Logical Consequence:
An Epistemic Outlook

The *1st-order thesis*, namely, the thesis that logical consequence is *standard 1st-order logical consequence*,[1] has been widely challenged in recent decades. My own challenge to this thesis in *The Bounds of Logic* (and related articles[2]) was motivated by what I perceived to be its inadequate philosophical grounding. The bounds of logic are, in an important sense, the bounds of logical constants, yet the bounds of the standard logical constants are specified by *enumeration*, i.e., dogmatically, without grounding or explanation. Of course, how a given collection of objects is specified may change in the course of time, but my analysis of the role logical constants play in producing logical consequences led me to arrive at a criterion of logical constanthood whose 1st-order extension far exceeds the standard selection. More specifically, I showed that if we characterize logical consequence as necessary, formal, topic neutral, indifferent to differences between individuals, etc., then this characterization, restricted to languages of the 1st-level, is not adequately systematized by the standard 1st-order system. A richer system (or family of systems), with new logical constants, is required to fully capture it.

The Bounds of Logic had as its goal a critical, systematic and constructive understanding of logic. As such it aimed at maximum neutrality *vis-à-vis* epistemic, metaphysical and meta-mathematical controversies. But a conception of logic does not exist in a vacuum. Eventually our goal is to produce an account of logic that answers the needs of, contributes to the development of, and is supported by, a broader epistemology. In this paper I would like to make a first step in this direction. I will begin with an outline of a model of knowledge whose basic principles are based on the early Quine. I will identify, and offer independent justification for, the special requirements this model sets on an adequate conception of logic. Finally, I will show how, by satisfying these requirements, the conception

of logic delineated in *The Bounds of Logic* (and the related papers) can naturally be incorporated in this epistemic model.[3]

I. A Model of Knowledge

In "Two Dogmas of Empiricism" Quine drafted a model of knowledge based on two central theses: *the negative analytic-synthetic thesis* (NAS), which says that the traditional division of concepts, statements, theories and disciplines into analytic and synthetic is epistemically detrimental, and *the center-periphery thesis* (CP), which structures our system of knowledge as a body with two distinguished zones—metaphorically, a center and a periphery. Other, related principles underlying the model are: *no sharp boundary between science and metascience or science and metaphysics, uniformity of epistemic norms throughout the model, universal revisability and considerable latitude in revision, inseparability of language and theory, interconnectedness of knowledge, holism, the Duhem-Quine thesis, antifoundationalism, antireductionism, underdetermination of theory by experience, the Neurath-boat principle, realism, pragmatism, empiricism (in the natural and social sciences), etc.*[4] In Sher (1999b) I argued that while Quine's model is superior to more conventional empiricist models, there is a deep tension (or even conflict) between its two major theses, CP and NAS: Some traditional tenets rejected by NAS are brought back by CP. This return to traditional epistemology is manifested in the fact that CP construes the center and periphery of our system of knowledge as *fixed*. Logic, mathematics and philosophy are permanently and absolutely located in the center, experimental science in the periphery. One important ramification of this fixity of structure is the rebifurcation of standards of knowledge: sciences located in the center are subject to one set of standards, sciences located in the periphery to another. To resolve the conflict I suggested a change in Quine's model. Instead of a static and absolutist model, I proposed a dynamic, contextual model. The new model still has two distinguished zones, center and periphery, but the distribution of sciences between them is no longer fixed, either in time or in context. Instead, sciences and theories change their position both contextually and temporally: In some contexts, from some perspectives, logic is located in the center, physics in the periphery; in other contexts their locations shift. During some periods,

logic's main contribution to knowledge takes place in the center, during others, in the periphery. Before turning to logic, however, I would like to further elaborate on certain general issues concerning the motivation for, and the distinctive characteristics of, the new Quinean model.

NAS. The analytic-synthetic thesis (AS) divides statements (and theories) into two kinds: those whose truth is grounded in meaning (convention, pragmatic decision), and those whose truth is grounded in fact (experience, correspondence with reality). This thesis is often distinguished from overtly epistemic theses (e.g., the *a priori-a posteriori* thesis) as linguistic or, more precisely, semantic, but its epistemic import is almost immediate. If two types of statement differ radically in their truth conditions, then the (appropriate) methods for discovering, arriving at, verifying, falsifying, justifying them etc. are also drastically different. Analytic statements are verified (falsified) by consulting a dictionary, examining a conceptual scheme, introducing a convention or making a pragmatic decision; synthetic statements, in contrast, are verified (falsified) by looking at the world, conducting an experiment, exercising our power of intuition,[5] or using any other method which reveals the state and nature of things in the world. The unavoidable outcome is epistemic dualism: Our system of knowledge is divided into two zones—an "analytic" zone that contains logic, mathematics, methodology of science, meaning postulates and definitions, and a "synthetic" zone containing the natural and social sciences (or, rather, their factual segments). Each zone is assigned its own set of norms: *veridical norms*—truth, factuality, agreement with reality, correctness, accuracy, evidence, justification, verification (falsification), predictive force—for the synthetic sciences; *pragmatic* (or more generally *extra-veridical*) standards—simplicity, economy, convenience, utility, aesthetic value, richness of applications, learnability, informativeness, explanatory and unificatory power—for the analytic sciences.[6]

NAS rejects this dualistic methodology. Instead of *bifurcation* of epistemic norms it requires *unity* of norms. This unity has two components: (i) all sciences are subject to the same epistemic standards, and (ii) each science is subject to both veridical and pragmatic standards. Quine's epistemic revolution consists in replacing a methodology characterized by *external dualism* (two sets of standards) and *internal monism* (each set consists of one type of standard) by a methodology characterized by

external monism and *internal dualism*. The result is a stricter code of knowledge throughout science: Natural and social sciences are subject not just to veridical norms but also to pragmatic norms, and logic, mathematics and philosophy are subject not only to pragmatic norms but also (and just as much) to veridical norms.

This, in my view, is the main epistemic innovation of NAS. The AS methodology divides the sphere of knowledge into two zones: a zone in which we do, and a zone in which do not, envision conflicts with reality. The former is characterized by a proactive attitude towards conflict (i.e., taking steps to avoid, dispel, overcome and, if need be, recover from conflict), the latter by a passive, complacent posture *vis-à-vis* the possibility of conflict. The assumption is that logic, mathematics, and philosophy (the analytic sciences) cannot be challenged by reality, therefore agreement with reality (getting things *right*) need not concern these sciences. The NAS methodology rejects this attitude. There is no way of determining in advance what fields of knowledge nature will or will not challenge, and therefore logic, like physics, must orient itself towards truth, fact, readiness for conflict. In short, logic, mathematics and philosophy cannot exempt themselves from the veridical norms.

CP. CP plays an altogether different role in Quine's theory. Its role is to shield the Quinean model from two pitfalls of holism: (i) being structureless, and (ii) being anchorless in reality. (i) *Structureless holism*: It is in the nature of holistic models that they deny the division of knowledge into strictly bounded, independent and self-sufficient units. In the extreme case, holism denies the division of knowledge into any identifiable parts. This leads to several interrelated difficulties: (a) "If a total theory is represented as indecomposable into significant parts, then we cannot derive its significance from its internal structure, since it has none; and we have nothing else from which we may derive it." (Dummett 1973a: 600) (b) An unstructured model is unexplanatory. To give an explanatory account of a broad and complex object (like our total system of knowledge) we have to break it into significant parts, delineate their interrelations and identify their distinctive contribution to the given whole. An unstructured model cannot do any of these. (c) Extreme holism renders the acquisition of knowledge impossible. Knowledge can be acquired by humans only one step at a time, but an unstructured body of knowledge can be grasped only

in its entirety or not at all, i.e., it cannot be acquired in ways available to humans. (ii) *Anchorless holism*: Holism can easily slide into coherentism, and coherentism has difficulty with anchoring knowledge in reality. In fact, coherentism is compatible with giving up the requirements of truth and evidence altogether, embracing a fictional story (satisfying the minimal requirement of inner coherence) as science.

CP safeguards Quine's model against these pitfalls by (i) introducing structure into his holism, and (ii) anchoring the model in experience. (i) *Structured holism*. CP introduces structure into Quine's model by dividing the sphere of knowledge into two (or three) zones: center, periphery (and intermediate region). The center is the center of interconnectedness, the periphery—the outer boundary of the system. Units in the center are connected to their counterparts throughout the system by a network of cognitive passageways, different units occupying different positions within the network. In this way each statement, theory, and discipline acquires a unique identity and relative autonomy, making up a learnable, explainable and significant whole. (ii) *Anchored holism*. CP's second task is linking our system of knowledge to the reality, and this it does by introducing the periphery. Our theories are tested for agreement with reality in the periphery, and these tests guarantee that our corpus of knowledge consists of not just any internally-consistent "theories", but of theories whose agreement with reality is checked by a battery of tests, utilizing (not exclusively, but significantly) some direct means of relating to reality, e.g., acts of sense perception.

An Inner Conflict. In spite of its indubitable merits, Quine's model is marred by inner conflict. The crux of the matter is the rigidity of the center-periphery dichotomy. I will point out three dimensions of the conflict, *extensional*, *conceptual*, and *normative*.

Extensionally, the division of statements, theories and disciplines into those located in the center, those located in the periphery and those located in the intermediate zones simulates their traditional division into analytic and synthetic. Logic, mathematics and philosophy reside in one area—the center; physics, biology, and other empirical sciences in another—the periphery and intermediate zones. This division is very similar to the logical-positivist division of our corpus of knowledge into science (proper) and meta-science, with logic, mathematics and philosophy allocated to the

latter, the natural and social sciences to the former. This division of the sciences, however, is rejected by NAS, whence its extensional conflict with CP.

Conceptually, the conflict arises (according to Dummett) as follows:

> In accordance with [NAS], the revision of truth-assignments to the sentences of the language which is elicited in response to a recalcitrant experience may not affect any of the peripheral sentences, but only those lying below the periphery. But, if this is so, then, it seems, experience does not impinge particularly at the periphery; rather, it impinges on the articulated structure . . . as a whole, not at any one particular point. In that case, it becomes difficult to see how we can any longer maintain a distinction between periphery and interior: the periphery was introduced as that part of the structure at which the impact of experience is first felt. (Dummett 1973b: 376–77)

Normatively, the main tenet of NAS is the uniformity of standards of knowledge: the standards for the acceptance, rejection, and revision of logical, mathematical and philosophical theories are essentially the same as those for theories of the natural and social sciences. CP reintroduces a bifurcated system of norms. Notwithstanding the possibility of overcoming conflicts in the periphery by changes in the center, the standards for the revision of statements in the center are essentially different from those for revising statements in the periphery. Revision of a peripheral statement P (say, an observation statement) is judged by its success in resolving conflicts between P *itself* (the claim made by P) and experience; revision of a central statement, C, (say, a logical statement,) is guided by its success in resolving conflicts between *other* statements (e.g., P) and experience. A peripheral statement is true due to its own direct link with reality, a central statement is true due to direct links between other statements, namely peripheral statements, with reality. To accept or reject P we set up tests concerning the objects and properties referred to by P. (In the simplest case, we check whether the observable objects referred to by P have the observable properties attributed to them by P.) But to accept or reject C we consider how this would affect the overall working of the system, including its ability to handle conflicts involving peripheral statements (like P). The standards for handling statements and theories in the periphery are factual and evidential, those for handling statements and theories in the center, pragmatic and instrumental.[7]

The contrast between NAS and CP is especially sharp with respect to logic: CP's conception of logic is essentially traditional, NAS's, icono-

clast; CP views logic as instrumental, NAS, as substantive and factual; CP regards logic as subject to pragmatic norms, NAS as subject to veridical as well as pragmatic norms.

Solution to the Conflict. My solution to the inner conflict in Quine's model (Sher 1999b) is to replace its fixed, stationary conception of the center and the periphery by a dynamic, contextual conception. The center and the periphery still exist as symbolic representations of complementing factors in knowledge: fact and convention, language and reality, mind and world, space of reasons and space of causes, evidence and pragmatic support, discovery and conceptual resources, truth and interconnection, front and rear of the battle for knowledge, fixed and changeable constituents of knowledge, veridical and non-veridical epistemic norms, etc; but concepts, statements, theories and disciplines are no longer located in fixed locations within the model. Instead, their position shifts from the center to the periphery, from the periphery to the intermediate zones, from those to the center, and so on. The movement of epistemic units takes place along two axes: the axis of *context* and the axis of *time*. In some contexts our interest in physics centers on what it tells us about the world, in others (e.g., that of Thagard's *Conceptual Revolutions*), on its generation of ever new systems of concepts; during certain periods in the history of science advancements (or, more neutrally, changes) in physics are mainly factual, during others (e.g., during scientific revolutions, according to Kuhn), largely conceptual. A similar duality holds for other sciences, including logic. Normatively, the mobility of disciplines enables CP model to meet the requirement that each branch of knowledge be subject to two types of standards: veridical standards, metaphorically associated with the periphery, and pragmatic standards, associated with the center. Thus, as a peripheral discipline, logic is bound by the norms of truth and evidence, as a central discipline—by those of economy and unification.

The new model sets two substantive tasks for an adequate philosophy of logic: (1) Explain and justify the claim that logic is world-oriented, subject to veridical standards, lies in the periphery, etc. (2) Explain and justify the claim that logic is interconnected to other branches of knowledge, plays a unificatory role in our system of knowledge, is subject to pragmatic norms, etc. In short, an adequate philosophy of logic must confront the deep questions of epistemology as they pertain to logic.

In Section II I will motivate the view that logic is world-oriented from a perspective internal to the philosophy of logic. In Section III I will

explain how the theory delineated in *The Bounds of Logic* can be regarded as a systematization (and vindication) of this view. In Section IV I will discuss one of logic's roles as a "central" discipline, namely, unification. I will show that far from undermining the claim that logic plays a central unificatory role in science, the view that logic is world-oriented and its systematization as recounted in Section III enable us to understand why and how it plays this role.

II. Logic and the World

The view that logic is "world-oriented" is based not just on general epistemic grounds but also on considerations pertaining to logic itself. I will now present a few considerations of this kind, starting with a simple, common-sensical observation and continuing with more abstract and methodological reflections.

A. *Logical theory, like physical theory, is correct or incorrect in the straightforward sense that it either "works" or "doesn't work" in the world.* It is a simple and straightforward observation that in the same way that the use of, say, defective aerodynamical principles may cause an airplane to crash (or fail to take off in the first place), so the use of defective logical principles may result in an airplane crashing. This is not to say that we have no latitude in constructing our logical (or physical) theories, but that there is a very real sense in which a logical (or a physical) theory either works or doesn't work in, e.g., sending a rocket into space, predicting the weather, raising crops, etc. For the sake of illustration, let us create a simplified scenario of how (in the absence of corrective measures) logic can bring down an airplane.

Let s be a state of an airplane of type t (say, a jetliner flying horizontally under normal conditions), and suppose we set out to formulate rules for getting a lift effect in airplanes of type t at state s. Using certain physical, logical and mathematical facts and laws, one of the rules we arrive at is:

> R1. *To achieve a lift effect in airplane of type t at state s, set the flaps at a **small** downward angle.*

One way to arrive at *R1* is by the following chain of reasoning: Let 'A', 'Sm', 'L', 'd' and 'l' stand for 'Airplane of type t is at state s', 'Flaps are set at a small downward angle', 'There is a lift effect', 'the drag coeffi-

cient' and 'the lift coefficient', respectively, and let 'm' and 'n' represent two real numbers—specifically: the values of d and 1, respectively, when A and Sm hold. We show that by bringing about Sm (formally, introducing 'Sm' as an assumption) we obtain L (formally, derive 'L').

1. A	*Given*
2. Sm	*Assumption*
3. d<1→L	*Physics*
4. A&Sm → (d=m & 1=n & m<n)	*Physics; Mathematics*
5. A & Sm	*Logic (CI: $\Phi, \Psi \Rightarrow \Phi \& \Psi$)*
6. d=m & 1=n & m<n	*Logic (MP: $\Phi, \Phi \to \Psi \Rightarrow \Psi$)*
7. d=m	*Logic (CE3: $\Phi \& \Psi \& \Xi \Rightarrow \Phi / \Psi / \Xi$)*
8. 1=n	*Logic (CE3)*
9. m<n	*Logic (CE3)*
10. d<n	*Logic (Id: $\Phi(a), a=b \Rightarrow \Phi(b)$)*
11. d<1	*Logic (Id)*
12. L	*Logic (MP)*

Assuming the physical and mathematical facts and laws used in this chain of reasoning are correct, the logical laws *CI, MP, CE*3 and *Id* lead us to a valid rule of aviation (for horizontally flying jets under normal conditions). But suppose instead of *Id*, we used a defective logical law, say, *Id**: $\Phi(a), a \neq b \Rightarrow \sim\Phi(b)$. Keeping the standard physics and mathematics unchanged, we would be led to accept a defective rule of flight:

> *R2. To achieve a lift effect in an airplane of type t at state s, set the flaps at a **large** downward angle.*

With 'Lr' standing for 'Flaps are set at a large downward angle', we would arrive at *R2* by the following chain of reasoning:

1. A	*Given*
2. Lr	*Assumption*
3. d<1 → L	*Physics*

4. A&Lr $\rightarrow$ (d $\neq$ m & l $\neq$ n & m<n)	*Physics; Mathematics*[8]
5. A&Lr	*Logic (CI)*
6. d $\neq$ m & l $\neq$ n & m<n	*Logic (MP)*
7. d $\neq$ m	*Logic (CE3)*
8. l $\neq$ n	*Logic (CE3)*
9. m<n	*Logic (CE3)*
10. ~(d<n)	*Logic (Id*)*
11. ~~(d<l)	*Logic (Id*)*
12. d<l	*Logic (DN: ~~$\Phi \Leftrightarrow \Phi$)*
13. L	*Logic (MP)*

In fact, however, lowering the flaps to a large angle has a drag effect. Everything else being equal, replacing one logical law by another could cause an airplane to plummet.
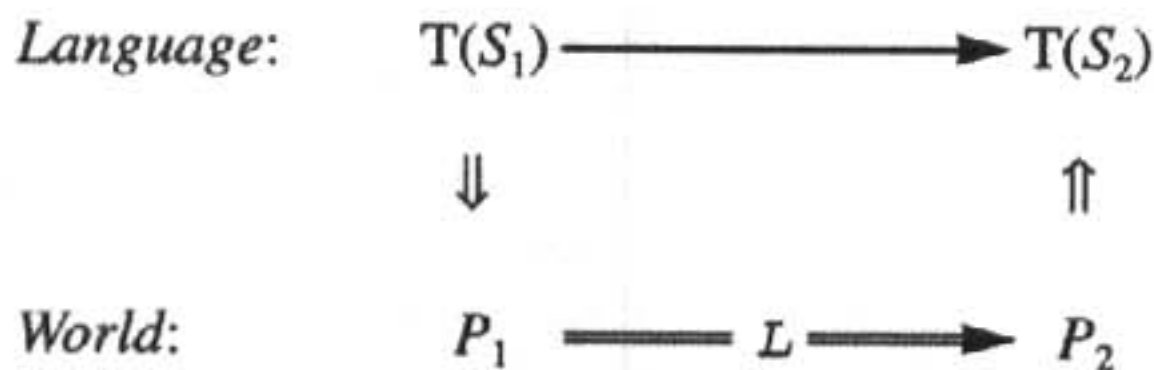
B. "*Logical theory . . . is . . . world-oriented rather than language-oriented; and the truth predicate makes it so.*" (Quine 1970: 97, my italics) Another, more theoretical reason for viewing logic as oriented towards the world is its intimate connection with truth. Truth, as Tarski, Quine and many others (including myself) understand it, is correspondence (of some kind) with reality; logical truth, as a particular type of truth, exhibits a particular kind of correspondence. To understand what kind of correspondence pertains to logic, I will proceed by analyzing the relation of logical consequence.

Logical consequence is a particular kind of consequence and consequence relations in general are relations of preservation, or transmission, of truth. To understand in what sense logical consequence is a correspondence relation we have to understand the principles governing the logical transmission of truth. I will progress in two steps: first I will discuss the principles governing consequence relations in general, then those governing logical consequence.

(a) *Consequence relations*. To understand the principles governing consequence relations in general as relations of transmission of truth, we have to start with a general understanding of truth. As I have indicated above, my starting point is the view that truth, in general, is based on cor-

respondence between language and the world. By this I mean not the iso-morphism thesis or the picture theory of language, nor commitment to Platonism or naturalism in mathematics, but just the simple Aristotelian principle that to say of what is that it is or of what is not that it is not is true, while to say of what is that it is not or of what is not that it is, is false. Part of our task is to figure out how this principle works in logic; for the time being, the general idea will suffice.

Based on this idea, I propose the following analysis of the connection between premises and conclusion in a valid argument (consequence). Let L be a law connecting phenomena (i.e., structures of objects, properties and relations) of types t_1 and t_2 within a certain space of possibilities, P, in such a way that the occurrence of a phenomenon of type t_1 guarantees (within the space P) the occurrence of a matching phenomenon of type t_2. Let P_1 be a phenomenon of type t_1, P_2 a matching phenomenon of type t_2, S_1 a sentence saying that P_1 occurs, and S_2 a sentence saying that P_2 occurs. Suppose S_1 is true. Then its truth is transmitted to S_2. In other words: If, in the world, phenomena of type t_1 are, as a matter of law (L) accompanied by phenomena of type t_2, then "the world", so to speak, licenses the passage from statements asserting the occurrence of phenomena of type t_1 to statements asserting the occurrence of (matching) phenomena of type t_2. What is the scope and force of this license?—The scope and force of L. Figuratively, we may represent the process of transmitting truth from S_1 to S_2 by:

$$\textit{Language:} \qquad T(S_1) \longrightarrow T(S_2)$$
$$\Downarrow \qquad\qquad \Uparrow$$
$$\textit{World:} \qquad P_1 \overbrace{} L \longrightarrow P_2$$

The truth of S_1 guarantees the occurrence of P_1; given P_1, L guarantees the occurrence of P_2; and the occurrence of P_2 guarantees the truth of S_2. The questions a philosophical theory of logic must answer are: (i) What kind

of law is L in the case of *logical* consequence? (ii) What is the process whereby truth is transmitted from sentences to sentences by laws of this kind?

(b) *Logical consequence.* Here the theory presented in *The Bounds of Logic* enters into our discussion. I will expound the basic principles of this theory in the next section, but briefly the answers to the above questions are: (i) The laws licensing logical inferences are *formal* or *structural*, i.e., laws governing the formal (structural) behavior of objects and structures of objects. (ii) The logical transmission of truth from a sentence (or set of sentences), S_1, to a sentence, S_2, involves four basic steps: (α) identifying the logical contents of S_1 and S_2, $1(S_1)$ and $1(S_2)$, respectively; (β) identifying the formal situations described by $1(S_1)$ and $1(S_2)$) and $F(1(S_1))$ and $F(1(S_2))$, respectively; (γ) connecting $F(1(S_1))$, and $F(1(S_2))$ by a formal law, and (δ) relating S_2 to S_1 by the relation of logical consequence.

C. *The normativity of logic.* Logic is a normative science and its normativity is greater than that of most other sciences. Physics, biology, psychology (as well as many other sciences) are all bound by the norms of logic, but logic is (for the most part) not bound by theirs.[9] This unique situation places a special burden on the philosophy of logic. It is impossible to understand our system of knowledge without understanding the source, force and scope of the logical norms; therefore a theoretical explanation of the normativity of logic is imperative. One of the advantages of viewing logic as world-oriented (along the lines suggested above) is the opportunity it provides of explaining the normativity of logic. A detailed account of this explanation must await the discussion in Section III below, but briefly, and non-specifically, the main points are these: (i) the normativity of logic has its roots in the truth and lawlikeness of the laws underlying logical consequence, i.e., formal laws; (ii) the scope of the logical norms is the scope of these laws; (iii) the power of the logical norms is due to the modal status of the formal laws.

The great force and scope of the logical norms saddles us with another burden: a critical approach to logic. The main point is this: An error in most disciplines would have relatively narrow repercussions for our system of knowledge, but an error in logic would threaten the entire system. Therefore, the development of critical tools for evaluating, establishing and improving the correctness of our logical theories is mandatory. Many, however, deem a critical outlook on logic to be impossible. Three popular views sharing this attitude are: (i) the view, due to the early Wittgenstein,

that there is no standpoint outside logic, therefore a critical examination of logic is impossible; (ii) the conventionalist view according to which logic is conventional, hence there is no possibility of *error* in logic (*Id**, for instance, is not erroneous but merely inconvenient), hence there is no possibility of detection or correction of error in logic; (iii) the view (wrongly associated with Quine, in my opinion) that the logical "laws" are sanctioned by their *obviousness* and as such are not open to rational criticism.

The wide appeal of these views is not surprising given the prevalence of foundationalist trends in philosophy. From a foundationalist perspective, logic lies at the base of the epistemic hierarchy, therefore a rational critique of logic is impossible. Such a critique would require conceptual resources more basic than those of logic, but foundationalism rules out the existence of such resources. The foundationalist view, however, is problematic. If logic lies at the bottom of the epistemic hierarchy, then a mistake or even an omission in logic is all the more likely to have dire consequences for the whole edifice; therefore, the impossibility of a critical approach to logic is a serious cause for concern. This, in my view, is the Achilles heel of foundationalism: The fields that, due to their position in the foundationalist hierarchy, are in greatest need of a critical underpinning, are just those that, as a matter of principle, cannot be given such an underpinning.

In contrast to the above views, the view that logical consequence is grounded in laws of objects allows the possibility of error in logic and, when combined with a holistic approach to knowledge, also the possibility of constructing a critical foundation for logic (a foundation not in the foundationalist sense, but in the sense of Shapiro's "foundation without foundationalism").[10] An error in logic, on this view, is an error in the underlying formal laws, and to avoid, detect, and correct such an error is to avoid, detect and correct the corresponding error in the putative formal law. To flesh out this and other points made in the last two subsections, let us now turn to the conception of logic delineated in *The Bounds of Logic*.

III. A Conception of Logic

In *The Bounds of Logic* I proposed a conception of logic whose main principles are:

1. Logical consequences are necessary and formal.

2. Logical constants rigidly refer to formal operators.

3. Formal operators are characterized by an invariance criterion which says that an operator is formal iff it does not distinguish the identity of individuals within and across universes. Using the resources of contemporary mathematics we can formulate this criterion as follows;

Formality Criterion:[11] *An operator is formal iff it is invariant under isomorphisms of argument-structures.* I.e.:

Let O be an operator, i.e., a function that to each universe (non-empty set of objects considered individuals), U, assigns a function O_U, such that for any element (or construct of elements) of U of a given type associated with O—a member of U, a pair of members of U, a triple of members of U, a subset of U, a pair of subsets of U, a binary relation on U, etc.—assigns a truth-value, T or F. O is formal iff for any structures $<U, \beta>$ and $<U', \beta'>$, where U and U' are universes and β and β' are arguments of O in U and U', respectively: if $<U, \beta>$ and $<U', \beta'>$ are isomorphic, then $O_U(\beta) = O_{U'}(\beta')$.[12]

4. It follows from the formality criterion that the collection of 1st-order formal operators far exceeds the standard collection of formal operators, i.e., the collection of operators corresponding to the standard logical constants. Some examples of non-standard formal operators are "finitely many", "indenumerably many", the 1- and 2-place "most" operators (appearing in "Most things are B" and "Most B's are C's", respectively), the "well-ordering" operator, etc.

5. The definition of logical consequence is the standard Tarskian definition; the notion of model is the standard notion.

6. Models represent formally-possible states of affairs (*vis-à-vis* a given language); the totality of models represents the totality of such states of affairs.

7. Models are governed by laws of formal structure, i.e., laws describing the formal behavior of objects and structures of objects in any formally-possible state of affairs.

8. The totality of models and the laws governing them are determined by a background theory of formal structure (part of mathematics).

9. A logic is a family of families of logical systems. Each family of logical systems consists of a logical unit and a family of extra-logical units. The logical unit consists of a complete set of logical connectives and a non-empty set of logical constants (other than connectives), their referents, i.e., truth-functional and formal operators, and a background system of formal laws governing these operators. Each extra-logical unit consists of a non-logical vocabulary and an apparatus of models in which the non-logical constants of this vocabulary receive all formally-possible denotations (of the type corresponding to their syntactic category). A logical system consists of a logical and an extra-logical unit.
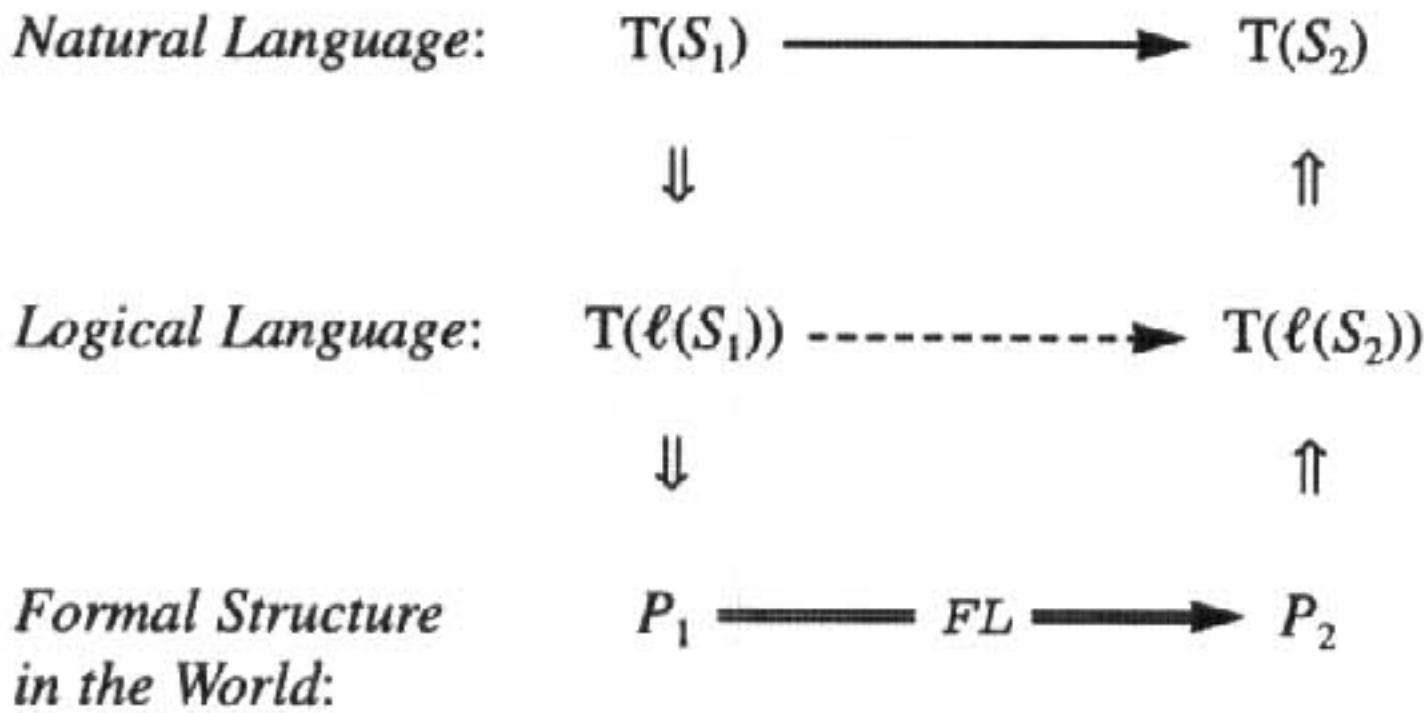
10. A family of logical systems may be assigned a sound-proof system.

This, of course, is a very rough and incomplete outline of the conception delineated in the *Bounds of Logic*, but it suffices to show how this conception systematizes the ideas expressed in Section II above, opens a critical venue to logic, and completes the explanations of (i) why and how logic works/fails to work in the world, (ii) how logical consequence transmits truth from premises to conclusion, (iii) what is the source of the normativity of logic and why the logical norms overpower most other norms. Briefly:

(i) *Logical axioms and rules of inference work in the world* because, and to the extent that, they are based on universal laws of formal structure, i.e., laws governing the formal behavior of objects in every formally-possible state of affairs (*vis-à-vis* a given language). When this condition is not fulfilled, the axioms and rules may fail. Thus, *Id* works in the construction of an airplane because the objects (including properties) involved in its construction all satisfy Leibniz's Law; *Id** fails in the construction because at least some of the relevant objects (properties) do not obey the putative law corresponding to it.

(ii) *Logical consequence transmits truth from premises to conclusion* in a process involving four basic steps: (α) identifying the logical "skeleton" of the premises and conclusion, (β) identifying the formal situations described by the respective skeletons of the premises and conclusion, (γ) connecting these formal situations by means of an appropriate formal law, and (δ) concluding that it is formally necessary that if

the premises are true, the conclusion is also true. Figuratively, let S_1 and S_2 be the premise and conclusion of a logically valid argument. Truth is transmitted from S_1 to S_2 as follows:

Natural Language: $T(S_1) \longrightarrow T(S_2)$

$\Downarrow$ $\Uparrow$

Logical Language: $T(\ell(S_1)) \dashrightarrow T(\ell(S_2))$

$\Downarrow$ $\Uparrow$

Formal Structure $P_1 \Longrightarrow FL \longrightarrow P_2$
in the World:

where $\ell(S)$ is the translation of S into a logical language and FL is a formal law connecting P_1 to P_2.

To see how this template is instantiated, consider the following two logical inferences:

(1) Every child deserves free education. Therefore: Every child either deserves free education or deserves a free trip to Disneyland,

and

(2) Some gangster is feared by most gangsters. Therefore: Most gangsters fear some gangster.[13]

The four basic steps in the logical transmission of truth are instantiated as follows:

Inference (1):

(α) S_1 = "Every child deserves free education", S_2 = "Every child either deserves free education or deserves a free trip to Disneyland".

(β) $\ell(S_1)$ = "$(\forall x)(Ax \supset Bx)$", $\ell(S_2)$ = "$(\forall x)[Ax \supset (Bx \lor Cx)]$". (Logical constants: "$\forall$", "$\supset$", "$\lor$".)[14]

(γ) P_1 = Situation in which a subset A of a universe U is included in a subset B of U.

   P_2 = Situation in which A is included in the union of B and a subset C of U.

   FL = The law saying that whenever a set A is included in a set B (in any given universe U), A is included in the union of B with any set C (in U).

(δ_1) $\{(\forall x)(Ax \supset Bx)\} \Rightarrow (\forall x)[Ax \supset (Bx \lor Cx)]$.

(δ_2) "Every child deserves free education" *logically implies* "Every child either deserves free education or deserves a free trip to Disneyland".

Inference (2):

(α) S_1 = "Some gangster is feared by most gangsters", S_2 = "Most gangsters fear some gangster."

(β) $\ell(S_1)$ = "$(\exists x)(My)Rxy$", $\ell(S_2)$ = "$(My)(\exists x)Rxy$". (Logical constants: "$\exists$" and "M".)[15]

(γ) P_1 = Situation in which a binary relation R over a universe U is such that $\{a \in U: Rng(a \upharpoonright R)$ is larger than its complement in U$\}$ is not empty.

   P_2 = Situation in which R over U is such that $\{b \in U: Dom(R \upharpoonright)$ is not empty$\}$ is larger than its complement in U.

   FL = The law saying that whenever a 2-place relation R in any universe U satisfies the condition C_1 below it satisfies the condition C_2 below, or that the set of relations R on U satisfying the condition C_1 is included in the set of relations R on U satisfying the condition C_2. Where:

   C_1 is the condition "$\{a \in U: Range (a \upharpoonright R)$ is larger than its complement$\}$ is not empty".

   C_2 is the condition "$\{b \in U: Dom(R \upharpoonright b)$ is not empty$\}$ is larger than its complement.

(δ_1) $\{(\exists x)(My)Rxy\} \Rightarrow (My)(\exists x)Rxy$

(δ_2) "Some gangster is feared by most gangsters" *logically implies* "Most gangsters fear some gangster".

In both examples, the truth of the premise (assuming it is true) is transmitted to the conclusion in virtue of a formal law connecting the respective situations described by them. Since formal laws, as laws holding in all formally possible states of affairs, have the modal status of formal necessity, so do the consequences based on them.

(iii) The source of *the normativity of logic* is the truth of the formal laws underlying logical consequences. Any field subject to these laws is subject to the norms based on them. For example, any field subject to the law that an object cannot be in the intersection of a set and its complement (in a given universe) is subject to the logical norm of non-contradiction. Now, following Tarski (1966), we can distinguish different types of objects by their invariance properties. The stricter the invariance conditions a given type of object satisfies, the more aspects of the world it is oblivious to, and the wider its domain. Formal objects satisfy stronger invariance conditions than, say, physical or biological objects, and this explains why their domain is larger than that of other objects and the laws governing them have a broader field of application. Speaking in terms of operators (any object can be viewed as an operator), we can explain the difference between formal and, say, physical or biological operators as follows: (i) While formal operators do not distinguish different ontologies, e.g., an ontology of numbers and an ontology of atoms, so long that they are formally alike (i.e., have the same number of elements), physical and biological operators do. Thus, biological operators are not invariant under replacements of living cells by atoms, and physical operators are not invariant under replacements of atoms by numbers, regardless of how formally similar the resulting structures are. But formal operators are invariant under such replacements, provided the resulting structures are formally identical (i.e., isomorphic). (ii) Even with respect to a single universe, physical and biological operators are not invariant under all replacements of isomorphic argument-structures, while formal operators are. Thus, let U* be a universe of oxygen and hydrogen atoms, and consider the physical predicate "$x_1, x_2,$ and x_3 form a water molecule". The operator denoted by this predicate, call it W, assigns a given triple of individuals (in any given universe) the value T if its elements form a

molecule of water, F otherwise. It is easy to see that W is not invariant under isomorphisms of argument-structures, even when they share the same universe, U^*. The argument-structures $<U^*, h_1, h_2, o_1>$ and $<U^*, h_1, o_1, o_2>$, where h_1 and h_2 are hydrogen atoms and o_1, o_2 are oxygen atoms, are isomorphic, yet $W_{U^*}(h_1, h_2, o_1) = T$ while $W_{U^*}(h_1, o_1, o_2) = F$. This explains why the scope of the logical norms—i.e., those based on formal laws—is so much broader than that of the physical or biological norms. Formal laws hold in any universe, regardless of whether its ontology consists of living cells, atoms or numbers, but physical and biological laws, hence the norms based on them, do not (in general) hold outside the physical domain (the former) and the biological domain (the latter).

IV. The Unifying Force of Logic

It is common to think of a theory's pragmatic virtues as divorced from its veridical virtues. In the case of logic, it is common to think that its success as a unifier has nothing to do with its truth, indeed, that it somehow marks it as a discipline for which the question of truth (agreement with reality) does not arise. Logic's pragmatic advantages are connected with its conventional nature, and conventionality rules out truth by correspondence.

The idea that unifying power is incompatible with correspondence is, however, illusory. Not only is logic's agreement with reality compatible with its having a considerable unifying power, but its special way of agreeing with reality explains why it has this power. The explanation is straightforward:

1. Logic provides a method of transmitting truth from sentences to sentences based on the formal laws connecting the situations described by these sentences.

2. Formal laws are universally applicable.

3. Therefore, the logical method is universally applicable.

4. A universal method of transmitting truth—i.e., a method of transmitting truth that applies within and across all fields of knowledge—has a strong unifying effect. This explains why (and how) logic makes a special contribution to the unity of knowledge.

So, logic's unifying power is due to the universality of the logical method, and the universality of the logical method is due to the universality of the formal laws on which it is based, i.e., the laws governing the formal behavior of objects and systems of objects in the world. But it is logic's grounding in these laws that constitutes, according to the explanation given in *The Bounds of Logic*, its grounding in the world. Thus, the explanation of logic's grounding in the world is, at the same time, an explanation of its unifying force in knowledge.

While logic's grounding in the world does not explain all its pragmatic virtues, it does not conflict with them either. Simplicity, economy, convenience, utility, aesthetic value, richness of theoretical results, practical applications, learnability, informativeness, explanatory power—are all norms that apply throughout science, notwithstanding the worldly orientation of many of its disciplines.

V. Conclusion

We have seen how the conception of logic delineated in *The Bounds of Logic* can be naturally incorporated in the Quinean model of knowledge outlined above: how logic, on this conception, is both central and peripheral, both guided by pragmatic norms and subject to veridical norms, both grounded in the world and playing a central unifying role in knowledge. Other ways in which this conception is compatible with the model require further discussion. I will conclude with three brief points:

(i) *The Revisability of Logic*. Logic, according to the Quinean model, is revisable on pragmatic as well as factual grounds. It is easy to explain the pragmatic element involved in the revision of a logical theory, but the factual element is more difficult to account for. The conception of logic delineated in *The Bounds of Logic* provides the ingredients for such an explanation. Briefly: A logical theory is a theory of transmission of truth based on laws governing the formal behavior of objects. The central factual question involved in the construction of a logical theory is, therefore: What laws govern the formal behavior of objects? Different answers to this question give rise to different logical theories, and given a logical theory, the question arises: Is its underlying system of formal laws adequate? This question, too, has both pragmatic and factual components. Its main factual component is two-faceted: (a) Are the underlying formal laws correct? I.e.: Do they correctly describe the formal behavior of objects

in the world? (b) Are they genuine laws? I.e., Do they hold in all formally possible structures of objects? A negative answer to either question would motivate a (factual) revision in logic.

This account enables us to explain a few well-known proposals for revision in logic as motivated by factual considerations: the proposal for replacing the (standard) 2-valued Boolean logic by a 3-valued logic, proposals for replacing Boolean logics by a non-Boolean logics (e.g., the standard 2-valued Boolean logic by the non-Boolean logic of quantum-mechanics), proposals for replacing the (standard) ZFC-based 1st-order logic by a non-ZFC-based 1st-order logic, etc. The factual considerations motivating these revisions may take the form of such conditionals as: "If the formal laws underlying the behavior of properties in the world are those of a 3-element algebra rather than a 2-element Boolean algebra, then we ought to adopt a 3-valued logic"; "If the formal laws underlying the behavior of properties are not Boolean, we ought to develop a non-Boolean logic"; "If structures of objects are not governed by laws gleaned from ZFC, the standard semantics of 1st-order logic should be changed". Tarski may have intended something along those lines when he said:

> I think that I am ready to reject certain logical premisses (axioms) of our science in exactly the same circumstances in which I am ready to reject empirical premisses (e.g., physical hypotheses). . . . It depends on the nature of experience what we reject—a rather special law which is an 'inductive generalization' of individual statements, or a more general and profound hypothesis, or even one of fundamental premisses of our science (e.g., the rejection of Newton's mechanics or Euclidean geometry). Axioms of logic are of so general a nature that they are rarely affected by such experiences in special domains. However, I don't see here any difference 'of principle'; I can imagine that certain new experiences of a very fundamental nature may make us inclined to change just some axioms of logic. And certain new developments in quantum mechanics seem clearly to indicate this possibility. (Tarski 1944: 31–32)

(ii) *A Holistic Conception of Logic.* The conception of logic delineated in *The Bounds of Logic* is holistic on two levels: (i) the conception of logic (as a philosophical theory) is holistic, and (ii) logic itself is conceived as a holistic discipline. Logic is conceived as a holistic discipline in two senses: (a) the acceptance (rejection, revision) of a given logical theory is partly based on pragmatic considerations, and (b) to the extent that it is based on factual considerations, these considerations

involve another discipline, namely, mathematics, or more specifically, that part of mathematics that deals with the formal behavior of objects in general. The conception of logic, as a piece of philosophy, is holistic in three ways: (a) It is formulated in terms taken from other philosophical theories, including epistemology, the theory of truth, ontology, the philosophy of mathematics and mathematics proper. (b) It is formulated in a meta-language equipped with a rich logical apparatus. (c) It is partly motivated by pragmatic considerations: explanatory power, contribution to a broader epistemology, richness of theoretical results, fruitfulness of applications, overall philosophical economy, etc. The last point deserves special consideration.

One way to achieve philosophical economy is by reducing the number of tasks philosophy has to perform. For example, by reducing mathematics to logic we reduce the two tasks of constructing a foundation for logic and constructing a foundation for mathematics to the one task of constructing a foundation for logic. Such an economic gain was promised by Logicism, but its promise did not materialize. Two obstacles that stood in the way (besides the well-known problems with the reduction itself) were: (i) the scarcity of foundational accounts of logic, (ii) the impossibility of providing a foundation for logic within a foundational epistemology. The second obstacle is more serious: by espousing a foundationalist epistemology and placing logic at the bottom of the foundational hierarchy, logicism ruled out a foundation for logic in principle.

In contrast, the account of logic in *The Bounds of Logic* offers a genuine reduction in the tasks of philosophy. This gain is achieved by reversing the direction of the logicist reduction and changing its orientation. Instead of reduction of mathematics to logic, it offers a reduction of logic to mathematics (the formal); and instead of a foundationalist reduction, it proposes a holistic reduction. Due to these changes the aforementioned obstacles are removed: a holistic reduction does not bar, in principle, a critical-explanatory account (i.e., a foundational account in the holistic sense) of the reducing field (here, mathematics); and there are, in the literature, a number of critical-explanatory accounts of mathematics compatible with *The Bounds of Logic*—for example structuralism, Platonism, and naturalism. (Needless to say, the proposed reduction is not threatened by either Russell's paradox or the incompleteness result, which undermine, or at least seriously weaken the prospects of, a logicist reduction.)

(iii) *Philosophy of Logic as a Substantive Discipline*. One of the popular approaches to philosophy today is deflationism, namely, the view that there are no deep philosophical questions to be asked and no substantive philosophical answers to be given. The epistemic model in which I have attempted to incorporate my view of logic repudiates this approach. By subjecting philosophy to the same epistemic norms as physics, biology and mathematics, it requires philosophy to ask hard questions, give genuine explanations, be informative, develop intricate theories, make discoveries, yield interesting theoretical results, etc. *The Bounds of Logic* aims at, if not necessarily succeeds in, meeting these standards. In this, at least, it is fully and unquestionably congruent with the Quinean model.

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NOTES

1. See Barwise (1985).

2. Sher 1996a,b, 1999a and 2001. Henceforth I will use *"The Bounds of Logic"* to refer to the book together with the articles.

3. Due to the openness of the model, incorporating the conception of logic in it does not completely obviate its neutrality towards metaphysical and mathematical questions.

4. The purpose of the list is to remind the reader of Quine's main tenets; the principles listed are not mutually independent.

5. As, e.g., in Kant's and Platonists' conceptions of mathematics.

6. While the veridical norms apply only in the synthetic zone, the pragmatic norms apply not only in the analytic zone but also in the synthetic zone. This is due to the special role the analytic zone plays in our system of knowledge.

7. The normative conflict holds even if we find a way to overcome the conceptual conflict. (Such a way is implicitly suggested in the account of the normative conflict itself.)

8. Sentence #4 says that when A and Lr hold, the values of d and l are no longer the real numbers m and n specified above, the first of which, as we recall, is smaller than the second.

9. The reader may note that on my conception, too, there is a sense in which logic differs from the natural and social sciences. This is correct. The new Quinean model does not claim that all sciences are the same in all ways. What it says is that sciences do not divide into those that are bound only by the pragmatic norms and those that are bound by the veridical norms (as well). Some sciences are more broadly interconnected than others, but all sciences are, in principle, both world- and concept-oriented.

10. See Shapiro (1991).

11. For the origins of this criterion see Sher 1991, Chapter 3. The formality criterion, as it is presented here, does not apply to sentential connectives. I commented on the relation between the present criterion and the logical connectives in Sher 1991 and the related articles.

12. I have omitted here the case in which O is an operator representing a function from objects to objects. In that case the condition is: if $<U, \beta>$ and $<U', \beta'>$ are isomorphic, then $<U, O_U(\beta)>$ and $<U', O_{U'}(\beta')>$ are isomorphic.

13. I understand the quantifier-scopes in this inference to be indicated by its surface grammar.

14. "$\forall$" denotes the formal operator **All**. Given a universe U, **All**(U) is a function, **All**$_U$, defined by: For any subset A of U, **All**$_U$(A) = T iff the complement of A in U is empty.

"$\supset$" denotes the operator **If**, defined both over the domain {T, F} of truth values and over all universes (of individuals), U. Its definition over {T, F} is the usual Boolean definition; given a universe U, **If**(U) is a function, **If**$_U$, such that for any subsets A and B of U, **If**$_U$(A, B) = T iff A is included in (or equal to) B.

"$\vee$" denotes the operator **Or**, defined both over {T, F} and over all universes U. Its definition over {T, F} is the usual one; given a universe U, **Or**(U) is a function, **Or**$_U$, such that for any subsets A and B of U, **Or**$_U$(A, B) = the union of A and B.

15. "$\exists$" denotes the formal operator **Some**. Given a universe U, **Some**(U) is a function, **Some**$_U$, defined by: For any subset A of U, **Some**$_U$(A) = T iff A is not empty.

"M" denotes the formal operator **Most**. Given a universe U, **Most**(U) is a function, **Most**$_U$, defined by: For any subset A of U, **Most**$_U$(A) = T iff A is larger than its complement in U.

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