

Torricelli's Correspondence on Ballistics

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Received 3 June 1983

Summary

Torricelli elaborated the theory of ballistics as part of Galileo's theory of motion. In 1647 he had an interesting exchange of letters with G. B. Renieri, from Genoa, who complained that some experiments he had made with guns contradicted Galileo's theory. The correspondence discloses some fundamental issues of the Seventeenth century Scientific Revolution, the main one being to what extent mathematics can be applied to physics. Torricelli's view on this issue is ambivalent. He defends Galileo's kinematics as the correct description of reality yet says that mathematics does not describe reality. It is suggested that this ambivalence is an outcome of the climate of uncertainty which followed Galileo's trial.

Evangelista Torricelli (1608–47), Galileo's best-known pupil, has become famous as the inventor of the barometer. In 1644 he performed an experiment in which, by means of a column of mercury, he measured the atmospheric pressure.¹ The most important aspect of this experiment is the fact that Torricelli succeeded, for the first time, in producing an artificial vacuum. Much has been written about Torricelli's experiment,² and I do not wish here to add to the subject. In this article I will deal with another interesting and little known exchange of letters with Torricelli, also dealing with experiments, this time on ballistics.³

The theory of ballistics is a particular case of the theory of motion, and many sixteenth- and seventeenth-century scientists occupied themselves with the theory. The pioneer in this field was Nicolò Tartaglia (c. 1500–57), who in his works *La nova scientia*

¹ This experiment is described in a letter Torricelli wrote to Michelangelo Ricci (1619–82) in Rome. See *Opere di Evangelista Torricelli*, edited by Gino Loria and Giuseppe Vassura, 4 vols in 5 (Faenza, 1919–44), hereafter *OT*, III, 186–88. The letter also appears in the more recent *Le opere dei discepoli di Galilei, Carteggio 1642–1648, Volume I*, edited by Paolo Galluzzi and Maurizio Torrini (Florence, 1975), hereafter *Carteggio*, 122–23. Since the *OT* has been subject to much criticism and regarded by some scholars as unreliable (e.g. by Paolo Galluzzi in 'Evangelista Torricelli. Concezione della matematica e segreto degli occhiali', *Annali dell'Istituto e Museo di Storia della Scienza di Firenze* (1976), I, 71–95; see p. 72, note 3), I will rely on the *Carteggio* rather than on the *OT*.

² See Cornelis de Waard, *L'Expérience barométrique, ses antécédents et ses explications; étude historique* (Thouars, 1936); Mario Gliozzi, 'Origini e sviluppi dell'esperienza torricelliana', *OT* (footnote 1), IV, 231–94; W. E. Knowles Middleton, *The History of the Barometer* (Baltimore, 1964).

³ The exchange of letters between Torricelli and G. B. Renieri has been mentioned by a few authors, but none has dealt with it thoroughly. Among them are: A. Rupert Hall, in *Ballistics in the Seventeenth Century. A Study in the Relations of Science and War with Reference Principally to England* (Cambridge, 1952), pp. 97ff.; Serge Moscovici, *L'Expérience du mouvement, Jean-Baptiste Baliani, disciple et critique de Galilée* (Paris, 1967), pp. 196ff.; Paolo Galluzzi, 'Evangelista Torricelli ...' (footnote 1), pp. 81ff., and 'Vecchie e nuove prospettive torricelliane' in *La scuola galileiana. Prospettive di ricerca* (Florence, 1979), pp. 13–51 (pp. 39–40).

(1537) and *Quesiti et inventioni diverse* (1546),⁴ studied the variation of the trajectory in relation to the angle of elevation of the gun and found, by theoretical considerations, that the range is maximal when the elevation is 45 degrees. Tartaglia also designed an elevation gauge for guns called a *squadra* (square or sector) which consisted of two arms at right angles, a plumb line and a quadrant arc divided into twelve points ('*punti*').⁵ One arm was longer than the other and was inserted into the gun so that the plumb line showed the angle of elevation on the graduated quadrant (see Figure 1).

After Tartaglia, several authors—Benedetti, Galileo, Cavalieri, Mersenne and Torricelli—all dealt with ballistics.⁶ Galileo and Torricelli, in particular, had to study the subject as Court Mathematicians and, in Galileo's case, as tutor of the Crown Prince; ballistics lay within the realm of the theoretical and mathematical knowledge that they were expected to teach, and at the same time within the practical knowledge that their employers had to know.

Galileo appears to have occupied himself with military engineering when he was in Padua, during the years 1592 to 1610. In the second volume of his collected works there are two treatises, 'Breve istruzione all'architettura militare' and 'Trattato di fortificazione' which bear no date and which Galileo did not publish.⁷ They are

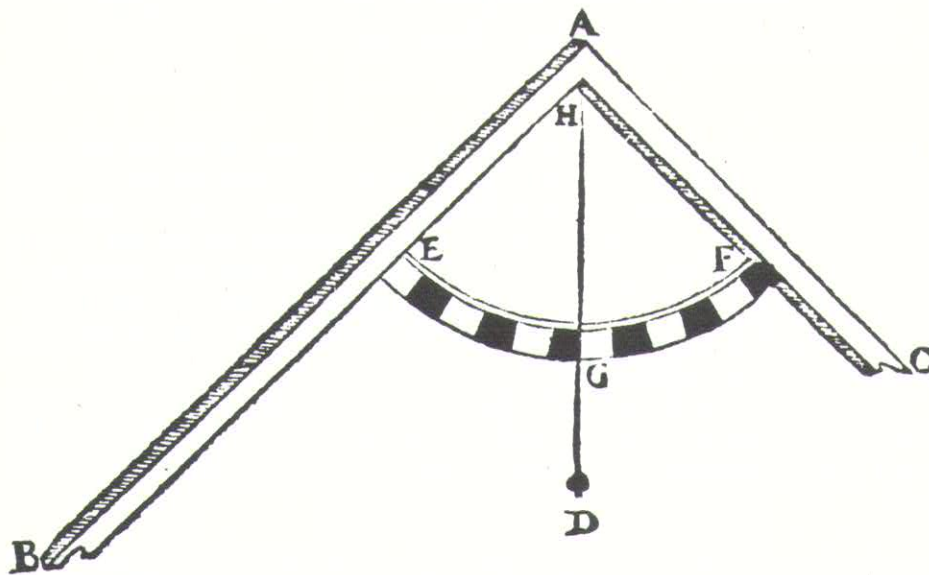


Figure 1.

Tartaglia's elevation gauge. (From *La nova scientia*; Pisa University Library.)

⁴ *La nova scientia* (Venice, 1537), *Quesiti et inventioni diverse* (Venice, 1546). Selections of these works have been translated and annotated by Stillman Drake in *Mechanics in Sixteenth-Century Italy; Selections from Tartaglia, Benedetti, Guido Ubaldo and Galileo* (Madison, 1969), pp. 61–143.

⁵ See Stillman Drake, 'Tartaglia's squadra and Galileo's compasso', *Annali dell'Istituto e Museo di Storia della Scienza di Firenze* (1977), II, 35–54.

⁶ See Hall (footnote 3).

⁷ *Le opere di Galileo Galilei. Edizione nazionale*, edited by Antonio Favaro, 20 vols in 21 (Florence, 1890–1909; reprinted in 1929–39, 1964–66 and 1968), hereafter *OG*, II, 7–146.

Pezzo elleuato alli.45.gradi sopra al orizzonte.

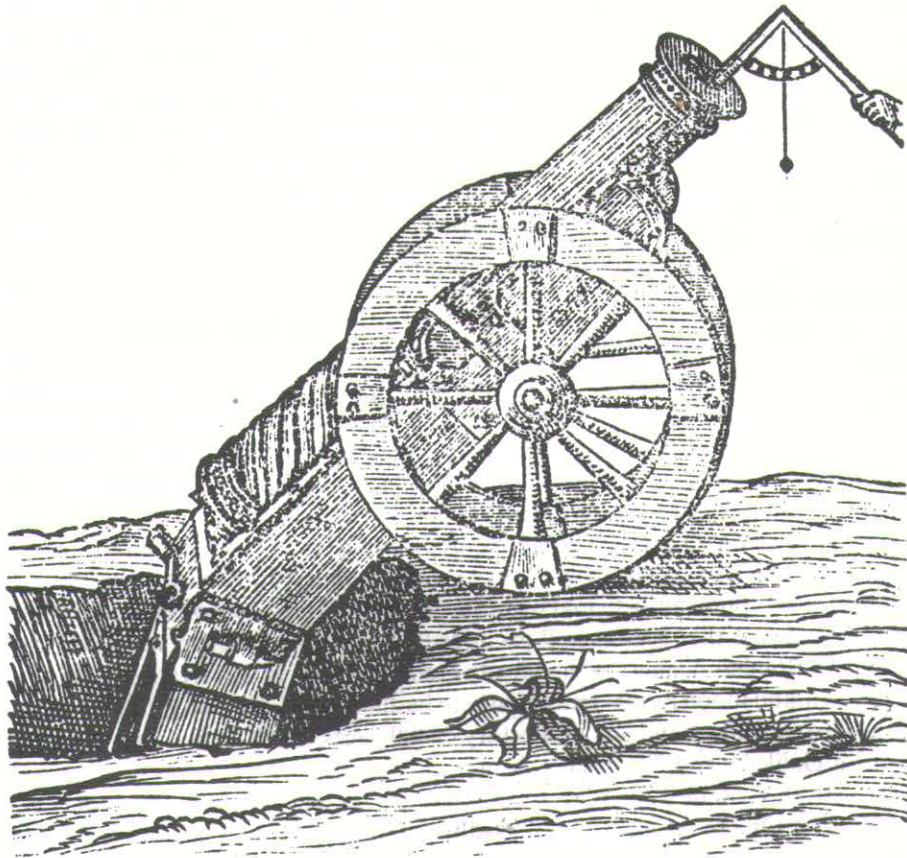


Figure 2.

The use of Tartaglia's gauge-piece elevated at 45°. (From *La nova scientia*; Pisa University Library.)

believed to be drafts of courses he delivered at the University of Padua, and the first treatise was probably written as early as 1592. Galileo had also designed, and in 1597 began producing for sale, a geometrical and military compass, which could be used, among other things, to measure the elevation of guns.⁸ Stillman Drake has shown that Galileo's compass was derived partly from Tartaglia's gauge.⁹ In 1606, Galileo published a work in which he explained how to use his instrument, *Le operazioni del compasso geometrico et militare*,¹⁰ which has also been included in the second volume of

⁸ On the various applications of Galileo's compass, see Guglielmo Righini, 'Sulla costruzione del compasso geometrico e militare di Galileo', *Physis. Rivista internazionale di Storia della Scienza*, 16 (1974), 201-22.

⁹ In 'Tartaglia's squadra and Galileo's compasso' (footnote 5).

¹⁰ (Padua, 1606).

Galileo's collected works.¹¹ Finally, in the fourth day of his *Discourses* of 1638,¹² Galileo dealt extensively with the theory of projectiles; he showed that the range increases in proportion to $\sin 2\alpha$, where α is the angle of elevation, and composed tables to aid gunners in calculating the trajectory of their guns at a given angle. Nevertheless, Galileo did not present an instrument indicating the relation between the angle of elevation and the range. This was done, apparently for the first time, by Torricelli.

In 1644 Torricelli published a work on geometry and kinematics entitled *Opera geometrica*,¹³. In the second part of this work, entitled 'De motu', Torricelli produces some of the work of Galileo on projectiles with a few additions and variations. From this treatise we understand that gunners did not use Galileo's tables since they assumed that the trajectory increased (or decreased, in case of elevations of more than 45 degrees) in *direct* proportion to the angle of elevation.¹⁴ This may have been the reason that induced Torricelli to design a new instrument and present it in the *Opera geometrica* (see Figure 3).

Three years after the publication of the *Opera geometrica*, Torricelli received a letter from one Giovanni Battista Renieri of Genoa,¹⁵ complaining that some experiments he had made with guns contradicted Galileo's theory. We do not know exactly who G. B. Renieri was. All we know is that he was the brother of Vincenzo Renieri (1606–47), a friar of the Olivetan order, who had made observations and calculations on the Medicean planets and communicated them to Galileo.

In his letter, dated 2 August 1647, Renieri complains: 'Your work on the motion of projectiles, which reveals your very sharp intellect, has reached Genoa and given our gentlemen the opportunity to make several experiments with various kinds of guns and I was astonished that such a well-grounded theory turned out to work so badly in practice.'¹⁶ Renieri relates that he set a two-ells-high gun at an elevation of 45 degrees (six points), and found its maximum range to be 2300 paces. He then shot at point blank (i.e. at zero elevation), and got a trajectory of 400 paces, much longer than the expected result of Galileo's and Torricelli's theory.

Regrettably, Renieri does not say what the expected result was, nor how he calculated it. One could easily calculate it by making use of the constant of acceleration g . But g was unknown in Torricelli's times, and the calculation, relying only on theorems known to Torricelli, is a little more complicated. I have done the calculation as follows: I began by assuming that the trajectory in the first case is described by the parabola ABC, where B is its vertex and highest point (Figure 4). I neglected the initial height, two ells, of the gun. According to a theorem quoted by Galileo in the

¹¹ OG, II, 363–424.

¹² *Discorsi e dimostrazioni matematiche, intorno à due nuoue scienze attenenti alla meccanica & i movimenti locali*... (Leiden, 1638), OG, VIII. There are several translations of this work into English. The two most recent ones are, *Dialogues Concerning Two New Sciences*, translated by Henry Crew and Alfonso de Salvio, with an introduction by Antonio Favaro (New York, 1914); *Two New Sciences, Including Centres of Gravity and Force of Percussion*, translated with introduction and notes by Stillman Drake (Madison, 1974).

¹³ (Florence, 1644). The various parts of this work are scattered in OT (footnote 1), I, pt 1, and II.

¹⁴ OT, II, 219–20.

¹⁵ Letter of Giovanni Battista Renieri to Evangelista Torricelli (in Florence), Genoa, 2 August, 1647, *Carteggio* (footnote 1), 388–90.

¹⁶ *Ibid.*, 388:

Essendo pervenuta a Genova la sua opera del moto de' proietti, nella quale così al vivo si scuopre il suo acutissimo ingegno, diede occasione a questi nostri signori di farne diverse esperienze con il tiro di varie sorti di cannoni, e veramente mi ha reso assai stupefatto che tal teorica, così ben fondata, abbi così malamente risposto alla pratica.

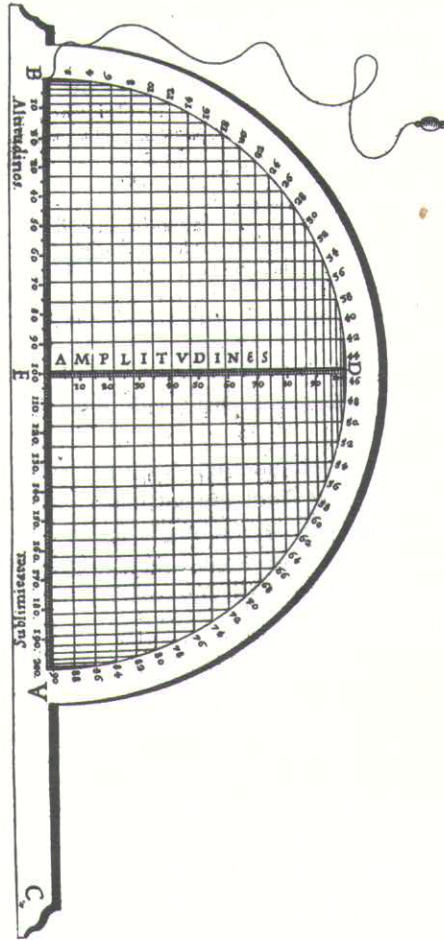


Figure 3.

Torricelli's *Squadra*. (From his *Opera geometrica*; Pisa University Library.) This instrument makes possible the correlation of the range and the elevation of a gun.

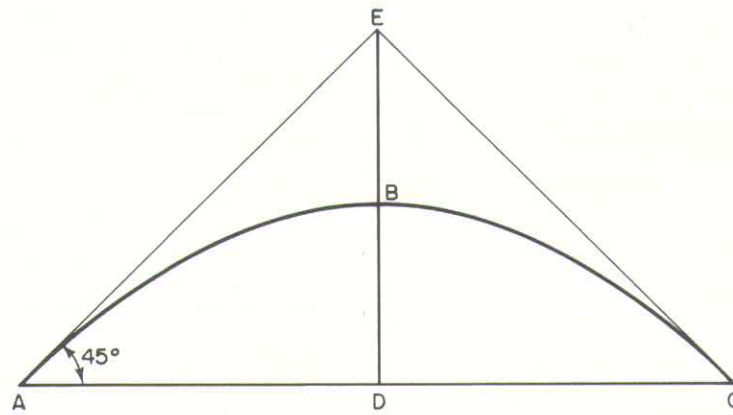


Figure 4.

In a parabolic trajectory ABC , where the angle EAD is 45° , the range AC is four times the maximum height BD .

Discourses,¹⁷ $EB = BD$, but $AD = DE$, and by symmetry $AD = DC = 2300/2 = 1150$, $BD = \frac{1}{2}ED = 575$. At B the projectile has only a horizontal velocity, equal to $v_0 \sin 45$ (v_0 being the initial velocity), or to $v_0(\sqrt{2}/2)$. Let us now look at the point blank shot: (Figure 5).

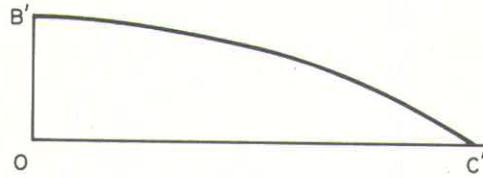


Figure 5.

$B'C'$ is the trajectory at point blank. The height of the falconet, $B'O$, is two ells.

In the previous case one could assume that a projectile was shot from B at point blank with a velocity of $v_0(\sqrt{2}/2)$, from a height of 575 paces, reaching a range of 1150 paces. To find what the range will be of a projectile shot at point blank with an initial speed v_0 , from a height of 2 ells (an ell can be taken as approximately equal to a pace¹⁸), let us assume x is the horizontal distance travelled by a projectile shot at point blank with an initial speed of $v_0(\sqrt{2}/2)$ from a height of 2 ells (2 paces). Then:

$$\frac{2}{575} = \frac{x^2}{1150^2},$$

$$x = 68 \text{ paces.}$$

If the same projectile is shot at a speed of v_0 from the same height, having the same times of flight, it will reach a range of $68 \times \sqrt{2} = \text{approx. } 96$ paces.

This result is very different from the measured range, 400 paces, and Renieri concludes: 'If not for the authority of Galileo, of whom I am a partisan, I would certainly feel some doubt concerning the motion of projectiles: whether it is parabolic or not, and if it is so, I would doubt that the axis of the said parabola is perpendicular to the horizon...'¹⁹ He then proposes an (*ad hoc*) variation of Galileo's theory: whereas, according to Galileo, the axis of the parabolic trajectory goes through the centre of the earth, Renieri suggests that the axis is slightly shifted away from the centre.

A few days later, presumably on 8 August, Torricelli replied with a long, remarkable letter.²⁰ He begins by emphasizing:

'... when writing that booklet on motion I never intended to make any assertion

¹⁷ *OG* (footnote 7), VIII, 284–85, Problem I, Proposition IV. English translation by Crew and de Salvio (footnote 12), 262–63.

¹⁸ I cannot tell exactly how long was the ell and pace used by Renieri. On the copy of Renieri's letter to Torricelli of 24 August 1647 (footnote 24), kept in the Central National Library of Florence, MSS. Gal. 151, 87–88, a diagram of a palm of Genoa has been added in the margin (folio 87^v) with the note 'The pace of Genoa makes 59 quattrini of ours'. According to W. E. Knowles Middleton in *The Experimenters: A Study of the Accademia del Cimento* (Baltimore, 1971), (xiii), the Tuscan ell was equal to 60 quattrini. Hence I assume that the Genoese pace (59 Tuscan quattrini) was approximately one Tuscan ell. I have also measured the palm of Genoa and found it to be approximately 24.5 cm.

¹⁹ *Carteggio* (footnote 1), 389:

Se l'autorità del Sig.r Galileo, del quale conviemmi esser parziale, non mi facesse resistenza, non mancherei d'aver qualche dubbio circa il moto de' proietti, se fusse parabolico o no, oppure, se è tale, non saprei accertarmi se l'asse della detta parabole debba essere perpendicolare all'orizzonte o no....

²⁰ Letter of Evangelista Torricelli to Giovanni Battista Renieri (in Genoa), Florence, c. 8 August 1647; *ibid.*, 391–94.

which is not *ex hypothesi*, in contrast to those who take for true those very well known two suppositions: that the spaces fallen by the body in equal times are *ut numeri impares ab unitate*; and that the spaces travelled horizontally in equal times are equal.²¹

In other words, Torricelli stresses that he wrote on hypothetical subjects and did not pretend, as some others may perhaps have thought, to describe truth. The first of Galileo's two suppositions is that the distance fallen by a body is proportional to the square of the time of fall, and Torricelli excludes, by logical reasoning, any other power of time. The second supposition is that in absence of impediments, the speed of a body moving horizontally remains constant. The motion of a projectile is a combination of these two motions and results in a parabola. Torricelli does not forget to mention that both Galileo and he 'speak the language of geometry'.

Next, Torricelli deals with the practical applications of the theory. He is well aware of possible sources of error:

... when experiments appear discordant, it will be worth speculating what could be the cause of the discordance. Many of these causes which can make experiment disagree with theory have been noticed by Galileo in his book on motion. The main one, however, is air resistance which resists any kind of motion but especially when the body is faster; and on the whole it will resist very strongly when the projectile is thrust by the supernatural fury of fire which is the highest of all our natural and artificial speeds. No wonder, then, if experiments, and mainly those made with fire machines, turn out different from theory. Our supposition is that the horizontal impetus remains unvaried; experience, however, shows that the horizontal impetus near the muzzle of machines is four or six times greater than at the end of the shot.²²

Nevertheless, Torricelli admits that, even accounting for due errors, Renieri's results are too far from what would be expected in theory. He conjectures that the discrepancy may have been caused by three factors. (1) The gun had an imperceptible elevation. (2) The plane of fire was not horizontal, i.e. the gun's site and the landing site were not at the same height. (3) At the moment of shooting, the gun tilted up, thus lengthening the shot.

Torricelli proposes to eliminate the sources of error by the following precautions. (1) To level the gun with several squares; a slight imprecision of a square may, in fact, alter the range considerably. (2) To shoot on the sea shore, or on a plain, so as to be sure that

²¹ Ibid., 391:

... quando io scrissi quel libretto del moto, di voler sostener le cose che io affermavo in esso, se non *ex hypothesi*, cioè contro coloro i quali mi concederanno per vere quelle due famosissime supposizioni: che le discese del grave in tempi eguali siano *ut numeri impares ab unitate*; e che gli spazi passati orizzontalmente in tempi eguali siano eguali tra di loro.

²² Ibid., 391-92:

... quando le esperienze appariranno discordanti, converrà specolare da che cagione possa nascere la discordanza. Molte di queste cause, le quali possono far discordare l'esperienza dalla dimostrazione, furono avvertite dal Galileo nel suo libro del moto. La principalissima però è l'impedimento dell'aria, la quale resiste ad ogni sorte di moto, ma molto più quando il mobile sarà più veloce; et in somma moltissimo quando il proietto verrà cacciato dalla furia soprannaturale del fuoco, che è la massima di tutte le nostre velocità e naturali et artificiali. Non è però meraviglia se l'esperienze, particolarmente quelle che si fanno con macchine da fuoco, riescano diverse dalla dimostrazione. Il supposto nostro è che l'impeto orizzontale si mantenga sempre il medesimo; la pratica però dimostra che l'impeto orizzontale presso alla bocca delle machine è quattro e sei volte maggiore che presso alla fine del tiro.

the gun's site and the landing site are the same height. (3) Not knowing how to prevent the tilting of the gun, Torricelli suggests assuring that the shot is indeed at point blank by the following arrangements. (a) Placing in front of the gun a square frame, made of four canes of two or three ells each, with paper stretched over it. The frame should be situated so that the ball goes through it, so as to permit tracing the trajectory. (b) Dropping a cannon ball from the same height as the muzzle to see whether it touches the ground simultaneously with the projectile.

Torricelli gives the impression of having a very subtle experimental skill. In reality it is evident that he never experimented with guns. Had he done so he would not have bothered to send such a detailed answer to Renieri, but would merely have stated that guns are imperfect instruments. As A. Rupert Hall rightly points out, 'The standard of engineering technology was not merely insufficient to make scientific gunnery possible, it deprived ballistics of all experimental foundation and almost of the status of an applied science, since there was no technique to which it could, in fact, be applied.'²³

There were many factors that rendered seventeenth-century guns imperfect. To mention a few, the gun barrels were irregular (grooved barrels had not yet been invented) and their shape could easily be altered by heat. The trunnions were not always symmetrical with respect to the axis of the piece, hence altering the angle of elevation. The elevation could also be altered, as Torricelli noticed, by the tilting or vibrations of the gun at the moment of shooting. The quantity of gunpowder used for each shot was not measured accurately, hence the thrust given to the ball, and as a consequence its initial velocity varied from shot to shot. The balls were not uniform in their shape or in their density, hence the effect of air resistance differed from one shot to another. Lastly, atmospheric factors, like air pressure, which greatly influence the trajectories of projectiles, were not taken into account. Seventeenth-century science was still not able to account for all these factors, and the most important role in shooting was still played by the individual experience and judgement of the gunner.

Toricelli's experimental instructions are, to a certain degree, worthless. In spite of this, Renieri did not spare efforts to carry them out. In a second letter, written on 24 August,²⁴ he described two sets of experiments he performed with a 3.5 palms high falconet, having a point blank range of 700 palms. Renieri placed three frames at distances of 100, 300 and 500 palms from the falconet. In theory the ball should have crossed the frames at heights of about 3.4, 2.9 and 1.7 palms respectively.²⁵ In practice it struck at 1.5, 1 and a little below 1 palm respectively. As the diagram of this experiment shows (Figure 6), the trajectory was not only not parabolic, it was partially concave to the horizon! Renieri then shot with an elevation angle of one minute, equivalent to 0.625 degrees (a minute being 1/12 of a point) and got a range of 1800 palms, instead of about 1530 palms as expected.

Torricelli replied with a letter written at the beginning of September.²⁶ He repeats the hypothetical principles of ballistics and emphasizes that to avoid controversies in his book on motion, he intentionally and clearly stated repeatedly that he was

²³ Hall (footnote 3), 16.

²⁴ Letter of Giovanni Battista Renieri to Evangelista Torricelli (in Florence), Genoa, 24 August 1647, *Carteggio* (footnote 1), 399–400.

²⁵ Renieri was measuring the fall of the ball from the height of the falconet (3.5 palms), and his results were 2, $2\frac{1}{2}$ and a little over $2\frac{1}{2}$ palms, i.e. 1.5, 1 and a little below 1 palm from the ground.

²⁶ Letter of Evangelista Torricelli to Giovanni Battista Renieri (in Genoa), Florence, beginning of September 1647, *Carteggio* (footnote 1), 405–7.

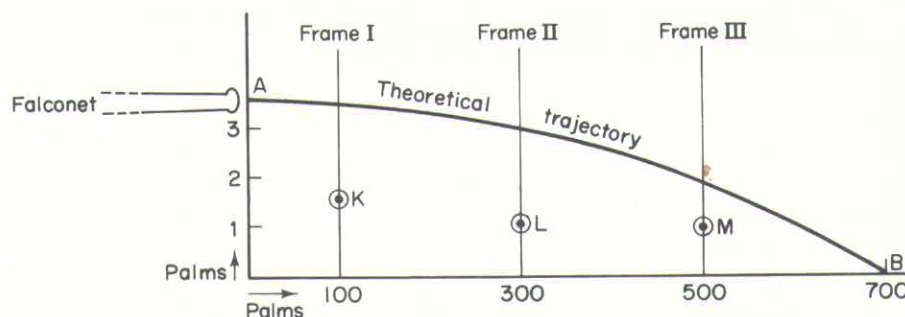


Figure 6.

Renieri's experiment: three frames, with paper stretched over them, were placed at distances of 100, 300 and 500 palms from the falconet. AB is the theoretical parabolic trajectory, and K, L, M are the points where the ball struck the paper.

addressing philosophers rather than gunners. As to Renieri's results, he illustrates that a concave trajectory is absurd.

From his theoretical point of view Torricelli may have been right. In practice, a trajectory like the one Renieri got is not impossible, especially if the cannon ball is uneven, a factor which Torricelli did not take into account. The exchange between Renieri and Torricelli stopped after this letter; the rainy season was going to prevent Renieri from carrying on with his experiments, and Torricelli died in October.

The Torricelli–Renieri correspondence is an occasion where the two main traditions of culture lying at the basis of the Scientific Revolution—that of natural philosophers and that of Renaissance artisans—interact. I do not intend to discuss this complex and little known interaction, especially on such a restricted basis.²⁷ Yet from the correspondence, the 'philosopher' (Torricelli) and the artisans (Renieri and his gunners), in spite of their goodwill, appear to have difficulty in holding a dialogue. This may hint that the new science had its roots, after all, in the traditional philosophical culture, although it was much encouraged by the technical revolution. We still know too little on the subject to be able to draw any conclusion. Moreover, Torricelli was a first rate scientist and can hardly represent all the scientists and natural philosophers of his age. Others, in his place, might have given different answers.

Torricelli's replies to Renieri are interesting, however, not only from their scientific and technical aspects, but also because they contain some of the main epistemological problems which characterized the Scientific Revolution in general and Galileo's and Torricelli's work in particular: the gradual assimilation of mathematics and physics.

What does Torricelli mean when he says in his first letter to Renieri, 'I never intended to make any assertion which is not *ex hypothesi*, i.e. in contrast to those who take for true those very well known two suppositions ...'? This seemingly unclear sentence could be explained by a very indicative passage in a letter Torricelli wrote a

²⁷ On this subject see Paolo Rossi, *I filosofi e le macchine (1400–1700)* (Milan, 1962, 1971), translated into English by Salvator Attanasio with the title *Philosophy, Technology and the Arts in the Early Modern Era* (New York, 1970).

year earlier to Michelangelo Ricci. Speaking of Galileo's two principles of motion, Torricelli says:

I do not care whether the principles of *De motu* are true or false. For if they are not true, let us feign they are true, as we have assumed, and then look at other speculations derived from these principles, not as mixed but as purely geometrical. I feign or suppose that a body or a point moves downwards or upwards with the known proportion and horizontally with uniform motion. In any case, I say that it will conform with what Galileo and I, after him, have said. If then, lead or iron or stone balls do not comply with these suppositions, too bad: we shall disregard them [the experiments].²⁸

Torricelli touches on a very fundamental question concerning the Scientific Revolution: to what extent can mathematics be applied to physics? His position is ambivalent. He appears at first sight to display an instrumentalist approach to science, that is, to hold that mathematics cannot describe physical reality. Nevertheless, some contradictions appear even in Torricelli's first letter, in which he shows by logical reasoning that only the Galilean law of free fall could possibly be true; he seems to be less instrumentalist than he claimed. In spite of what he says very explicitly, it is hard to believe that he thought science should be entirely detached from reality. As A. R. Hall points out, Torricelli's book has all the apparatus of reality. 'It is pardonable for the reader' says Hall, 'to suppose that when he talked of guns he meant real guns, that when printing tables giving measurements in paces he was not merely computing lines on a diagram, that when he designed a gunner's quadrant it was not to be used in the geometrical barrels of hypothetical guns.'²⁹ Furthermore, in his 'De motu', as long as Torricelli deals with the theory of ballistics he writes in Latin, the language of philosophers. But when he turns to applied ballistics he writes in Italian so that uncultured gunners may understand him.³⁰

Why then did Torricelli insist so much on presenting Galilean science as a hypothetical science? In my opinion Torricelli was compelled to do so by his position as Court Mathematician. Both he and his patrons, the Medici, could not disregard the fact that although Galileo was officially condemned for having held that the Copernican system gave a true and not merely a hypothetical description of the world, his condemnation had more far-reaching implications. It was a warning to all men of science that the Church would not tolerate the claim for truth of any hypothesis, whether a mere conjecture or a mathematical model, which contradicted its official doctrines. Hence, if a scientist or his patron wished to keep out of trouble he should

²⁸ Letter of Evangelista Torricelli to Michelangelo Ricci (in Rome), Florence, 10 February 1646, *Carteggio*, 276-77:

Che i principii della dottrina *de motu* siano veri o falsi a me importa pochissimo. Poiché, se non son veri, fingasi che sian veri conforme habbiamo supposto e poi prendansi tutte le altre specolazioni derivate da essi principii, non come cose miste, ma pure geometriche. Io fingo o suppongo che qualche corpo o punto si muova all'ingiu et all'insu con la nota proporzione et horizontalmente con moto equabile. Quando questo sia, io dico che seguirà tutto quello che ha detto il Galileo et io ancora. Se poi le palle di piombo, di ferro, di pietra non osservano quella supposta proporzione, suo danno: noi diremo che non parliamo di esse.

²⁹ Hall (footnote 3), 98-99.

³⁰ See *OT* (footnote 1), II, 209ff.

avoid applying mathematics to physical reality.³¹ Paolo Galluzzi has recently noticed³² that on the front page of Torricelli's *Opera geometrica*, Torricelli is not presented as Court Mathematician and Philosopher (the title held by Galileo) but merely as the Grand Duke's Mathematician ('Serenissimi Magni Ducis Mathematico'). This may lead one to think (as Galluzzi hints) that Grand Duke Ferdinand II appointed him Court Mathematician only, to meet the instrumentalist demands of the Church, and that the idea behind this demotion was to separate mathematics as far as possible from philosophy and hence physics. Torricelli for his part was certainly cautious. He concentrated on mathematics and did not write anything on astronomy. He did not even publish the result of the quicksilver experiment in spite of its importance, most probably because the existence of a vacuum was contrary to Aristotle, limiting himself to describing the experiment in a letter to his friend Michelangelo Ricci.³³ The cautious phrasing of his replies to Renieri, his emphasizing that he speaks *ex hypothesi*, that is, that Galileo's laws of motion are only mathematical speculation which cannot 'dictate' the truth, are to be understood in the same way.

The Torricelli–Renieri correspondence thus contains the core of Galileo's contribution to science: the mathematization of nature. It is surprising, therefore, that it has been given so little attention by historians of science.

Acknowledgment

I am indebted to Professor Eric Mendoza for the careful reading of an early draft and for his useful comments.

³¹ No systematic research has been made on the restrictions imposed on mathematical sciences after Galileo's trial. So far, the most comprehensive account of science in Tuscany after Galileo's trial is by Giovanni Targioni Tozzetti, *Notizie degli aggrandimenti delle scienze fisiche, accaduti in Toscana nel corso degli anni LX. del secolo XVII*, 3 vols in 4 (Florence 1780). Other, more recent works which deal with this subject are *La scuola galileiana. Prospettive di ricerca* (footnote 3); Paolo Galluzzi's 'Libertà scientifica, educazione e ragion di stato in una polemica universitaria pisana del 1670', *Atti del XXIV Congresso Nazionale di Filosofia (L'Aquila 28 aprile–2 maggio 1973)* (Rome, 1974), II, pt 2, 404–12, and his 'Evangelista Torricelli. Concezione della matematica a segreto degli occhiali' (footnote 1); Maurizio Torrini, *Dopo Galileo: una polemica scientifica (1684–1711)* (Florence, 1979).

³² See Galluzzi's 'Vecchie e nuove prospettive torricelliane' (footnote 3), 46.

³³ (Footnote 1.) Torricelli's letter was published for the first time only in 1663 by Carlo Dati (1619–79), a member of the Accademia del Cimento, under the pseudonym of Timauro Antiato, in a work entitled *Lettera ai Filaleti di Timauro Antiato della vera storia della cicloide, e della famosissima esperienza dell'argento vivo* (Florence, 1663), *OT*, I, pt. 2, 441–82.