The other de Broglie wave

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Abstract

In his famous doctoral dissertation, de Broglie assumed that a massive particle is surrounded in its rest frame by a standing wave. He argued that as observed from another inertial frame this wave becomes the superluminal wave now known as the de Broglie wave. It is shown here that under a Lorentz transformation, such a standing wave becomes, not the de Broglie wave, but a modulated wave moving at the velocity of the particle. It is the modulation that has the superluminal velocity of the de Broglie wave and should be recognized as the true de Broglie wave. De Broglie's demonstrations relied, variously, on his "theorem of the harmony of phases", on a mechanical model, and on a spacetime diagram. It is shown that in each case the underlying wave was inadvertently suppressed. Identified as a modulation, the de Broglie wave acquires a physically reasonable ontology, avoiding the awkward device of recovering the particle velocity from a superposition of such waves. The deeper wave structure implied by this de Broglie wave must also impinge on such issues in quantum mechanics as the meaning of the wave function and the nature of wave-particle duality.

1 Introduction

There is an understanding of the de Broglie wave in which it is not the independent wave usually supposed, but the modulation of an underlying wave structure that moves with the velocity of the particle[1]. In the inertial frame of the particle itself, this underlying structure has the form of a standing wave.

It is argued here, essentially from a reconsideration of de Broglie's famous doctoral dissertation of 1924 [4], that this "other de Broglie wave" is indeed the true de Broglie wave.

The matters to be now discussed were dealt with by de Broglie in two brief sections (Sects. I and III) of his opening chapter, which is readily accessible in English translation (Haslett [5], and Kracklauer [6]), as also in German (Becker

[1] Following the publication of Ref. [1], which listed several such proposals, others were made known to the author, of which the earliest are Mellen [2] and Horodecki [3].

[7]). Quotations below are from those two sections of the dissertation, and generally follow in translation, Haslett [5].

De Broglie saw that if the Planck-Einstein relation,

$$E = h\nu, \tag{1}$$

for the photon were extended to matter, and equated with Einstein's statement,

$$E = mc^2, \tag{2}$$

of the equivalence of mass and energy, a massive particle could be associated in its rest frame with a characteristic frequency ν_0 or, when expressed as an angular frequency,

$$\omega_0 = 2\pi\nu_0 = \frac{mc^2}{\hbar},$$

where m, c, and $\hbar$ are respectively the rest mass of the particle, the speed of light, and the reduced Planck constant.

De Broglie rejected the possibility that this frequency could be the measure of some wholly internal or fictitious oscillation. Observing that the energy of an electron "spreads throughout all space", he argued that a massive particle must be surrounded in its rest frame by a "periodical phenomenon" which he assumed has the form of a standing wave. De Broglie provided no mathematical description of this standing wave, nor analysis of how a standing wave changes under a Lorentz transformation. He concluded nonetheless that to an observer for whom the particle is moving at a velocity v, the standing wave becomes a superluminal wave of velocity c^2/v.

De Broglie referred to this superluminal wave as a phase wave. He was later to call it the pilot wave, and it is of course the wave more usually now known as the de Broglie or matter wave.

But de Broglie was incorrect in concluding that under a Lorentz transformation a standing wave can simply become the de Broglie wave. As will be shown here, a standing wave, whatever its form or frequency, becomes to an observer for whom the particle is moving at velocity v, a carrier wave of that velocity, subject to a phase modulation having the wave characteristics and superluminal velocity c^2/v of the de Broglie wave. It is this modulation that will be identified here as the true de Broglie wave.

In demonstrating the existence of the de Broglie wave, de Broglie relied variously, on what he called "the theorem of the harmony of phases", on a toy model simulating the distribution of energy around a massive particle, and on the transformation of a standing wave in a spacetime diagram. But in each case, the existence of the underlying carrier wave was overlooked, either because what was transformed was not the spatially extended wave that de Broglie had supposed, but rather a single point in that wave or, when the extended wave was considered, because the analysis proceeded in a way that allowed the resulting carrier wave to be ignored.

De Broglie's insight that a massive particle has wave properties similar to those of the photon played a crucial role in the formulation of quantum mechanics. Einstein remarked that de Broglie had "lifted a corner of the great veil"

[8], and it was the dissertation that led Schrödinger to wave mechanics (see, for example, Raman and Forman [9]). The de Broglie wavelength was itself soon confirmed by the electron diffraction experiments of Davisson and Germer [10], and Thompson [11].

But Schrödinger was less successful in constructing from the de Broglie wave, a thoroughly wave-based theory of matter and radiation. He could neither confine the individual wave to an atomic orbit, nor preclude the spreading of the superposition that de Broglie had supposed would satisfactorily localize a particle.

As conventionally understood, the de Broglie wave is indeed very curious. It is superluminal, of unknown ontology, and strangely disembodied from the subluminal particle that it is forever overtaking but never outruns.

Identified as a modulation, much of the mystery disappears, but what is then to be considered is the significance of the underlying carrier wave thus implied. We refer briefly below to some obvious implications of this deeper wave structure, but the main concern of this paper is to establish how the modulation arises, and to consider why the underlying wave was not revealed by de Broglie's own derivations.

De Broglie's interpretation of his wave varied over time, but it was always a wave in its own right, or a superposition of such waves, as described in the dissertation, and it is on the dissertation that we concentrate here.

2 The modulation

But we first establish that the de Broglie wave does indeed emerge as a modulation from the Lorentz transformation of a spatially extended standing wave. We also show, conversely, that the de Broglie wave cannot arise from the corresponding transformation of an oscillating point or point particle.

Consider a standing wave of general form,

$$R(x, y, z)\, e^{i\omega t}, \tag{3}$$

where $R(x, y, z)$ describes the spatial variation of the wave, and $e^{i\omega t}$ is its development in time.

We want to know the form that this wave will take when observed from a frame of reference in which the particle is moving with velocity v. Assuming a boost in the x direction, and applying the Lorentz transformation,

$$\begin{aligned} x' &= \gamma\,(x - vt)\,, \\ y' &= y, \\ z' &= z, \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right), \end{aligned}$$

where γ is the Lorentz factor,

$$(1 - \beta^2)^{-\frac{1}{2}} \;\; and \;\; \beta = v/c,$$

standing wave (3) becomes the moving wave,

$$R(\gamma\,(x - vt)\,,\, y,\, z)\, e^{i\omega\gamma\left(t - vx/c^2\right)}. \tag{4}$$

Moving wave (4) has two factors. The first,

$$R(\gamma\,(x - vt)\,,\, y,\, z), \tag{5}$$

is a carrier wave, which is moving at the velocity v and, as indicated by the inclusion in (5) of the Lorentz factor γ, has suffered the slowing of time and contraction of length predicted by special relativity.

The second factor,

$$e^{i\omega\gamma\left(t - vx/c^2\right)} \tag{6}$$

is a plane wave, which is evidently moving through the carrier wave (5) at the superluminal velocity c^2/v. Identifying the frequency ω with the characteristic frequency ω_0 of a massive particle, wave factor (6) can be written in terms of the Einstein frequency,

$$\omega_E = \gamma\omega_0 = \frac{E}{\hbar}, \tag{7}$$

and de Broglie wave number,

$$\kappa_{dB} = \gamma\omega_0 \frac{v}{c^2} = \frac{p}{\hbar}, \tag{8}$$

(where p and E are respectively the energy and momentum of the particle), as,

$$e^{i(\omega_E t - \kappa_{dB} x)}, \tag{9}$$

and can be seen to be the de Broglie wave. However, it is not here an independent wave but the modulation of the carrier wave (5), defining the dephasing of that wave (and the failure of simultaneity) in the direction of travel.

It is thus the full composite wave,

$$R(\gamma\,(x - vt)\,,\, y,\, z)\, e^{i(\omega_E t - \kappa_{dB} x)}, \tag{10}$$

rather than the de Broglie wave (9), that displays the full complement of changes in length, time and simultaneity contemplated by special relativity. It is suggested that it would be anomalous if any spatially extended phenomenon, wave or otherwise, moving at a velocity less than c, could be Lorentz transformed into something that did not incorporate all these changes. Yet, as will be seen in Sects. 3 to 5, the carrier wave is effectively suppressed in de Broglie's derivations.

Now let us suppose that instead of a spatially extended wave, we have the oscillating point particle,

$$\delta(x_0, y_0, z_0)\, e^{i\omega_0 t}, \tag{11}$$

where $\delta(x, y, z)$ is the Dirac delta function and the particle is thus located in its rest frame at the point (x_0, y_0, z_0). Under a boost in the x direction, (11) becomes,

$$\delta(x_0 - vt,\, y_0, z_0)\, e^{i(\omega_E t - \kappa_{dB} x)},$$

which describes not a wave, but a moving and oscillating point.

It is not physically possible for a point to become, by Lorentz transformation, an extended wave. Under a Lorentz transformation, a point remains a point, and a wave, although changed in form, remains a wave. That this is so will be of some importance when considering de Broglie's theorem of the harmony of phases in the next section (Sect. 3).

Several other features of composite wave (10) will also be of relevance in the discussions to follow. Notice, firstly, that it was nowhere stipulated that the original standing wave (3) should comprise underlying influences propagating at the velocity c of light. Yet it is that velocity (together with relative velocity v) that determines the velocity c^2/v of the modulation. It is of course the Lorentz transformation itself that imposes the velocity c, and it does so because, as assumed implicitly in special relativity, all underlying influences develop ultimately at that velocity (see, for example, Ref. [1], Sect. 6).

Thus the velocity of the associated de Broglie wave would have been c^2/v even if we had insisted on a standing wave formed from counter-propagating influences of some velocity other than c, for instance counter-propagating sound or water waves, or light waves of velocity c/n in a medium of refractive index n. It follows from Eqn. (4) that a modulation of velocity c^2/v must arise whatever the form of the underlying wave structure.

But notice, secondly, that for the de Broglie wave to emerge, the antecedent structure must be oscillatory. The possibility of an entirely inelastic body that would transmit a change at one end instantaneously to the other end is denied by special relativity. It is not necessary that the oscillation be sinusoidal, but a non-sinusoidal oscillation will result in a superposition of de Broglie waves, all of the same velocity c^2/v, but having differing frequencies (and wave numbers). A particle with a well-defined characteristic frequency will thus have a well-defined sinusoidal oscillation as contemplated in wave (3).

Thirdly, a modulation of velocity c^2/v will emerge from the Lorentz transformation of a wave of form (3) even when its spatial variation $R(x, y, z)$ is unphysical. Consider, for example, a standing wave that has everywhere the same phase. Such a wave would comprise underlying influences of infinite wavelength and thus infinite velocity, contrary to special relativity. But a wave of this kind might nonetheless be simulated, at least in principle, by an array of identical oscillators.

Such a simulation will be encountered in the second of de Broglie's demonstrations (Sect. 4), while the unphysical wave simulated will itself be seen in the third (Sect. 5).

3 The harmony of phases

After proposing that a massive particle should be associated in its rest frame with a "periodic phenomenon", de Broglie asked,

> Should we assume that the periodic phenomenon is localized in

> the interior of a particle of energy? Not necessarily .... undoubtedly it is distributed throughout an extended region of space.

But he then concluded that under a Lorentz transformation, this distributed periodic phenomenon, which he assumed must be a standing wave, simply becomes the superluminal wave that we know as the de Broglie wave. It was shown in the preceding section that this is not so, and we will attempt to explain, as we now consider de Broglie's derivations, how the underlying carrier wave could have been overlooked. We consider in this section de Broglie's main argument, which was based on his theorem of the harmony of phases.

From the standpoint of a "fixed observer" (for whom the particle is moving at velocity v), the particle is perceived to have in its rest frame the reduced frequency,

$$\omega_{red} = \frac{\omega_0}{\gamma}, \tag{12}$$

yet in the frame of that same observer, the moving particle has an increased energy, and thus an increased frequency,

$$\omega_{inc} = \omega_0 \gamma. \tag{13}$$

But a wave can have only one phase at any point of space and time, and all observers must agree on that phase. There must be, as de Broglie put it, "a harmony of phases". He said[2],

> The periodic phenomenon connected to a moving body whose frequency is for the fixed observer equal to $[\omega_0/\gamma]$ appears to him to be constantly in phase with a wave of frequency $[\omega_0\gamma]$ emitted in the same direction as the moving body, and with the velocity $V =[c^2/v]$.

But in each frame of reference, what is observed is a spatially extended waveform. When de Broglie went on to derive the velocity $V = c^2/v$ of the wave observed by the fixed observer, he confined his consideration of phase to a single point, the position of the particle, which is stationary in one frame and moving in the other, but is nonetheless the same point, developing along the same world line.

De Broglie's derivation was as follows: As the particle travels the distance x in the time t, it is observed by the fixed observer to experience in its rest frame, from (12), the change of phase,

$$\omega_{red} t = \frac{\omega_0}{\gamma}\frac{x}{v}, \tag{14}$$

while in the fixed observer's own frame, the change of phase that occurs is given by, from (13),

$$\omega_{inc}\,(t - \frac{x}{V}) = \omega_0\gamma\,(\frac{x}{v} - \frac{x}{V}). \tag{15}$$

[2]In the quoted passage, we have simplified de Broglie's expressions for the frequencies and avoided here as elsewhere in this paper the practice of describing the velocity of the de Broglie wave as c/β.

These changes must be equal. Thus, equating (14) and (15),

$$\frac{\omega_0}{\gamma}\frac{x}{v} = \omega_0\gamma(\frac{x}{v} - \frac{x}{V}),$$

from which,

$$V = \frac{c^2}{v}.$$

But while this analysis delivers the velocity of the de Broglie wave, it can say nothing of the nature of that wave. De Broglie did not ask what happens to a spatially extended standing wave when it is Lorentz transformed. He confined his analysis to the transformation of an oscillating point which, as observed in Sect. 2, results only in a *moving* oscillating point, which might vary sinusoidally as it moves, but is not itself a sinusoidal wave.

By seizing upon that single point, and ignoring the disposition of phase across the remainder of the extended wave, de Broglie suppressed the carrier wave of velocity v, which as was also shown in Sect. 2, must result from the Lorentz transformation of a standing wave. If the de Broglie wave were the only wave associated with the moving particle, harmony of phase could be guaranteed only at the position of the particle. It is the full modulated wave that harmonizes phase at all points in the wave for all observers.

De Broglie's two other demonstrations did involve the transformation of an extended wave. But from his theorem of the harmony of phases, he had already concluded that the de Broglie wave is a wave in its own right.

4 The toy model

Faced with the potentially embarrassing superluminality of his "phase wave", de Broglie invoked a simple mechanical model to illustrate how a velocity greater than c might yet be consistent with special relativity provided the actual velocity of energy transport were less than c.

Thus de Broglie was not seeking to derive his wave from this model, merely to justify its superluminal velocity. Yet the de Broglie wave emerges in the model, not as an independent wave, but as the modulation of an underlying carrier wave.

As described by de Broglie, the model comprises a horizontal disk of very large diameter, from which are suspended many identical spring weights oscillating in phase and at the same amplitude, but with the number of springs per unit area diminishing with distance from the centre of the disc in "very rough analogy", as de Broglie explained, to the distribution of energy around a particle.

An observer in the inertial frame of the disk observes these weights to be oscillating in unison. But a second observer, for whom the disk is moving at velocity v, observes (along with other relativistic effects) what de Broglie described as the "dephasing of the movements of the various weights", that is to say, the failure of simultaneity in the direction of travel. To the first observer, the weights define a horizontal plane moving up and down. But to the observer for whom the disc is moving, this surface is not planar but sinusoidal, with the crests of this sinusoidal surface moving in the same direction as the disk, but at the superluminal velocity c^2/v of the de Broglie wave.

De Broglie did not provide a separate derivation of the velocity c^2/v of this sinusoidal effect. He merely said of the moving sinusoidal surface that:

> It corresponds, in the particular case under consideration, to our phase wave; according to the general theorem, the surface has a speed [c^2/v] parallel to that of the disk ... With this example we see clearly (and this is our excuse for such protracted insistence on it) how the phase wave corresponds to the transport of the phase and not at all to that of the energy.

In other words, the sinusoidal wave defined by the springs is an instance of the de Broglie wave and thus moves at the velocity c^2/v, while the "general theorem" de Broglie relies upon is the theorem of the harmony of phases.

De Broglie seems not to have noticed that the standing wave (the array of oscillating weights) has not become the de Broglie wave. It has become a structure moving at velocity v (a moving array of oscillating weights) modulated by a wave moving at velocity c^2/v. In treating the modulation as an independent wave, de Broglie has ignored the structure that it modulates.

In its rest frame, de Broglie's toy model is not strictly speaking of course a standing wave. It is a simulation of a standing wave - indeed the simulation of a physically unreasonable standing wave that has the same phase at every point of space. As in the example discussed in Sect. 2, it is a standing wave of infinite wave length comprising underlying influences of infinite velocity. But it is nonetheless of the general form described by Eqn. (3), and as shown in Sect. 2, the Lorentz transformation of such a structure results not in an independent wave of velocity c^2/v but in a wave of velocity v subject to a modulation of velocity c^2/v.

De Broglie's model thus becomes under a Lorentz transformation, not the simulation of an independent de Broglie wave, but the simulation of a modulated wave of velocity v in which the de Broglie wave is the modulation.

If de Broglie had supposed an assemblage of springs that varies sinusoidally in space as well as time, he would have had in this model, the essence of an explanation of the de Broglie wave that would have been physically reasonable and consistent with special relativity. But when he sought to recover the classical velocity of the particle from his superluminal wave, the analogy he drew ([4], Chap 1, Sect II) was with the group velocity of a wave packet formed from the superposition of de Broglie waves of nearly equal frequency.

It was from that very different physical effect that the difficult concept of a particle wave packet was carried into quantum mechanics.

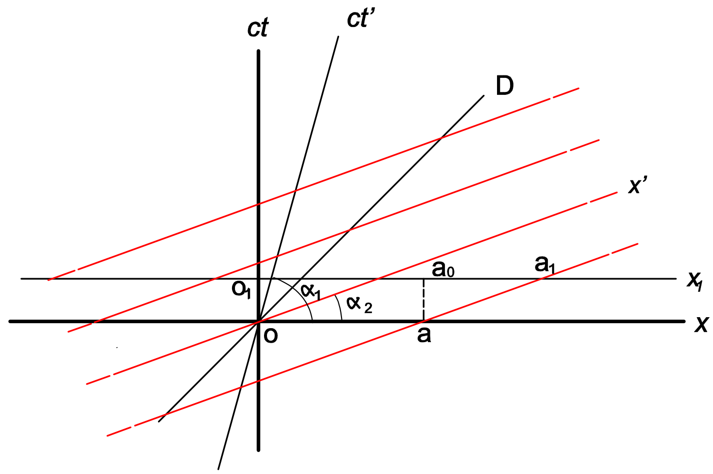

Figure 1: The world line of the particle follows the $ct^{'}$ axis. Surrounding the particle in its rest frame is a standing wave represented by four parallel equiphase planes. These are inclined at $\alpha_1 = \arctan v/c$ to the x axis, and thus define in the unprimed frame, a dephasing of velocity c^2/v. This dephasing is the de Broglie wave, considered by de Broglie an independent wave, but explained here as the relativistically induced modulation of the standing wave. (adapted from de Broglie [4], Chap. 1, Sect III).

5 In spacetime

By representing a standing wave in a spacetime diagram (see Fig. 1) de Broglie was able to derive the velocity of the de Broglie wave, while demonstrating in an intuitive manner, the loss of phase and consequent failure of simultaneity defined by this wave.

In Fig. 1, the unprimed (x, ct) coordinates are those of the fixed observer, and the primed ($x^{'}$, $ct^{'}$) coordinates, those for the rest frame of the particle. The $ct^{'}$ and $x^{'}$ axes are inclined at $\alpha_1 = \arctan c/v$, and $\alpha_2 = \arctan v/c$, respectively, to the x axis, while the world line OD at 45° to that axis defines one edge of the light cone. The particle itself is moving to the right at the velocity v, and its world line thus follows the primed $ct^{'}$ axis.

The standing wave is represented in the diagram by what de Broglie referred to as "equiphase spaces", these being the four equally spaced lines drawn parallel to the $x^{'}$ axis. De Broglie also referred to these as "planes", presumably hyperplanes of three dimensions in the four dimensions of spacetime. Each such plane defines a sub-space in which the wave has reached a particular phase. In each inertial frame, these planes thus repeat after a time equal to the period of oscillation in that frame.

The equiphase planes are parallel to the $x^{'}$ axis, leaving no doubt that it

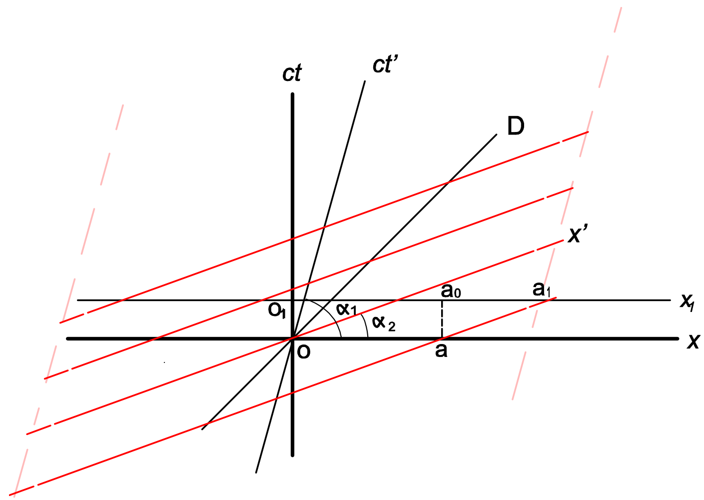

Figure 2: As Fig 1, except that the four equiphase planes are now centred on the world line of the particle, and a world tube has been defined by dashed lines enclosing the world line of the particle.

was assumed by de Broglie that in the rest frame of the particle, the periodic phenomenon comprises some form of standing wave. What we want to know is what these planes mean in the unprimed frame of the fixed observer.

But there is a feature of de Broglie's original drawing that could hinder any intuitive appreciation of how these equiphase planes transform between frames of reference. As drawn by de Broglie (as in Fig. 1), the planes are not centred, as would be natural, on the particle itself. Indeed they are so positioned that they could possibly suggest a wave propagating to the left of the diagram. In Fig. 2, the planes have thus been redrawn so that they are centred on the world line of the particle.

To complete the picture, the roles of primed and unprimed frames have been interchanged in a further diagram (Fig. 3). In this, it is the formerly fixed observer that now moves to the right.

However, we concentrate now on Fig. 2. In the unprimed frame, the equiphase planes are inclined to the x axis and thus display the asymmetry in phase and failure of simultaneity in the direction of travel predicted by special relativity. From this asymmetry, de Broglie calculated the velocity of the de Broglie wave. He first explained that as the line O_1x_1 represents the frame of the fixed observer at $t = 1$, the distance aa_0 is exactly c. He then says that at $t = 1$,

> The phase which at time $t = 0$ was at a is now at a_1. For the fixed observer it is then displaced in space by an amount a_0a_1 in the x direction, during one unit of time. One can then say that the

speed is:

$$V = a_0a_1 = aa_0 \cot \alpha_2 = \frac{c^2}{v}.$$

This is again the velocity of the de Broglie wave, but when de Broglie refers in this passage to the speed at which phase has become "displaced in space .... in the x direction", he is describing the velocity of a dephasing, that is to say, of a modulation.

To see that the transformed standing wave and the de Broglie wave are not the identical wave, it is only necessary to notice they do not have the identical representation in the diagram. In a spacetime diagram, such as those included here, a Lorentz transformation is a *passive* transformation under which an event or world line retains its location within the diagram[3]. Instead of the event or world line changing position, its coordinates differ according to the frame to which it is referred. Thus, if the de Broglie wave and the transformed standing wave were the same wave they would have the same world line in Fig. 2 (and Fig. 1).

But that is not so. Consider the standing wave. In its rest frame, a standing wave is, by definition, stationary. The world line of any point in that wave thus follows the time axis of its rest frame, which in this case is the ct' axis of the primed frame. Or we could consider instead some extended region within the wave, let us say a spherical region enclosing the particle. Instead of a world line, the passage through spacetime of such a region defines a "world tube" (see Misner, Thorne and Wheeler, p. 473), which in this case, and as shown in Fig. 2, encloses the world line of the particle and similarly follows the ct' axis of the primed frame .

On the other hand, the superluminal velocity c^2/v of the de Broglie wave implies a world line (or tube, but it will suffice now to speak of a line) lying beyond the light cone and parallel to the x' axis. Clearly then, the standing wave does not simply become the de Broglie wave, as was assumed by de Broglie.

That the world line of the de Broglie wave is parallel to the x' axis means that in the rest frame of the particle, this wave is of infinite velocity. That velocity would be anomalous in an independent wave, but acquires a natural explanation once the de Broglie wave is understood as a modulation. At rest, the crests of the underlying wave are no longer peaking in sequence, but in unison, simultaneity has been restored, alignment of phase has become instantaneous, and the velocity of the modulation describing the progress of that alignment has thus become infinite.

In effect, modulation and carrier have merged, and the de Broglie wave has disappeared.

[3] To show the Lorentz transformation as *active*, the diagram would include two depictions of the equiphase planes, that shown in Fig. 2 and that shown in Fig. 3.

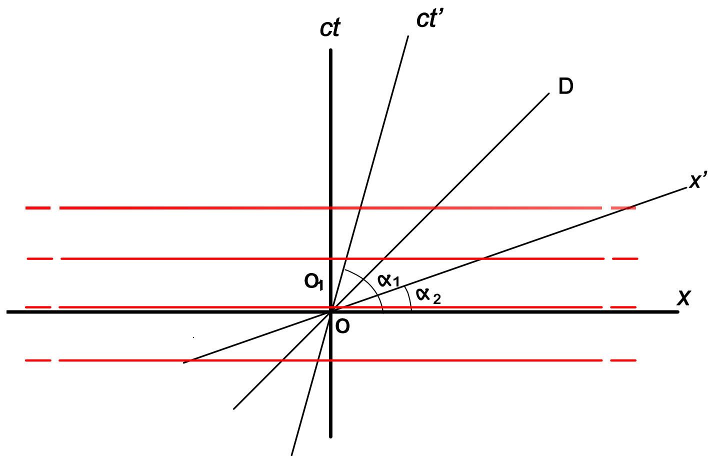

Figure 3: As Fig. 2, but with the roles of primed and unprimed frames interchanged. The formerly fixed observer is moving to the right with velocity v, while the unprimed frame is now the inertial frame of the particle. The four equiphase planes representing the standing wave are thus parallel to the x axis.

6 Discussion

It was shown in Sect. 2, the de Broglie wave cannot arise from the Lorentz transformation of an oscillating point particle. Thus de Broglie was correct in insisting that the particle is surrounded in its rest frame by some form of periodic phenomenon.

But it was also shown, in Sects. 3 to 5, that de Broglie was mistaken in concluding that this extended waveform becomes the de Broglie wave when Lorentz transformed. In none of de Broglie's three demonstrations does the de Broglie wave emerge as an independent wave. In the first, harmonizing of phases occurs only for a point within each wave and not for the wave as a whole. In the second (the toy model) and the third (the spacetime diagram), the de Broglie wave emerges from the antecedent standing wave, not as a wave in its own right, but as the modulation of an underlying carrier wave.

De Broglie did not explain the basis of his assumption that the periodic phenomenon that surrounds the particle is a standing wave. Perhaps he thought that this was obvious. Nor, in the dissertation, did he prescribe the form of this standing wave, although he was later to suggest that in the rest frame of the particle the de Broglie wave becomes a superposition of incoming and outgoing waves [13]. But clearly, this phenomenon is oscillatory and wave-like, and centred upon the particle, and as demonstrated in Sect 2., a standing wave of any form does produce as modulation the required de Broglie wave.

Once identified as a modulation, much that has seemed anomalous in the de Broglie wave is explained immediately. The superluminal velocity of the wave is no longer that of energy (or information) transport. Consistency with special relativity is thus achieved and there is no need to equate the velocity of the particle with the group velocity of a superposition of such waves. It can also be seen why the superluminal wave does not outrun the subluminal particle. A modulation must remain forever coextensive with the wave it modulates. As discussed in the preceding section, it is only natural that the velocity of this modulation should increase as the particle slows, and become infinite as the particle comes to rest.

But it is the suggestion of a deeper wave structure underlying the de Broglie wave that is likely to have the greater significance for quantum theory. In the mystery of wave-particle duality, the role of wave has been played solely by the de Broglie wave. If the de Broglie wave emerges from an antecedent wave, with which it must therefore share an ontology, it becomes possible to speculate that for a massive particle, this underlying wave, moving at the velocity of the particle, might explain both the wave and the particulate properties of the particle.

The existence of the underlying wave also questions the meaning of the wave function. The wave equations for massive particles - importantly, the Schrödinger, Klein-Gordon, and Dirac equations - were contrived from the wave characteristics of the de Broglie wave (see Ref. [1]). If the de Broglie wave is a modulation, so also must be, in some sense, the wave functions that emerge as solutions of these equations. And for a massive particle, the wave function is also of course the state vector or probability wave of standard quantum mechanics.

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