

## MR. MAGOO'S MISTAKE

ASSAF SHARON AND LEVI SPECTRE

### ABSTRACT

Timothy Williamson has famously argued that the (KK) principle (namely, that if one knows that  $p$ , then one knows that one knows that  $p$ ) should be rejected. We analyze Williamson's argument and show that its key premise is ambiguous, and that when it is properly stated this premise no longer supports the argument against (KK). After canvassing possible objections to our argument, we reflect upon some conclusions that suggest significant epistemological ramifications pertaining to the acquisition of knowledge from prior knowledge by deduction.

### 1. AN ARGUMENT AGAINST (KK)

In his *Knowledge and its Limits*,<sup>1</sup> Timothy Williamson introduces a situation he takes to be revealing of the nature of knowledge. The situation involves a subject – Mr. Magoo – who is looking at a somewhat distant tree. By looking, Mr. Magoo can give a good estimate of the tree's height although he cannot tell it to an inch. So, for no natural number  $i$ , does Mr. Magoo know that the tree is  $i$  inches tall. However, since the tree is in fact 666 inches tall (though he does not know this), his ability to judge heights from a distance is good enough to allow him to know that it is not 60 inches tall and that it is not 6000 inches tall. Reflection on his discriminatory and evaluative abilities allows Mr. Magoo to state the following for each relevant natural number  $i$ :

- (1) Mr Magoo knows that if the tree is  $i+1$  inches tall, then he does not know that the tree is not  $i$  inches tall.

By contraposing the imbedded conditional, we get a more intuitive formulation of what we will call ‘Williamson’s Proposition’ (WP),<sup>2</sup> a proposition relating to Mr. Magoo’s knowledge of the tree’s height (we use both formulations in this text).

(WP) Mr Magoo knows that (if he knows the tree is not  $i$  inches tall, then the tree is not  $i+1$  inches tall).<sup>3</sup>

The reason to think (WP) is true is that, knowing the limitations of his own visual discriminatory abilities, Mr. Magoo knows that if the tree is  $i+1$  inches tall he cannot observe that it is not  $i$  inches tall. He therefore knows that, if he can indeed tell that the tree is not  $i$  inches tall, it must be neither shorter nor taller by one inch from the height he knows it not to be. His reflective knowledge regarding his perceptive abilities enables Mr. Magoo to expand his merely perceptual knowledge.

Having supported (WP) for the Magoo story, Williamson proceeds to articulate his argument against (KK), the principle according to which second order knowledge follows from first order knowledge. The argument is to show that (KK), together with (WP), and other very plausible assumptions about Mr. Magoo, lead to a contradiction.

Williamson’s argument proceeds as follows.<sup>4</sup> For *reductio ad absurdum*, we are to assume:

(KK) For any pertinent proposition  $p$ , if Mr Magoo knows  $p$  then he knows that he knows  $p$ .<sup>5</sup>

The idea that knowledge is closed under known entailment (AKA: the principle of epistemic closure) is, according to Williamson, intuitively valid. So Mr. Magoo can assume:<sup>6</sup>

- (C) If  $p$  and all members of the set  $X$  are pertinent propositions,  $p$  is a logical consequence of  $X$ , and Mr Magoo knows each member of  $X$ , then he knows  $p$ .

Williamson also assumes that knowledge is factive (i.e., that if  $S$  knows that  $p$  then  $p$ ),<sup>7</sup> and that Mr. Magoo can reasonably judge and therefore know, for instance, that the tree is not 3 inches tall. He then argues that, from these premises, we can derive that Mr. Magoo knows that the tree is not 666 inches tall (or, for that matter, any other height), even when it actually is. The argument proceeds as follows: If Mr. Magoo knows that the tree is not 3 inches tall (knowledge he can acquire just by looking at the 666 inch tall tree), he also knows that he knows that it is not 3 inches tall (by (KK)). If so, since he knows that if he knows that it is not 3 inches tall it is not 4 tall (by (WP)), he also knows that it is not 4 inches tall (through (C)). He knows, therefore, that the tree is not 4 inches tall and thus also knows (by (KK)) that he knows it is not 4 inches tall. But now he can use this knowledge again to conclude that the tree is not 5 inches tall. By repeating these steps using (WP) and the other premises, Mr. Magoo can deduce from his knowledge, according to Williamson, that he knows that the tree is not 666 inches tall (which it is). Given the factivity of knowledge, we have a contradiction.<sup>8</sup>

Something, therefore, is inconsistent in the Magoo scenario. To regain consistency, Williamson suggests, we must abandon the least plausible of the premises, namely (KK). Once (KK) is rejected, the reiteration of the inferential move is blocked and the contradiction is avoided.<sup>9</sup> It is the aim of this paper to show that (WP) in its present form is ambiguous and that once the ambiguity is resolved, it no longer supports the argument against (KK).

## 2. WILLIAMSON'S PROPOSITION

As we have seen, the Magoo argument clearly hinges on the introduction of (WP) or (1<sub>j</sub>). Let us, therefore, look closer at this proposition. As it stands (WP) cannot be true as a general claim. It is only true for the specific circumstances of the hypothetical tree-observer, Mr. Magoo. It is crucial to note that this restriction is not only a matter of stylistic or argumentative preference. Stating (WP) as a general claim, even as a general claim about Mr. Magoo, unconstrained by the specific conditions of the case as specified by Williamson, leads to the absurd claim that Mr. Magoo can never know the height of a tree. This becomes evident when the following deduction is considered.

Williamson, recall, phrases (1<sub>j</sub>) thus:

- (1) Mr Magoo knows that if the tree is  $i+1$  inches tall, then he does not know that the tree is not  $i$  inches tall.<sup>10</sup>

Let us formalize it as follows:<sup>11</sup>

- (1)  $K_M(t_{i+1} \supset \sim K_M \sim t_i)$       ( $'K_M'$  denotes *Mr. Magoo knows that* and  $'t_i'$  denotes *the tree is  $i$  inches tall*).

Since knowledge is factive, from (1), we can deduce:

- (2)  $t_{i+1} \supset \sim K_M \sim t_i$       (If the tree is  $i+1$  inches tall, then Mr. Magoo does not know that it is not  $i$  inches tall.)

The following is an epistemic truism in connection with tree heights:<sup>12</sup>

- (3)  $K_M t_{i+1} \supset K_M \sim t_i$       (If Mr. Magoo knows that the tree is  $i+1$  inches tall, then he knows that it is not  $i$  inches tall).

Contraposing (3) we get:

(4)  $\sim K_M \sim t_i \supset \sim K_M t_{i+1}$  (If it is not the case that Mr. Magoo knows that the tree is not  $i$  inches tall, then it is not the case that Mr. Magoo knows that the tree is  $i+1$  inches tall).

From (2) and (4) we deduce, by *Hypothetical Syllogism*:

(5)  $t_{i+1} \supset \sim K_M t_{i+1}$  (If the tree is  $i+1$  inches tall, then it is not the case that Mr. Magoo knows that the tree is  $i+1$  inches tall).

Contraposing this we get:

(6)  $K_M t_{i+1} \supset \sim t_{i+1}$

This contradicts the following instance of the factivity principle:

(7)  $K_M t_{i+1} \supset t_{i+1}$

From (6) and (7) it follows that:

(8)  $\sim K_M t_{i+1}$

So, simply put, if the proposition applies with no restrictions, Mr. Magoo never knows the height of the tree. Clearly an unacceptable result.

This simple argument shows that (WP) cannot be true generally. It relates only to Mr. Magoo as he is looking at the tree and has no other source of knowledge about its height. Had his knowledge been based on an actual measurement of the tree, for example, the conditional codified in (WP) would not be true. He could then know that the tree is not  $i$  inches tall, but it would not follow that the tree is not  $\neq 1$  inches. This much is clear from the way Williamson sets up the argument. But there's more. For Mr. Magoo to go through with the inferences, he must know that he knows by looking and has no other source of information. To see why, just imagine that this is not the case. Say Mr. Magoo knows that the tree is not 50 inches tall because he was so told by his reliable

friend, Ms. Magee. And imagine that he does not know how she knows this. Clearly, in this case, since he does not know *how she knows*, he cannot infer that the tree is not 51 inches tall, even though he knows it is not 50. The point is that the proposition is applicable only to cases in which Mr. Magoo knows that he has no other sources of knowledge and that the ones he does have do not allow him to know the precise height of the tree.

This formal observation is also supported by the rationale underlying (WP), i.e. the reason we are to accept that Mr. Magoo knows the quantified conditional proposition. According to Williamson:

*Looking* out of his window, Mr Magoo can see a tree some distance off. He wonders how tall it is. Evidentially, he cannot tell to the nearest inch *just by looking*. His eyesight and ability to judge heights are nothing like that good. Since *he has no other source of relevant information* at the time, he does not know how tall the tree is to the nearest inch. (Williamson, 114, our italics)<sup>13</sup>

To know that the tree is  $i$  inches tall, Mr Magoo would have to judge that it is  $i$  inches tall; but even if he so judges and in fact it is  $i$  inches tall, he is merely guessing; for all he knows it is really  $i-1$  or  $i+1$  inches tall. He does not know that it is not. Equally, if the tree is  $i-1$  or  $i+1$  inches tall, he does not know that it is not  $i$  inches tall. Anyone who can tell *by looking* that the tree is not  $i$  inches tall, when in fact it is  $i+1$  inches tall, has much better eyesight and a much greater ability to judge heights than Mr Magoo has. (Williamson, 115, our italics)

Note that it is in virtue of the fact that he can know *just by looking* that the tree is not  $i$  inches tall that Mr. Magoo knows it is not  $i+1$  nor  $i-1$  inches tall. It would be wrong for Mr. Magoo to apply (WP) in cases where he has just measured the tree, for instance, or in cases where he has some other form of evidence regarding the height of the tree. (WP) relates only to knowledge of the tree's height that is actually available to him in the specific conditions of the case. In the next section we state (WP) so as to explicitly reflect these limitations and check if, so stated, it maintains its force via Mr. Magoo's argument.

### 3. MR. MAGOO'S ARGUMENT

The gist of the argument, as we have seen, is the following: Since obviously Mr. Magoo can know that the tree is not 2 inches tall, and since (C) and (WP) are intuitive and justified (respectively), what we must opt for is the rejection of (KK) as an epistemic principle. Let us now examine the argument with the restrictions on (WP) identified in the previous section in mind.

When Mr. Magoo first looks at the tree, he can tell just by looking that it is not  $i$  inches tall. He then applies (WP) and, together with other epistemic principles ((KK) and (C)), concludes that the tree is not  $i+1$  inches tall. Note, however, that Mr. Magoo has not acquired this latter piece of knowledge just by looking at the tree. From his observational knowledge that the tree is not  $i$  inches tall, he *inferred* by (WP) and the other principles that it is not  $i+1$  inches tall. Now remember, the full and precise proposition Mr. Magoo holds is:

if I can know *just by looking* at the tree that it is not  $i$  inches tall, it must not be  $i+1$  (nor  $i-1$ ) inches tall either.

Since his knowledge that the tree is not  $i+1$  inches tall is not something he knows just by looking but rather also *by inference* from what he knows just by looking, the proposition cannot be used for further inferences. The iteration is not possible since Mr. Magoo's knowledge relates to, and is justified by, limitations of his visual discriminatory abilities and, therefore, pertains only to what he knows just by looking. In other words, if Mr. Magoo can tell by looking that the tree is not 3 inches tall, he knows well enough that it is not 4 inches tall (or he *can* know it if he takes the time to figure it out). However, with the tools that Williamson gives him, he cannot know that the tree is not 5 inches tall since his knowledge that the tree is not 4 inches tall is not based merely on mere looking.

The point can also be made this way: Although (WP) is true for all  $i$  (that is, for all  $i$ , if Mr. Magoo can know *directly by looking at the tree* that it is not that height in inches, then it is not within an inch of that height either), it does not follow that, for all  $i$ , if Mr. Magoo knows that the tree is not  $i$  inches tall, then the tree is not  $i \pm 1$  inches tall either. This inference has not been shown to be valid and, as we shall see, it should not convince those who want to hold on to (KK). (WP) does not pertain to deductive knowledge, nor to knowledge by measurement, reliable procedure, self-reflection, clairvoyance, testimony, and so forth.<sup>14</sup> It pertains solely to knowledge suffering from Mr. Magoo's perceptual limitations. Mr. Magoo does not know that the tree is not 5 inches tall if he does not know *by looking* that it is not 4 inches tall. This important detail gets lost in Williamson's formulation:

- (1<sub>*i*</sub>) Mr Magoo knows that if the tree is  $i+1$  inches tall, then he does not know that the tree is not  $i$  inches tall.

(Williamson, 115)

As we have seen, if (1<sub>*i*</sub>) is a general non-restricted claim, it is clearly false. Imagine that the tree is 101 inches tall and Mr. Magoo measures it. He then certainly knows that the tree is not 100 inches tall. (1<sub>*i*</sub>) should, therefore, be amended in the following way:

- (1<sub>*i*</sub>-KK) Mr Magoo knows that, if the tree is  $i+1$  inches tall, then he does not know *just by looking* at the tree that it is not  $i$  inches tall.<sup>15</sup>

In other words, returning to (WP), Mr. Magoo knows the tree is not  $i+1$  inches tall since he can tell, *by looking*, that it is not  $i$  inches tall. But this is as far as he can go with the tools he has been given by Williamson. Mr. Magoo knows that the tree is not 4



inches tall since he knows *just by looking* that it is not 3 inches. But this knowledge is constituted by inference and is therefore not suitable as an antecedent in (WP). He may also know by mere looking that it is not 4 inches tall, and then, go on to infer that it is not 5 inches tall. But again, if this latter piece of information is not itself known by Mr. Magoo on the basis of looking alone, it *cannot* be further used as an antecedent in (WP). The consequences of Mr. Magoo's argument are therefore avoided. An advocate of the (KK) principle can hold on to his favored principle by stating the premises more precisely. Attending to the modes of knowledge and deducing accordingly, one can make proper inferences without reaching contradiction. Let us formalize the proposition so that its application is now appropriately restricted by modifying the knowledge operators:

$$(WP-KK) K_M^{reflection} (K_M^{looking} \sim t_i \supset \sim t_{i+1})$$

Now think again of the Magoo argument. In our version of the story, Mr. Magoo knows by looking that the tree is not 3 inches tall. We therefore have:

$$(9) K_M^{Looking} \sim t_3 \quad (\text{Mr. Magoo knows by looking that the tree is not 3 inches tall}).$$

$$(10) K_S p \ \& \ K_S (p \supset q) \ \therefore \ K_S q \quad (\text{If } S \text{ knows that } p \text{ and knows that if } p \text{ then } q \text{ it follows that } S \text{ knows that } q).^{16}$$

Assume (KK) for *reductio*:

$$(11) K_S p \supset K_S K_S p \quad (\text{If } S \text{ knows that } p, \text{ then } S \text{ knows that } S \text{ knows that } p).$$

Now let us introduce the new and improved (WP-KK):

$$(12) K_M^{reflection} (K_M^{looking} \sim t_i \supset \sim t_{i+1}) \quad (\text{Mr. Magoo knows by reflection that if he knows just by looking that the tree is not } i \text{ inches tall,}$$

then it is not  $i+1$  inches tall).

From 9-12 we may infer 13 (notice that (10) and (11) are not restricted here, these principles are applicable (supposedly) to all forms of knowledge):

(13)  $K_M K_M^{Looking} \sim t_3$  (Mr. Magoo knows that he knows by looking, that the tree is not 3 inches tall).<sup>17</sup>

By (12) and (9) we can get:

(14)  $K_M^{reflection} (K_M^{looking} \sim t_3 \supset \sim t_4)$  (Mr. Magoo knows by reflection that if he knows by looking that the tree is not 3 inches tall then the tree is not 4 inches tall).

Applying (10) to (13) and (14) we can infer:

(15)  $K_M^{reflection\&looking} \sim t_4$  (Mr. Magoo knows by looking and inferring that the tree is not 4 inches tall).

From (15), and (11) we may infer that:

(16)  $K_M K_M^{reflection\&looking} \sim t_4$  (Mr. Magoo knows that he knows that the tree is not 4 inches tall).

Notice that (16) is a result of reflection, perception and inference via epistemic principles, namely, (KK) and (C). For the argument to continue, the knowledge described in (16) must be posited in (WP-KK) so as to construct the second conjunct of (10). Although ' $K_M K_M^{reflection\&looking} \sim t_4$ ' can still serve as the first conjunct in (10), it cannot be posited in (WP-KK) since the antecedent in this conditional only accommodates knowledge by mere looking. In other words, the conclusion of the deductive move via (WP-KK) – for the very reason that it is the result of a deductive process – is not fit to be posited in (WP-KK) in order to get the desired (17):

(17)  $K_M (K_M \sim t_4 \supset \sim t_5)$  (Mr. Magoo knows that if he knows that the tree is not 4 inches tall, then the tree is not 5 inches tall).

Without (17), we cannot infer from (9) that the tree is not 5 inches tall. For this Mr. Magoo would have to look at the tree and know on *that* basis that the tree is not 4 inches tall. The premises of Mr. Magoo's argument do not carry him any further than one deductive sequence, i.e. to the conclusion that the tree is not 4 inches tall. This is a reasonable consequence leading to no contradiction. So, inasmuch as Williamson's argument was standing in their way, it seems that with the correct modifications of the knowledge operators advocates of (KK) can safely iterate their knowledge and live happy epistemic higher order lives.

#### 4. ANOTHER CANDIDATE FOR THE JOB?

The argument thus far conclusively shows that (WP) does not support an argument leading to the rejection of (KK). Some may suggest, however, that a similar proposition may do the job. We have shown in section two that the key premise of Williamson's argument cannot be stated too broadly. Section three showed, conversely, that if modified too narrowly, the argument does not entail the desired conclusion. Can a middle ground be found? Can (WP) be modified in a way that is not unacceptably liberal and, at the same time, is not too narrow for the argument to proceed?

A possible candidate that steers clear of both extremes may be a proposition in which the embedded knowledge operator is modified to accommodate knowledge attained either just by looking or by reflection on what is known just by looking (or a mixture of the two).<sup>18</sup> This proposition we will label '(WP-dis)', to reflect its disjunctive nature.

(WP-dis)      Mr. Magoo knows that (if by looking or by reflecting on what he knows by looking, Mr. Magoo knows that the tree is not  $i$  inches tall, then the tree is not  $i+1$  inches tall)

And more formally:

$$(WP\text{-dis}) \quad K_M^{reflection} (K_M^{reflection \vee looking} \sim t_i \supset \sim t_{i+1})$$

(WP-dis) is designed to accommodate knowledge that is attained through deduction by reflective epistemic principles from what is known by looking. “Once this modification is in place,” it can be suggested on Williamson’s behalf, “the forced choice between (KK), (C), factivity of knowledge, and the correct reading of (WP), is reinstated.”

But (WP-dis) seems *ad-hoc*ish - an artificial construct, tailored for the argument. The questions is, therefore, are there considerations supporting the disjunctively modified (WP)?

For (WP-dis) to be motivated it must be shown to be supported either by the considerations supporting (WP) or by some independent considerations. As for the first option, there seems to be no room for optimism. As Williamson says: “the premises of the argument are ... justified ... by limits on Mr Magoo’s eyesight and his knowledge of them” (118).<sup>19</sup> This supports the idea that from knowledge that depends on his eyesight and ability to judge heights (by eyesight) Mr. Magoo can deduce knowledge about heights close to those he knows about by looking. Yet this reasoning does not support similar claims about knowledge attained by perception and reflection on his perceptual abilities.<sup>20</sup> The rationale behind (WP) stems from the limitations of Mr. Magoo’s visual discriminatory abilities. Surely, this rationale applies only to cases suffering from these limitations. Knowledge Mr. Magoo attains by means other than mere visual discrimination does not (necessarily) suffer from the same deficiencies as knowledge attained by these means alone. Furthermore, even if they do suffer from these (or similar) deficiencies, it must be shown that Mr. Magoo can know of them and can formulate a proposition that correctly reflects them. Clearly, the motivation for (WP) - namely, that Mr. Magoo knows that he cannot tell the tree’s height to an inch just by

looking - do not apply to situations where he has other means, reflective or otherwise, to assess the tree's height, or alternatively and more realistically, it does not apply to cases where he does not know how he knows. Justifying a (WP)-type proposition does not, therefore, provide justification for a disjunctively modified proposition in which one of the disjuncts is the modifier in the original (WP)-type proposition. This point is also true of the other cases presented by Williamson as cases regarding which an equivalent argument holds (assessing loudness in decibels, heat in degrees centigrade, the number of sugar grains in one's tee, etc. See p. 119). In all of these cases, for the argument to work the (WP)-type proposition must be stated in the disjunctively modified form, and in all these cases the knowledge attained after carrying out the deduction no longer suffers from the limitations underlying the original (WP).

We believe it can be shown that (WP-dis) is not justifiable, since, as we articulate in detail in the appendix, disjunctive propositions of this sort lead to contradictions and inconsistencies. But be that as it may, the considerations raised so far suffice to establish a rejection of Mr. Magoo's argument against (KK). Remember, the argument is a *reductio* argument showing that the proposed premises lead to a contradiction and therefore one of them must be forsaken. The attack on (KK) was complete only with the support of the claim that it is the weakest of all the premises. Once (WP) has been substituted by (WP-dis) this claim loses its force. Even if it is not shown that (WP-dis) cannot be justified, there is no reason to suppose it stronger than (KK); surely not for one who is an advocate of (KK). In this sense, the argument devised by Williamson can be turned against (WP-dis). By showing that together with other rather plausible assumptions (WP-dis) leads to a contradiction, the argument provides good reason for dismissing (WP-dis). Pending new justification for (WP-dis), then, the Magoo argument should not convince or bother the (KK) proponent.

## 5. CONCLUSION

What is the source of (WP)'s initial appeal? Perhaps it was the assumption that knowledge is unchanged. That knowing that  $p$ , no matter how you come by it, has the same limitations and implications. Sections two and three undermine this thought. Knowing that the tree is not 5 inches tall by measurement has different implications from knowing this on the basis of mere observation. As the title of this paper suggests, we think Mr. Magoo has made a mistake in his reasoning. His mistake is in failing to distinguish between the types of knowledge that result from looking directly at the tree, on the one hand, and from deduction from perceptual knowledge, on the other. In the way the scenario is constructed, it is tacitly assumed that both methods carry the same results. But, as we have shown, this assumption is false. The argument, therefore, fails to recognize the inherent limitations of (WP) and utilizes it with no regard to the distinction between the types of knowledge to which it applies and those to which it does not.

Careful attention to the specific modes by which knowledge is attained proves crucial in the present context. Specifying the mode of knowledge in the premises and preserving it in the course of logical deduction prevents inappropriate application of (some) epistemic principles, thereby preventing contradictions and confusions. The lesson does not relate merely to perceptual knowledge, as is evident when we consider Williamson's application of the Magoo story to a regeneration and solution of the "surprise examination" paradox and to cases of knowledge by testimony, by perception, and mixed cases involving both. Since the argument for these cases has the same structure as the Magoo argument,<sup>21</sup> the same considerations hold. It should be clear then, that the problem is not in perception, or in testimony, or in the mixture of both. The problem is more general, and hence revealing. This case exemplifies the tendency to forget that (sometimes) what matters is not only whether you know that  $p$  but also *how* you know it. The underlying theme of this paper is that the differences between the

modes by which knowledge is acquired have more implications than epistemologists tend to recognize. Gaining knowledge by deduction from prior knowledge is sometimes (as in the present case) constrained by the particularities of the mode on which the prior knowledge is grounded. Some contents, although logically following from a known content, do not follow epistemically, as items of knowledge.

It should be made clear that we are not arguing in favor of (KK),<sup>22</sup> nor in favor of its rejection. We are merely urging that the Magoo argument does not support the rejection of (KK). It may also be suggested (in line with our argument) that the validity of (KK) ought to be determined separately for different modes of knowledge. Knowing that  $p$  by mathematical proof, for example, arguably entails second-order knowledge of  $p$ . This, however, may not carry over to other forms of knowledge. In other words, our argument supports the thought that at least in some contexts, the full specification of an item of knowledge should include the way in which this knowledge was attained. This is particularly important when assessing what can be epistemically derived from an item of knowledge. As we have shown, the derivation of second-order knowledge from knowledge of the first-order is (at least) sometimes restricted by the mode by which the latter is attained. This suggests that the restricting knowledge entailment differently for different modes of knowledge may be a more general phenomenon.<sup>23</sup>

#### APPENDIX: WHAT ABOUT (WP-DIS)?

Is the disjunctively modified (WP) justifiable? Well, what was it that made (WP) appealing in the first place? Two features of Williamson's presentation of the argument contribute to the intuitive plausibility of (WP) and obscure the matter at hand. One feature relates to the range for which the story is told and the other to the intervals of heights for which (WP) is stated. We take these points in turn.

First, Mr. Magoo's story begins with his observation that the tree is not 0 inches tall. This makes the following steps intuitively compelling, since for normal trees it is reasonable that Mr. Magoo can correctly and directly judge that they are not 5, 6 or even 20 inches tall. But for this no proposition such as (WP-dis) is needed. It seems that we can easily go along with Williamson in the initial stages since we know that any reasonable evaluator of heights will not only know that the 666 inch tall tree, is not 0 inches tall, but will also know that the tree is not 20 inches tall. In normal conditions a person can know just by looking at a tree that it is not 20 inches tall, independent of his observations that it is not 3 or 4 inches tall. At these stages the appeal to (WP) is idle.

In fact, in stating the rationale for (WP) Williamson himself says that Mr. Magoo knows that the height of the tree is not 60 inches (p. 114). That is part of the reason why it seems evident that if Mr. Magoo knows that the tree is not 3 inches tall he can come to know that it is not 4 inches tall – he *already* knows it is taller than 4 inches. That much was already implicit in the setup. We easily play along with the first stages since we know already that Mr. Magoo knows that the tree is not 3, 4, or even 50 inches tall. But once we accept the first steps, the game seems to be over. We find no acceptable reason to stop the argument from proceeding past the limit that is directly discernable by a normal observer. To be more accurate, once the reader has accepted the first two steps, it is hard to see how a stopping rule can be devised. But what if we start the argument at a height that is at the limit of what Mr. Magoo can know by direct observation? For instance, let us say that 500 inches is the closest height to the actual height of the tree that Mr. Magoo can reliably judge that the tree is not.<sup>24</sup> Can Mr. Magoo proceed to infer that the tree is not 501 inches tall?

A negative answer to this question is in effect an unreasonable rejection of WP (and of (WP-KK) as well). It seems that if Mr. Magoo knows how poor his judgment of heights is, he may also legitimately infer from his knowing by looking at the tree that it is



not 500 inches tall, that it is not 501 inches tall either. The point to think about, however, is the next step. Granted that Mr. Magoo knows that the tree is not 501 inches tall, can he then infer correctly from this knowledge and from his knowledge of his limited abilities to tell tree heights by looking, that the tree is not 502 inches tall? Here intuitions may diverge. It seems that when the case is laid open in this way, the intuition that he can so infer is no longer as compelling. To clothe this intuitive weakness in reasons, we suggest that this is so because the inference exceeds the evidence available to Mr. Magoo. No matter whether he knows that he knows the tree is not 501 inches tall or merely knows that it is not of that height, the evidence available to him does not support the claim that the tree is not 502 inches tall, at least not on the basis of this margin for error; and this brings us to the other point.

Both (WP) and (WP-dis), gain intuitive appeal from the density of the intervals chosen. The interval of one inch is very modest relative to the evaluative abilities of normal subjects. But, once again, this is not material to (WP). Clearly, the small interval of one inch, although it may very well be the source of our tendency to accept (WP) in the initial stages, is not essential to the proposition. (WP) should be true for any interval of which Mr. Magoo knows that he cannot discriminate just by looking. If (WP) is correct for intervals of one inch, it should be good for 5, or 10 inches as well (provided that Mr. Magoo cannot discriminate between heights differing by 10 inches). It seems, however, that when the interval is sufficiently large, only one or two applications of the principle are intuitively convincing.<sup>25</sup>

Combining these two observations regarding the source of (WP)'s intuitive appeal, defeats repeated applications of the principle and therefore undermines the validity of the multiple-application-enabling (WP-dis). If we start the Magoo argument at the closest height that Mr. Magoo can know that the tree is not in inches and we take the interval to be the largest that Mr. Magoo can know that he cannot discern by mere

looking, then, it seems to us, allowing for more than one application of (WP) is rather *unintuitive*. In any case, the intuitive appeal of (WP-dis), at least of its recurring application, is not maintained once the contingent features of the story are changed.

But, regardless of intuitive force, is there an argument to show that (WP-dis) is not acceptable?

There are two strategies to argue against the acceptability of (WP-dis). The first is a direct rebuttal, viz. showing that it leads to unwelcome consequences, or is in conflict with some other, well-based, principle. The second strategy, and the one we pursue here, is an indirect argument. Our rejection of (WP-dis) is motivated by an analysis of the faults of propositions similar in both form and function. The general strategy is to show that the unacceptable conclusion reached by Mr. Magoo's argument can be reached without the (KK) principle. We introduce two propositions similar to (WP-dis), and articulate two Magoo-type arguments. We show how these new disjunctive propositions lead to the same contradiction as does (WP-dis) without assuming (KK). This shows that the appeal to knowledge (or to its known limits) is not essential to the argument. In other words, it seems that just as in these cases the contradiction is reached through the introduction of the disjunctive propositions and independently of (KK), in the Magoo argument too the disjunctive (WP-dis) is responsible for the mess and not necessarily (KK). So, if the argument is sound, there's reason to suspect that disjunctive propositions like (WP-dis) should be rejected.

Assume that for a normal tree Mr. Magoo can tell that if he recognizes it from a distance of 100 yards, it must be taller than one foot. Sitting at one end of a football field, Mr. Magoo sees a tree at the other end of the field. He therefore knows that the tree at the far end of the field is not 12 inches tall. He also knows that it is not taller than 10,000 inches, since presumably, no tree is.<sup>26</sup>

(A)  $K_M t_{12-10,000}$  (Mr. Magoo knows that the tree's height in inches is between 12 and 10,000)

Since knowledge entails belief (as will be assumed throughout) we may also suppose that:

(B)  $B_M t_{12-10,000}$  (Mr. Magoo formed the belief that the tree's height in inches is between 12 and 10,000).<sup>27</sup>

Now think of a principle quite similar to (WP-dis):

(1)  $K_M (B_M^{looking \vee reflection} \sim t_i \supset \sim t_{i+1})$

The idea behind this principle is that, knowing the limitations of his own visual ability Mr. Magoo knows that if, upon looking and reflecting on his abilities alone, he has formed a belief that the tree is not of a certain height, then the tree must not be one inch taller than that. Now contrast (1) with:

(1')  $K_M (B_M^{looking} \sim t_i \supset \sim t_{i+1})$

We will demonstrate that while the non-disjunctive (1') does not lead to a contradiction, its disjunctive analogue, (1), does, and this without recourse to (KK).

Assume that after looking at the tree Mr. Magoo believes it is not 12 inches tall:

(2)  $B_M^{looking} \sim t_{12}$

Now assume that Mr. Magoo performs a series of deductions, similar to those Williamson has him perform. To avoid possible failure of transparency of beliefs, assume that he is writing the steps of the deduction on paper, so that in this case:

(3)  $B_M p \supset K_M B_M p$

From (2) and (3) it follows that Mr. Magoo knows that he believes the tree is not 12 inches tall:

(4)  $K_M B_M \sim t_{12}$

Positing (2) as the antecedent in (1) we get:

$$(5) K_M(B_M \sim t_{12} \supset \sim t_{13})$$

From (4) and (5) it follows by closure that Mr. Magoo knows that the tree is not 13 inches tall:

$$(6) K_M \sim t_{13}$$

Since knowledge entails belief, from (6) it follows that:

$$(7) B_M \sim t_{13}$$

At this point Mr. Magoo can posit (7) in (1) and get:

$$(8) K_M(B_M \sim t_{13} \supset \sim t_{14})$$

Once again, since he is fully conscious and writing the steps of his deduction as he is going through them, Mr. Magoo knows that he believes that the tree is not 13 inches tall:

$$(9) K_M B_M \sim t_{13}$$

and therefore, again by closure we get:

$$(10) K_M \sim t_{14}$$

These iterations can be continued for the full expanse between 12 and 10,000 inches. Looking at his notes, Mr. Magoo concludes, and therefore knows, that the tree is not any height within this interval (or at least knows that he believes this):

$$(11) K_M \sim t_{12-10,000}$$

(11) clearly contradicts (A) thus ascribing to Mr. Magoo knowledge that he holds contradictory beliefs. Note that this conclusion amounts to possible inconsistency within Magoo's cognitive system, regardless of the tree's actual height (so no appeal to factivity is required). But more importantly, this time the blame cannot be laid on (KK), since the troubling result is reached without recourse to this principle. In the present argument, (KK) has been replaced with (3), which is, under the circumstances of the case, very plausible.

Now, the point to notice is that the knowledge in (6) is knowledge by direct perception and inference by reflection on Mr. Magoo's visual abilities. The belief in (7) therefore, must also be modified as belief by looking and reflection. (7) is then plugged into (2) which - modified to accommodate belief by looking *or reflection* - allows the argument to proceed. So the full formulization of (7) should be:

$$(7^*) B_M^{\text{looking\&reflection}} \sim t_{13}$$

Let us remind ourselves that our inquiry into this case has emerged from the question of the adequacy of the disjunctively modified (WP-dis). Our question was whether this disjunctive modification, allowing for knowledge gained by both mere looking and inference from reflective truths to appear as the antecedent in the conditional, is acceptable. Proposition (1) of the present argument has been modified in exactly the same way as (WP-dis). We now want to suggest that it is this liberal formulation which is the source of trouble. Think of the less permissive (1') (structurally analogous to (WP-KK)):

$$(1') K_M(B_M^{\text{looking}} \sim t_i \supset \sim t_{i+1})$$

It is easy to see that (7\*) cannot be posited in (1'). To put it explicitly, had the belief operator in (1) been modified to accommodate only belief by mere looking, it would have been impossible to posit (7) in (1) and the move to (8) and (11) would have been blocked. It seems then, that the inappropriate modification of the belief operator in (1) is to blame for the unwanted results. Since this argument and proposition (1) on which it turns were designed to mirror Mr. Magoo's argument in its (WP-dis) formulation, the failure of the former argument can be instructive regarding the latter. Although this is not a direct argument against the coherence of (WP-dis), the present argument does suffice to cast doubt on the plausibility of disjunctive conditionals like (WP-dis). The argument shows that a disjunctive proposition similar to (WP-dis) in both form and

intent leads to a contradiction independent of (KK). It is therefore reasonable to suspect that it is also the disjunctive modification in (WP-dis) that leads into trouble and not (KK).<sup>28</sup>

Let us take another disjunctive principle of the (WP-dis)-type, this time, one pertaining not to knowledge but rather to the norms of belief-formation. Once again, we show, the disjunctive character of the principles leads to inconsistency regardless of (KK) or any analogous principle.

Say Mr. Magoo is deliberating about his own perceptual beliefs. Which of the following belief forming norms, structurally analogous to (WP-KK) and (WP-dis), should Mr. Magoo adopt?

(BFN-dis) If Mr. Magoo believes by looking or reflecting (on what he believes by looking) that a tree is not  $i$  inches tall, then he may also form a belief that a tree is not  $i+1$  inches tall.

Or, alternatively, is he justified in adopting:<sup>29</sup>

(BFN-KK) If Mr. Magoo believes by looking alone that a tree is not  $i$  inches tall, then he may also form the belief that it is not  $i+1$ .

Notice that (BFN-dis) is more permissive than (BFN-KK). Like (WP-dis), it applies to other sources of belief, besides the purely visual. As we will now demonstrate, this permissiveness makes (BFN-dis) unacceptable since it can lead one to an incoherent belief system. In other words, the permissiveness of this norm leads to incoherence just as its analogue, (WP-dis), leads to a contradiction.

Imagine that, looking at a tree, Mr. Magoo forms the true belief that it's between 5 and 10,000 inches tall. We therefore have the following:

(12)  $B_M^{looking} t_{5-10,000}$  (Mr. Magoo believes by looking that the tree is between 5 and 10,000 inches tall).

From (12) we can infer:

(13)  $B_M^{looking\&reflecting} \sim t_4$  (Mr. Magoo believes by reflecting and looking that the tree is not 4 inches tall).

Now recall our candidate for belief-forming norm:

(BFN-dis)<sup>30</sup>  $B_M^{looking\vee reflecting} \sim t_i \rightarrow B \sim t_{i+1}$

Following the direction of (BFN-dis), Mr. Magoo forms on the basis of (13) the belief:

(14)  $B_M^{looking\&reflecting} \sim t_5$  (Mr. Magoo believes by reflecting and looking that the tree is not 5 inches tall).

He then iterates this move, advancing one inch at a time. Imagine that he is recording these steps by writing them in succession on paper. Therefore, at least in this case, if he has a belief ( $Bp$ ) he also knows that he has it ( $KBp$ ). He repeats these steps again and again, forming his beliefs according to the direction of his accepted norm (BFN-dis).

Finally, after surveying his records, he concludes that:

(15)  $B_M^{looking\&reflecting} \sim t_{5-10,000}$  (Mr. Magoo believes by reflecting on what he believes by looking and reflecting that the tree is not of any height between 5 and 10,000)

But (15) is in direct contradiction with (12). Although a person may (arguably) hold incompatible beliefs, the fact that (BFN-dis) inevitably leads to contradiction is good reason not to adopt it as a norm of belief-formation. This conclusion is strengthened by the fact that there is another candidate available that has the virtues of (BFN-dis) (it is justified by the same considerations) without its vices, namely (BFN-KK):

(BFN-KK)  $B_M^{looking} \sim t_i \rightarrow B_M^{looking\&reflecting} \sim t_{i+1}$

To see the advantage of this norm, imagine that we start with the visually formed belief:

(13\*)  $B_M^{looking} \sim t_4$  (Mr. Magoo believes by looking that the tree is not 4 inches tall.)

Under the guidance of (BFN-KK) he can form the belief that the tree is not 5 inches tall.

(14')  $B_M^{\text{looking \& reflecting}} \sim t_5$  (Mr. Magoo believes by looking *and reflecting*, that the tree is not 5 inches tall).

But this belief-forming norm can carry Mr. Magoo no further than this, since (16') cannot be posited as the antecedent in (BFN-KK). The more moderate character of (BFN-KK) prevents one from getting into contradictions, by adopting it. Unlike its more radical disjunctive counterpart: (BFN-dis).

Notice that the problematic result of (BFN-dis) does not involve (KK). It seems then that it is just the disjunctive nature of (BFN-dis) that is to blame for the inconsistency. And so, by analogy it is plausible to claim that the disjunctive nature of (WP-dis), rather than (KK), should be identified as responsible for Magoo's contradiction. It is this principle's insensitivity to the source of knowledge it operates on (which, ironically, was the reason for accepting this principle in the first place) that entitles us to reject it.<sup>31</sup>

---

<sup>1</sup> Williamson, T. (2000). *Knowledge and its limits*. (New York: Oxford University Press). Unless noted otherwise, references are to this text and by name and page number only.

<sup>2</sup> We follow Williamson in calling (1<sub>i</sub>) a proposition rather a principle (e.g. "let q be the proposition that the tree is not i+1 inches tall" p. 116). Strictly speaking it is a propositional function from numbers to propositions or a proposition provided that it is quantified. It would be more apt to call it a principle (though a limited one) since the conditionals embedded in (1<sub>i</sub>) or (WP) are known for a uniform reason pertaining to Mr. Magoo's perceptual and evaluative abilities. We thank to Karl Karlander here.

<sup>3</sup> Strictly speaking, this is not a straightforward contraposition since the conditional is within the range of the knowledge operator. However, since, Mr. Magoo has deduced all that follows from his knowledge (Williamson, 116), and assuming closure and factivity (see (C) below), this change is legitimate. Apart from this the same justification supporting (1<sub>i</sub>) can be given as justification to (WP).

<sup>4</sup> For ease of exposition we present Williamson's argument in a simplified form. Nothing of substance will turn on the differences between the original argument and the one we present here.



---

<sup>5</sup> This formulation of (KK) is meant to avoid the obvious counter-examples to the better known versions. Furthermore, it is clear that Williamson is out to refute the more plausible formulation of (KK), according to which, if one knows  $p$ , one is thereby in a position to know that one knows that  $p$ . The way he does this is by imagining that Mr. Magoo has made all the deductions: “If he is in a position to know that he knows  $p$ , he does know that he knows  $p$ ” (Williamson, 115). He then goes on to present what he takes to be a counter instance assuming that Mr. Magoo knows all that is deducible from his position thereby (allegedly) refuting the weaker less demanding formulation. That this is the relevant formulation is evident from the way chapter (5) begins. For more detail, see (Williamson, 115).

<sup>6</sup> The case is so construed that Mr. Magoo has deduced all pertinent conclusions about the tree and its height: “Statement (C) is simply a description of Mr Magoo’s state once he has attained reflective equilibrium over the propositions at issue, by completing his deductions.” (116) See also previous note.

<sup>7</sup> Williamson formalizes factivity as follows:  $\forall p \Box (Kp \supset p)$  (Williamson, 271). For simplicity and clarity, we have omitted throughout this paper quantification and necessity operators which play no essential role in the arguments.

<sup>8</sup> He could reason that, although the tree is definitely taller than 3 inches and shorter than 6000 inches, it cannot be of any height  $n$ , for any natural number  $n$  between 3 and 6000. Absurdly, he could therefore deduce that the tree has no height whatsoever even though he can see that it does. In this case, even without factivity, the result would be absurd.

<sup>9</sup> “The crude point is that iterating knowledge is hard, and each iteration adds a layer of difficulty.” (Williamson, 122). “The iteration of knowledge operators leads sooner or later to falsity through a process of erosion resulting from the need for margins for error.” (Williamson, 140-1)

<sup>10</sup> This is Williamson's formalization in page (Williamson, 115). He repeats it in pages 117, 118, 120, 121, 139, 140, as well as in appendix 2 (Williamson, 305-6).

<sup>11</sup> The scope of the knowledge operator is explicitly presented by Williamson: “Mr. Magoo knows that (for all natural numbers  $m$  (if the tree is  $m+1$  inches tall then he does not know that it is not  $m$  inches tall) and (the tree is not 0 inches tall)).” (Williamson, 117-8; and in logical notation on page 139 and in appendix 2 (305-6)).

<sup>12</sup> Assuming (C), it can be deduced from Mr. Magoo’s knowledge of the following truism about tree heights:  $t_i \supset \sim t_{i+1}$ . From this it follows that, if he knows the tree is  $i$  inches tall, he knows it is not  $i+1$  inches tall. Note that on the theoretical level it does not make any difference how wide the margin for error is.

<sup>13</sup> Notice that Williamson leaves out an important condition. As the Magee scenario shows, for the principle to be valid, Mr. Magoo must know what his source of knowledge is, and that it is exclusive.

<sup>14</sup> There is textual evidence suggesting that Williamson concedes a similar point, namely that the margin of error changes in accordance with what he calls “the basis of knowledge” (Williamson, pp. 128, 132). When, for instance, one knows of an event by seeing it, the margin, according to Williamson may be different from what it would have been had one known it by hearing. He does not explain why the change in the “basis” of knowledge does not influence the margin of error in the Magoo case.

---

<sup>15</sup> We call this proposition '(1<sub>i</sub>-KK)' since, as we intend to show, the advocates of (KK) can accept it.

<sup>16</sup> For our purposes this is similar enough to Williamson's (C).

<sup>17</sup> If we are correct, a further assumption has been left tacit in Williamson's argument, i.e., for the argument to work Mr. Magoo must not only know *that* he knows, but also *how* he knows. This is one more premise the (KK) advocate can deny (we thank Anders Nes for pointing this out). We will not follow this line of reasoning, however, since it seems that in special cases this problem can be overcome.

<sup>18</sup> This response has been suggested by Hagit Benbaji and Peter Pagin independently. We take this opportunity to thank Peter Pagin for correcting a mistake in a draft of this paper.

<sup>19</sup> This is mentioned in response to a possible claim according to which it is not (KK) that is the source of the contradiction but rather vagueness.

<sup>20</sup> Or if you want, it supports (WP-KK) as much as it supports (WP-dis).

<sup>21</sup> See, e.g., (Williamson, 135, 139).

<sup>22</sup> To the extent that they are independent of the Magoo argument, Williamson's other considerations against luminosity are not debated here.

<sup>23</sup> We believe that the general lesson emerging from our criticism of Williamson's argument, supports inquiring about the valid scope, or the constraints on the proper application, of the principle of epistemic closure. As discussing this principle clearly exceeds the scope of this paper, we pursue this project elsewhere (Sharon and Spectre, "Towards a New Principle of Epistemic Closure" (unpublished manuscript)).

<sup>24</sup> Some subjects will get closer to the actual height of the tree by first judging in centimeters and then converting to inches. This is interesting since it seems that you get more knowledge from the same evidence (assuming that looking at the tree and thinking in centimeters is the same as looking at it and wondering how many inches it is in height). With a smaller tree, by looking, one can come much closer to its height than with a mammoth tree such as the one Williamson has chosen as his example. A reasonable evaluation of a tree that is 166 inches tall is that it is more than 0 inches and less than 332 inches.

<sup>25</sup> Note that it makes a big difference how tall the tree is. The smaller the tree, the smaller the interval it is reasonable to assume is indiscernible by a subject (by merely looking). (WP) will not apply to a "tree" 10 inches tall if the interval is 10 inches, but it will if the tree is 666 inches tall (as in the Magoo story).

<sup>26</sup> A *Eucalyptus Regnans* at Mt. Baw Baw, Victoria, Australian, is believed to have measured 143 m (470 ft.). The reports and measurements are from 1885. Another Australian eucalyptus, at Watts River, Victoria, had probably been over 150 m (492 ft.) tall. It is safe to assume that Mr. Magoo knows that in his area there are no 10,000 inch (833 ft.) trees.

<sup>27</sup> Given the reasoning above, it is quite straightforward to assume that Mr. Magoo should hold this belief. For the argument to work, however, this is not necessary, since, as we show later, it can be stated as an argument about what Mr. Magoo *ought* to believe.

---

<sup>28</sup> There is another formulation that perhaps comes closer than (WP-KK) to what was intended in formulating (WP), which also does not lead to contradiction. It turns on a distinction between reflective content and perceptual content. The general idea that this new proposition is justified by, is that the interval in the imbedded conditional (i.e. the interval in inches between what the subject knows the tree not to be and the height that it follows from this that the tree is not) decreases by an amount directly proportional to the increase in reflective content. This is one way to formalize this idea (when  $n$  is a positive natural number and  $m$  is a number such that  $0 < m < 1$ ):

$$(WP-int) \quad K_S^{reflection} (K_S^n \text{ reflections on what S knows by looking } \sim t_i \supset \sim t_{i \pm m^n})$$

This would hold generally and even with (KK) (given the correct value for  $m$ ) no contradiction will result.

What this demonstrates is that even if (WP-KK) is somehow shown to be too restrictive, there are yet other alternatives to (WP-dis) (at least one, namely, (WP-int)) which would also have to be dismissed for the Magoo argument to work. It is hard to see how (WP-dis) can be shown to be superior to (WP-int) since both can be applied indefinitely and both are disjunctive. Although (WP-int) is sensitive to the number of times it has been used it admits both reflective and perceptual content. It is therefore similar to (WP-dis) and should be expected to be more resilient in this context.

In order for Williamson's argument to work, then, it must be shown that the margin does not decrease and that Mr Magoo can know that it does not decrease. That is a tall order indeed.

<sup>29</sup> A third alternative is that both are unjustified norms. We claim, however, that no special problem results from adopting (BFN-KK).

<sup>30</sup> The arrow is not to be understood as an implication but rather as a conditional permission operating on beliefs. This applies to (BFN-KK) as well.

<sup>31</sup> We are indebted to Hagit Benbaji, Yitzhak Benbaji, David Enoch, Patrick Girard, Sandy Goldberg, Sören Häggqvist, John Hawthorne, Mikael Janvid, Karl Karlander, Igal Kvart, Avishai Margalit, Anders Nes, Peter Pagin, Itamar Pitowsky, James Simmons, Åsa Wikforss, and Jonathan Yaari for many helpful comments. We have also benefited from the responses of participants in the Graduate Student Workshop at Stanford University, the Philosophy of Science Seminar at Stockholm University, and the Rutgers-Princeton Graduate Philosophy Conference (2007). In addition we would like to thank Timothy Williamson for helpful discussion. In private conversation Prof. Williamson and Josh Snyder independently mentioned that an argument along somewhat similar lines is being developed by Jérôme Dokic and Paul Egré. We cannot comment on similarities or differences. Let it be noted that this is not to imply that our argument is endorsed by Williamson.