Evaluation of post-Einsteinian gravitational theories through parameterized post-Newtonian formalism

Nicolae Sfetcu

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Email: nicolae@sfetcu.com

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Proliferation of post-Einsteinian gravitational theories

Right after the elaboration and success of general relativity (GR), alternative theories for gravity began to appear, which can fall into four broad categories: [1]

- Bifurcated theories (with the Lakatosian[2] hard core identical or very similar to that of general relativity) or directly related to general relativity but not bifurcated, such as Cartan, Brans-Dicke and Rosen bimetric theories.
- Unifying theories that try to unify quantum mechanics with general relativity (theories of quantum gravity), such as loop quantum gravity.
- Unifying theories that try to unify gravity with other forces, such as Kaluza-Klein.
- Unifying theories that try to unify several theories simultaneously, such as the M theory.

In developing these theories, many different strategies (positive heuristics) have been tried, by adding new hypotheses to GR, using a spacetime for which the universe is static, hypotheses that eliminate gravitational singularities, etc. In this competition, so far has won the Einstein's GR theory, proving by far a greater heuristic power than his rivals. Some of these theories have been abandoned, others are still being developed by various communities of researchers, trying to eliminate the anomalies found in GR, or to expand GR by bifurcation or as unifying theories.

After 1980, when the scientific community agreed that GR is confirmed, generally, only theories that include GR as a particular case have survived. Particular attention began to be paid to theories of quantum gravity, in particular string theory. Most of the newer non-quantum gravity theories try to solve various cosmological anomalies, such as cosmic inflation, dark matter, dark energy, and so on. The proliferation of GR anomalies lately, including in the Pioneer case, has led to a revival of alternatives to this theory.

Most of the theories in the first category listed above include a Lagrangian density, an "action" (which guarantees the existence of conservation laws, and whose gravitational component is deduced from the Lagrangian density by integration) [3], and a metric.

Metric theories can be classified into (from the simplest to the most complex):

- Theories using scalar fields (including conformally flat theories and stratified theories with conformally flat space slices)
- Quasilinear theories (including linear fixed gauge)
- Tensor theories
- Scalar-tensor theories

[1] Timothy Clifton et al., "Modified Gravity and Cosmology," *Physics Reports* 513, no. 1–3 (March 2012): 1–189, https://doi.org/10.1016/j.physrep.2012.01.001.

[2] Imre Lakatos, *The Methodology of Scientific Research Programmes: Volume 1: Philosophical Papers* (Cambridge University Press, 1980).

[3] Franz Mandl and Graham Shaw, *Lagrangian Field Theory, in Quantum Field Theory* (John Wiley & Sons, 2013), 25–38.

- Vector-tensor theories
- Bimetric theories
- Other metric theories

More important *non-metric theories* include

- Belinfante-Swihart
- Einstein-Cartan theory
- Kustaanheimo
- Teleparallelism
- Gravity based on gauge theory

Some of these theories are based on Mach's principle (the frame of reference comes from the distribution of matter in the universe[4], considered to be an intermediary between Newton (absolute space and time) and Einstein (there is no absolute frame of reference). Experimental evidence shows that the Mach's principle is wrong, but the related theories were not entirely excluded.

In order to verify and classify all these theories, specific tests have been developed, based on self-consistency (among the non-metric theories includes the elimination of theories that allow tachyons, ghost poles and higher order poles, and those that have problems with the behavior at infinite), and on completeness (to allow the analysis of the result of each experiment of interest). For example, any theory that cannot predict from the first principles the motion of the planets or the behavior of atomic clocks is considered incomplete.

Three *tests* are considered "classics" for the ability of gravity theories to manage relativistic effects:

- gravitational redshift
- gravitational lenses (around the Sun)
- abnormal advance of the perihelion of the planets.

To these tests was added, in 1964, the fourth test, called the Shapiro delay. Each theory should confirm these tests.

The *Einstein equivalence principle* (EEP), which is also tested for relativistic theories of gravity, has three components:

- uniqueness of the free fall (weak equivalence principle): the inertial mass is equal to the gravitational mass;
- Lorentz invariance: in the absence of gravitational effects, the speed of light is constant;
- local position invariance: the result of any local non-gravitational experiment is independent of where and when it is performed.

[4] Alfred North Whitehead, *The Principle Of Relativity With Applications To Physical Science* (Whitefish, Mont.: Kessinger Publishing, LLC, 2008).

Schiff's conjecture states that any complete, self-consistent theory of gravity that checks the principle of weak equivalence necessarily also checks the principle of Einstein's equivalence (if the theory has a complete conservation of energy).

Metric theories satisfy the EEP. Only some non-metric theories satisfy the EEP.

The main general non-quantum post-relativity theories are Brans-Dicke theory, Fifth force, and Geometrodynamics.

Brans-Dicke theory is a scalar-tensor theory, in which gravitational interaction is mediated by a scalar field, and by the tensor field of general relativity[5]. The theory is considered to be in general agreement with the observations. The source of the gravitational field is, as in RG, the stress-energy tensor or the matter tensor. In Brans-Dicke theory, in addition to metrics (a rank two tensor field), there is a scalar field that changes the actual gravitational constant depending on the location (this is a key feature of the theory, being part of the Lakatosian hard core). The Brans-Dick theory, compared to GR, admits several solutions. It predicts the deflection of light and the precession of the perihelion of the planets, and general relativity can be derived from the Brans-Dicke theory as a particular case, but Faraoni argues that this is not valid in all situations allowed by the theory[6], and some physicists claim that it does not meet the powerful principle of equivalence.

The fifth force is a theory that involves, in addition to gravitational, electromagnetic, strong nuclear and weak nuclear forces, a fifth force to explain various anomalous observations that do not match existing theories. One hypothesis of this theory is that dark matter could be related to an unknown fundamental force. Others speculate that a form of dark energy called quintessence might be a fifth force[7]. Such a new, weak, fundamental force is difficult to test. In the late 1980s, some researchers[8] reported that they discovered this force while re-analyzing Loránd Eötvös results from the turn of the century, but other experiments failed to replicate this result. One of the tests that can be undertaken to prove the theory is supposed to be based on the strong principle of equivalence (the fifth force would manifest through an effect on the orbits of the solar system, called the Nordtvedt effect) with Lunar Laser Ranging and very long baseline interferometry. Other tests may consider additional dimensions; the mantle of the earth as a giant particle detector,

[5] C. Brans and R. H. Dicke, "Mach's Principle and a Relativistic Theory of Gravitation," *Physical Review* 124, no. 3 (November 1, 1961): 925–935, https://doi.org/10.1103/PhysRev.124.925.

[6] Valerio Faraoni, "Illusions of General Relativity in Brans-Dicke Gravity," *Physical Review D* 59, no. 8 (March 22, 1999): 084021, https://doi.org/10.1103/PhysRevD.59.084021.

[7] Michele Cicoli, Francisco G. Pedro, and Gianmassimo Tasinato, "Natural Quintessence in String Theory," *Journal of Cosmology and Astroparticle Physics* 2012, no. 07 (July 23, 2012): 44, https://doi.org/10.1088/1475-7516/2012/07/044.

[8] E. Fischbach et al., "Reanalysis of the Eotvos Experiment," *Physical Review Letters* 56 (January 1, 1986): 3–6, https://doi.org/10.1103/PhysRevLett.56.3.

focusing on geoelectrons[9]; pulsation rate of cepheid variable stars in 25 galaxies[10]; and so on. Various additional hypotheses have been proposed in recent years to strengthen the theory, but no results have been conclusive so far.

Geometrodynamics is an attempt to describe spacetime and associated phenomena in terms of geometry. This is a unifying theory, trying to unify the fundamental forces and reformulate the general relativity. It's a theory initiated by Einstein but still active. In a way, the term geometrodynamics is synonymous with general relativity, in which case it is more precisely referred to as Einstein's geometrodynamics to denote the initial formulation of the value of general relativity. John Wheeler promoted this theory in the 1960s, trying to reduce physics to geometry in a fundamental way, with a dynamic geometry with a variable curve over time. Basically, Wheeler tried to integrate three concepts: mass without mass, charge without charge, field without field. [11]

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9 Jacob Aron, "Earth's Mantle Helps Hunt for Fifth Force of Nature," New Scientist, 2013, https://www.newscientist.com/article/dn23202-earths-mantle-helps-hunt-for-fifth-force-of-nature/.

10 Bhuvnesh Jain, Vinu Vikram, and Jeremy Sakstein, "Astrophysical Tests of Modified Gravity: Constraints from Distance Indicators in the Nearby Universe," *The Astrophysical Journal* 779, no. 1 (November 25, 2013): 39, https://doi.org/10.1088/0004-637X/779/1/39.

11 John A. Wheeler, "On the Nature of Quantum Geometrodynamics," *Annals of Physics* 2, no. 6 (December 1, 1957): 604–14, https://doi.org/10.1016/0003-4916(57)90050-7.

Post-Newtonian parameterized formalism (PPN)

In the field of experimental gravity, one of the important applications is formalism. For the evaluation of gravity models, several sets of tests have been proposed. Post-Newtonian formalism considers approximations of Einstein's gravity equations by the lowest order deviations from Newton's law for weak fields. Higher terms can be added to increase accuracy. At the limit, post-Newtonian expansion is reduced to Newton's law of gravity. The post-Newtonian parametric formalism (PPN) details the parameters that differentiate the theories of gravity, in the weak gravitational field and speeds much lower than the speed of light. PPN can be applied to all gravity metric theories in which all bodies satisfy the Einstein equivalence principle (EEP). The speed of light remains constant in the PPN formalism and it is assumed that the metric tensor is always symmetrical. PPN is used to compare and classify alternative metrics of gravity. With the help of this formalism, many theories previously considered viable have been eliminated.

In *beta-delta notation*, the behavior of the weak gravitational field in general relativity is completely characterized by ten post-Newtonian parameters. In the notation of Will's[12], Misner et al. [13], they have the following values:

- γ: g_{ij} space curvature produced by unit rest mass
- β: nonlinearity in superposition law for gravity g_{00}
- β_1: gravity produced by unit kinetic energy $\rho_0 v^2/2$
- β_2: gravity produced by unit gravitational potential energy ρ_0/U
- β_3: gravity produced by unit internal energy $\rho_0 \Pi$
- β_4: gravity produced by unit pressure p
- ζ: difference between radial and transverse kinetic energy on gravity
- η: difference between radial and transverse stresses on gravity
- Δ_1: dragging of inertial frames g_{0j} produced by unit momentum $\rho_0 v$
- Δ_2: difference between radial and transverse momentum on dragging of inertial frames

Here, $g_{\mu\nu}$ is the symmetrical 4x4 metric tensor with indices μ and ν taking values between 0 and 3. An index 0 will indicate the time direction and the indices i and j with values from 1 to 3 will indicate the spatial directions. In general relativity, the values of these parameters are chosen so that within the limits of velocity and small mass they coincide with Newton's law of gravity, to ensure the conservation of energy, mass, momentum and angular momentum; and to ensure the independence of the equations from the frame of reference.

For general relativity, $\gamma = \beta = \beta 1 = \beta 2 = \beta 3 = \beta 4 = \Delta 1 = \Delta 2 = 1$ and $\zeta = \eta = 0$.

[12] C. M. Will, "Theoretical Frameworks For Testing Relativistic Gravity. Ii. Parametrized Post-Newtonian Hydrodynamics, And The Nordtvedt Effect.," *Astrophys. J. 163: 611-28(1 Feb 1971).*, January 1, 1971, 163, 611–28, https://doi.org/10.1086/150804.

[13] Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler, *Gravitation* (W. H. Freeman, 1973).

In the most recent *alpha-zeta notation* of Will & Nordtvedt[14] and Will[15], a different set of ten PPN parameters is used:

- $\gamma = \gamma$
- $\beta = \beta$
- $\alpha_1 = 7\Delta_1 + \Delta_2 - 4\gamma - 4$
- $\alpha_2 = \Delta_2 + \zeta - 1$
- $\alpha_3 = 4\beta_1 - 2\gamma - 2 - \zeta$
- $\zeta_1 = \zeta$
- $\zeta_2 = 2\beta + 2\beta 2 - 3\gamma - 1$
- $\zeta_3 = \beta 3 - 1$
- $\zeta_4 = \beta_4 - \gamma$
- ξ is calculated from $3\eta = 12\beta - 3\gamma - 9 + 10\xi - 3\alpha_1 + 2\alpha_2 - 2\zeta_1 - \zeta_2$

Parameters γ and β are used to describe "classical" GR tests and are the most important, the only non-zero parameters in GR and scalar-tensor gravity. Parameter ξ is non-zero in any theory of gravity that predicts the effects of preferred location; α_1, α_2, α_3 measures whether or not the theory predicts effects of the preferred post-Newtonian framework; α_3, ζ_1, ζ_2, ζ_3, ζ_4 measure whether or not the theory predicts violation of global conservation laws for total momentum.

In this notation, general relativity has the parameters PPN $\gamma = \beta = 1$ and $\alpha_1 = \alpha_2 = \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = \xi = 0$

There is a mathematical relationship between the metric, the metric potential, and the PPN parameters for this notation, with ten metric potentials (one for each PPN parameter) to ensure a unique solution. The methodology of applying the PPN formalism to the alternative theories of gravity is a nine-step process[16]. The limits of PPN parameters[17] are determined from experimental tests.

The only gravitational field entering the equations of motion is the metric *g*. Other fields will only contribute to the generation of spacetime curvature associated with the metric. Matter can create

[14] Kenneth Nordtvedt Jr. and Clifford M. Will, "Conservation Laws and Preferred Frames in Relativistic Gravity. II. Experimental Evidence to Rule Out Preferred-Frame Theories of Gravity," *The Astrophysical Journal* 177 (November 1, 1972): 177, 757, https://doi.org/10.1086/151755.

[15] Clifford M. Will, "The Confrontation between General Relativity and Experiment," *Living Reviews in Relativity* 17, no. 1 (December 2014): 4, https://doi.org/10.12942/lrr-2014-4.

[16] Clifford M. Will, *Theory and Experiment in Gravitational Physics, Revised Edition*, Revised edition (Cambridge England ; New York, NY, USA: Cambridge University Press, 1993).

[17] Will, "The Confrontation between General Relativity and Experiment."

these fields, and they together with matter can generate metrics, but they cannot act directly on matter. Matter responds only to the metric[18]. It turns out that the metric and the equations of motion for matter are primary entities for calculating observable effects, and the only distinction between two metric theories is the particular way in which matter and possibly other gravitational fields generate the metric.

At the post-Newtonian limit (slow motion, weak field), comparing a theory with gravitational experiments and theories between them is accurate enough for most tests, especially those involving the solar system. The differences appear in the numerical values of the coefficients in front of the metric potential (parameters in the PPN formalism).

Carlton Morris Caves concluded that laboratory experiments for investigating nonlinear characteristics of the gravitational field, as well as laboratory measurements of gravity produced by internal energy, are difficult and inconclusive[19]. The most accessible laboratory experiments from the point of view of post-Newtonian effects are the effects of preferred frame and preferred orientation (they can be modulated by rotating the entire laboratory apparatus relative to the inertial space) and the magnetic gravity effects (the effects associated with components of metric *g*: dragging inertial frames through rotating bodies, Lens-Thirring gyroscopic precession, gravitational accelerations produced by spin-spin interactions of rotating bodies, and gravitational accelerations due to spin-orbit coupling). Magnetic effects are much more sensitive to the direction of rotation or to the movement of a source or laboratory detector than other laboratory experiments. As a source, a rapidly rotating, symmetrical axial body can be used, and its angular velocity can be slowly modulated. [20]

[18] Will.

[19] Carlton Morris Caves, "Theoretical Investigations of Experimental Gravitation" (phd, California Institute of Technology, 1979), http://resolver.caltech.edu/CaltechTHESIS:03152016-161054898.

[20] Caves.

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