Questions and Answers about Oppositions

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Abstract

A general characterization of logical opposition is given in the present paper, where oppositions are defined by specific answers in an algebraic question-answer game. It is shown that opposition is essentially a semantic relation of truth values between syntactic opposites, before generalizing the theory of opposition from the initial Apuleian square to a variety of alternative geometrical representations.

In the light of this generalization, the famous problem of existential import is traced back to an ambiguous interpretation of assertoric sentences in Aristotle's traditional logic. Following Abelard's distinction between two alternative readings of the **O**-vertex: *Non omnis* and *Quidam non*, a logical difference is made between negation and denial by means of a more fine-grained modal analysis.

A consistent treatment of assertoric oppositions is thus made possible by an underlying abstract theory of logical opposition, where the central concept is negation. A parallel is finally drawn between opposition and consequence, laying the ground for future works on an abstract operator of opposition that would characterize logical negation just as does Tarski's operator of consequence for logical truth.

Key words: affirmation, denial, existential import, negation, **NOT**, opposition, **QAS**

1. An Abstract Theory of Opposition

1.1. A Question-Answer Game for Oppositions

To define logical oppositions, we developed a question-answer game in which the logical value of a formula is not a truth-value but an answer to the corresponding question. We refer to this conceptual framework hereafter as Question-Answer Semantics (**QAS**).

1.1.1 What is a Logical Opposition?

While the concept of opposition occurs in various places and senses within Aristotle's works[1], the theory of logical opposition is the resulting group of four logical relations: contradiction, contrariety, subcontrariety, and subalternation. Let us symbolize by $\mathrm{Op}(\alpha, \beta)$ the 2-place relation between the two relata α and β, where Op is to be read as " – is opposed to …". If these are strictly concerned with the combination of truth and falsity, then only *propositions* can serve as values of α and β. Let us symbolize by T and F the logical values of truth and falsity, and let v be the interpretation function that assigns a logical value to propositions. Then $v(\alpha) = \mathrm{T}/\mathrm{F}$ means that the proposition α is true/false.

Recalling that the Aristotelian oppositions obey the Principle of Bivalence (**PBV**), namely: every proposition is either true or false but not both, every opposition $\mathrm{Op}(\alpha, \beta)$ could be seen as a 2-ary relation belonging to the set of ordered couples $\{\langle \mathrm{T}, \mathrm{T}\rangle, \langle \mathrm{T}, \mathrm{F}\rangle, \langle \mathrm{F}, \mathrm{T}\rangle, \langle \mathrm{F}, \mathrm{F}\rangle\}$. In other words, oppositions could be rendered by the perfect disjunctive normal forms (DNF) characterizing the relation $\mathrm{R}(\alpha, \beta)$: $(\alpha \wedge \beta) \vee (\alpha \wedge \sim\beta) \vee (\sim\alpha \wedge \beta) \vee (\sim\alpha \wedge \sim\beta)$, where every affirmative proposition (α or β) is read as T and every negative proposition ($\sim\alpha$ or $\sim\beta$) is read as F.

1 See Aristotle (b), Δ10; and Aristotle (a), Chapter 10 for a general examination of the oppositions: by contrariety, by relation, by privation (and possession) and by affirmation (and negation).
While the "metaphysical" sense of opposition is stated as a set of relations between *concepts*, the logical sense only concerns relations between *propositions* and the subcase of opposition by affirmation and negation. As for the metaphysical oppositions of contrariety and contradiction, they are included into logical oppositions by reformulating their conceptual relata into propositions.

Definition 1. An *opposition* is a set of combined truth-values involving both truth and falsehood. Each opposition $\mathrm{Op}(\alpha, \beta)$ is characterized by its specific DNF.

Definition 1.1. α and β are said to be *contradictory* to each other (Op: CD) if and only if (hereafter: iff) they cannot be T or F together.
$\mathrm{CD}(\alpha, \beta) = \{\langle T, F\rangle, \langle F, T\rangle\}$
DNF: $(\alpha \wedge {\sim}\beta) \vee ({\sim}\alpha \wedge \beta)$

Definition 1.2. α and β are said to be *contrary* to each other (Op: CT) iff they cannot be T together but can be F together.
$\mathrm{CT}(\alpha, \beta) = \{\langle T, F\rangle, \langle F, T\rangle, \langle F, F\rangle\}$
DNF: $(\alpha \wedge {\sim}\beta) \vee ({\sim}\alpha \wedge \beta) \vee ({\sim}\alpha \wedge {\sim}\beta)$

Subcontraries are also presented as "contradictories of contraries", that is: each component of the DNF for subcontraries is the negation of a corresponding component for contraries.

Definition 1.3. α and β are said to be *subcontrary* to each other (Op: SCT) iff they cannot be F together but can be T together.
$\mathrm{SCT}(\alpha, \beta) = \{\langle T, T\rangle, \langle T, F\rangle, \langle F, T\rangle\}$
DNF: $(\alpha \wedge \beta) \vee (\alpha \wedge {\sim}\beta) \vee ({\sim}\alpha \wedge \beta)$

The last opposition of subalternation is more difficult to define for two main reasons: it is not a *symmetric* relation; the proper DNF of subalternation cannot be so easily obtained from one of the three preceding oppositions.

Definition 1.4. α is said to be *subalternate* to β (Op: SB) iff β cannot be F whenever α is T.
$\mathrm{SB}(\alpha, \beta) = \{\langle T, T\rangle, \langle F, T\rangle, \langle F, F\rangle\}$
DNF: $(\alpha \wedge \beta) \vee ({\sim}\alpha \wedge \beta) \vee ({\sim}\alpha \wedge {\sim}\beta)$

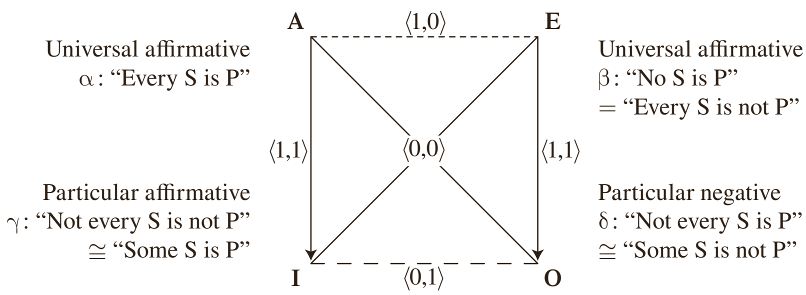
Such definitions may strike as rather artificial, or at least incomplete: why not exhaust the set of possible DNFs? And why isn't SB a symmetric relation like the other oppositions? It can be observed that a complete list of possible DNFs does not consist in a set of 4 oppositions but, rather, in a set of 16 logical connectives for classical logic. A reformulation of oppositions in terms of a question-answer game enhances their understanding.

Definition 2. An *opposition* is a set of answers to questions about the com-possible truth-values of α and β.

To the $\mathbf{Q}=2$ following questions $\mathbf{Q}_1$: "Possibly $v(\alpha)=v(\beta)=\mathrm{F}$?" and $\mathbf{Q}_2$: "Possibly $v(\alpha)=v(\beta)=\mathrm{T}$?", oppositions reply with an ordered set of two yes-no answers. Let $\mathbf{P}$ be symbolized by 1 for affirmative answers: "yes", and by 0 for negative answers: "no". Only one answer is to be given to each question, so that oppositions result in an ordered 2-tuple of answers: $\mathrm{Op}(\alpha,\beta)=\langle\mathbf{P}_1,\mathbf{P}_2\rangle$.

The Aristotelian theory of quantified oppositions (see Aristotle (a), Ch.7) occasioned the famous Apuleian square **AEIO**, with a set of four sentential schemata $\{\alpha,\beta,\gamma,\delta\}$ and a set of four oppositions $\mathrm{Op}=\{\mathrm{SB},\mathrm{CT},\mathrm{SCT},\mathrm{CD}\}$. The two horizontal lines stand for CT (the line **AE**) and SCT (the line **IO**); the two diagonals correspond to CD (the lines **AO** and **EI**); as for the two vertical lines, the arrows indicate the asymmetric relation of SB from α to β (the lines **AI** and **EO**)[2].

Figure 1: Oppositions between Quantified Propositions



Definition 2.1. SB yields an affirmative $\mathbf{P}_1$ and an affirmative $\mathbf{P}_2$.
$(\alpha,\gamma)=(\beta,\delta)=\langle 1,1\rangle$

2 The vertices **A**, **E**, **I**, **O** are to be traced back to medieval times, with **AI** (**A**ff**I**rmo) for affirmative propositions and **EO** (n**E**g**O**) for negative propositions. It means that the capital letters have to do with the *quality* of the opposed formula, and not the *quantity*; consequently, they can and will be used hereafter for any applied oppositions beyond the single case of quantified oppositions.

Definition 2.2. CT yields an affirmative $\mathbf{P}_1$ and a negative $\mathbf{P}_2$.
$(\alpha, \beta) = \langle 1, 0 \rangle$

Definition 2.3. SCT yields a negative $\mathbf{P}_1$ and an affirmative $\mathbf{P}_2$.
$(\gamma, \delta) = \langle 0, 1 \rangle$

Definition 2.4. CD yields a negative $\mathbf{P}_1$ and a negative $\mathbf{P}_2$.
$(\alpha, \delta) = (\beta, \gamma) = \langle 0, 0 \rangle$

1.1.2 How Many Oppositions are There?

The reason why there are 4 oppositions in Aristotle's theory seems to be due to the cardinality of **Q** and **P**. Aristotle's theory includes only 2 truth-values T and F (by **PBV**). No further truth-value can be introduced to increase the number *n* of questions, and no question can be answered both affirmatively and negatively; hence there are $m = 2$ sorts of available answers to each question, so that the number of the Aristotelian oppositions is $m^n = 2^2 = 4$ in the light of **QAS**.

Now some objections could be raised against the meaning and the number of "genuine" oppositions. Against this algebraic characterization of **QAS**, Béziau (2003) argues that opposition is synonymous with "incompatibility": a propositional relation is an opposition only if its relata cannot be both true, in which case the appropriate question for characterizing Op is not $\mathbf{Q}_2$ but only $\mathbf{Q}_1$. And given that the answer to $\mathbf{Q}_1$ must be affirmative accordingly, opposition is not defined any longer as a combinatorial game of questions-answers and should exclude both SCT and SB. Béziau however accepts SCT as a "verbal" opposition because of its derivation from CD and CT[3]; but he clearly excludes SB from his geometrical representations of opposition[4].

3 "Let us recall that Aristotle does not introduce explicitly the notion of "subcontraries", but refers to them only indirectly as "contradictories of contraries"; moreover he does not really consider them as opposed: '*Verbally* four kinds of opposition are possible, viz. universal affirmative to universal negative, universal affirmative to particular negative, particular affirmative to universal negative and particular negative to particular negative. But *really* there are only three: for the particular negative is only verbally opposed to the particular negative. Of the genuine opposites I call those which are universal contraries, e.g. "every science is good", "no science is good"; the others I call contradictories.' (Aristotle, *Prior Analytics*, 63b21–30)" (Béziau 2003; italics added.)

4 "I think that neither subalternation nor superalternation can be considered as relations of opposition. For example P is subaltern of $P \wedge Q$, and it does not really make sense to consider them as opposed." (Béziau 2003, 224)

Conversely, Sion (1996) increases the set of oppositions by including two further elements: implicance, and unconnectedness[5]. But he can do so only by modifying the content of the questions **Q**. Sion's view of oppositions proceeds from a set of $\mathbf{Q}=4$ alternative questions about whether the relata are possibly T or F, rather than compossibly. We thus have a new set of basic questions-answers (where $\mathbf{Q}=4$ and $\mathbf{P}=2$): $\mathbf{Q}_1$: "$v(\alpha)=\mathrm{T}$, $v(\beta)=\mathrm{T}$?", $\mathbf{Q}_2$: "$v(\alpha)=\mathrm{T}$, $v(\beta)=\mathrm{F}$?", $\mathbf{Q}_3$: "$v(\alpha)=\mathrm{F}$, $v(\beta)=\mathrm{T}$?", and $\mathbf{Q}_4$: "$v(\alpha)=\mathrm{F}$, $v(\beta)=\mathrm{F}$?", where to the additional oppositions of implicance and unconnectedness correspond the 4-tuples of answers $\langle 1,0,0,1\rangle$ and $\langle 1,1,1,1\rangle$.

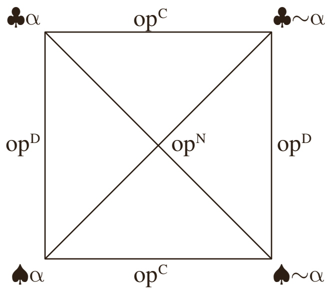
But to do so is actually to characterize the meaning of a classical propositional connective. Should any such connective be considered as a proper opposition? It is well-known that each logical opposition corresponds to exactly one logical connective among the $m^n=2^4=16$ ordered answers $\langle \mathbf{P}_1,\mathbf{P}_2,\mathbf{P}_3,\mathbf{P}_4\rangle$. Thus CT amounts to incompatibility ($\uparrow(\alpha,\beta)=\langle 0,1,1,1\rangle$), CD to exclusive disjunction ($\underline{\vee}(\alpha,\beta)=\langle 0,1,1,0\rangle$), SCT to inclusive disjunction ($\vee(\alpha,\beta)=\langle 1,1,1,0\rangle$), and SB to conditional ($\rightarrow(\alpha,\beta)=\langle 1,0,1,1\rangle$). However, a geometrical representation of Sion's alleged oppositions does not result in the Apuleian square but a slightly different set of geometrical objects, namely: a logical *quatern*, depicted syntactically with a theory of quaternality by Gottschalk (1953) and semantically with a theory of reversibility (or INRC group) by Piaget (1949).

Sion's alleged oppositions of implicance and unconnectedness refer to the connectives of biconditional ($\leftrightarrow(\alpha,\beta)=\langle 1,0,0,1\rangle$) and tautology ($\mathrm{T}(\alpha,\beta)=\langle 1,1,1,1\rangle$), respectively; but he stops the process there, instead of exhausting the combinatorial list of his answers. Thus there is a gap

We'll see later that Moretti (2009) presents a constructive argument against Béziau's present point: the geometry of oppositions cannot be expanded without using subalternation in its representations.

5 "By the 'opposition' of two propositions, is meant: the exact logical relation existing between them – whether the truth or falsehood of either affects, or not, the truth or falsehood of the other. In this context, note, the expression "opposition" is a technical term not necessarily connoting *conflict*. We commonly say of two statements that they are "opposite", in the sense of *incompatible*. But there, the meaning is *wider*; it refers to any mental confrontation, any logical face-off, between distinguishable propositions. In this sense, even forms which imply each other may be viewed as 'opposed' by virtue of their *contradistinction*, though to a much lesser degree than contradictories. Thus, the various relations of opposition make up a continuum." (Sion 1996, Ch.6, 1; italics added.)

Figure 2: The Logical Quatern



♣/♠: dual constants (♣α ⇔ ∼♠∼α)
α/∼α: affirmative sentences/negative sentences

op^C: contraduality
op^N: negation
op^D: duality

between the compossible truth-values of our own characterization of Op and the possible truth-values of Sion's characterization.

Given that Sion's view of opposition is not only unjustifiably incomplete but also gives rise to a geometrical object that is different from Aristotle's square, we argue that the meaning of Op eventually relies on a combination of compossible truth-values. Roughly speaking, our own characterization in **QAS** constitutes a trade-off between two views of opposition: *intensive* (according to its meaning by **Q**), and *extensive* opposition (according to its extensions by **P**). As a minimal precondition, **Q** must be about the compossibility of truth-values in order to follow the traditional theory of opposition. Such a precondition can make room for a generalization of the initial square by changing the parameters **Q** and **P**.

1.2 Generalization of Oppositions: Syntactic and Semantic Views

1.2.1 Syntactic Generalization

A syntactic generalization of the theory of opposition relies upon a structural extension of the relata in Op(Φ, Ψ), without any semantic reference to truth-values. As pointed out in the Figure 2, dual constants can be applied to propositions to give rise to a range of syntactic transformations.

Blanché (1953) noted this structural point, arguing that Aristotle's theory of opposition still prevails beyond the sole case of quantified propositions (see Figure 1) and includes modal oppositions. Furthermore, the formulas Φ and Ψ serve as structured formulas $\Phi = \clubsuit\alpha$ and $\Psi = \spadesuit\alpha$, where the modal functions $\{\clubsuit,\spadesuit\}$ can be variously interpreted: as null modality $\{\varnothing\}$, quantifiers $\{\forall / \exists\}$, alethic modalities of necessity and possibility $\{\Box / \Diamond\}$, deontic modalities of obligation and permission {op / P}; and so on[6]. An additional, purely syntactic definition of opposites op (where op $\neq$ Op) can thus be given as a set of construction rules for quaterns, where op^X means "operates as a X".

Definition 3. A logical *opposite* op is a 1-ary function on one formula Φ, where op is an operation from Φ to Ψ. To the two initial questions $\mathbf{Q}_1$: "Is the dual constant interchanged?" and $\mathbf{Q}_2$: "Is the sentence interchanged?", each sort of opposition op^X is a 1-ary operation that yields a 2-tuple of yes-no answers $\langle \mathbf{P}_1,\mathbf{P}_2 \rangle$.

Definition 3.1. Duality (X = D) yields an affirmative $\mathbf{P}_1$ and a negative $\mathbf{P}_2$.
$op^N = \langle 1, 1 \rangle$

Definition 3.2. Negation (X = N) yields an affirmative $\mathbf{P}_1$ and an affirmative $\mathbf{P}_2$.
$op^D = \langle 1, 0 \rangle$

Definition 3.3. Contraduality (X = C) yields a negative $\mathbf{P}_1$ and an affirmative $\mathbf{P}_2$.
$op^C = \langle 0, 1 \rangle$

Definition 3.4. Identity (X = I) yields a negative $\mathbf{P}_1$ and a negative $\mathbf{P}_2$.
$op^I = \langle 0, 0 \rangle$

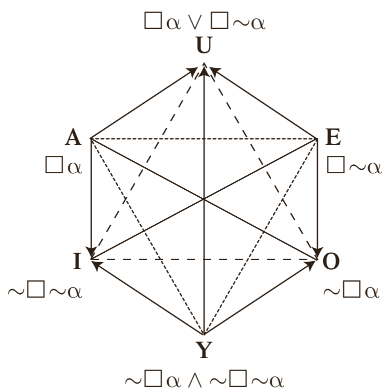
6 Φ and Ψ stand for 1-ary functions in (Blanché 1953, von Wright 1951) (where $\clubsuit = \Box$ and $\spadesuit = \Diamond$); but they can also stand for 2-ary functions, as in (Piaget (1949), Gottschalk (1953)) (where $\clubsuit = \wedge$ and $\spadesuit = \vee$).
Unlike von Wright (1951), who strictly referred to 'modalities' as the modes in which a sentence is said to be true or false (necessarily, obligatorily, and so on) and distinguished them from the truth-operators (truth and falsity), we use "modalities" to refer only to a 1-ary sentential function throughout the present paper.

Since Ψ results from Φ by applying one of the four preceding operations to Φ to get $\mathrm{op}(\Phi) = \Psi$, any opposition relation $\mathrm{Op}(\Phi, \Psi)$ amounts to the opposition of Φ and one of its opposites $\mathrm{op}(\Phi)$.

Definition 4. For any opposition relation $\mathrm{Op}(\Phi, \Psi)$ and any oppositional operation op^X on Φ (where $X \in \{N, D, C, I\}$), we have: $\mathrm{Op}(\Phi, \Psi) = \mathrm{Op}(\Phi, \mathrm{op}^X(\Phi))$.

A famous expansion of the Apuleian square is the hexagon of alethic modalities in (Blanché 1953, Sesmat 1951) where $\clubsuit = \Box$. The difference between logical squares and hexagons does not reside in their relation Op, which is the same for the two; but rather in their relata: they are related by the same sorts of opposition. Thus Blanché's hexagon results from the union of two triangles of contrariety and subcontrariety by their subaltern opposites.

Figure 3: The Hexagon of Modal (Alethic) Oppositions



Y amounts to the non-basic modality of *contingency* (two-sided possibility), and its contradictory **U** is introduced in order to respect the central symmetry for any paired opposites in the geometrical representation. The above transition from 4 to 6 vertices has been generalized by Moretti (2009) with his theory of *n*-opposition (**NOT**): a logical square is a bi-segment resulting from the subalternate connection of 2 segments or "simplexes" (viz. contrariety and subcontrariety); a logical hexagon amounts to a bi-triangle resulting from the subalternate connection of 2 triangles; and so on, where each expanded logical polygon goes from one to an indefinite number of simplexes.

Aristotle's square is thus only one geometrical opposition within a series of increasing cases, and the syntactic structure of opposed terms accounts for the complexity of logical polygons in **NOT**.

1.2.2 A Syntactic Key Notion for Oppositions: Negation

A distinction has just been made between an opposition Op and an opposite op: opposition is a relation, whereas opposites are relata. The previous figures have shown that Aristotle's square was a geometrical representation of opposition to be characterized by *semantic* questions about truth-values, while Gottschalk's and Piaget's quatern was a geometrical representation of opposites encoded by *syntactic* questions about structured formulas.

An intermediary position between semantic oppositions and syntactic opposites would be a syntax of oppositions, all the more that a central topic for **QAS** is the syntactic concept of *negation*. In contradistinction to Moretti's semantic approach, opposition has been thought in Béziau (2006) as a theory about classical and non-classical negations in the controversy against Slater's objection to paraconsistent negation (see Slater (1995)). For this very reason, oppositions can be viewed as a relation between any formula and one of its negated counterparts.

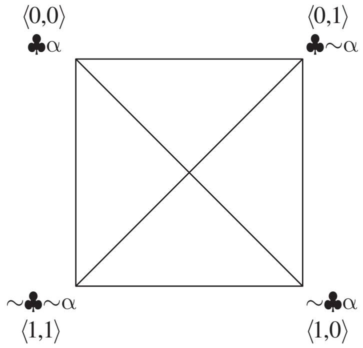
Definition 5. A logical *opposite* is a set of answers to questions about negating the components of an initial formula $\clubsuit\alpha$.
To the $\mathbf{Q} = 2$ following questions $\mathbf{Q}_1$: "Is $\clubsuit$ negated?" and $\mathbf{Q}_2$: "Is α negated?", oppositions reply by an ordered set of $\mathbf{P} = 2$ yes-no answers (where the 2-tuple $\langle 0, 0\rangle$ is the trivial case of self-identity).

Definition 5.1. A subaltern opposite (op^D) yields an affirmative $\mathbf{P}_1$ and an affirmative $\mathbf{P}_2$.
$op^D(\clubsuit\alpha) = ({\sim}\clubsuit{\sim}\alpha) = \langle 1, 1\rangle$

Definition 5.2. A contradictory opposite (op^N) yields an affirmative $\mathbf{P}_1$ and a negative $\mathbf{P}_2$.
$op^N(\clubsuit\alpha) = ({\sim}\clubsuit\alpha) = \langle 1, 0\rangle$

Definition 5.3. A (sub-)contrary term (op^C) yields a negative $\mathbf{P_1}$ and an affirmative $\mathbf{P_2}$.
$op^C(♣\alpha) = (♣{\sim}\alpha) = \langle 0, 1\rangle$

Figure 4: The Logical Square



The advantage of such questioning is that it characterizes quaterns without any reference to duals, and defines ♠α syntactically as ∼♣∼α. It also explains why subalternation is seen as a contradictory of contraries, given that the subaltern ∼♣∼α results from the successive application of op^C and op^N to the initial formula. Finally, it provides a relevant explanation for why subalternation is not a proper opposition: if any opposition includes an opposite-forming operator that proceeds by negating one or several components in a formula, then subalternation amounts to a double negation and, thus, yields an affirmation from a classical perspective.

The disadvantage is, again, that a *sub*contrary opposite cannot be gathered from any unnegated formula: the basic ♣α has no subcontrary opposite, but only a contrary one ♣∼α. Likewise, subcontrariety cannot be gathered by reading its expression 'contradictory *of* contraries' as a function of function: to say that SCT is the relation $Op(♣\alpha,(op^N(op^C(♣\alpha))$ is not a correct translation, because if $♣\alpha = \langle 0, 0\rangle$ then $op^N(op^C(♣\alpha)) = op^N(♣{\sim}\alpha)$ $= ({\sim}♣{\sim}\alpha) = op^D(♣\alpha)$. Hence $Op(♣\alpha, op^D(♣\alpha) = SB$, and not SCT. Moreover, this syntactic characterization cannot be successfully applied to structurally more complex formulas Φ such as non-basic modal sentences: the **Y**- and **U**-vertices of a logical hexagon cannot be specified by means of

$\mathbf{Q}_1$ and $\mathbf{Q}_2$ above, which means that a semantic characterization eventually prevails for a complete account of oppositions.

1.2.3 Semantic Generalization

Another way to generalize the geometry of oppositions is to turn what Moretti (2009) calls "logical bi-simplexes" into logical p-simplexes, including $p > 2$: a syntactic generalization has to do with the structure of opposed formulas, whereas a semantic generalization concerns the number of their compossible truth-values. It is clear that Aristotle's theory of opposition is entirely based on the **PBV**. But there is no reason not to introduce $p > 2$ truth-values in a **QAS** abstract characterization, thereby turning the classical bi-simplexes into non-classical p-simplexes. By doing so, **NOT** sheds a new light on the controversial meaning of negational principles such as the excluded middle (**PEM**) or the non-contradiction (**PNC**).

The device should sound appealing for any champion of many-valued logics, especially in the variety of philosophical logics where "standing propositions" are superseded by mere "variable sentences". Furthermore, Aristotle himself seemed to contest the universal applicability of **PBV** as regards the case of future contingents[7]. Despite this controversy about bivalence, no attempt was made to expand the theory of opposition to embrace $p > 2$ truth-values before Moretti (2009), who overcomes Slater's objection to paraconsistent negation in the light of a non-classical reading of **PNC**[8].

From our abstract perspective of **QAS**, a semantic generalization of oppositions entails a modification in the content of **Q** or **P**: either the questions are to be asked about the compossibility of alternative truth-values beyond truth and falsity, so that $n > 2$; or the answers may be extended beyond "yes" and "no", so that $m > 2$. Let us take Łukasiewicz's 3-valued logic of indeterminacy $Ł_3$ as an example, where $v(\alpha) = ½$ means that α is

7 See Aristotle (a), Chapter 9. The customary interpretation is that Aristotle renounced to the universal application of **PBV** while maintaining **PEM**. Some other commentaries blamed this view for confusing on *sentence* and a *proposition*, however. Such differences will be returned to in the next section about illocutionary modalities.

8 The question is whether **PNC** is about contradiction in particular, or negation in general. It is argued in Béziau (2006) that a negation needn't be a contradictory-forming operator: not every negation op^X verifies $Op(\Phi, op^X(\Phi)) = CD$. But since **PNC** is about CD, it is hardly relevant to say that $Op(\Phi, op^R(\Phi)) = SCT$ when the point is op^N and not op^C. Consequently, a further challenge would be to find a non-classical version of op^N for which $Op(\Phi, op^N(\Phi)) = CD$ is verified while departing from **PNC**.

indeterminate. Indeterminacy can be interpreted in two different ways in **QAS**. Either it is added as a further truth-value besides truth and falsity, in which case a new question $\mathbf{Q}_3$: "Possibly $v(\alpha) = v(\beta) = ½$?" is introduced; or it is interpreted as an indefinite answer beyond affirmation ("yes") and denial ("no"), so that a new answer $\mathbf{P}_3$: "maybe" is introduced. The resulting set of questions-answers will amount to either $m^n = 2^3 = 8$ oppositions, or $m^n = 3^2 = 9$ oppositions.

These non-classical oppositions give rise to three problems: how does one account for the difference in the number of oppositions when $m = 3$ or n = 3, since the results in the two cases are supposed to have the same meaning? How should one interpret the non-classical oppositions? Isn't it possible to reduce the non-classical oppositions to classical ones? It is out of the scope of the presented paper to examine these questions.

2. Application of the Theory of Opposition

After describing the abstract properties of opposition, we now apply the logical analysis presented to settle several problems from philosophical logic. Among these are the traditional problem of *existential import*, and the intermediary case of epistemic oppositions.

It is argued in the sequel that epistemic modalities help to settle the problem of existential import by making a distinction between two senses of the underlying concept of *truth*, namely: an ontological sense of "being true", and an epistemological sense of "being held to be true". Opting for the latter allows the construction of the so-called "illocutionary modalities", a linguistic trade-off between epistemic modalities and truth-claims. The results of Moretti (2009) will be used as a way to represent our solution to existential import in an alternative geometry of oppositions.

2.1 The Core Problem of Opposition: Existential Import

It has been generally claimed that Aristotle's theory of opposition was flawed unless one implicitly assumed the non-emptiness of its component terms. The reason for this claim is that Aristotle did not address the specific problem of the alleged existential import for negative propositions **E**-**O**. Our answer to the initial question of knowing whether negative propositions are existentially committed is clearly no. But before developing our own diagnosis, let us first consider the issue.

Hypothesis 1. Universal affirmatives have existential import.
Let **A** be: "Every griffin is an animal", and let v be an interpretation function for every proposition in **AEIO**. Granted there are no griffins at all, it is naturally assumed that $v(\mathbf{A}) = \mathrm{F}$: nothing can be truly predicated about whatever does not exist, which implies that $v(\mathbf{A}) \neq \mathrm{T}$; now by **PBV**, $v(\mathbf{A}) \neq \mathrm{T}$ entails $v(\mathbf{A}) = \mathrm{F}$.

O is the contradictory of **A**, hence $v(\mathbf{O}) = \mathrm{T}$. **A** is a universal affirmative, hence **O** is a particular negative stating that there is at least one griffin that is not an animal. Hence there is at least one griffin and there are no griffins at all (by assumption), which is self-contradictory. Such a consequence is absurd, by **PNC** and so is its premise, which in turn must be refuted: $v(\mathbf{A}) \neq \mathrm{F}$, so $v(\mathbf{A}) = \mathrm{T}$.

The initial assumption that griffins don't exist is compatible with $v(\mathbf{A}) = \mathrm{T}$, provided that universal propositions don't presuppose the existence of their subject terms. Modern logic meets this proviso, by encoding a universal proposition in the form of a conditional whose antecedent needn't be true. If so, then **A** means: "If x is a griffin, then x is an animal". Given that the universal affirmative is the negation of the corresponding particular negative: "Every A is B" = "Not-(Some A is not B)", $v(\mathbf{A}) = \mathrm{T}$ iff there is no griffin which is not an animal. Now there is no griffin at all. Hence $v(\mathbf{A}) = \mathrm{T}$.

Hypothesis 2. Universal affirmatives don't have existential import.
That $v(\mathbf{A}) = \mathrm{T}$ entails that $v(\mathbf{E}) = \mathrm{F}$. If $v(\mathbf{E}) = \mathrm{F}$, then its contradictory is T, *i.e.* $v(\mathbf{I}) = \mathrm{T}$. **I** is a particular affirmative, thus meaning that there is at least one griffin that is an animal. Now the initial assumption claims that there are no griffins, whatever the interpretation of the universal may be (whether existential or not). Hence $v(\mathbf{I}) = \mathrm{F}$ and $v(\mathbf{I}) = \mathrm{T}$. This is self-contradictory, by **PNC**. Hence the initial assumption must be rejected again.

Conclusion.
Both hypotheses $v(\mathbf{A}) = \mathrm{T}$ and $v(\mathbf{A}) = \mathrm{F}$ result in an inconsistent theory of oppositions when applied to propositions with empty terms.

In the light of the foregoing *reductio ad absurdum*, two general reactions are conceivable.

In order to avoid inconsistency in the theory of opposition, one either assumes the non-emptiness of subject terms as an *ad hoc* hypothesis. This was Russell's stance on the semantic paradox, when he proposed his ramified type theory as a way out.

Or one sustains that propositions with empty terms (without referent) are as much meaningful as any other, thus opening the way for improving Aristotle's theory of opposition without begging the point. We will endorse this second attitude in the sequel.

There is a rich literature on existential import. Horn (1989, p. 24) lists four different positions on the subject as follows:

There are, as it happens, at least four distinct ways of answering such questions:

(i) Existential import is determined by the *quality* of the proposition; affirmative (**A** and **I**) propositions entail existence, while negative ones (**E** and **O**) do not.
(ii) Existential import is determined by the *quantity* of the proposition: universals (**A** and **E**) have no existential import, while particulars (**I** and **O**) do.
(iii) Existential import corresponds to a *presupposition* associated with **A**, **E**, **I** and **O** propositions.
(iv) The question of existential import is entirely absent from the Square of Opposition.

The present paper rejects (ii) and (iv) while endorsing (i) and (iii): **E** and **O** entail (don't entail) existence whenever the latter is presupposed (is not presupposed), and such a presupposition depends upon the speaker's intention. The problem is not of a syntactic order but has to do with the ambiguous interpretation of **O**, in the sense that not every contradictory of a universal affirmative amounts to a negative *particular*. The difficulty thus lies in the natural translation of the **O**-vertex, which is customarily rendered as a particular negative in the Apuleian square. Our approach squares with Abelard's position, as described by (Horn 1989, p. 26):

> For Aristotle, *Not every man is white* was indeed taken to be the canonical contradictory of *Every man is white* (*De Int.*, 24b6), but there is no suggestion that it is not considered to be equivalent to *Some man is not white*; for Apuleius and Boethius, these

> two forms were explicitly taken to be notational variants. Abelard's results, despite the consistency of his argumentation, were apparently too counterintuitive to be taken seriously; later medieval (and modern) logicians almost without exception rejected this distinction between *non omnis* and *quidam non*[9].

Such a distinction will be taken seriously and made intuitive in the sequel, on the basis of a twofold reading of Aristotle's assertoric propositions: an *ontological* reading, which concerns the truth and falsity of propositions; an *epistemological* reading, which concerns the attitude of speakers toward such propositions. The source of the problem seems to reside in a deep confusion between the "affirmation" and "negation" of *assertoric* oppositions; the suggested way out is a more fine-grained analysis of negation, especially the **O**-vertex of the initial logical square.

2.2 *Epistemic Modalities*

An epistemological reading of assertoric oppositions refers to epistemic modalities, where a propositional attitude is attached to a sentential content. The most familiar epistemic concepts are knowledge and belief, symbolized by two modal operators K and B. Now three main difficulties arise if we want to construct a polygon of epistemic oppositions; these concern the logical relations between knowledge and belief, knowledge and truth, and the assumption of mere consideration.

2.2.1 Knowledge and Belief

Firstly, knowledge and belief don't seem to be included in the same polygon of opposition. A tentative theory of epistemic oppositions was proposed by Engel (2007), with the square of Figure 5.

Although the affirmative subalternation **AI** matches with the "entailment thesis": $K\alpha \Rightarrow B\alpha$, it is easily seen that it leads to some counterintuitive results: **IE** states that whatever is ignored is not believed (and conversely), while **EO** claims that no ignored proposition can be believed. Furthermore, ignorance is currently viewed as the negation of knowledge ($\sim$K) and should thus be opposed to knowledge contradictorily rather than contrarily. As for

9 About Apuleius' conflation of *non omnis* and *quidam non*, see also Monteil (2003) and Parsons (2006). Expansion from the Apuleian square to a new logical polygon, as proposed in the sequel, is actually aimed at making Abelard's results more intuitive.

disbelief, the ambiguous meaning between B$\sim\alpha$ and $\sim$Bα cannot be made sufficiently clear in a polygon of opposition with only 4 vertices. The crucial point is that K and B are equally strong modalities paralleling necessity ♣ = □, so that both of them should be located at one and the same vertex **A**. There are two alternative ways to settle the problem.

A first move is to turn Engel's logical square into a logical hexagon of (strictly) epistemic and doxastic modalities, where K and B occur in two separate polygons and are contrary to a common epistemic counterpart of contingency, namely: doubt (the **Y**-vertice). The following figure depicts similar oppositions for ♣ = K or B, while recalling the apparent impossibility to combine these into one and the same polygon (Figure 6).

Figure 5: A Square for Epistemic Oppositions

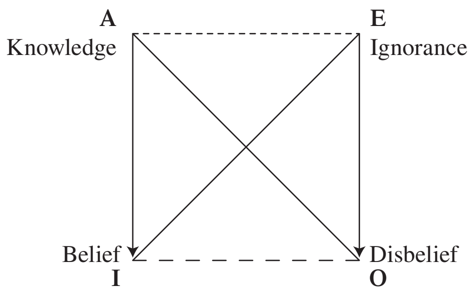
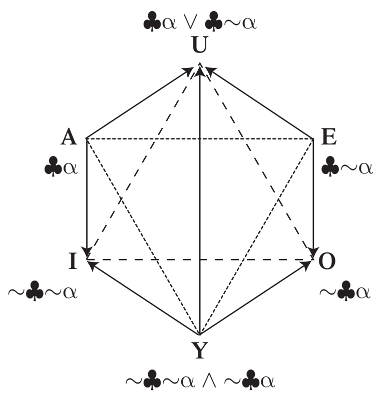


Figure 6: A Hexagon for Epistemic Oppositions



2.2.2 Knowledge and Truth

A second move is to merge knowledge and belief into one and the same modality, in order to produce a common modality of strong belief or subjective certainty.

Although some logics of knowledge as strong belief already occurred in the literature of epistemic logic, the mainstream view of epistemic modalities follows Plato's definition of knowledge as justified true belief (*Theaetetus*, 200d-e). The concept of truth makes the substantial difference between belief as a psychological attitude and knowledge as an objective state: such an "ontological" difference means that every proper knowledge is a belief that is made true by a state of affairs. In accordance with this philosophical tradition, the logical framework of Kripke's relational semantics gave rise to a series of modal systems with specific accessibility relations between "possible worlds"; the conspicuous domination of Kripke structures thus requires an examination of several modal systems for characterizing K and B.

Epistemic modalities are currently characterized as **KT4**(**5**) or **KD45** modalities, depending on whether the modality at hand is knowledge or belief. This means that the following axiom schemata variously hold for $\clubsuit = \mathrm{K}$ or B: (**K**): $(\clubsuit\alpha \wedge (\alpha \Rightarrow \beta)) \Rightarrow \clubsuit\beta$, (**T**) $\clubsuit\alpha \Rightarrow \alpha$, (**D**) $\clubsuit\alpha \Rightarrow {\sim}\clubsuit{\sim}\alpha$, (**4**) $\clubsuit\alpha \Rightarrow \clubsuit\clubsuit\alpha$, (5) ${\sim}\clubsuit\alpha \Rightarrow \clubsuit{\sim}\clubsuit\alpha$. The reduction laws of modalities for the iterative systems **S4** and **S5** help to introduce corresponding polygons of opposition, as illustrated by Moretti (2009).

But it can also be noted that such a characterization of epistemic modalities can be made intuitive by an alternative, illocutionary interpretation of epistemic modalities where "truth" is only synonymous with "truth-claim"[10]. So is the case with the *Moore's Paradox*, according to which it is absurd but not inconsistent to say "It rains, but I don't believe it" $(\alpha \wedge {\sim}\mathrm{B}\alpha)$. The prominent diagnosis argues that its *statement* departs from the sincerity condition for every uttered statement: any speaker is supposed to believe what it states. In other words, a correct formalization of the Moorean statement is not $(\alpha \wedge {\sim}\mathrm{B}\alpha)$ but its iterated version $\mathrm{B}(\alpha \wedge {\sim}\mathrm{B}\alpha)$. The distributivity of epistemic operators entails that the Moorean sentence amounts to a statement of the logical form $(\mathrm{B}\alpha \wedge \mathrm{B}{\sim}\mathrm{B}\alpha)$, thus violating the axiom schema (**4**) and making it explicitly inconsistent. More generally, Fitch (1963) has shown that $\clubsuit(\alpha \wedge {\sim}\clubsuit\alpha)$ is inconsistent (logically false) for any

10 See Schang (2007) for a detailed account of epistemic paradoxes and the motivation of their illocutionary treatment.

value concept ♣, whether it be instantiated by belief, knowledge, or even truth. The same inconsistency occurs in the subsequent *Fitch's Paradox of Knowability*, where any epistemic modality ♣ turns its sentential content α into the content of a statement $♣\alpha$ with illocutionary clauses.

In short, an illocutionary reading of epistemic modalities consists in merging knowledge and belief into one and the same attitude of *truth-claim* and replacing the criterion of justification for K by its degree of force: knowledge is then synonymous with certainty, or strong truth-claim, in such a logic of statements where no realistically-minded difference is maintained any longer between the concepts of knowledge and truth.

Whether the reader favours a realist or anti-realist stance about knowledge, the point is that such a reduction of truth (of a sentential content) to truth-claim (of its corresponding statement) will be endorsed in the next section about existential import; this will be done after considering another difficulty with epistemic oppositions and their formulation.

2.2.3 Knowledge and Mere Consideration

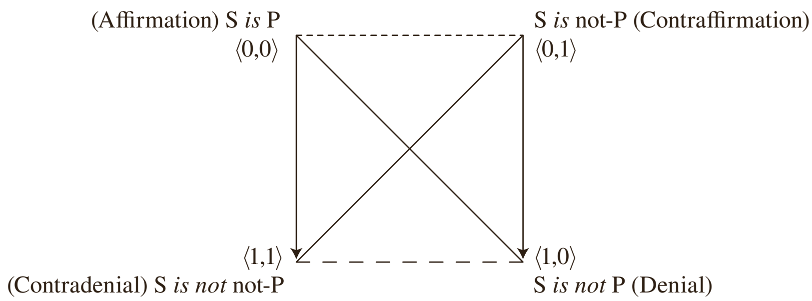
The statement of doubt (in symbols: D) was characterized above as an epistemic counterpart of contingency. That is, whoever has doubts about α also disbelieves both α and $\sim\alpha$: $D\alpha =_{df} \sim B\alpha \wedge \sim B\sim\alpha$, where disbelief is rendered as the negation of belief.

But there is a difference between non-belief, as a mere absence of belief, and disbelief. As noted in Englebretsen (1969) and Hart (1980), not every sentential content α can be doubted by anyone. It is so, because disbelief assumes a self-conscious activity by a subject who considers, *i.e.* thinks about the sentential content. Thus Socrates couldn't have been said to doubt (or to have doubts about) Obama's victory during the US elections 2008, because he wouldn't have been able to think about such a referent. This results in a more complex picture of epistemic oppositions, where every such attitude towards a sentential content α requires its consideration as a minimal precondition.

In order to construct a modal logic of doubt, Hart (1980) claims in that the latter is to be defined as a believed non-belief, so that we obtain the compound characterization $D\alpha =_{df} B(\sim B\alpha \wedge \sim B\sim\alpha)$. A **S5**-modal logic of belief would help to include the non-basic modality of doubt by reducing $B\sim B$ to $\sim B$; but such a characterization would conflate whatever is not believed self-consciously or not. In a nutshell, non-beliefs are to be opposed to disbeliefs; the problem is thus to find the right vertex in the right polygon of epistemic oppositions supplemented with doubt.

For this purpose, Englebretsen (1969) takes the basic notion of consideration into account by adding a new scope of negation to the epistemic oppositions. It is to be noticed that Englebretsen proceeds by returning to the sources of Aristotle's tradition or "term logic", where negation could be applied either to the copula or to the predicate term.

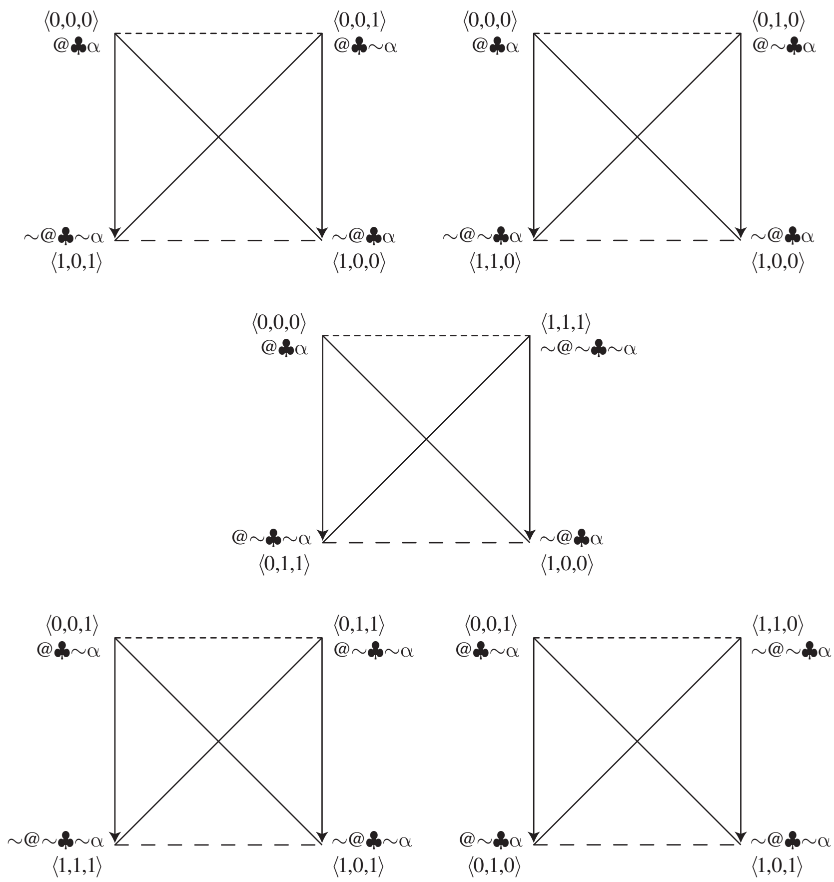
Figure 7: The Four in Term Logic



By analogy with the abstract structure of quaterns (see Figure 2), we fall here again on a compound structure ♣α where ♣ = S and α = P. A relevant difference should be made between a *predicate* negation "not" and a *predicate term* negation "not-": the former is attached to the whole predication "S is P" and proceeds like sentential negation; the latter uses a hyphen that restricts the scope of negation to the predicate term P. The first results in contraffirmation (♣$\sim\alpha$) and the second in mere denial ($\sim$♣α). No wonder the opposites are as numerous in term logic (Figure 6) as in quantified propositions (Figure 1): their structured formulas result from the same question-answer game with $m^n = 2^2 = 4$, Q_1: "Is ♣ negated?", and Q_2: "Is α negated?".

In addition to the foregoing more fine-grained scope of negation, the introduction of the component of consideration turns the structure of epistemic modalities from $\Phi =$ ♣α into the more complex structure @(Φ) = @♣α, where @ is a consideration-operator that applies to the whole epistemic statement ♣α and means that α is considered. Accordingly, Englebretsen's conflation of term and epistemic logic results in a

Figure 8: Epistemic Multi-Modal Oppositions



multi-modal logic with $n = \mathbf{P}^{\mathbf{Q}} = 2^3 = 8$ available opposites, given $\mathbf{Q}_1$ = "Is @ negated?", $\mathbf{Q}_2$ = "Is ♣ negated?", and $\mathbf{Q}_3$ = "Is α negated?". The 8 subsequent answers are described in Englebretsen (1969) by a group of 5 different squares, where ♣ = K but could also stand for B in our merged perspective of epistemic modalities.

The **NOT** framework should shed theoretical light on the construction of a complete polygon including all of these epistemic oppositions to replace Englebretsen's five separate squares. Pending such a construction for multi-modalities, the following semantics for assertoric oppositions suggests an alternative reading of epistemic mono-modalities where consideration is vacuously included.

2.3 Illocutionary Modalities

2.3.1 Assertoric Statements

The *illocutionary* import of epistemic attitudes stems from Austin's seminal distinction between a locutionary, illocutionary and perlocutionary feature of speech-acts (symbols: Fα): the locutionary aspect refers to the sentential content α and its semantic properties; the illocutionary aspect refers to the intended aim of its utterance by a speaker; the perlocutionary aspect refers to its expected effect upon the hearer.

A formal pragmatics has been proposed accordingly by Searle & Vanderveken (1985), where every speech-act Fα essentially includes an "illocutionary force" F with six main components among five classes of speech acts. Assertive acts are one case in point: a statement such as "Obama won the US elections 2008" is an assertion, where the speaker intends to tell the truth and succeeds in doing so only if its sentential content α describes how things really are.

Without going into further details on the framework of illocutionary logic, we need two of its main components in what follows, namely: the sincerity condition of an assertive act and its degree of strength. Indeed, the so-called "direction of fit" between a sentential content and the state of affairs expressed by it assumes a correspondence theory of truth that couldn't be endorsed from an antirealist perspective, so that assertive acts occur hereafter as truth-claims that don't really depart from belief. Moreover, the logical status of denial is not made clear by Searle & Vanderveken: it is not clearly argued whether denial belongs to the specific class of assertive acts or serves as a general negation for any illocutionary force. While such an *illocutionary negation* is compared with intuitionistic negation[11], it will be argued below that the latter is just a subcase of denial.

11 "Illocutionary negation is like intuitionistic negation from this perspective since, in intuitionistic logic, it is equally invalid that $\alpha \vee \sim\alpha$ and $\sim\sim\alpha \rightarrow \alpha$." (Searle & Vanderveken, p. 154; also quoted in Vernant (2003, footnote 22 p. 5).

2.3.2 An Assertoric Logic of Affirmation and Denial

In order to give a more fine-grained analysis of denial (illocutionary negation), the technical use of relational semantics by Vanderveken's formal pragmatics is replaced here by an algebraic semantics for affirmation and denial. Here is the characterization of assertive or assertoric statements $F=\{A,R\}$, where each speech-act $\clubsuit=F$ is the expression of a corresponding epistemic attitude.

Definition 6. An *assertoric statement* $S\alpha$ (with $F=S$) is a value-claim or logical judgment about the truth-value of a sentence α, where a truth-value $\{T,F\}$ is predicated of the sentential content.
To the $\mathbf{Q}=2$ initial questions about a sentential content α: "Do you hold $v(\alpha)=T$?" and "Do you hold $v(\alpha)=F$?", assertoric modalities reply by an ordered set of $\mathbf{P}=2$ yes-no answers.

Definition 6.1. An *affirmation* (or acceptance) (A) yields an affirmative $\mathbf{P}_1$, irrespective of $\mathbf{P}_2$.
$(\alpha)=\langle 1,-\rangle$
The epistemic counterpart of affirmation is belief: $B\alpha$.
There are two subspecies of acceptance.

Definition 6.1.1. A *conjecture* (A^-) yields an affirmative $\mathbf{P}_1$ and a affirmative $\mathbf{P}_2$.
$(\alpha)=\langle 1,1\rangle$
The epistemic counterpart of conjecture is weak belief: $B\alpha\wedge B{\sim}\alpha$.

Definition 6.1.2. An *assertion* (A^+) yields an affirmative $\mathbf{P}_1$ and a negative $\mathbf{P}_2$.
$(\alpha)=\langle 1,0\rangle$
The epistemic counterpart of assertion is strong belief: $B\alpha\wedge{\sim}B{\sim}\alpha$.

Definition 6.2. A *denial* (or rejection) (R) yields a negative $\mathbf{P}_1$, irrespective of $\mathbf{P}_2$.
$(\alpha)=\langle 0,-\rangle$
The epistemic counterpart of denial is disbelief: ${\sim}B\alpha$.
There are two subspecies of rejection.

Definition 6.2.1. A *neg-assertion* (R^-) yields an affirmative $\mathbf{P}_1$ and a negative $\mathbf{P}_2$.
$(\alpha) = \langle 0, 1\rangle$
The epistemic counterpart of neg-assertion is strong disbelief: $\sim B\alpha \wedge B{\sim}\alpha$.

Definition 6.2.2. A *doubt* (R^+) yields an affirmative $\mathbf{P}_1$ and a negative $\mathbf{P}_2$.
$(\alpha) = \langle 0, 0\rangle$
The epistemic counterpart of doubt is weak disbelief: $\sim B\alpha \wedge {\sim}B{\sim}\alpha$.

A formal semantics for assertoric modalities can be characterized as follows, where these are taken to be 1-ary statement-forming operators upon sentences (in the vein of Bochvar's external operators or von Wright's truth logics). In addition to truth-values for sentences, truth-claim values are introduced for statements such that $v(A^+\alpha) = 1$ iff $v(\alpha) = \langle 1, 0\rangle$ and $v(A^+\alpha) = 0$ otherwise.

Definition 7. A statement $\clubsuit\alpha$ is *right* (1) or *wrong* (0) if it expresses the speaker's epistemic attitude about α.

Definition 7.1. $v(A^-\alpha) = 1$ iff $v(\alpha) = \langle 1, 1\rangle$, and $v(A^-\alpha) = 0$ otherwise.

Definition 7.2. $v(A^+\alpha) = 1$ iff $v(\alpha) = \langle 1, 0\rangle$, and $v(A^+\alpha) = 0$ otherwise.

Definition 7.3. $v(R^-\alpha) = 1$ iff $v(\alpha) = \langle 0, 1\rangle$, and $v(R^-\alpha) = 0$ otherwise.

Definition 7.4. $v(A^+\alpha) = 1$ iff $v(\alpha) = \langle 0, 0\rangle$, and $v(A^+\alpha) = 0$ otherwise.

Illocutionary oppositions are reformulated correspondingly in terms of compossible truth-claim values rather than truth-values.

Definition 8. An *opposition* is a set of answers to questions about compossible truth-claims for the statement $\clubsuit\alpha$ and its opposite $op^X(\clubsuit\alpha)$ (where $\clubsuit \in \{A, R\}$ and $X \in \{I, N, C, D\}$).

To the $\mathbf{Q} = 2$ following questions $\mathbf{Q}_1$: "Possibly $v(\clubsuit\alpha) = v(op(\clubsuit\alpha)) = F$?" and $\mathbf{Q}_2$: "Possibly $v(\clubsuit\alpha) = v(op(\clubsuit\alpha)) = T$?", oppositions reply by an ordered 2-tuple of answers: $Op(\clubsuit\alpha, op(\clubsuit\alpha)) = \langle \mathbf{P}_1, \mathbf{P}_2\rangle$.

The same ordered 2-tuples hold here as in Definition 2, with $Op = \{SB, CT, SCT, CD\}$.

It is easily seen that such a logic of statements departs from classical logic, in the light of the answers $v(\alpha) = \langle 1, 1\rangle$ and $v(\alpha) = \langle 0, 0\rangle$: the truth of a sentence classically entails the falsehood of its negation, but not here. This is due to the formulation of **Q**, in our corresponding question-answer semantics: the point about statements is not whether their sentential content *is* true or false, but whether it *is held to be* so.

Each of the four answers refers to a special assertoric force, whose classification depends both upon its quality (affirmative or negative) and its degree of force (X^+ or X^-).

On the one hand, it is to be noted that "affirmation" and "negation" refer here to **P** (and not α): an affirmative answer may be given about whether a negative sentence is true, (e.g., $v(\sim\alpha) = \langle 1,-\rangle$), and conversely (e.g., $v(\alpha) = \langle 0,-\rangle$). Let us recall that *locutionary* affirmation and negation concern the quality of a sentential content α, whereas *illocutionary* affirmation or "yes"-answer ($A = \{A^+, A^-\}$) and negation or "no"-answer ($R = \{R^+, R^-\}$) concern the quality of an answer **P**[12].

On the other hand, assertion shouldn't be assimilated to affirmation in our non-classical semantics: it is the strongest assertoric force, because it is a one-sided affirmation that is closer to truth-claim than any of its counterparts in $\{A, R\}$. In other words, the degree of force of an assertoric statement is determined by a linear ordering relation $>$ between definite and indefinite answers **P**: $\langle 1, 0\rangle > \langle 1, 1\rangle > \langle 0, 0\rangle > \langle 0, 1\rangle$. The previous modality of consideration @ is vacuously assumed in every aforementioned speech act, in the sense that any answer **P** is self-consciously given by the speaker.

As regards the existential import, an important point about assertoric statements is that only two of them are mentioned as usual speech acts: assertion A^+ and denial R, whose utterances are rendered by such ambiguous statements as "S is P" and "S is not P" respectively. The reason is that assertoric statements ordinarily aim at telling the truth and nothing but the truth, whether it concerns affirmative sentences or their negation. Hence the unique judgment stroke $\vdash$ in Frege's *Begrisschrift*, which means that the

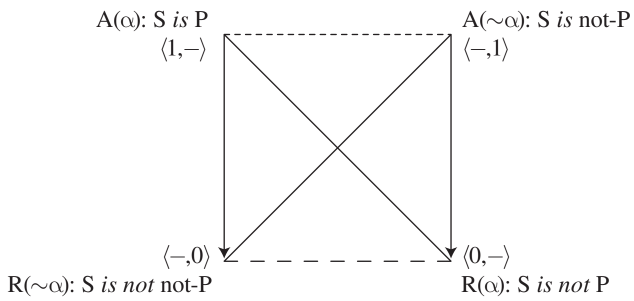
12 A way to argue that denial (illocutionary negation) is prior to sentential (locutionary) negation is that the former essentially occurs as a *definiens* of the latter. Firstly, no "$\sim$" occurs in the formulation of **Q** and **P** but only the expressions of falsity (F) or denial (0). Secondly, locutionary negation cannot occur without a corresponding illocutionary negation, but the converse does not hold: not every "no"-answer to $\mathbf{Q}_1$ entails a "yes"-answer to $\mathbf{Q}_2$, as witnessed by $\mathbf{P} = \langle 0, 0\rangle$. In other words, every negative statement proceeds from an illocutionary denial before being rendered by a locutionary negation.

following judgeable content α is held to be true by the speaker and holds as an axiom or derived theorem. But while Frege (1952) equated the opposite act of denial with an assertion of the negative content: $\vdash\sim\alpha$, denial may mean a mere rejection of α without any corresponding affirmation of $\sim\alpha$. Now any mere refusal doesn't stand for a particular speech act but a negative psychological attitude, and the indefinite statements of conjecture or doubt are just hinted by the tone of voice or qualifying clauses like "It seems that" (for conjecture) or "I don't say that" (for doubt). Hence any utterance of α is naturally understood as the expression of $\langle 1,0\rangle$ or $\langle 0,1\rangle$, and the problem of existential import is symptomatic of such a narrow understanding of affirmative and negative statements.

2.3.3 Back to the Existential Import

Recalling Aristotle's term logic, the modality of neg-assertion amounts to a contraffirmation and differs from denial. Thus Figure 6 can be partly reconstructed by means of assertoric modalities with the same structure $\clubsuit\alpha$, where $\clubsuit = \mathrm{A}$ or R.

Figure 9: The Four in Assertoric Logic



It clearly appears that not every assertoric statement is represented in the above figure: A and R are expressed by means of corresponding affirmative or negative statements, but nothing is said about their degree of force. This is so because Aristotle's traditional logic was uniquely composed of categorical assertions, where **PBV** restricts the range of **P** to $\langle 1,0\rangle$ and

$\langle 0, 1\rangle$. Now the case of empty subject terms created a difficulty in Aristotle's semantics: as argued by Horn (1989), Aristotle generally accepted that $v(\sim\alpha) = \mathrm{T}$ whenever the subject term does not exist; this entails that any denial of such a sentential content was held to be true by Aristotle[13]. Thus $v(\alpha) = \langle 0, 1\rangle$ seems to be the right answer for any statement whose sentential content includes an empty subject term.

But this is not the case, in the light of the relevant distinction between contraffirmation (neg-assertion) and denial: a sentence can be denied without its sentential negation being thereby affirmed, and an appropriate translation for Aristotle's attitude about empty terms should be rather $v(\alpha) = \langle 0, 0\rangle$. After all, Aristotle was said to accept the truth of denials such as "Not every griffin is an animal" while denying the truth of "Some griffin is not an animal". This means that only negative predications were accepted by the Stagirite in such cases, but not negated predicate terms; and given that only the first case corresponds to an illocutionary negation in our logic of assertoric statements, it follows from this term distinction that what was accepted by Aristotle is not the truth of a negative sentence $\sim\alpha$ but the denying attitude of any speaker about α: $\mathrm{R}(\alpha)$ is right, not $\mathrm{A}(\sim\alpha)$.

Does this illocutionary interpretation of negation give a consistent picture of the theory of opposition embracing the case of empty terms? On the one hand, the preceding figure shows that the logical square is insufficient for a geometrical representation of illocutionary oppositions; on the other hand, the syntactic generalization due to Moretti (2009) yields a group of polygonal oppositions for a logic of assertoric statements that takes account of their degree of strength (see Figure 10).

There are not $\mathrm{CT} = 2$, as in Aristotle's square, but $\mathrm{CT} = 4$ sorts of contrary statements: any two assertoric modalities exclude from each other (e.g., whatever is asserted is not conjectured), and the previous definition of subcontraries as contradictories of contraries gives rise to an identical number for subcontrary terms.

We thus obtain a gathering of 3-dimensional bisimplexes, according to the terminology of Moretti. An exhaustive geometrical representation of these illocutionary oppositions is composed of two tetrahedra of contrariety

13 In connection with footnote 13, (Horn 1989, p. 24) says that the "qualitative approach (i) has its roots in Aristotle (…) The existence of Socrates, as we have seen, is a necessary condition for the truth of any singular proposition concerning him (e.g., *Socrates is ill*), while his nonexistence is a sufficient condition for the truth of the corresponding (contradictory) negation (*Socrates is not ill*)".

Figure 10: Two Tetrahedra of Simplexes and One Cube of Subalternation

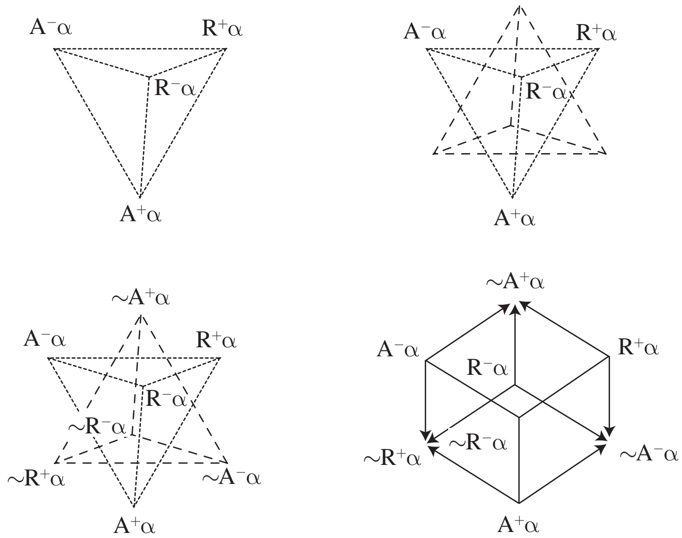
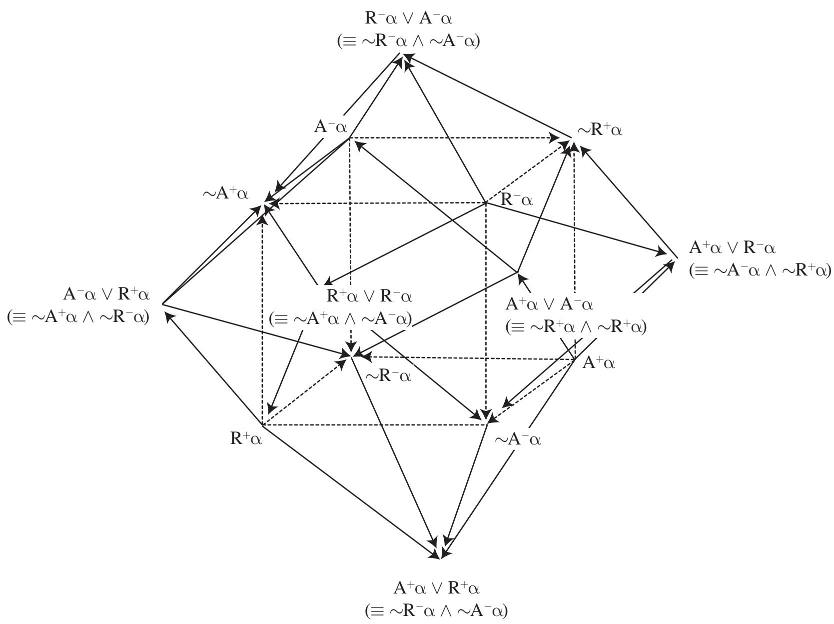


Figure 11: A Tetraicosahedron of Assertoric Oppositions



and subcontrariety (connected by contradictories) together with one cube of subalternation, where the four vertices of Aristotle's square (see Figure 8) occur as partial cases (see Figure 11).

The difference between the neg-assertion A^- and the denial R was thus rendered using Abelard's medieval distinction between the two alternative expressions of **O**: "Not every S is P", and "Some S is not P". We stick here to the first interpretation of **O** as an *illocutionary* negation of **A**, without following Apuleius' and Boethius' translation as a particular negative: "S is not-P". In other words, nothing is to be affirmed about non-existing entities like griffins: neither $v(\alpha) = T$, nor $v(\alpha) = F$ (*i.e.* $v(\sim\alpha) = T$), so that the proper formalization of **O** is not $A^-(\sim\alpha)$ but $R(\alpha)$. To be more precise, the initial sentence α: "Every griffin is an animal" (or, equivalently: "Griffins are animals") is as much denied as its locutionary negation: "Some griffin is not an animal", in the sense that no affirmation ("– *is* …") could be rightly uttered about a non-existing entity. If so, then α and $\sim\alpha$ are equally denied and the corresponding statement is not neg-assertion but doubt: $R^+(\alpha)$, or $v(\alpha) = \langle 0, 0\rangle$.

In the light of the preceding figure highlighting assertoric oppositions, Aristotle's **A**- and **O**-vertices turn out to be contradictory to each other in a restrictive sense of negation: "Not every S is P" appears as the *illocutionary* negation of "Every S is P", in accordance to the equivalence $\sim A(\alpha) \Leftrightarrow R(\alpha)$ (whatever is not affirmed is denied, and conversely). By contradistinction, any affirmation about griffins will be *contrary* to a neg-assertion about them: both are incorrect, *i.e.* $v(A^+(\alpha)) = v(A^+(\sim\alpha)) = v(R^-(\alpha)) = 0$. It amounts to saying that **O** doesn't mean the illocutionary affirmation of a negative sentential content (viz. a particular negative) but rather the illocutionary negation of an affirmative sentential content (viz. a universal affirmative). In other words: while the Abelardian "Not every S is P" attaches the negation to the universal quantifier, its (Apuleian or Boethian) variant "Some S is not P" is misleading in the light of our modal structuration. For the contradictory of $\clubsuit\alpha$ is not $\clubsuit\sim\alpha$ but $\sim\clubsuit\alpha$, which appears clearly only if the illocutionary aspect of an utterance is made explicit by $\clubsuit$.

3. Conclusion: the Future of Logical Opposition

Logical opposition has been considered throughout this paper in its main properties and its various applications. It has mainly been argued that:

(1) A question-answer game **QAS** properly used supports a complete theory of opposition, beyond Aristotle's initially quantified theory; a semantic generalization helps to characterize oppositions as 2-ary relations between any propositions, while a syntactic generalization helps to define logical opposites as its relata.

(2) The theory of opposition can be made totally consistent and settle the case of existential import by adopting an illocutionary view of oppositions that relates statements rather than mere sentences and distinguishes two senses of negation.

The theory of logical opposition may thus be completely overhauled, in the light of **NOT** and its formulation within the conceptual framework of **QAS**. Why not think about logic as a theory of opposition, rather than as a theory of consequence?

Is it because of the imprecise meaning of an opposition as compared to the precise characterization of logical consequence as truth-preservation? This paper has attempted to advance the perfectible theory of opposition by placing the emphasis on the concept of negation rather than on truth.

The incompleteness of Aristotle's theory of opposition has left a crucial gap between traditional and modern logic.

Our solution to existential import is an attempt to show that the theory of opposition is beyond traditional logic but can adequately be dealt with in purely modern logic terms such as relations, negation, and truth-values.

Given the prominent role of logical consequence in modern logic, the author will present opposition Op (and its opposite-forming operator op) as a counterpart of abstract consequence Cn (and its associated conclusion-forming operator) in a future work.

In a nutshell, opposition does not deserve to be considered as a mere tourist curiosity with old-fashioned properties. Given a proper facelifting, logical opposition can clearly emerge as a thick-skinned concept. This motivates our interest in **NOT** and **QAS**. The ageless concept of logical opposition needs to be clarified if the concern of logic is not to be just about consequence and truth but also about opposition and negation. The ultimate goal of our future works is to show that a theory of opposition comes to a logical theory of negation just as a theory of consequence comes to a logical theory of truth:

$$\frac{\text{Consequence}}{\text{Truth}} = \frac{\text{Opposition}}{\text{Negation}}$$

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