantifiers give us a ready way of disentangling the two available meanings of (1).

We owe these variable-binding quantifiers to Frege. Later Fregeans thought that Frege had found and solved a problem of which contemporary Aristotelian logic—as developed by Boole and his followers—was unaware, with which it was ill-equipped to cope. If Boolean logic is the latest and most sophisticated version of Aristotelian logic, then by implication problems with it are problems with its Scholastic Aristotelian predecessor as well.

Some writers are, however, historically aware, and note the accomplishments of the Scholastics in this regard. For example, Michael Dummett tells us that:

> Scholastic logic had wrestled with the problems posed by inferences depending on propositions involving multiple generality—the occurrence of more than one expression of generality. In order to handle such inferences, they developed ever more complex theories of different types of '*suppositio*' (different manners in which an expression could stand for or apply to an object): but these theories, while subtle and complex, never succeeded in giving a universally applicable account, either from the standpoint of syntax (the characterization of valid inferences in formal terms) or from that of semantics (the explanation of the truth conditions of propositions involving multiple generality).[3]

Dummett's account is accurate and fair: while Scholastic logic attained a high level of complexity in its analysis of multiply-general propositions, it never provided a general account to compete with Fregean variable-binding quantifiers.

Admittedly, Dummett's account still has a whiff of whiggishness to it: were Scholastics searching, as he seems to suppose, for a universally applicable account, of the sort furnished by Frege? Or were they thinking of the problem quite differently? I doubt that Scholastic logicians, whose object of analysis was reasoning in ordinary language, would have

[3] Michael Dummett, *Frege: Philosophy of Language* (London: Duckworth, 1981), 8. For a more recent formal discussion of the applicability of Scholastic notions of supposition to contemporary problems of multiple generality, see Gyula Klima and Gabriel Sandu, "Numerical Quantifies in Game-Theoretical Semantics", Theoria 56 (1990), 173–92.

wanted recursively defined quantified propositions of arbitrary complexity even if we could go back and offer it to them. Past the few layers of multiple generality the human mind is capable of sustaining in a single line of reasoning, it is not clear what advantage an analysis of arbitrarily long strings of quantifiers provides—at least, as a tool for the solitary reasoner, or for debaters in the forum.

At any rate, whether or not Scholastic logic could (or would) do what Frege did, its achievements are remarkable, and merit study in their own right. Moreover, understanding how these forms work can tell us more general things about Scholastic logicians' views on form and logical foundations—something I take up in §3.

In outline, a Scholastic solution to the ambiguity of (1) turns on the fact that Latin, as a synthetic language, has relatively free word order. Given this flexibility, Latin word order can be regimented, by convention, to express (1a) and (1b), as follows:

($1a_L$) Omnes aliquem amant.
($Everyone_{SUBJ}$ $someone_{OBJ}$ $loves$)

($1b_L$) Aliquem omnes amant.
($Someone_{OBJ}$ $everyone_{SUBJ}$ $loves$).[4]

So long as we agree—as we did a moment ago with ∀ and ∃—that whichever quantificational term is leftmost has widest scope, the problem is solved. But beyond differentiating the two readings of (1), we might wonder what further things we can express using this regimented Latin—and what other sorts of propositions the medievals themselves had in mind. Happily, medieval writers love a catalogue—"that most medieval of constructions," as M. J. Toswell memorably puts it[5]—and the Scholastics are no exception: later medieval logicians produced extensive lists of multiply quantified propositions, and cataloged these propositions' logical interrelations.

[4] For this example, I am borrowing the technique of using subscripts to denote grammatical function set out by Terence Parsons, *Articulating Medieval Logic* (Oxford: Oxford University Press, 2014).

[5] M. J. Toswell, *Borges the Unacknowledged Medievalist: His Life and Work* (New York: Palgrave Macmillan, 2014), 8.

To date, studies have focused on the expressive power of certain constructions, or their role in syllogistic logic. E. J. Ashworth has explored the use of multiple quantifiers in sixteenth-century Aristotelian logic in terms of the theory of supposition.[6] As Terence Parsons has set out at length, later medieval logicians had the tools to disambiguate propositions like (1), using a regimented Latin syntax and semantics.[7] Stephen Read has discussed their role in syllogistic logic in two papers on these non-normal forms.[8] And Paul Thom has given a detailed account of how propositions with oblique terms can be run through the syllogistic machinery.[9]

The present paper supplements these studies by presenting two huge figures (*magnae figurae*) displaying multiply-general propositions and their logical interrelations. These appear in a fourteenth-century MS of John Buridan's *Summulae de propositionibus*.[10] The syntax of one of these *figurae* (that of §2, below) has been discussed by Juan Manuel Campos Benítez,[11] using the formal language L developed by Walter Redmond for the logical systems of the Spanish Golden Age and New Spain.[12] Yet there is no systematic treatment of the semantics of these propositional forms, as they appear in the *magnae figurae*, in Buridanian

[6] E. J. Ashworth, "Multiple Quantification and the Use of Special Quantifiers in Early Sixteenth-Century Logic," *Notre Dame Journal of Formal Logic* 19 (1978), 599–613.

[7] Parsons, *Articulating Medieval Logic*, esp. 81ff. See also Parsons, "The Power of Medieval Logic," in C. Bolyard and R. Keele (eds.), *Later Medieval Metaphysics: Ontology, Language, and Logic* (New York: Fordham University Press, 2013), 188–205.

[8] Stephen Read, "John Buridan's Theory of Consequence and His Octagons of Opposition," in J.-Y. Béziau and D. Jacquette (eds.), *Around and Beyond the Square of Opposition* (Basel: Springer, 2012), 93–110; Stephen Read, "Non-Normal Propositions in Buridan's Logic," in L. Cesalli et al. (eds.), *Formal Approaches and Natural Language in Medieval Logic* (Barcelona: Brepols, 2016), 453–68. 'Non-normal forms' is Read's translation of *modi loquendi inconsueti*, and I have adopted this translation here.

[9] Paul Thom, "Termini Obliqui and the Logic of Relations," *Archiv für Geschichte der Philosophie* 59 (1977), 143–55.

[10] While Buridan's term for such eight-noded diagrams is *magna figura*, modern commentators have tended to represent and accordingly speak of them as octagons. The manuscript where these *magnae figurae* appear is Vatican ms. Pal. Lat. 994, ff. 7r (propositions with oblique terms) and 11v (multiply-quantified propositions). The same manuscript also features, at f. 6r, an expanded Square, considered below. It is not clear whether these *figurae* for multiply-general propositions are themselves Buridan's, or merely constructed using the rather terse discussion Buridan gives in *Summulae* 1.4.2 (see below, §2.a.i). Here, I remain neutral on this question: I merely assume that, whether or not the *figurae* are Buridan's, they are at any rate Buridanian.

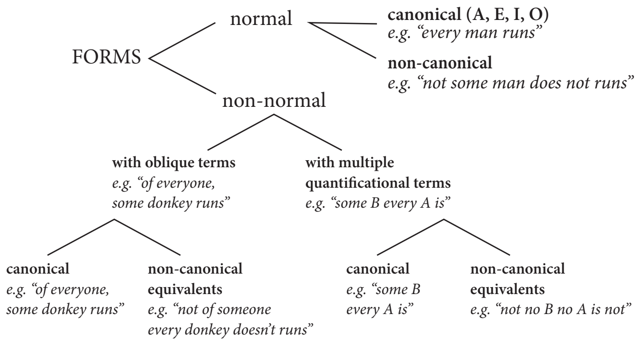
[11] Juan Campos Benítez, "The Medieval Octagon of Opposition for Propositions with Quantified Predicates," *History and Philosophy of Logic* 35 (2014), 354–68.

[12] Walter Redmond, "Quantified Inference in 16th-Century Mexican Logic," *Vivarium* 39 (2001), 87–118.

terms. This is the purpose of the present paper, though as we'll see, the observations we set out here have more general ramifications for our understanding of the development of logic in the fourteenth century.

These forms are not all reducible to the **normal** forms of the traditional Square of Opposition, which we'll see in a moment. Buridan calls the forms in these *magnae figurae* "unusual ways of speaking" (*modi loquendi inconsueti*). Following Stephen Read, I call them **non-normal** forms.[13] Both normal and non-normal forms have **canonical** forms as well as **non-canonical** variants. The former are generally the simplest. The latter are (often much more complicated) equivalents, which make use of iterated (and in one case, even *quintuple*) negation, plus alteration of quantities, and so on, in a way analogous to the equivalences worked out by Ockham and later De Morgan.

Here is a diagram to clarify these relations among the forms:



These interrelations will become clearer as we go.

Now, the use of the term 'form' here to cover propositions like "of everyone, some donkey runs" as well as logical schemata like "some B every A is" might seem a bit suspect. But I am trying to work as closely as possible with what the diagrams give us. For instance, there are no examples of what terms we might plug into the B and A of "every B every A is" in order to get an ordinary proposition. (This fact poses special problems for interpretation of this form, as we'll see in §2.a.i, below). And, while a proposition like "of everyone, some donkey runs" obviously has a form, this form is not dealt with schematically in the diagrams or

[13] Read, "Non-Normal Propositions in Buridan's Logic."

the text—e.g., as "of every C, some B is A." So I am using the term 'form' here somewhat loosely, to cover all of the propositions and schemata we'll look at below, both normal and non-normal.

The non-normal forms themselves antedate Buridan—as we will see, they are under discussion nearly two centuries before him. Yet the *magnae figurae* contribute something new: in addition to cataloguing the forms, they also *arrange* them according to their logical interrelations. To my knowledge, this is the first attempt to do so with this level of complexity.

Buridan's own account of these forms is relatively sparse. In contrast, the modal propositions ("every B is necessarily A," "some B is not possibly A") of a third *magna figura* receive extensive treatment across Buridan's logical works, including about half of his *Tractatus de consequentiis*. What Buridan does have to say about the semantics of the non-normal forms is said in passing. And, on the whole, he seems more concerned with their syntax—the role, for instance, they play in syllogisms, and whether an oblique term should be analyzed as part of the subject.[14] In terms of semantics, Buridan himself nowhere gives a thorough and exhaustive account of the truth conditions of these non-normal forms, especially in terms of their existential requirements.[15]

Faced with these difficulties, I have adopted the following approach: I assume that the semantics of these non-normal propositional forms can be analyzed in light of what Buridan says about the truth conditions of normal forms of propositions, vacuous truth, and so forth. Building on this basis, we can fill in the details of his passing observations about these non-normal forms. As far as possible, I draw on what Buridan says, though a good deal of the semantics of these forms has to be extrapolated, and even worked out *a priori*.

In what follows, I present each diagram, along with a schematization of it, before setting out truth conditions for each non-normal form in turn. As we will soon see, establishing these truth conditions is frequently non-trivial and even counterintuitive. But first, since the non-normal forms are built up on the basis of the normal Aristotelian forms (A, E, I, and O), let's begin with these.

[14] See especially his discussions in *Summulae de syllogismis* (5.8.2–4), and *Quaestiones longe super librum Perihermeneias* (1.6).

[15] However, he does examine the modal propositions, which have their own similar *magna figura*, in great detail. In this, most modern commentary has followed him.

0. Normal Aristotelian Forms

In *De Interpretatione* 7 (17^b17–37), Aristotle recognizes four propositional forms, and sets out their interrelations (though not diagrammatically). Traditionally, these propositions have the following designations, forms, symbolizations and names:[16]

A	All S is P	S*a*P	Universal Affirmative
E	No S is P	S*e*P	Universal Negative
I	Some S is P	S*i*P	Particular Affirmative
O	Some S is not P	S*o*P	Particular Negative

These forms are arranged in the Square of Opposition, which displays their logical interrelations:[17]

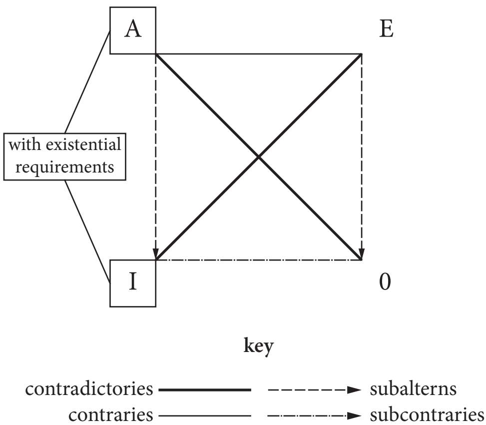


Fig. I The traditional Square of Opposition

Note that the relation of subalternation is asymmetric: the truth of I (O) follows from A (E), but not vice versa. Hence the arrow.

[16] For his part, Aristotle uses the much less natural predicate-subject form, e.g. "P belongs to every S." But by and large, this approach fell by the wayside in the Aristotelian tradition, in part because it is so unnatural. Most medieval logicians follow the forms as they are set out here, that is in subject-predicate form.

[17] For clarity, I use roman numerals to number the *figurae* in the texts, and Arabic numerals for the individual diagrams displaying the truth conditions of the various forms, below.

Buridan gives us the following summary of the interrelations among the nodes of this square, represented here by different lines. These interrelations will be indispensable in our discussion of the non-normal forms. To begin:

> **Contradiction**: the rule governing contradictory pairs is that if one is true, then the other is false, and vice versa. So in no matter can the two be either both true, or both false.[18]

Every proposition has a contradiction, and by the Rule of Contradiction the two parts of a contradictory pair take opposite truth values. On the Square of Opposition, this relation holds between A and O, and between E and I. Buridan's examples are the following:

(2)	Every man runs[19]	[A-type]
(3)	Some man does not run	[O-type]

If (2) is not true—that is, if it is not the case that everyone runs—then it must be the case that at least one man is not running, in which case (3) is true.[20] Conversely, if (3) is false—that is, if it is not the case that someone does not run—then everyone runs. Hence it must be the case that one of (2) or (3) is true, from which it follows that the other of the pair is false. So every contradictory pair will have a true and a false proposition. It is common to express this in medieval logical texts by saying that contradictory pairs have no intermediate (*medium*).[21] Jointly, (2) and (3) exhaust all possibilities. This fact sets contradictory pairs apart from pairs of contraries:

[18] "Lex contradictoriarum talis est quod si una est vera, reliqua est falsa, et e converso; in nulla enim materia possunt simul esse verae vel falsae" (*Summulae* 1.4.4).

[19] All these examples of canonical forms, set out here in (2)–(11), are from *Summulae* 1.4.2.

[20] As we'll see in a moment, (3) has no existential requirements, and so is capable of vacuous truth.

[21] *Summulae* 1.4.2.

> **Contrariety**: the rule and nature (*lex et natura*) of contraries is such that, if one is true, then the other is false, but not the other way around.[22]

If one of a contrary pair is true, then the other is false. But, unlike contradictory pairs, the falsity of one does not entail the truth of the other. This relation forms the A–E axis of the Square. Here is Buridan's example:

(4) Every man runs [A-type]
(5) No man runs [E-type]

In contrast with (2) and (3), above, (4) and (5) *do* have an intermediate (*medium*): they can both be false in a case in which some men are running, and some are not. Hence from the falsity of one of them, the truth of the other does not follow. Nevertheless, if one of them is true, then the other must be false. This is, in a sense, parallel but opposite to the Law of Subcontrariety, to wit:

> **Subcontrariety**: The law of subcontraries is such that if one is false, then the other is true, but not the other way around.[23]

So from the falsity of one proposition of a subcontrary pair, the truth of the other follows. This relation holds between I and O propositions with the same terms. Buridan gives the following example:

(6) Some man runs [I-type]
(7) Some man does not run [O-type]

If (6) is false, then (7) is true, and vice versa. But from the truth of either, the falsity of the other cannot be inferred, since it is possible that some man runs, and some other man does not, in which case both are true.

[22] *Summulae* 1.4.4. Note that Buridan's phrase *lex et natura* suggests that his accounts here are both rules, and also descriptions of how these pairs of propositions work logically.
[23] *Summulae* 1.4.4.

All of the foregoing relations are symmetric: if a proposition φ contradicts a proposition ψ, then ψ contradicts φ. The same holds *mutatis mutandis* for contraries and subcontraries. But there remains a final relation which governs the inference of particulars (I, O) from universals (A, E), and which stands apart from the rest as asymmetrical:

> **Subalternation**: the rule governing subalterns is that if the universal is true, the corresponding particular is also true—but not vice versa, for it is possible that the particular be true and the universal false. And if the particular is false, so is the universal, but not vice versa.[24]

Buridan gives the following:

(8)	Every man runs	[A-type]
(9)	Some man runs	[I-type]

Here (9) follows from (8), but not the other way around, and the falsity of (8) follows from the falsity of (9), but not the other way around. Likewise, these relations hold among negative universals and particulars, like the following:

(10)	No man runs	[E-type]
(11)	Some man does not run	[O-type]

These four relations exhaust the possibilities on the Square of Opposition, and will be extended to the two irregular *magnae figurae* in the next two sections.

On the foregoing square (Fig. I), I have noted that the affirmatives (A and I) have existential requirements, whereas the negatives (E and O) do not. Buridan is clear that, propositions like "Every S is P" ("Some S is P") are only true when there actually exists some S, all (some) of which is P. Conversely, the E/O axis has no such requirements, and indeed can be vacuously true. That is, "No S is P" and "Some S is not P" can be true either when there is some S, none (or some) of which is not P, or when

[24] *Summulae* 1.4.4.

there is no S whatsoever. Buridan explicitly acknowledges this fact, and even goes so far to tell us that the following proposition is true:

(12) No chimaera is a chimaera.[25]

There are no chimaeras, and so the universal negative proposition (12) is true.[26] Accordingly, the contradiction of (12), namely,

(13) Some chimaera is a chimaera

is false. By the Law of Subcontrariety, if the particular affirmative (13) is false, then its subcontrary particular negative, namely,

(14) Some chimaera is not a chimaera[27]

is true. Hence (14) must likewise not have existential requirements. Accordingly it can, like (12), be vacuously true. And since it is true, its contradiction—a universal affirmative like the following—is false:

(15) Every chimaera is a chimaera.

This point is worth lingering on for a moment, for two reasons. The first is, because the existential requirements of the propositions in the Square are carried over to those of the *magnae figurae*. For our exposition, we have to get them right. The second is because, in my experience, the existential requirements of Scholastic logic often confuse those versed in modern logic—or anyway, puzzlement about these requirements comes up fairly often following conference presentations. This is in large part because

[25] In his discussion of truth, Buridan gives the particular "illa est vera: 'chimera non est chimera'" (*Quaest. Metaphys.* VI.6, fol. 37rb). But the universal negative "*nulla chimaera est chimaera*" is, in his semantics, true, as he confirms elsewhere, in a discussion not of truth but of syllogistic form (*Quaest. AnPr* II.13). The examples here, from (12)–(15), are based on the *Quaest. Metaphys.* passage just cited.

[26] Chimaeras are stock examples of impossible objects in medieval textbooks, much like the round squares of modern ones. For a lively and entertaining overview of this role of the chimaera, see Sten Ebbesen, "The Chimaera's Diary," in S. Knuuttila and J. Hintikka (eds.), *The Logic of Being* (Dordrecht: Springer, 1986), 115–43.

[27] Alternatively, we could derive (14) from the falsity of (13), by the rule of subcontraries.

these requirements differ significantly from the way we are used to thinking of them in terms of modern predicate logic (MPL). In MPL, we read universal affirmatives like "Every S is P" as follows:

(16) $\forall x(S(x) \rightarrow P(x))$
(*For all* x, *if* x *is S, then* x *is P*)

In MPL, (16) can be vacuously true, thanks to the truth conditions for material conditionals. A material conditional is false only when it has a true antecedent and a false consequent. Accordingly, it is always true when it has a false antecedent. And if there is no S whatsoever, then the antecedent is false.

Conversely, in the formulation of MPL a particular negative—O-type propositions of the form "Some S is not P"—*does* presuppose the existence of its subject matter. This is because we write these with an existential quantifier, as follows:

(17) $\exists x(S(x) \wedge {\sim}P(x))$
(*There exists an* x, *such that* x *is S and* x *is not* P)

In MPL, (17) is true only when there exists at least one S which is not P.[28] Hence MPL groups the existential requirements along the I/O axis, and allows A/E to be vacuously true. This is a defining aspect of MPL, and it is about as ingrained in the thinking of many researchers and teachers as it is perplexing to students in introductory logic classes. Indeed, it is so ingrained in the minds of the former that it is even possible to find

[28] In case you were wondering, Scholastic logic can express (17), too. As Gyula Klima points out, a proposition with equivalent truth conditions can be constructed using predicate negation. So to get an existentially-committed particular-negative proposition, we would accordingly read (3) as follows: "Some S is *non*-P." This is all well and good, though I worry it looks a tad weird when we want to deny a predicate of a subject which would itself be categorically erroneous—i.e. of a predicate that does not apply to a predicate *simpliciter*, rather than one which merely fails to do so. For example, from "Earth's orbit is not tragic" we could generalize to "Some planet's orbit is non-tragic." But Earth's orbit does not seem to be the kind of thing to which contrary predicates like *tragic* and *non-tragic* even *can* apply (or fail to apply), and such a reading will indeed cause trouble for any category theory that defines categories in terms of contrary predicates, as Fred Sommers's does. See Gyula Klima, "Existence and Reference in Medieval Logic," in A. Hieke and E. Morscher (eds.), *New Essays in Free Logic* (Dordrecht: Springer, 2001). Also see Fred Sommers, "Structural Ontology," *Philosophia* 1 (1971), 21–42.

putative arguments against the traditional Square on the grounds of symbolizations like (16), which do not align with the traditional assessment of particular negatives.[29] Since MPL is the true logic (*ex hypothesi*), it follows that the Square is wrong.[30] These arguments are a bit embarrassing: criticizing the traditional Square for not accommodating Fregean apparatus is a bit like criticizing the Appian Way for lacking gas stations.

For our present purposes, we just have to bear in mind that Scholastic logic clusters the existential requirements around the affirmative axis of the Square, whereas it is now fashionable to cluster them around the particular axis. This fact will be so significant when we turn to the non-normal forms that it merits setting down here for future reference:

Fact 1: On traditional Aristotelian logic, affirmative propositions (i.e. those with A/I form) have existential requirements, whereas negative (E/O form) ones do not.

The appeals to form in the statement of Fact 1 call for a closer look at the irreducible forms of the square. Getting the syntax of these is vital for our exposition, because many non-normal forms like the following are not obviously either affirmative or negative:

(18) Of everyone, some donkey does not run
(19) Every S some P is not

Propositions like (18) and (19) look negative, but are they more like E-type, or more like O-type negatives? And can they, like the normal negative categoricals, be vacuously true? Whatever happens, half of each *magna figura*, like one half of the Square, must be capable of vacuous truth. Otherwise, things would fall apart: annihilation of everything in question would render more or less than half the propositions false; there would thus be a *medium* between contradictory propositions;

[29] See, for instance, Terence Parsons, "The Traditional Square of Opposition," in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy* (2017), §1.2.
[30] See e.g. Parsons, "The Traditional Square of Opposition," esp. §1.2.

and the Law of Contradiction would not hold. Accordingly, if we are to assign truth conditions to these non-normal forms, we need to be clear what makes negative propositions negative.

Buridan's general term for categorical propositions' negation and affirmation is 'quality' (*qualitas*). What accounts for quality, he says, is the predicative verb or copula involved. The copula, for Buridan, is the principal logical part of any categorical proposition.[31] Indeed, he goes so far as to call the copula a proposition's "most formal part" (*pars formalior*).[32] Copulae come in two flavors: affirmative and negative. A negative proposition is one whose principal part is a special kind of copula, where the negative sign acts on (*cadat super*, literally "falls on") the copula. Buridan is very careful to distinguish negation which acts on the copula, producing proposition-wide negation, from narrower term-negation. Hence the mere presence of a negative particle is not sufficient to render a proposition negative. Otherwise, the following affirmative would be negative:

(20) Socrates is a non-donkey

Since the negation in (20) only applies to the term 'donkey,' and not to the copula, (20) is an affirmative proposition of I-type form. Accordingly, (20) has very different existential requirements and truth conditions from a corresponding proposition with negation that affects the copula—namely, one of O-type. In sum:

Fact 2: The quality of a categorical proposition is determined by the negation (or lack thereof) of its copula.

As you can imagine, the notion of quality becomes more difficult in the case of propositions with multiple negations. And such propositions abound: in addition to the *magnae figurae*, one manuscript of Buridan's *Summulae de dialectica* (Vatican ms. Pal. Lat. 994, f. 6r) also includes an expanded Square (Fig. II).

[31] As opposed to hypotheticals, which are made up of categorical-like parts: these include conditionals, but also conjunctions, disjunctions, and so forth.
[32] *Summulae* 1.3.4.

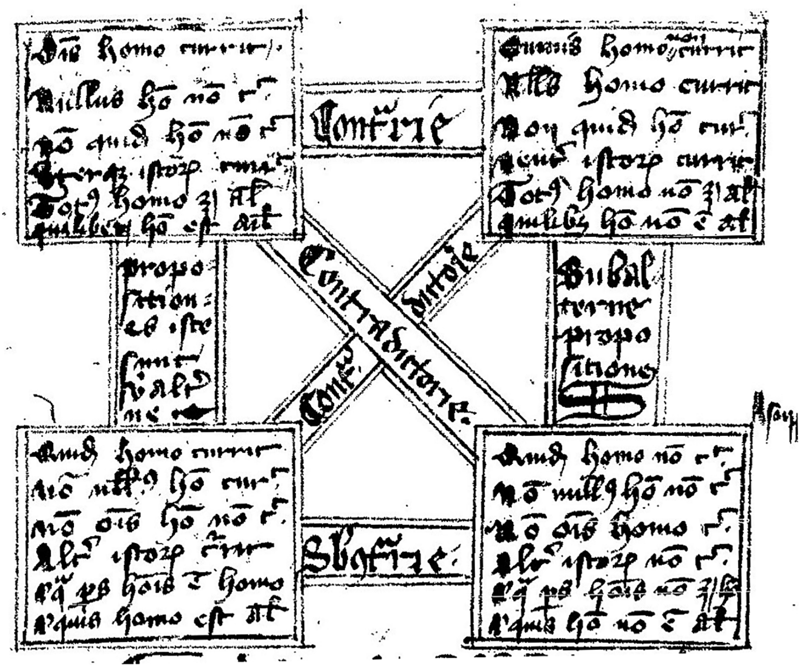


Fig. II The expanded Square in Pal. Lat. 994, f. 6r

Fig. II, as well as Figs. III and V, are here reproduced with permission from the Vatican Library.

This square includes non-canonical forms of the canonical A, E, I and O forms discussed above, and schematized in Fig. I. There are also non-canonical forms, listed in the squares of the above diagram with similar relations to consider:[33]

A: canonical form:

0.A.1 *omnis homo currit*
"every man runs"

A: Non-canonical forms:

0.A.2 *nullus homo non currit*
"no man does not run"

E: canonical form:

0.E.1 *omnis homo non currit*
"every man does not run"

E: Non-canonical forms:

0.E.2 *nullus homo currit*
"no man runs"

[33] Here and below, I call the first listed form the *canonical* form, and the remainder *non-canonical*. The propositions we will consider below are non-normal, and have both canonical and non-canonical forms, in contrast with the normal forms set out in the traditional Square. I'll clarify these things as we go.

0.A.3 *non quidam homo non currit* “not some man does not run”	0.E.3 *non quidam homo currit* “not some man runs”
0.A.4 *uterque istorum currit* “both of the men run”	0.E.4 *neuter istorum currit* “neither of the men runs”
0.A.5 *totus homo est homo* “The whole of man is man”	0.E.5 *totus homo non est homo* “The whole of man is not man”
0.A.6 *quilibet homo est asinus* “every man is a donkey”[34]	0.E.6 *quilibet homo non est asinus* “every man is not a donkey”
I: canonical form:	**O: canonical form:**
0.I.1 *quidam homo currit* “some man runs”	0.O.1 *quidam homo non currit* “some man does not run”
I: Non-canonical forms:	**O: Non-canonical forms:**
0.I.2 *non nullus homo currit* “not no man runs”	0.O.2 *non nullus homo non currit* “not no man does not run”
0.I.3 *non omnis homo non currit* “not every man does not run”	0.O.3 *non omnis homo currit* “not every man runs”
0.I.4 *alter istorum currit* “one of the men runs”	0.O.4 *alter istorum non currit* “one of the men does not run”
0.I.5 *aliquis pars hominis est homo* “some part of man is man”	0.O.5 *aliquis pars hominis non est homo* “some part of man is not man”
0.I.6 *aliquis homo est asinus* “some man is a donkey”	0.O.6 *aliquis homo non est asinus* “some man is not a donkey”

Now the penultimate propositions in each of these groupings incorporate mereological language that may seem strange at first blush. What does it mean to say that a part (or the whole) of humankind is (or is not) a human? This language is perhaps a bit embarrassing for a committed anti-realist about universals like Buridan. But it is also much older than

[34] I have opted for ‘every’ to translate *quilibet*, rather than ‘any,’ in part because it makes the meaning of the oblique propositions below much clearer: “anyone’s donkey runs” is harder to parse than “everyone’s donkey runs,” and the semantics for the Latin *cuiuslibet asinus currit* aligns nicely with the latter.

he is. For example, Aristotle remarks that "a universal is a kind of whole, comprehending many things within it like parts" (*Physics* 1.1, 184a3–5). Boethius echoes this language in *De divisione* (887d), telling us that "we may also call a universal like man (*homo*) or horse (*equus*) a whole."[35]

Even so, this language need not hint at some deeper conceptual tension in Buridan. After all, he is often happy to adopt traditional formulations, even while he develops new concepts. As Sten Ebbesen memorably puts it, Buridan is like a renovator of old houses: he keeps the Aristotelian façade, but updates the interior to suit his purposes.[36] Accordingly, such language of parts and wholes of universals should, when it comes up in Buridan, be taken to be conventional (*ad placitum*), and little else.[37] So we can set this worry aside.

Importantly, the forms displayed in Fig. II cannot all be taken to be equivalent. For example, the fifth proposition at each node ("the whole of humankind is a human," etc.) clearly has logical relations to every other fifth proposition in the other nodes, but cannot be taken to be logically equivalent to the other ones in its node. In contrast, all the propositions in each node of the *magnae figurae* are indeed logically equivalent. This makes Fig. II an outlier, though the difference in terms incorporated into the propositions does a lot to prevent confusion: no one would take, for example, "both of them run" (0.A.4) to be logically equivalent with "every man runs" (0.A.1), at least for any domain with more than two people in it.

Let me give one caveat before we turn to the non-normal forms which go beyond those of the Square: in what follows, I avoid metaphysical talk wherever possible, and focus on the syntax and semantics. Gabriel Nuchelmans has already written at length on the metaphysics of propositions in medieval logic, including Buridan's: in what way they exist, to

[35] Note however that Boethius is careful to distinguish the relations of priority that obtain between wholes and their parts, on one hand, and genera and species, on the other: the parts of a whole are naturally prior to the whole itself, whereas a genus is prior to its species (879c).

[36] Sten Ebbesen, "Proof and Its Limits according to Buridan *Summulae* 8," in *Topics in Latin Philosophy from the 12th–14th Centuries: Collected Essays of Sten Ebbesen*, vol. 2 (New York: Routledge, 2016), 210.

[37] I am grateful to Peter King for bringing these aspects of medieval mereology about universals to my attention. For a clear and concise discussion of this aspect of medieval mereology, see Andrew Arlig, "Medieval Mereology," in E. N. Zalta (ed.), *Stanford Encyclopedia of Philosophy* (2019).

what (if anything) they refer, what their truthmakers are, and so on.[38] Gyula Klima has already analyzed Buridan's anti-realism or tokenism about propositions.[39] Curious readers should check them out. It is not that I have anything against metaphysics—far from it—but I want to limit my scope as far as possible to the truth conditions and syntax of these non-normal forms.[40] With these things in mind, let's turn to the first set of propositions.

1. Plural and Singular Genitives

The first set of propositions we will look at are the non-normal forms incorporating an oblique term in the subject—in this case, a genitive universal or particular composite term. For example, "of every man, every donkey runs" (*cuiuslibet hominis quilibet asinus currit*), which is true when every man has at least one donkey, and every donkey any man owns is running. In part because these are given as sample propositions and not forms, they are easier to think about than the second set, which are presented only schematically (e.g. as "every B every A is") and face special difficulties. That's why I am starting with the forms incorporating these oblique terms.

As with the normal forms of the Square, I divide these non-normal forms into canonical forms—that is, the first form listed in each node of each *figura*—and non-canonical forms—that is, the other equivalents, also listed in the nodes, which are usually more syntactically complex. Each node contains nine forms: one canonical, and eight non-canonical, for a total of nine per node. This prompts a series of questions, which I will take up right now.

[38] Gabriel Nuchelmans, *Theories of the Proposition: Ancient and Medieval Conceptions of the Bearers of Truth and Falsity* (Amsterdam: North Holland, 1973). See also Gyula Klima, "Consequences of a Closed, Token-Based Semantics: The Case of John Buridan," *History and Philosophy of Logic* 25 (2004), 95–110.

[39] Gyula Klima, *John Buridan* (Oxford: Oxford University Press, 2009), esp. chs. 9–10.

[40] Even "truth conditions" might itself be a bit fraught. I ask the reader to bear in mind that I do not mean to use this term in any ontologically-laden truth-maker-y sense. Nor do I mean to deny that such a sense makes sense. It is hard enough to do justice to the subject at hand; to speak generally of related subjects in medieval metaphysics at the same time is to court disaster.

Why nine? In the *Summulae de propositionibus*, Buridan gives instructions for producing non-canonical variants which are logically equivalent equivalent (*aequipollentes*), by altering negation signs, and changing the quantities of the terms. He then remarks that:

> from these combinations, there results a *magna figura* with eight vertices, and at each vertex there are nine propositions.[41]

Yet it seems that more non-canonical variants of these non-normal forms could easily be listed and analyzed, for instance by iterative applications of negation. I see no reason—apart from shortages of space and the inherent limitations of the human mind—why this process could not go on indefinitely. After all, Buridan's list even includes a whopping quintuple-negation (see §2.a.ii, 8). Accordingly, the *figurae* we will consider are themselves not complete even for the non-canonical variants of the non-normal forms they set out. But at any rate, we'll stick to those listed in the *figurae*.

Why these forms, and not others? Buridan certainly recognizes other, non-normal forms, and discusses their logical properties. Yet these forms are nowhere arranged diagrammatically. For example, Buridan's texts contain discussions of propositions like "a horse belonging to the king is seen by some man (or other)" (*equum regis homo est videns*) and "any animal belonging to a king is a horse" (*quodlibet regis animal est equus*).[42] Likewise, we find the relatively simple "a man's donkey runs" (*hominis asinus currit*).[43] And there are many other forms to be found in his *Sophismata*, e.g. "there is a donkey that every man sees" (*aliquem asinum omnis homo vidit*).[44] Clearly, such forms are logically related to others by contradiction, subalternation, and so forth. Why then do we not find additional *magnae figurae* dedicated to their interrelations as well? I'll admit: I do not know. For my part, I have selected the forms we

[41] *Summulae* 1.5.1.

[42] *Summulae de syllogismis* 4.2.6 and 4.3.8.1, respectively. Here, I follow Klima's lead in supplying an indefinite, rather than definite article (see his translation of the *Summulae*, 274, n. 76).

[43] *Quaestiones super libros Analyticorum Priorum* (1.17).

[44] *Sophismata* 3 ad. 6, ed. Scott p. 56.

will examine here simply because they appear in the *magnae figurae*, and because their truth conditions have not yet been exhaustively analyzed.

Where do these propositions about running-donkey ownership come from? Difficulties posed by terms like '*cuiuslibet*' ("of each man") were noted and discussed well before Buridan, particularly in the context of invalid inferences. L. M. De Rijk has found evidence that these forms were already current toward the end of the twelfth century, in discussions of *sophismata*.[45] The forms, and the supposition of their terms, even receive passing treatment in the (in)famous Condemnation of 1277 which includes, holus-bolus, fourteen condemnations strictly about logic and grammar, and not theology.[46] All these facts suggest that these forms were current and much discussed.

William of Sherwood (c. 1200–72) gives a careful treatment of a related *sophisma*, and his treatment has received considerable attention. Here is the argument that has Sherwood worried:

> Suppose that each man owns one donkey, and that it is running. And suppose that Brownie is a donkey all men own in common, and that Brownie is not running. Then each man's donkey is running. But whatever is each man's donkey is Brownie. Therefore, Brownie is running.[47]

The difficulty, as Kretzmann helpfully points out in a footnote, is that there are two senses of "each man's donkey is running": (i) "for each man x there is a donkey y such that x owns y and y is running," and (ii) "there is a donkey y such that each man owns y and y is running." It is also pretty easy to see how failure to distinguish these senses could lead one to mistakenly infer that, from the fact that each man has (his or her own) running donkey, it follows that there is some running donkey owned by

[45] L. M. De Rijk, "Each Man's Ass Is Not Everybody's Ass," *Historiographia Linguistica* 7 (1989), 225.

[46] De Rijk, "Each Man's Ass," 228–29. There was also some overlap between certain of the theses condemned and the logic of the day, especially modal and temporal logic. For an overview, see Sara Uckelman, "Logic and the Condemnations of 1277," *Journal of Philosophical Logic* 39 (2010), 201–27.

[47] *Syncategoremata*, p. 20 (O206 vb).

every man.[48] Indeed, we must keep these things in mind when we set out the truth conditions for related forms in (§1.b.i).

Such forms also pose serious syntactic problems, and indeed seem to have provoked a crisis for medieval logicians in the thirteenth century. As Angel D'Ors shows, thirteenth-century logicians struggled to account for these three-term propositional forms in terms of the two-term logical paradigm they inherited from Aristotle.[49] Treating oblique terms like '*cuiuslibet*' ("of each man") simply as part of the subject, like any other determiner ('blue,' 'rectangular') introduces serious problems, because it paves over the logical role the oblique terms themselves play. On the other hand, introducing a whole new logic of three-term propositions flies in the face of many established grammatical criteria, as well as the authoritative pronouncement of Aristotle that the subject term of a categorical proposition must invariably be in the nominative case.[50] For these and other reasons, it is not at all clear how such forms should work in syllogistic logic.

Problems, then, abound. Here, let's limit ourselves to two: the truth conditions for these non-normal forms, and their logical interrelations, specifically that of contradiction.

Before we begin, a word on this grammatical case: the most basic function of the genitive is possession: <u>*alicuius hominis*</u> *asinus* (is the donkey <u>of some man</u>, i.e. belonging to some man). More generally, the genitive case makes an otherwise stand-alone noun relational. As E. C. Woodcock points out, this has the effect of turning nouns into quasi-adjectives: compare the genitive phrase *mors fratris* with the adjectival phrase *mors fraterna* (the death of a brother).[51]

Why do I bring this up? Because the non-normal forms we are about to consider, like "of every man every donkey runs," are not multiply quantified in the same way as propositions like "every B is every A." Rather, the multiple generality of these propositions depends on a singular or plural genitive which acts as a determiner of the subject term. This construction, as a kind of adjective, does not alter the

[48] For an overview of this sophism in the thirteenth century, see De Rijk, "Each Man's Ass."
[49] Angel D'Ors, "*Hominis Asinus/Asinus Hominis*," in S. Read (ed.), *Sophisms in Medieval Logic and Grammar* (Dordrecht: Springer, 1993), 382–97.
[50] *Prior Analytics* 1.36 (49b39–a5).
[51] E. C. Woodcock, *A New Latin Syntax* (Mundelein, IL: Bolchazy-Carducci, 2013), 50.

supposition of the term in the same way that a simple quantificational term like 'some' or 'every' would. Such quantificational terms render normal propositions particular or universal, respectively. Yet, as we just saw, these oblique terms are not logically neutral determiners like 'blue' or 'rectangular.' Accordingly, we should be careful not to confuse the quantity of the genitive term with the main quantificational term of the proposition. But we should also be aware that these oblique terms do play a logical role. For instance, a genitive plural like *omnium* ("of all") in the subject does not render a proposition universal. The complex subject term "of every man, some donkey" is particular, not universal: its kernel is "some donkey," to which the adjective-like genitive phrase "of every man" attaches.

Buridan is keenly aware of these distinctions. At one point in the *Summulae de propositionibus*, he worries about the following proposition:

(21) Of every man, a donkey runs[52]

How, Buridan wonders, should we contradict (21)? If we take the 'every' in the subject to be the main quantificational term, then—following the example of the Square—we can create a contradiction by changing the quantity and adding negation. This gives us the following:

(22) Of some man, a donkey does not run[53]

The analogy between (21) and A-type forms on one hand, and (22) and O-type on the other, is clear. But there's a problem: as Buridan points out, (21) and (22) do not actually contradict each other: "both can be true at the same time, for instance in a case in which each man has two donkeys: one which runs, and the other which does not."[54] Clearly this is unacceptable: by the definition of contradiction, set out above, contradictory propositions have to take opposite truth-values. So (22) cannot be a contradiction of (21).

[52] *Summulae* 1.4.2. [53] Ibid. [54] Ibid.

The correct way to contradict (21), Buridan tells us, is with the following proposition, which changes not merely the quantificational term of the oblique case, but also the quantification governing 'donkey':

(23) Of some man, no donkey runs[55]

Thus a proposition like (21) is, in spite of the universal possessive construction, singular.[56] It is about *some* donkey, first and foremost, and every man only secondarily.

Accordingly, Buridan gives us instructions for constructing contradictions of these non-normal forms:

> a rule should be given for contradiction: every term, be it nominative or oblique, which is taken universally (that is, distributively) in one proposition should be taken indefinitely or as a particular in the other (that is, determinately), and vice-versa. Accordingly, the following two propositions contradict each other: 'of every man, a donkey runs' and 'of some man, no donkey runs'; likewise 'of every man, every donkey runs' and 'of a man, a donkey does not run'; and further still, the contradiction of 'of each contradictory pair, one or the other is true' is 'of some contradictory pair, neither one is true'. And similarly, the contradiction of 'every man an animal is not' is 'some man every animal is.'[57]

Hence in order to contradict (21), we changed the negation, and the signs of quantity, to get (23). This rule applies to the forms we'll see in §2, too, as the final example Buridan gives here makes clear.

Let's turn to the diagrams I will use here to display truth conditions for these forms. To clarify how they work, I'll run through a few examples. As we saw, in the following proposition, the quantificational term with greatest scope (to use a helpful if slightly anachronistic notion) is the singular one, not the plural:

[55] Ibid. [56] Buridan makes this point earlier on, in *Summulae* 1.3.3.
[57] *Summulae* 1.4.2.

(24) Of every man, some donkey runs

A proposition like (24) is accordingly particular, and not universal. But it is a particular of a universalized sort—a universalized I-type, if you will. (This terminology is mine). We can represent this fact with subscripts: it is of I_U-form (set out fully in 1.b.i, below). Its truth conditions can be displayed as follows:

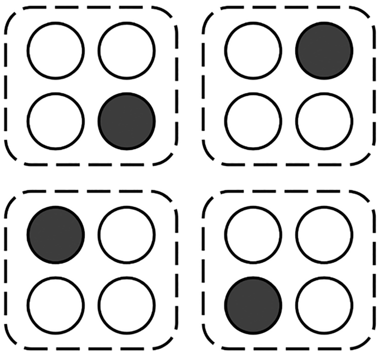


Fig. 1.3

Here the dotted lines demarcate groups of owned donkeys. I find it helps to think of groups of donkeys owned by a given man as being placed in a pen, which is enclosed by a dotted line.[58] The circles within the 'pens' represent individual donkeys: shaded ones are running, and unshaded ones are not.

Compare universalized-I forms with particularized-A (A_P) type (set out fully in 1.c.i, below):

(25) Of some man, every donkey runs

This is true just when there are donkey owners, and when at least one of them owns some donkeys that are all running. We can diagram the truth conditions for A_P-type propositions as follows:

[58] Notice however that there is nothing in what follows to prevent overlap in pens: one donkey could belong to multiple people. But I have chosen not to add this to the diagram, since it would complicate things unnecessarily, and indeed in some cases be misleading.

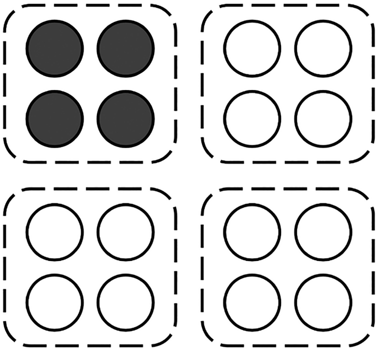


Fig. 1.5

Since an A_P-type proposition like (25) is localized to one pen, its truth depends only on what is going on in one of them (namely, the upper left-hand pen).

Consider, finally, a universalized O-type (O_U) proposition like the following, which has a universal genitive in the subject, but an O-type copula:

(26) Of every man, some donkey does not run

That is, either every man owns a donkey and at least one of the donkeys in each pen does not run, or no man owns a donkey (since, recall, O-type propositions can be vacuously true). For display of vacuous truth conditions here and below, I leave it to the reader to picture a blank diagram. Assuming this proposition is *not* vacuously true (i.e. assuming that everyone owns at least one donkey), we can display its truth conditions this way.

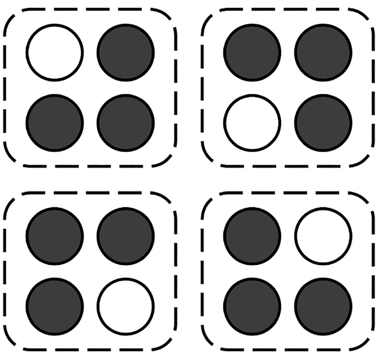


Fig. 1.6

Just as particularized propositions deal with what goes on in at least one pen, universalized propositions deal with all the pens. So (26) is true here because there is one donkey in each pen which does not run.

Now four pens of four donkeys each is a pretty limited domain, but it is certainly sufficient to illustrate the minimal truth conditions for the propositions with oblique terms set out in this *magna figura*. It is also important to stress that the truth conditions displayed in these figures are *minimal*: many of these propositions can be true under many other conditions. For example, (26) is true also of the following:

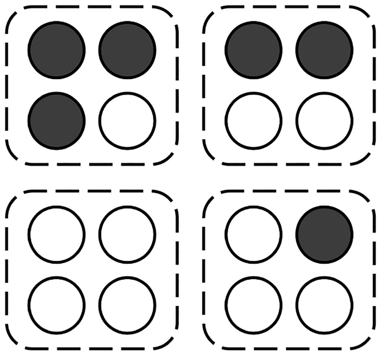


Fig. 1.6a

Since in each pen there is at least one donkey at rest, (26) is true. But these are not the minimal conditions for the truth of (26): we might say that the above figure *exceeds* (26)'s truth conditions. In what follows, I will stick to the minimal conditions for truth, since these are easier to read off, and are more visually appealing.

All the proposition-forms set out in this *magna figura* have relations of contradiction, contrariety, subalternation, and subcontrariety discussed in the previous section. I will list these for each one, but focus on contradictory pairs, which I group together. I do this for five reasons: (i) because each proposition has a unique canonical contradictory form, whereas many propositions have multiple canonical subalternate forms, and so grouping them by subalternation will not do; (ii) because every proposition has a contradictory, whereas some propositions have no contraries, subcontraries or subalterns, so these relations will not do, either; (iii) because grouping contradictory pairs allows direct contrast between mutually exclusive truth conditions; and (iv) because contradictory pairs always have exactly one affirmative and one negative, and all negatives can be vacuously true—a fact that becomes clearer as we consider them together. Finally (v) the opposition of contradictory pairs is in an important sense logically primary, as C. W. A. Whitaker has discussed at length.[59] These pairs can, accordingly, serve as opposite poles to a compass in an unfamiliar landscape.

Before we jump in, there is one final notion to clarify: unlike the forms of the Square of Opposition considered above, the following non-normal forms also include *disparate* pairs. Of these, Buridan tells us that they "are not related by any law of opposition."[60] Here, then, is our rule for disparates:

> **Logical disparity**: the truth (falsity) of one of a pair of disparates tells us nothing about the truth (falsity) of the other.

We will note disparates, too, though in a strict sense they do not stand in any logical relation at all.[61]

Buridan's *magna figura* for these propositions appears in Vatican Pal. lat. 994, f. 7r, and is here reproduced as Fig. III and schematized in Fig. IV. Conventionally, Buridan's *magna figura* is represented as an octagon, and I have followed this convention. The most elegant and clear of these diagrams is Gyula Klima's, which is featured in his translation of Buridan's *Summulae*.[62] I have consciously modeled the above diagram, and its cousin below, on Klima's octagon, though the arrangement of the nodes is Read's.[63] Granted, Buridan never calls his figure an octagon, as we now often do, nor does he arrange the nodes as such. But rearranging it in this way allows us to place the subalternate relations within the figure, rather than pushing them out like flying buttresses, as Fig. III does.

[59] C. W. A. Whitaker, *Aristotle's "De Interpretatione": Contradiction and Dialectic* (Oxford: Oxford University Press, 1996).

[60] *Summulae* 1.8.6.

[61] Campos Benítez has given a detailed treatment of logical disparity, and the role it plays as intermediate between certain nodes of the *figura*, in his "Medieval Octagon," 361ff.

[62] John Buridan, *Summulae de dialectica*, tr. G. Klima (New Haven, CT: Yale University Press, 2001), 42.

[63] Read, "Octagons," 104.

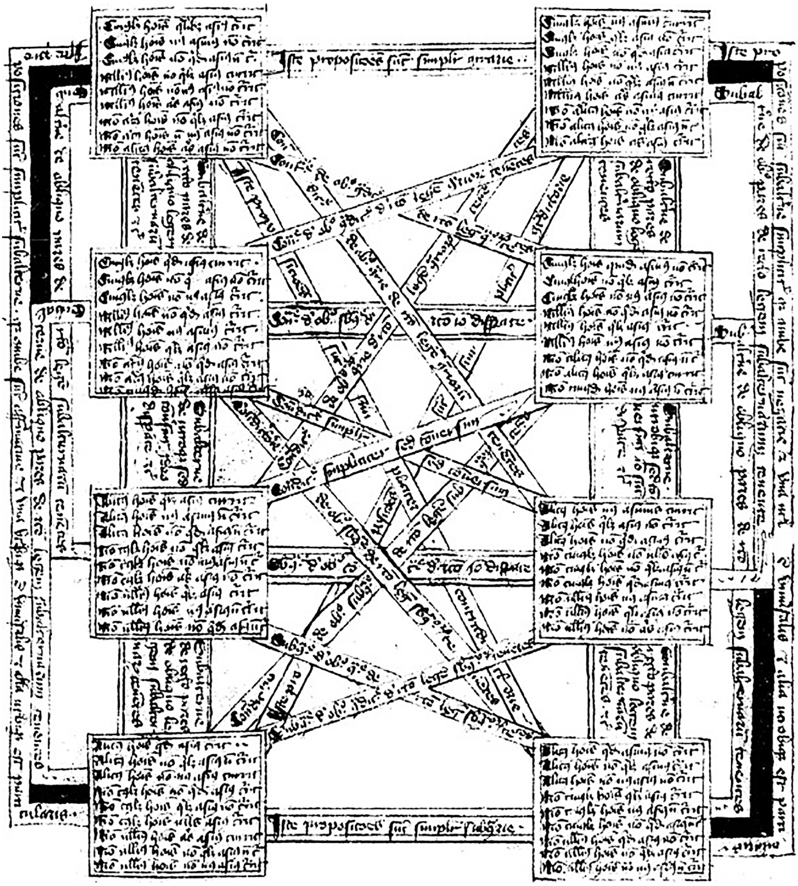


Fig. III The *magna figura* for obliques
Each node lists the canonical form first, followed by eight non-canonical variants. All of these are transcribed and translated in what follows.

For clarity and elegance, I have modified the standard A, E, I, and O forms with subscripts along the lines discussed above (U for 'universalized,' P for 'particularized'[64]), and given the type rather than the propositions themselves. As can be seen in the original *magna figura*, Buridan lists several equivalent variants, as he does with the Square of Opposition. These

[64] The terms 'particularized' and 'universalized' were suggested to me by Stephen Read.

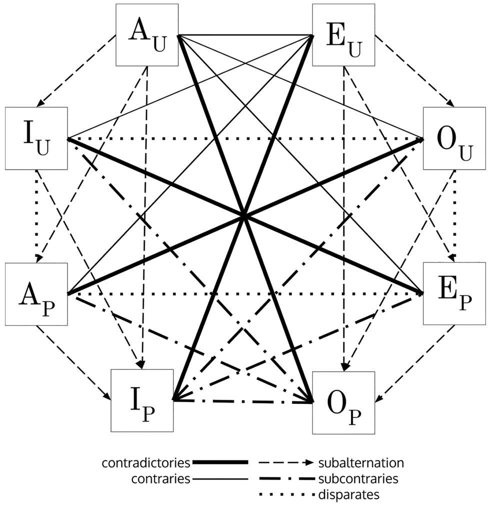


Fig. IV

variants will appear in the individual treatments of each type of proposition. For now, here are the canonical forms, arranged in contradictory pairs:

A_U: universalized A: *cuiuslibet hominis quilibet asinus currit*
"Of every man, every donkey runs"
O_P: particularized O: *alicuius hominis quidam asinus non currit*
"Of some man, some donkey does not run"

I_U: universalized I: *cuiuslibet hominis quidam asinus currit*
"Of every man, some donkey runs"
E_P: particularized E: *alicuius hominis quidam asinus non currit*
"Of some man, some donkey does not run"
A_P: particularized A: *alicuius hominis quilibet asinus currit*
"Of some man, every donkey runs"
O_U: universalized O: *cuiuslibet hominis quidam asinus non currit*
"Of every man, some donkey does not run"

I_P: particularized I: *alicuius hominis quidam asinus currit*
"Of some man, some donkey runs"
E_U: universalized E: *cuiuslibet hominis nullus asinus currit*
"Of every man, no donkey runs"

Let's look at each of these pairs. In what follows, I will set each one out in a dedicated section, for a total of eight sections in all.

a. Contradictory Pair I: A_U and O_P

i. Of every man, every donkey runs (A_U)
This proposition is true under the following conditions:

1A. Every man owns at least one donkey, and
1B. Every donkey owned by any man is running

Clause (1B) is straightforward, though (1A) calls for some clarification. Readers versed in MPL might be inclined to read propositions of A_U-form conditionally: that *if* every man owns a donkey, *then* every donkey owned by a man runs. But for Buridan, this will not do. In Buridan's interpretation of the Square, there are no vacuously true affirmative propositions, universal or otherwise. So it cannot be that no one owns a donkey, and yet that this A_U proposition is true, the way it is on the conditional reading. Rather, an A_U like this one is true just in case at least one man exists, and at least one donkey exists, and every man owns a donkey, and every one of those donkeys is running.

A_U-form propositions have an important ambiguity, however, a bit like that of the ambiguous proposition we saw at the outset:

(1) Everyone loves someone

Similarly, does a proposition of A_U-form tell us that any donkey that belongs to every man is running? Or rather that every man owns some donkeys (or other), all of which run? If it is the former, then this A_U-proposition is true in a case in which (i) there is only one donkey that is running, (ii) all other donkeys are at rest, (iii) the running donkey is jointly owned by every man, and (iv) the running donkey is the only such communal donkey. Here, as in the Sherwood passage considered above, the possibility of communal ownership introduces semantic puzzles. In the case under consideration, every donkey that is owned by everyone is, itself, running, even though there is only one such donkey, and other owned donkeys are at rest. Following this reading such an A_U-form proposition can be true even if not every donkey owned by someone or other is running.

But clearly this will not do: if we check the *magna figura*, we'll see that A_U-forms contradict O_P-forms, namely "Of some man, some donkey does not run." On the reading of A_U we're now entertaining, an O_P-proposition would be true as well, since some man also owns a donkey that does not run. So the former reading of A_U is incorrect, and the latter is the one we will adopt: this A_U is true where every man owns at least one donkey, and all the donkeys which any man owns are running. Hence the ambiguity of A_U-forms can be solved by appealing to their logical relations with the other propositions on the *figura*, in a way that A_U-form, on its own, could not. Given the already noted paucity of discussion of these forms in Buridan, we'll let the *a priori* be our guide.

With this, we can see the advantage to the admittedly unnatural rendering of such propositions with their Latin word order. Following the lead of Klima and others, I have placed the genitive *cuiuslibet* ("of every man") at the front of the translation of the proposition into English, to get "of every man, every donkey runs." Doing so clarifies the scope of the terms in a way a more natural rendering like "every donkey belonging to any man is running" does not. Admittedly, this method of translation makes up in clarity what it lacks in style: hewing this closely to the Latin word order in this way gives us some pretty clunky English. But at least it's good logic.

Here is a diagram that represents a truth condition for the proposition "of every man, every donkey runs," where again dotted lines denote ownership or 'donkey-pens,' and shaded circles represent running donkeys:

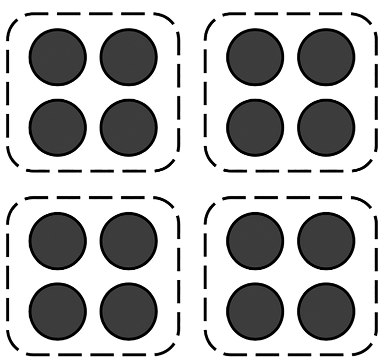


Fig. 1.1

The *magna figura* lists both canonical and non-canonical forms. In contrast with the forms listed in the expanded Square (Fig. II), the forms at each node of the *magnae figurae* are equivalent with all the other forms listed in the same node. Here are the A_U-forms displayed in the *figura*:

Canonical form:

1.a.i, 0) *cuiuslibet hominis quilibet asinus currit*
"of every man, every donkey runs"

Non-canonical forms:

1.a.i, 1) *cuiuslibet hominis nullus asinus non currit*
"of every man, no donkey does not run"

1.a.i, 2) *cuiuslibet hominis non quilibet asinus non currit*
"of every man, not any donkey does not run"

1.a.i, 3) *nullius hominis non quilibet asinus currit*
"of no man, not every donkey runs"

1.a.i, 4) *nullius hominis non nullus asinus non currit*
"of no man, not no donkey does not run"

1.a.i, 5) *nullius hominis quidam asinus non currit*
"of no man, some donkey does not run"

1.a.i, 6) *non alicuius hominis non quilibet asinus currit*
"not of some man, not every donkey runs"

1.a.i, 7) *non alicuius hominis non nullus asinus non currit*
"not of some man, not no donkey does not run"

1.a.i, 8) *non alicuius hominis nullus asinus non currit*
"not of some man, no donkey does not run"

A_U propositions contradict O_P ones:

ii. Of some man, some donkey does not run (O_P)

That is, of all the donkeys owned by a man, at least one of them is not running. This proposition is true just when either:

1C. No man owns a donkey, or

1D. At least one man owns a donkey, and at least one donkey owned by a man is not running.

Requirement (1D) is straightforward: if we read 'some' as "at least one," then if anyone owns a donkey at all, at least one man-owned donkey is not running. But (1C) might strike readers as puzzling: how can it be true of some donkey belonging to some man that it does not run, when there is no such donkey? Here, however, we see the same existential requirement rule at play for O-type propositions, which we saw in the preceding section: negative propositions, even negative particulars, can be vacuously true.[65]

Assuming that at least one man owns at least one donkey, we can display truth conditions for such a claim is as follows:

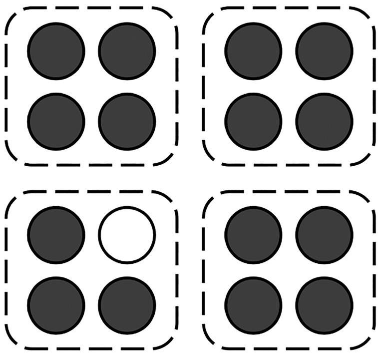


Fig. 1.2

Here the blank circle represents one donkey which is not running, whether or not the others are.

Canonical form:

1.a.ii, 0) *alicuius hominis quidam asinus non currit*
"of someman, some donkey does not run"

Non-canonical forms:

1.a.ii, 1) *alicuius hominis non quilibet asinus currit*
"of someman, not every donkey runs"

1.a.ii, 2) *alicuius hominis non nullus asinus non currit*
"of someman, not no donkey does not run"

[65] For a clear discussion of this fact about medieval logic, and a way of rendering propositions of the form "Some S is not P" with existential requirements in a terminist logic, see Klima, "Existence and Reference in Medieval Logic," §4.1.

1.a.ii, 3) *non cuiuslibet hominis quilibet asinus currit*
"not of everyman, every donkey runs"
1.a.ii, 4) *non cuiuslibet hominis nullus asinus non currit*
"not of everyman, no donkey does not run"
1.a.ii, 5) *non cuiuslibet hominis non quidam asinus non currit*
"not of everyman, not some donkey does not run"
1.a.ii, 6) *non nullius hominis quidam asinus non currit*
"not of no man, some donkey does not run"
1.a.ii, 7) *non nullius hominis non quilibet asinus currit*
"not of no man, not every donkey runs"
1.a.ii, 8) *non nullius hominis non nullus asinus non currit*
"not of no man, not no donkey does not run"

b. Contradictory Pair II: I_U and E_P

i. Of every man, some donkey runs (I_U)
An I_U-form proposition like this one is true just when:

1E. At least one donkey exists, and belongs to some man, and
1F. Every man has at least one donkey that is running.

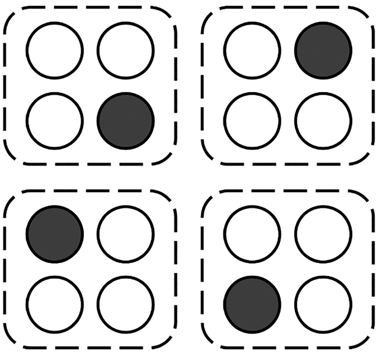


Fig. 1.3

Canonical form:
1.b.i, 0) *cuiuslibet hominis quidam asinus currit*
"of every man, some donkey runs"

Non-canonical forms:

1.b.i, 1) *cuiuslibet hominis non quidam asinus non currit*
"of every man, not some donkey does not run"

1.b.i, 2) *cuiuslibet hominis non nullus asinus currit*
"of every man, not no donkey runs"

1.b.i, 3) *nullius hominis non quidam asinus currit*
"of no man, not some donkey runs"

1.b.i, 4) *nullius hominis nullus asinus currit*
"of no man, no donkey runs"

1.b.i, 5) *nullius hominis quilibet asinus non currit*
"of no man, every donkey does not run"

1.b.i, 6) *non alicuius hominis non quidam asinus currit*
"not of some man, not some donkey runs"

1.b.i, 7) *non alicuius hominis quilibet asinus non currit*
"not of some man, every donkey does not run"

1.b.i, 8) *non alicuius hominis nullus asinus currit*
"not of some man, no donkey runs"

Propositions of I_U-form contradict those of E_P-form:

ii. Of some man, every donkey does not run (EP)

An E_P-form proposition like this one is true just when:

1G. No man owns a donkey, or

1H. There is at least one donkey owned by some man, and none of that man's donkeys is running.

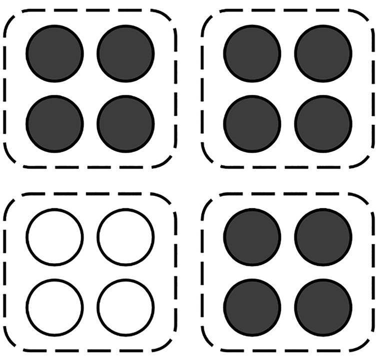


Fig. 1.4

Canonical form:

1.b.ii, 0) *alicuius hominis nullus asinus currit*
"of some man, no donkey runs"

Non-canonical forms:

1.b.ii, 1) *alicuius hominis quilibet asinus non currit*
"of some man, every donkey does not run"
1.b.ii, 2) *alicuius hominis non quidam asinus currit*
"of some man, not some donkey runs"
1.b.ii, 3) *non cuiuslibet hominis non nullus asinus currit*
"not of every man, not no donkey runs"
1.b.ii, 4) *non cuiuslibet hominis non quilibet asinus non currit*
"not of every man, not every donkey does not run"
1.b.ii, 5) *non cuiuslibet hominis quidam asinus currit*
"not of every man, some donkey does not run"
1.b.ii, 6) *non nullius hominis nullus asinus currit*
"not of no man, no donkey runs"
1.b.ii, 7) *non nullius hominis quilibet asinus non currit*
"not of no man, every donkey does not run"
1.b.ii, 8) *non nullius hominis non quidam asinus currit*
"not of no man, not some donkey runs"

c. Contradictory Pair III: A_P and O_U

i. Of Some man, every donkey runs (A_P)

This proposition is true just in case:

1I. At least one donkey exists, and belongs to some man, and
1J. All the donkeys that belong to at least one specific man are running.

Displaying truth conditions for such a proposition is relatively easy, as we saw above.

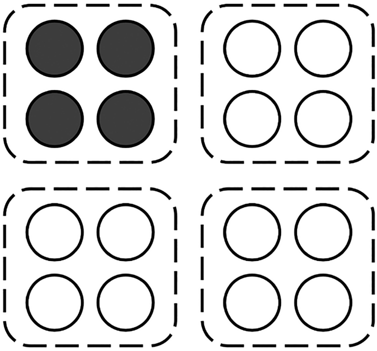


Fig. 1.5

Canonical form:

1.c.i, 0) *alicuius hominis quilibet asinus currit*
"of some man, every donkey runs"

Non-canonical forms:

1.c.i, 1) *alicuius hominis nullus asinus non currit*
"of some man, no donkey does not run"
1.c.i, 2) *alicuius hominis non quidam asinus non currit*
"of some man, not some donkey does not run"
1.c.i, 3) *non cuiuslibet hominis non quilibet asinus currit*
"not of every man, not every donkey runs"
1.c.i, 4) *non cuiuslibet hominis non nullus asinus non currit*
"not of every man, not no donkey does not run"
1.c.i, 5) *non cuiuslibet hominis quilibet asinus non currit*
"not of every man, every donkey does not run"
1.c.i, 6) *non nullius hominis quilibet asinus currit*
"not of no man, every donkey runs"
1.c.i, 7) *non nullius hominis nullus asinus non currit*
"not of no man, no donkey does not run"
1.c.i, 8) *non nullius hominis non quidam asinus non currit*
"not of no man, not every donkey does not run"

A proposition of this form contradicts the following:

ii. Of every man, some donkey does not run (O_U)

Such a proposition is true just when:

1K. There are no donkeys owned by any man, or
1L. At least one of the donkeys each man owns does not run.

Assuming there are donkeys that are owned by some man or other, then in order for the proposition to be true, at least one donkey in each man's possession must be at rest.

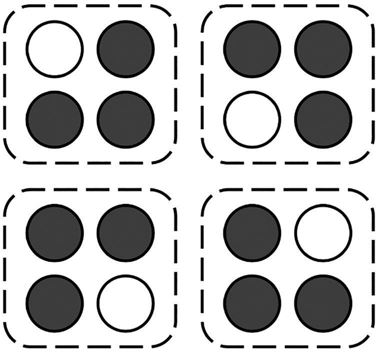


Fig. 1.6

Canonical form:
1.c.ii, 0) *cuiuslibet hominis quidam asinus non currit*
"of every man, some donkey does not run"

Non-canonical forms:
1.c.ii, 1) *cuiuslibet hominis non quilibet asinus currit*
"of every man, not every donkey runs"
1.c.ii, 2) *cuiuslibet hominis non nullus asinus non currit*
"of every man, not no donkey does not run"
1.c.ii, 3) *nullius hominis non quidam asinus non currit*
"of no man, not some donkey does not run"
1.c.ii, 4) *nullius hominis quilibet asinus currit*
"of no man, every donkey runs"
1.c.ii, 5) *nullius hominis nullus asinus non currit*
"of no man, no donkey does not run"
1.c.ii, 6) *non nullius hominis non quidam asinus non currit*
"not of no man, not some donkey does not run"

1.c.ii, 7) *non alicuius hominis quilibet asinus currit*
"not of some man, every donkey runs"
1.c.ii, 8) "*non cuiuslibet hominis nullus asinus non currit*
"not of every man, no donkey does not run"

d. Contradictory Pair IV: I_P and E_U

i. Of some man, some donkey runs (I_U)
This proposition is true just when:

1M. At least one donkey exists, and belongs to some man, and
1N. At least one such donkey is running.

We can display its truth conditions as follows:

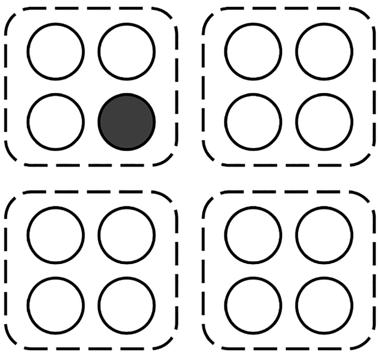


Fig. 1.7

Canonical form:
1.d.i, 0) *alicuius hominis quidam asinus currit*
"of some man, some donkey runs"

Non-canonical forms:
1.d.i, 1) *alicuius hominis non quilibet asinus non currit*
"of some man, not every donkey does not run"
1.d.i, 2) *alicuius hominis non nullus asinus currit*
"of some man, not no donkey runs"

1.d.i, 3) *non cuiuslibet hominis non quidam asinus currit*
"not of every man, not some donkey runs"
1.d.i, 4) *non cuiuslibet hominis quilibet asinus non currit*
"not of every man, every donkey does not run"
1.d.i, 5) *non cuiuslibet hominis nullus asinus currit*
"not of every man, no donkey runs"
1.d.i, 6) *non nullius hominis quidam asinus currit*
"not of no man, some donkey runs"
1.d.i, 7) *non nullius hominis non quilibet asinus non currit*
"not of no man, not every donkey does not run"
1.d.i, 8) *non nullius hominis non nullus asinus currit*
"not of no man, not no donkey runs"

Such propositions contradict those of E_U-form:

ii. Of every man, every donkey does not run (E_U)
That is, no donkey belonging to anyone runs. This can only be true when either:

1O. No man owns a donkey, or
1P. At least one donkey is owned by some man, but no such donkey is running.

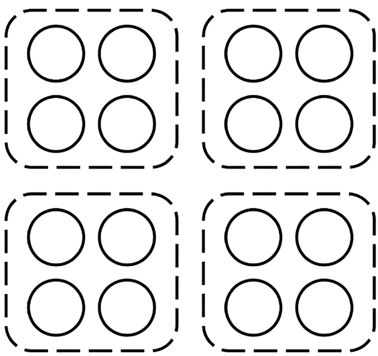


Fig. 1.8

Canonical form:

1.d.ii, 0) *cuiuslibet hominis nullus asinus currit*
"Of every man, no donkey runs"

Non-canonical forms:

1.d.ii, 1) *cuiuslibet hominis quilibet asinus non currit*
"of every man, every donkey does not run"
1.d.ii, 2) *cuiuslibet hominis non quidam asinus currit*
"of every man, not some donkey runs"
1.d.ii, 3) *nullius hominis non quilibet asinus <non> currit*
"of no man, not some donkey runs"
1.d.ii, 4) *nullius hominis non quilibet asinus non currit*
"of no man, not every donkey does not run"
1.d.ii, 5) *nullius hominis quidam asinus currit*
"of no man, some donkey runs"
1.d.ii, 6) *non alicuius hominis non nullus asinus currit*
"not of some man, not no donkey runs"
1.d.ii, 7) *non alicuius hominis non quilibet asinus non currit*
"non of some man, not every donkey does not run"
1.d.ii, 8) *non alicuius hominis quidam asinus currit*
"not of some man, some donkey runs"

This is the last of the forms in this *figura*.

2. Multiple Quantification

Now, all of the foregoing have incorporated a quasi-multiple quantification using genitive plurals and singulars. But there are many other options: we can take the normal forms of the square, and modify their predicate terms with a second quantificational particle, like *all* (*omnis*) or *some* (*quidam*).

The second *magna figura* includes propositions constructed on these lines, like "Every B every A is" (*omne B omne A est*) and "Some B some

A is not" (*quiddam B quiddam A non est*). As above, I set these out in contradictory pairs, commenting as I go, and providing diagrams displaying truth conditions for each. As we will see, the diagrams for the present section have to be altogether different from those above. Yet as with the forms set out above, Buridan does not thoroughly discuss the syntax and semantics of these, the way he does for the modal propositions of the third *magna figura*. We therefore have to keep our *a priori* approach, supplementing it with the texts where we can.

Here is a simple key to the diagrams in the present section:

B ———⊸ A
"B is A"

B ---------⊸ A
"B is not A"

For these diagrams, I've opted for '⊸' rather than '→' for the predicative copula 'is.' The arrow ('→'), while familiar, is commonly used for material implication, and material implication is not the way to think about any sort of medieval predication, as we saw in §0. As with cross-linguistic homophones, so too with notation: false friends abound, and familiar is not always better: '⊸' is clearly not "if… then," and so serves well for 'is.'

Note also that, in some diagrams that follow, I have added subscript numbers to the Bs and As. Where present, these numbers are meant to make clear that the Bs (As) in the diagram are discrete and jointly exhaustive: for some diagrams, there are multiple Bs and As, and when they are numbered, it is to indicate that those Bs and As are all there are. If a proposition does not determine that all the Bs (As) are in question, as happens with *some*, I will omit the numerical subscripts. I'll discuss this in greater detail when it comes up, first in connection with propositions of A_I-form (§2.b.i, below).

Fig. V displays the next *magna figura* as it appears in Vatican Pal. lat. 994, f. 1v, and which, like the other, includes non-canonical forms. As above, I list these forms in dedicated subsections.

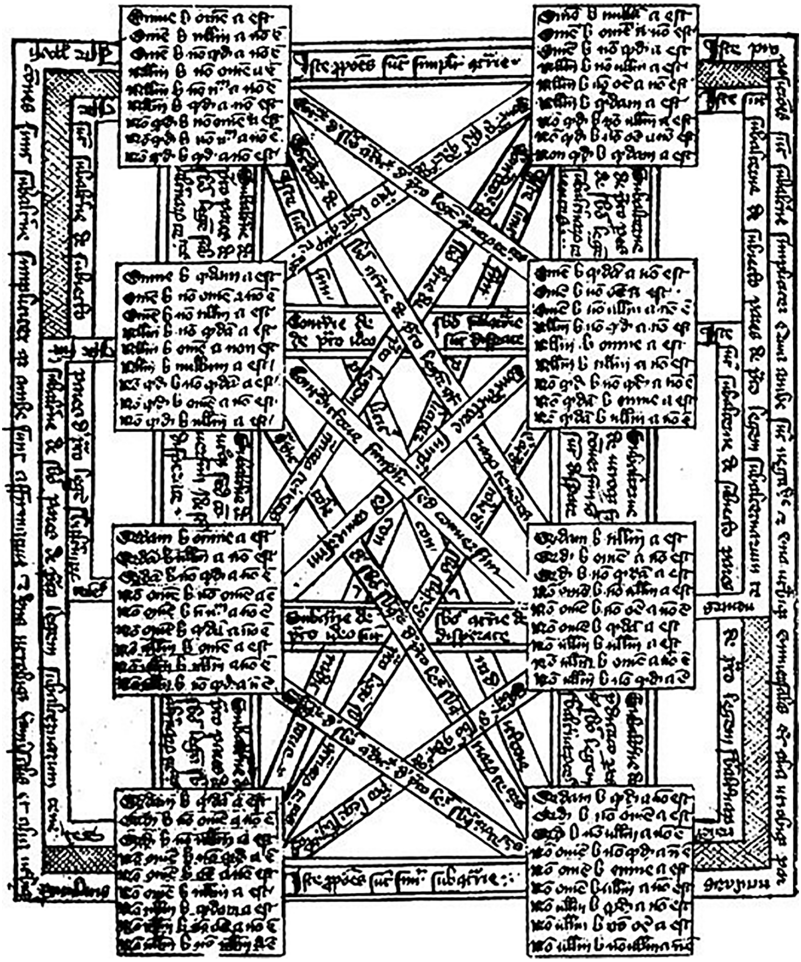


Fig. V The *magna figura* for multiply-quantified propositions

As in Fig. III, each node lists the canonical form first, along with eight non-canonical variants.

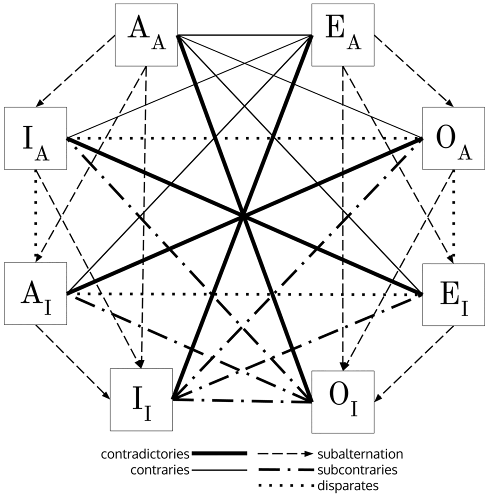


Fig. VI The multiply-quantified magna figura schematized

Here is a summary of the canonical forms, arranged in contradictory pairs:

$\mathbf{A_A}$: A-A type:	*omne B omne A est* "every B every A is"
$\mathbf{O_I}$: I-O type:	*quiddam B quiddam A non est* "some B some A is not"
$\mathbf{I_A}$: A-I type:	*omne B quiddam A est* "every B some A is"
$\mathbf{E_I}$: I-E type:	*quiddam B nullum A est* "some B no A is"
$\mathbf{A_I}$: I-A type:	*quiddam B omne A est* "some B every A is"
$\mathbf{O_A}$: A-O type:	*omne B quiddam A non est* "every B some A is not"
$\mathbf{I_I}$: I-I type:	*quiddam B quiddam A est* "some B some A is"
$\mathbf{E_A}$: A-E type:	*omne B nullum A est* "every B no A is"

In each of these, affirmatives (A- and I-types) have existential import, whereas their contradictory negatives (O- and E-types) do not. Below, we'll look at these forms one by one. Because, *per* Fact 2 (p. 229, above), the only propositions which are negative are those which have a negation sign which falls on (*cadat super*) the copula, I have followed the Latin in placing these copulae and their negative signs at the end. I find this practice especially useful for untangling the non-canonical forms, such as "not every B not some A is not" (*non omne B non quiddam A non est*; 2.a. ii, 3). This way of writing them is at least a little clearer, even though it makes for some pretty wacky English.

As has already been remarked by Campos Benítez, the original Square of Opposition is contained in this second *magna figura*.[66] At least, this is semantically true: though the following forms differ syntactically, the following equivalences hold:

A: every B is A	*iff*	**I_A**: every B some A is
E: no B is A	*iff*	**E_A**: every B no A is
I: some B is A	*iff*	**I_I**: some B some A is
O: some B is not A	*iff*	**E_I:** some B no A is

In what follows, I will set out the truth conditions for these diagrammatically, along with those of the other four.[67] As above, I present these in contradictory pairs.

a. Contradictory Pair I: A_A and O_I

i. Every B every A is (A_A)
A proposition of this form is true just in case:

2A. There are Bs (and As), and
2B. For each and every B, that B is every A.

[66] Campos Benítez, "The Medieval Octagon," 358.
[67] Walter Redmond has, in his studies of multiple quantification in 16th-century logic, given these forms different names: in addition to A, E, I, and O, he lists F (=A_A) and R (=A_I), N (=O_A) and G (=O_I). These letters he has adopted from *affirmo* and *nego*, respectively. See Walter Redmond, *La lógica del Siglo de Oro: una introducción histórica a la lógica* (Pamplona: University of Navarra, 2002), 53.

Of all the forms on this *figura*, A_A-forms are the hardest to think about.[68] At first, it seemed to me that such forms expressed something like a bijection from B to A. Accordingly, a visual representation of the truth conditions for propositions of this form would look like this:

$$\begin{array}{l} B_1 \longrightarrow A_1 \\ B_2 \longrightarrow A_2 \\ B_3 \longrightarrow A_3 \\ B_4 \longrightarrow A_4 \end{array}$$

Fig. 2.1a

Indeed, this looks like at least one available reading of "every B every A is." And this is an attractive way of reading the A_A-form, in part because it is much easier to come up with real-world examples. Here, for example, is the sample proposition of A_A-form in Klima's translation of the *Summulae de dialectica*:

(27) Every man every runner is.[69]

That is, every man is running, and everything running is a man. This proposition is constructed along similar lines to Buridan's examples of some of these forms—thought not this one—in *Summulae* 1.4.2. Now (27) calls for truth conditions that, at minimum, meet those presented in Fig. 2.1a, above. But are these enough? It seems that if we strengthen them, we would be forced to admit that, if there were more than one man and runner, each man would be multiple runners, and each runner multiple men. Which is hard to picture, really.

Even so, the reading of A_A-form suggested by (27) for a domain with more than one man or runner, and codified by Fig, 2.1a, is wrong. We can see this by looking at the contradiction of A_A, namely O_I (to be dealt with in §2.a.ii, below):

(O_I) some B some A is not.

[68] Indeed, it is precisely this form that Aristotle uses in his summary dismissal of multiply-quantified forms in De Interpretatione 7 (17b16).

[69] John Buridan, *Summulae de dialectica*, 45.

That is, for some B, there is some A which it is not.[70] We can represent this as follows, using a broken line to represent 'is not,' the way we used a solid line to represent 'is':

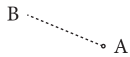
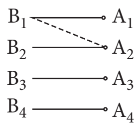


Fig 2.2

Hence whatever relations hold among the Bs and the As, it is true that some B is not some A. But then these two figures display compatible truth conditions, as we can see by overlaying them:



Figs. 2.1a + 2.2

This is unacceptable: since A_A and O_I contradict each other, any displays of their truth conditions should be incompatible. As we can see, reading off Fig. 2.2 gives us no sense of whether 2.1a can be true or not. So Fig. 2.1a is not an apt diagram for propositions of A_A-form.[71]

The correct way to diagram proposition of this sort is, therefore, the following:

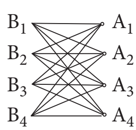


Fig. 2.1

Now if we suppose that all four Bs on the left are all the Bs there are, and likewise for the As on the right, then we have a nice visual model of "every B every A is," which is true just in case for every B, that B is every A.

[70] Or there are no Bs, or there are no As—we will get into the existential commitments in just a moment.

[71] In fact, the figure for which 2.1a *is* appropriate is I_A, to be dealt with in §2.b.i, below. Note that, by the relations of the nodes in Fig. IV, I_A and O_I are disparates, as Fig. 2.1a + 2.2 clearly displays.

I mentioned above that Klima's "every man is every runner" example for A_A-form propositions is, while syntactically available, false in any domain in which there is more than one man or runner. If there is more than one man, and every man is every runner, then each man is multiple runners, and each runner is multiple men. Still, we can give a Buridanian example of a true proposition constructed along these syntactic lines. Buridan gives examples of general terms like *sun*, *moon*, and *God* which, in spite of their generality, only have one item in their extension.[72] For instance, "every God is every omnipotent being," or "every sun is every greatest celestial body"—both of which are true, and both of which have A_A-form.

Here are the forms:

Canonical form:

2.a.i, 0) *omne B omne A est*
"every B every A is"

Non-canonical forms:

2.a.i, 1) *omne B nullum A non est*
"every B no A is not"

2.a.i, 2) *omne B non quiddam A non est*
"every B not some A is not"

2.a.i, 3) *nullum B non omne A est*
"no B not every A is"

2.a.i, 4) *nullum B non nullum A non est*
"no B not no A is not"

2.a.i, 5) *nullum B quodlibet A non est*
"no B every A is not"

2.a.i, 6) *non quiddam B non omne A est*
"not some B not every A is"

2.a.i, 7) *non quiddam B non nullum A non est*
"not some B not no A is not"

2.a.i, 8) *non quiddam B quiddam A non est*
"not some B some A is not"

[72] *Questions on Aristotle's De anima* 3.8.28.

Propositions of this form contradict those of O_I-form, namely:

ii. Some B some A is not (O_I)

A proposition of O_I-form is true just in case:

2C. There are no Bs (or As), or

2D. There is at least one B which is not an A.

Here is a diagram slightly different from the one above, following our rule that any numbered Bs (As) should be taken to be all the Bs (As) there are. For now, we'll show that the Bs (As) in question are just some Bs (As) or other, by eliminating the numerical subscripts. Since we only need one B which is not an A in order to make a proposition of this form true, we can represent the truth condition (2D), above, as follows:

B ·············∘ A

Fig. 2.2

Of course, by condition (2C), a proposition of this form is also true if there are no Bs (or As) to speak of. But in what follows I will only give diagrams displaying truth conditions on the assumption that what the subject and predicate terms stand for really do exist—that is, for non-vacuous truth.

Canonical form:

2.a.ii, 0) *quiddam B quiddam A non est*
"some B some A is not"

Non-canonical forms:

2.a.ii, 1) *quiddam B non omne A est*
"some B not every A is"

2.a.ii, 2) *quiddam B non nullum A non est*
"some B not no A is not"

2.a.ii, 3) *non omne B non quiddam A non est*
"not every B not some A is not"

2.a.ii, 4) *non omne B omne A est*
"not every B every A is"

2.a.ii, 5) *non omne B nullum A non est*
"not every B no A is not"
2.a.ii, 6) *non nullum B quiddam A non est*
"not no B some A is not"
2.a.ii, 7) *non nullum B non omne A est*
"not no B not every A is"
2.a.ii, 8) *non nullum B non nullum A non est*
"not no B not no A is not"

b. Contradictory Pair II: I_A and E_I

i. Every B some A is (I_A)

2E. At least one B (A) exists, and
2F. For every B, there is some A (or other) which it is.

We noted above, following Campos Benítez, that propositions of this form are true under the same conditions as traditional A-type propositions with corresponding terms. Such forms, while semantically—though not syntactically—equivalent to those of the Square, pose fewer problems than the novel four.

Here is a diagram displaying a case in which such a proposition is true:

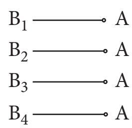


Fig. 2.3

Notice that, while Fig. 2.3 gives us a nice and tidy way to display truth conditions for I_A-form propositions, the conditions (2E) and (2F) do not require that for each of the Bs, there be a *unique* A which that B is. Nor do they require that there be *only one* A which every B is. There are, therefore, other ways to display the truth conditions for I_A. Here is one of them:

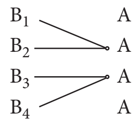


Fig. 2.3a

Conversely, if we wanted to say about some A that every B was the same as it, we would have to express this in A_I-form (see §2.c.i, below), with A as the subject term.

Canonical form:

2.b.i, 0) *omne B quiddam A est*
"every B some A is"

Non-canonical forms:

2.b.i, 1) *omne B non omne A non est*
"every B not every A is not"

2.b.i, 2) *omne B non nullum A est*
"every B not no A is"

2.b.i, 3) *nullum B non quiddam A est*
"no B not some A is"

2.b.i, 4) *nullum B omne A non est*
"No B every A is not"

2.b.i, 5) *nullum B nullum A est*
"no B no A is"

2.b.i, 6) *non quiddam B non quiddam A est*
"not some B not some A is"

2.b.i, 7) *non quiddam B omne A non est*
"not some B every A is not"

2.b.i, 8) *non quiddam B nullum A est*
"not some B no A is"

Sentences of this form contradict the following:

ii. Some B no A is (E_I)

This is true just when:

2G. There are no Bs (As), or

2H. There are Bs (As), but at least one B is none of the As.

Propositions of this form are true under the same conditions as corresponding O-type propositions of the Traditional Square.

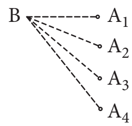


Fig. 2.4

Canonical form:

2.b.ii, 0) *quiddam B nullum A est*
"some B no A is"

Non-canonical forms:

2.b.ii, 1) *quiddam B omne A non est*
"some B every A is not"

2.b.ii, 2) *quiddam B non quiddam A est*
"some B not some A is"

2.b.ii, 3) *non omne B non nullum A est*
"not every B not no A is"

2.b.ii, 4) *non omne B non omne A non est*
"not every B not every A is not"

2.b.ii, 5) *non omne B quiddam A est*
"not every B some A is"

2.b.ii, 6) *non nullum B nullum A est*
"not no B no A is"

2.b.ii, 7) *non nullum B omne A non est*
"not no B every A is not"

2.b.ii, 8) *non nullum B non quiddam A est*
"not no B not some A is"

c. Contradictory Pair III: A_I and O_A

i. Some B every A is (A_I)

A proposition with this form is true just in case:

2I. There is at least one B, and

2J. Some B is every A.

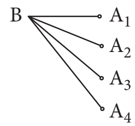


Fig. 2.5

Canonical form:

2.c.i, 0) *quiddam B omne A est*
"some B every A is"

Non-canonical forms:

2.c.i, 1) *quiddam B nullum A non est*
"some B no A is not"

2.c.i, 2) *quiddam B non quiddam A non est*
"some B not some A is not"

2.c.i, 3) *non omne B non omne A est*
"not every B not every A is"

2.c.i, 4) *non omne B non nullum A non est*
"not every B not no A is not"

2.c.i, 5) *non omne B quiddam A non est*
"not every B some A is not"

2.c.i, 6) *non nullum B omne A est*
"not no B every A is"

2.c.i, 7) *non nullum B nullum A non est*
"not no B no A is not"

2.c.i, 8) *non nullum B non quiddam A non est*
"not no B not some A is not"

This proposition contradicts the following:

ii. Every B some A is not (O_A)

A proposition of this form is true just in case:

2K. There are no Bs (or no As), or

2L. For each B, there is some A (or other) which that B is not.

Here is one way of representing truth condition (2L) visually:

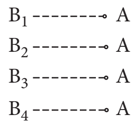


Fig. 2.6

As we noted above in connection with A_I, there are several ways of representing A_O, and we should not interpret the above diagram as requiring that, for each B there must be a unique A that it is not. We could, instead, represent it as follows:

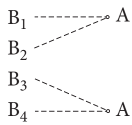


Fig. 2.6a

If, conversely, we wanted to say about some A that every B was not it, we would use a proposition of E_I-form, with A as the subject term.

Like Contradictory Pair I, Pair III does not match any of the truth conditions on the original Square, and so presents new exegetical difficulties. Luckily, Buridan discusses a related form in his *Sophismata*:

> Fourth sophism: every man an animal is not.
>
> Proof: Socrates is not the animal which Plato is. Therefore, Socrates an animal is not. Similarly, Plato is not the animal that Socrates is. Therefore Plato an animal is not. And so on for every other man. Therefore, every man an animal is not.[73]

Buridan later determines that this argument is not a *sophisma*: it is, rather, valid.[74] Hence it is true that, if there is more than one man, every man is not some animal or other, as propositions of this form claim. Propositions of this form are disparate with I_A-types (and semantically equivalent A-types of the traditional Square); they can, accordingly, both be true at the same time. We could

[73] *Sophismata* 3, 4th sophism, ed. Scott, p. 55.

[74] *Sophismata* 3, ad 4, ed. Scott, p. 57.

modify the above diagrams to make clear that, while every B is not some A, it nevertheless can be the case that every B is some A, too. But this would make our diagrams messy.

This textual evidence is reassuring: if we get one of the pair of these novel contradictory propositions, we can work out the truth conditions of the other. Since what Buridan says is consistent with our diagrams, we can rest assured that we are on the right track in our analysis of Contradictory Pair III.

Canonical form:

2.c.ii, 0) *omne B quiddam A non est*
"every B some A is not"

Non-canonical forms:

2.c.ii, 1) *omne B non omne A est*
"every B not every A is"

2.c.ii, 2) *omne B non nullum A non est*
"every B not no A is"

2.c.ii, 3) *nullum B non quiddam A non est*
"no B not some A is not"

2.c.ii, 4) *nullum B omne A est*
"no B every A is not"

2.c.ii, 5) *nullum B nullum A non est*
"no B no A is not"

2.c.ii, 6) *non quiddam B non quiddam A non est*
"not some B not some A is not"

2.c.ii, 7) *non quiddam B omne A est*
"not some B every A is"

2.c.ii, 8) *non quiddam B nullum A non est*
"not some B no A is not"

d. Contradictory Pair IV: I_I and E_A

i. Some B some A is (I_I)

This is true just in case:

2M. There exists at least one B (A), and
2N. At least one B is A.

Propositions of I_I-form are true under the same conditions as the traditional I-type propositions of the Square. Here is a visual representation of conditions in which (2M) and (2N) hold:

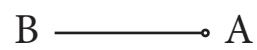


Fig. 2.7

Now granted, a proposition of this form is also true if there are many Bs, many (or even all) of which are A. In fact, it is subalternated to A_A—namely, "every B is every A"—as a quick check of Figs. V–VI makes plain. So Fig. 2.7 merely displays the *minimal* truth conditions for I_I.

Canonical form:
2.d.i, 0) *quiddam B quiddam A est*
"some B some A is"

Non-canonical forms:
2.d.i, 1) *quiddam B non omne A non est*
"some B not every A is not"
2.d.i, 2) *quiddam B non nullum A est*
"some B not no A is"
2.d.i, 3) *non omne B non quiddam A est*
"not every B not some A is"
2.d.i, 4) *non omne B omne A non est*
"not every B every A is not"
2.d.i, 5) *non omne B nullum A est*
"not every B no A is"
2.d.i, 6) *non nullum B quiddam A est*
"not no B some A is"
2.d.i, 7) *non nullum B non omne A non est*
"not no B not every A is not"
2.d.i, 8) *non nullum B non nullum A est*
"not no B not no A is"

ii. Every B no A is (E_A)

A proposition of this form is true iff:

2O. There are no Bs (or As) whatsoever, or

2P. There are Bs (and As); but for each B, there is no A which that B is.

Propositions of this form are true under the same conditions as E-type propositions of the Traditional Square. Such E_A-form propositions are, moreover, contrary to A_A-types, and have a similar-looking diagram to the one we saw at the beginning of the present discussion, in connection with A_A:

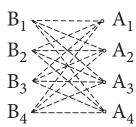


Fig. 2.8

Canonical form:

2.d.ii, 0) *omne B nullum A est*
"every B no A is"

Non-canonical forms:

2.d.ii, 1) *omne B omne A non est*
"every B every A is not"

2.d.ii, 2) *omne B non quiddam A est*
"every B not some A is"

2.d.ii, 3) *nullum B non nullum A est*
"no B not no A is"

2.d.ii, 4) *nullum B non omne A non est*
"no B not every A is not"

2.d.ii, 5) *nullum B quiddam A est*
"no B some A is"

2.d.ii, 6) *non quiddam B non nullum A est*
"not some B not no A is"

2.d.ii, 7) *non quiddam B non omne A non est*
"not some B not every A is not"

2.d.ii, 8) *non quiddam B quiddam A est*
"not some B some A is"

With that—*laus deo*—we've reached the end of the non-normal forms listed in the two *magnae figurae*. Now let's draw some conclusions.

3. What Does All This Tell Us About Buridan's Logic?

Readers who have made it this far will have a better intuitive grasp of the truth conditions of the foregoing non-normal forms. They will also, I hope, have found the observations made along the way philosophically illuminating. But there is a good deal more to be said about these forms, in particular to situate them in Buridan's overall logical project. The sheer number of these forms, plus the overwhelming number of valid arguments—syllogistic and otherwise—that they can figure in, tells us something important about Buridan's own views on logical foundations. In these last few pages, I will sketch these in outline.

It is an historical mystery mystery that medieval logicians of the fourteenth century suddenly began writing stand-alone treatises on logical foundations. These texts deal with the concept of logical consequence—that is, on what accounts for the validity of all and only valid arguments. Certainly this intense research focus was not present before, and there seems to be little to suggest it was coming. And there is no prior commentary tradition for these treatises, since there were no dedicated treatises written by authorities like Aristotle and Boethius. For my part, I am not sure what accounts for the trend overall, which at any rate must have been the product of many factors.[75]

Still, I think we can catch sight of something that may well have been on Buridan's own mind while he wrote his *Tractatus de consequentiis*, especially in light of all the forms set out above. As we have seen here, Buridan's logic has sufficient expressive power to deal with multiple

[75] For an overview of this development, plus a concise and systematic discussion of possible factors, see Catarina Dutilh Novaes, "Medieval Theories of Consequence," in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy* (2020), §3.1.

generality and with relational terms—at least as far as these doubly-general non-normal sentences go. But this power comes at a price, and that price is reduced simplicity: as we've seen, the non-normal forms are many, and their non-canonical variants are legion.

Further still, there is no reason to suppose that Buridan's catalogues of the non-normal forms (and their non-canonical variants), which I have reproduced here, are exhaustive. For example, if we're willing to countenance the quintuply-negated whopper like (2.a.ii, 8), "not no B not no A is not" (*non nullum B non nullum A non est*), then why not add two more negative terms to (2.b.i, 7), "not some B every A is not" (*non quiddam B omne A non est*)? So there are more forms lurking out there, in case we care to look for them. Many more. And many of them may bring new and unanticipated problems for Aristotelian logic, which go beyond the ones we have just enumerated.

For instance, the non-normal forms have significant ramifications for syllogistic logic. Stephen Read has pointed out that the non-normal proposition-forms Buridan presents render, thanks to their novel distribution of terms, new valid syllogisms. Here, for example, are two especially interesting ones:

(28)	Some P is not M	(29)	Some P is M
	Some S is M		Some S is not M
∴	Some S (some) P is not	∴	Some S (some) P is not.[76]

Syllogisms like (28) and (29), which conclude to O_I-forms, are valid. Yet they buck one of Aristotle's main metatheoretical rules: that no valid syllogism concludes from two particular premises.[77] We saw already that the first set of forms undermines Aristotle's claim that subjects must be nominative, and not in any of the oblique cases.[78] And now this. Already, then, long-standing pillars of syllogistic logic are cracking under the strain of these new forms.

Further still, if we also consider all the non-canonical forms Buridan lists for the non-normal forms in the *figurae* (to say nothing of the

[76] Read, "Non-Normal Propositions," 12.
[77] *Prior Analytics* 1.4 (26b21–25).
[78] *Prior Analytics* 1.36 (49b39–50a5).

non-canonical forms in the traditional square), our catalogue of syllogistic forms will become huge and unwieldy. After all, many of these will belong to altogether new syllogistic forms. To give just one example, take Read's syllogism (28), with its canonical non-normal O_I-form conclusion. We can swap it out with a non-canonical form of the same, say (2.a.ii, 8). Then we get the following:

(30) Some P is not M
Some S is M
∴ not no B not no A is not

—and many others like it, for a total of eight non-canonical non-normal syllogisms *just for this one form*, plus a ninth canonical non-normal one. And that's just counting the conclusions we can swap out. We haven't even yet considered changing up premises in the same way, not to mention all those additional non-normal forms we could come up with by iterative applications of negation. Our catalogue of syllogistic forms will rapidly balloon, and it is not obvious how or where this process will end.

We might try to arrest this process by treating all the non-canonical variants of the normal forms as somehow formally the same. For example, canonical A_A could be taken to include formally all the non-canonical A_A-variants like "every B no A is not" (2.a.i, 1) and "every B not some A is not" (2.a.i, 2), and so on for the rest of the forms listed or discoverable by doubling up negation.[79]

But this approach will not do, because it clashes with Buridan's own account of what accounts for logical form:

> the copulae—both of categorical and hypothetical propositions—pertain to form, as do negations, signs of quantity, and the number both of propositions and of terms, and the order (*ordo*) all these have to each other, and the relations (*relationes*) of relative terms, and the modes of signification that pertain to the quantity of the proposition in question—like discreteness and commonness of terms—and many things that those who are attentive will be able to spot if they come up.[80]

[79] This approach to a different but related problem was suggested to me by an anonymous reviewer of another paper for another journal. Whoever you are, if you're reading this, thank you.

[80] *Tractatus de consequentiis* 1.7. Buridan makes the same point at *Summulae* 1.6.1.

Negations, order, signs of quantity, and so forth are often completely different between the non-canonical variants of the non-normal forms. For example, here are two (A_I-type) propositions we have already seen, which differ in negations, order, and signs of quantity:

(2.c.i, 0) *quiddam B omne A est*
"some B every A is"
2.c.i, 4) *non omne B non nullum A non est*
"not every B not no A is not"

Though these two forms are logically equivalent, they are no more formally identical than the equivalences Ockham and De Morgan set out, e.g.:

(31) Some A is B
Not every A is not B

So again, our forms will proliferate, and Buridan's logic will start to look like just another medieval catalogue of valid arguments. Indeed, it is not clear that it will constitute a systematic logic at all, any more than hobbyist butterfly collecting counts as a systematic science.

For its part, modern logic can avoid these sorts of problems by furnishing recursive definitions of well-formed formulae, and then making use of them in generalizations. For example, in discussions of modern, mathematically-defined systems of logic, somewhat tedious—but mercifully brief—lists like the following are commonplace:

If φ is a wff, so is $\sim\varphi$.
If φ, ψ are wffs, so is $\varphi \vee \psi$.
If φ is a wff, so is $\forall x\varphi$.

Such definitions give us a sort of rule-book for the construction of forms. Thanks to such definitions, these modern systems are pretty well-behaved, and proving things about them is much easier than it would be for a (non-exhaustive) list of valid forms.

For example, let's return to the concept of logical consequence. More recently, Alfred Tarski has taught us to think of it in two ways: one syntactic, the other semantic. Syntactically, ψ follows from φ (symbolically, $\varphi \vdash \psi$) just in case there is a deductive system D, such that ψ follows from φ on D ($\varphi \vdash_D \psi$). Semantically, ψ follows from φ (symbolically, $\varphi \vDash \psi$) just in case there is no interpretation (that is, a *model* M) of the non-logical symbols in φ and ψ such that M makes φ true and ψ false. Using these definitions and the associated techniques, we can generalize across a very broad swath of arguments in a systematic and relatively straightforward way. Notice one thing, however: both these approaches are formal. The first turns on the rules governing logical syntax; the second turns on replacement of non-logical terms, while keeping the logical form intact.

Recursive definitions are not available to Buridan. Neither are the impressive apparatus of model theory. In fact, even if they were, I suspect Buridan would reject them: to discuss logic in terms of non-existent formulae which *could* be constructed following a set of rules would, after all, undermine his anti-realism about propositions (and about everything else). So too would a discussion of interpretations that aren't being presently thought, of formulae that are presently unformulated—not to mention unformulated deductions in an idealized system. For a committed anti-realist like Buridan, such talk is hogwash.

Even so, I do not think the boundless plurality of valid argument forms is a problem for Buridan. Showing why will take us on a brief trip through the opening chapters of his *Tractatus de consequentiis*. At the outset, Buridan gives us a succinct statement of his metalogical program:

> In this book, I wish to discuss consequences, dealing—as far as I can—with their causes, about which many things have already been said by others. But perhaps they have not yet been reduced to their first causes, through which they are said to hold.[81]

The latter sentence here tells us that Buridan's program is *reductive*: he does not aim to produce a mere catalogue of all the valid

[81] *Tractatus de consequentiis* 1.1.

arguments. Rather, Buridan wants to give a unified account of *why* valid arguments hold.

In virtue of what, then, do valid arguments hold—that is, what relates two propositions to one another as antecedent and consequent? After considering and modifying a series of proposals, Buridan gives us the following:

> one proposition is antecedent to another which is related to it in such a way that it is impossible that things should be as the former signifies, and not be as the latter signifies, when they are formulated at the same time.[82]

Now for reasons that need not detain us here, Buridan does not literally endorse the talk about propositional signification. In general, Buridan rejects any talk that presupposes or implies that propositions signify proposition-like things. Anti-realism strikes again. Truth for Buridan, as we saw above (§0), is a matter of the things for which the terms involved stand.

Nevertheless, what's significant about this passage for our present purposes is that it defines logical consequence in terms of necessity and truth, not of form: if the antecedent is true, it is *impossible* for the consequence to be. If such conditions obtain, then a consequence is valid.

Immediately after the foregoing discussion, Buridan distinguishes valid arguments along the following lines: there are those that are formally valid, and those that are valid *materially*—that is, valid by virtue of the meaning of their terms, not of their logical form. To clarify this distinction, contrast the following two arguments:

(32)	A man is running	(33)	A man is an animal
∴	An animal is running	∴	An animal is a man

As Buridan points out, we can uniformly replace all the non-logical terms ('man,' 'animal,' 'running') in (32), to get the following invalid argument:

[82] *Tractatus de consequentiis* 1.3.

(34) A horse is walking
∴ Wood is walking.[83]

Manifestly, (34) is not valid, even though it was constructed using the same form as (32) which, nevertheless, is valid. What this tells us about (32) is that it owes its validity to the meaning of its terms—its matter—rather than its form.

How does all this rescue Buridan from the ballooning logical forms, both propositional and syllogistic? The distinction between formally and materially valid arguments is downstream from the definition of consequence: materially valid arguments, like formally valid ones, are perfectly valid. Hence formal validity is merely a fragment of Buridanian validity—by no means the whole story. Unlike Tarski's approaches, both of which, (deductive and model-theoretic) are formal, the foundations of Buridan's logic are not, fundamentally, formal. Instead, Buridan undertakes to give an account of logical foundations more fundamental than the syllogistic forms, valid conversions, and so on, and more fundamental even than the metalogical rules set down by Aristotle. Thus, precisely because Buridan does not reduce validity to form, the uncontrolled proliferation of non-normal forms, variants, and syllogisms does not threaten his overall logical project.[84]

The Saxo Institute
University of Copenhagen

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[83] *Tractatus de consequentiis* 1.4.

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