Kappa calculus and evidential strength: A note on Åqvist's logical theory of legal evidence

SOLOMON EYAL SHIMONY1 and EPHRAIM NISSAN2

¹Department of Computer Science, Ben-Gurion University of the Negev, Beer-Sheva, Israel E-mail: shimony@cs.bgu.ac.il; URL: http://www.cs.bgu.ac.il/~shimony;

E-mail: E.Nissan@gre.ac.uk; URL: http://www.gre.ac.uk/~E.Nissan; http://www.gre.ac.uk/~E.Nissan/CmsWeb/ai_and_law_group.html

Abstract. Lennart Åqvist (1992) proposed a logical theory of legal evidence, based on the Bolding-Ekelöf of degrees of evidential strength. This paper reformulates Åqvist's model in terms of the probabilistic version of the kappa calculus. Proving its acceptability in the legal context is beyond the present scope, but the epistemological debate about Bayesian Law is clearly relevant. While the present model is a possible link to that line of inquiry, we offer some considerations about the broader picture of the potential of AI & Law in the evidentiary context. Whereas probabilistic reasoning is well-researched in AI, calculations about the threshold of persuasion in litigation, whatever their value, are just the tip of the iceberg. The bulk of the modeling desiderata is arguably elsewhere, if one is to ideally make the most of AI's distinctive contribution as envisaged for legal evidence research.

1. Introduction

In his paper, "Towards a logical theory of legal evidence", Lennart Åqvist (1992) provides a grading mechanism on worlds, and then defines the degrees of evidential strength on the basis of the grading mechanism. The idea of using a probability measure as a grading mechanism is then explored in the paper. Åqvist's model draws its notion of degrees of evidential strength from two authors that are likewise Swedish legal scientists: Bolding (1960) and Ekelöf (1964).

In this short paper, instead, we would like to draw the reader's attention to an already existing probabilistic grading mechanism, the so-called *kappa calculus*, that bears several similarities to Åqvist's scheme of degrees of evidence. The kappa calculus was originally developed by Spohn (1988), but then adapted for probabilistic reasoning in artificial intelligence, particularly for diagnostic reasoning and a qualitative decision theory for actions (Pearl 1993; Henrion et al. 1994). In the present paper, a semantic comparison between the schemes is made, followed by a discussion of each mechanism's advantages and deficiencies as relating to degrees

²School of Computing and Information Systems, The University of Greenwich, Queen Mary Court, Maritime Greenwich Campus, Old Royal Naval College, 30 Park Row, Greenwich, London SE10 9LS, UK

of evidence in evidentiary reasoning. We then suggest ideas for overcoming deficiencies existing in both schemes. More generally, we offer cautionary comments relating to legal evidence.

We are not taking sides, in the debate about Bayesianism in legal evidence. Important objections have been pointed out by various scholars, about the adequacy of probabilistic models in modeling the decision-making processes of fact-finding: not only about the probatory value of such calculations in litigation (which even Bayesians usually do not argue for, other than for such restricted applications as DNA frequencies), but even as analytic tools for grasping what happens in the courtroom from the coign of vantage of academic discussion. On the other hand, on occasion even decided Bayesiosketics have commended specific analyses from Bayesian Law, e.g., Johan Bring vis-à-vis Kadane and Schum's analysis of the *Sacco and Vanzetti* evidence, in Bring's commentary in the 1997 special issue of the *International Journal of Evidence and Proof* on "Bayesianism and Juridical Proof", reviewed in this volume.

Our purpose here, instead, is to show how elegantly a given model of legal evidentiary weight can be reformulated, by means of a formalism from the forefront of theoretical AI. Moreover, both Åqvist's paper (which appeared in Martino's collection, *Expert Systems in Law*), and the disciplinary background of the present co-authors, are in artificial intelligence, instead of Law or forensic statistics. Arguably, it is an interesting exercise for its own sake, to rethink problems across the disciplines (see also Nissan 2001).

Entries from recent debate over the Internet, two years ago, pondered, or rather wondered, about whether and how (if at all) AI approaches grounded in Judea Pearl's probabilistic belief networks dovetails with Bayesian Law's (if clearly not Bayesioskeptic) goals. Whereas, within that public debate, Pearl briefly commented on a specific methodological point (without these authors intervening), we feel it is appropriate for us to point out - especially as Shimony's formative disciplinary affiliation within AI research has been, indeed, closely associated with belief networks and uncertainty as well as with abductive reasoning (e.g., Charniak and Shimony 1990 sqq.; Shimony and Charniak 1990; Shimony 1993; Santos and Shimony 1994) – that on the face of it, mutual amenability is not inconceivable, in terms of representation syntax. Even if ancillary evidence in the legal context was to require special treatment, then nevertheless (short of unnaturally making explicit such causal relations that still defy precise pinpointing) one could figure out some filter within an architecture, enabling such evidentiary contribution to be parametrically attuned; or then, enabling such evidentiary contribution to be parametrically attuned; or then, perhaps, one could well envisage the adoption of a convenient probabilistic theory: to Nissan, Igal Kvart's (1994) theory of overall positive causal *impact* (or opci) occurs as a possible candidate: the theory is concerned with token causal relations (i.e., such that hold between particular actual event tokens), instead of type (i.e., generic) causal relations; and, most importantly, opci implies causal relevance, but is a weaker notion that causing or being a cause; furthermore, Kvart distinguishes *opci* from both "purely positive causal impact", and *spci*, i.e., "some positive causal impact". (In the rest of the paper, we'll not be considering again Kvart's approach).

Yet, representational sophistication by itself does not clear the way of epistemological concerns with whether, in principle, such a probabilistic approach would be (technical self-gratulation apart) true to its purpose of capturing likelihood in as complex a domain as legal evidence. Answering that question is beyond our present scope. What we think, all camps would eventually agree about, is that probability (in the Bayesian Law sense), or even plausibility (in Ron Allen's sense) is anyway part of a much broader picture: the commonsense of narrative conceptualization, for example, is an enticing, perhaps exciting challenge for AI & Law, a challenge to which the research community will hopefully respond across traditional disciplinary compartments. In this sense, when we approached the task of rethinking Åqvist's paper, we were fully aware that it is merely concerned with the tip of an iceberg. How to calculate the threshold of persuasion, in a legal burdens of proof perspective, is not as interesting as the entire process of conceptualising and handling whatever turns out in argumentation about evidence. Pinpointing truth values in a legal probatory context would not be the right ambition, for AI modeling of legal evidence (even if we were to correctly distinguish between legal truth and factual truth; on truth and verdicts, see, e.g., Jackson 1998b, c). Supporting – by otherwise empowering – human judgment at such tasks is arguably the proper practical one, in the set of appropriate goals that the new evidentiary subdiscipline within Al & Law should (Nissan 1999; Martino and Nissan 1998b).

1.1. REVIEW OF ÅQVIST'S SCHEME

Several mechanisms were suggested in Åqvist's paper, but rather than start with his 4-level scheme and proceed to the general mechanism, we begin with his generalized notion of a grading mechanism, in the interest of conciseness. The scheme uses a set W of disjoint, internally consistent, "possible worlds", consisting of all possible courses of events. Every fact X (a propositional logical formula) can be either true or false in each of the possible worlds. A formula thus corresponds to the set of worlds in which X is true. A grading mechanism is defined over the possible worlds. We begin with the formal definition of the grading mechanism, paraphrased from Åqvist's paper, and correcting a number of typographical errors found there in the relevant passages:

DEFINITION 1. A k-level grading mechanism on a set of possible worlds is a structure $G = (W. \ge, \{C_1, C_2, \dots, C_k\})$ where

- 1. W is a non-empty, finite set of possible worlds.
- 2. \geq is a weak ordering, i.e., a transitive and total binary relation on W. For any worlds $w, w' \in W$, the relation $w \geq w'$ means that w is at least as probable as w'.

3. The set $\{C_1, C_2, \dots, C_k\}$ is a partition of W, such that, if $w \in C_i$ and $w' \in C_j$, then $w \ge w'$ if and only if $i \ge j$.

Relative to the grading mechanism, Åqvist defines a notion of evidential strength, in support of any formula X, with P_i denoting positive evidence:

$$P_i X$$
 iff X is true in all the worlds in $\bigcup_{j \geq i} C_j$,

with P_1X denoting strongest possible evidence for X, and R_i denoting negative evidence:

$$R_i X$$
 iff X is false in all the worlds in $\bigcup_{j>k-i} C_j$,

this time with $R_k X$ denoting *strongest* possible evidence *against* X.

Åqvist shows that P_iX if and only if $R_{k-i+1}\neg X$, that P_iX is inconsistent with R_jX for any i, j, and that P_iX implies $P_{i+1}X$ for positive integer i < k. Åqvist proceeds to define a "non-vacuous" version of the above definition, where C_k must be non-empty.

In Aqvist's paper the initial definitions are in terms of k=4, where the C_i are identified with legal evidential grades, as follows. C_1 is identified with the set of "non approved" (non approbatur) members of W, C_2 with "just approved" (approbatur), C_3 with "approved, not without distinction" (non sine laude approbatur), and C_4 with "approved with distinction" (cum laude approbatur). The P_iX in this case are named "obvious" for i=1, "certain" for i=2, "probable" for i=3, and "presumable" for i=4.

A probability measure p is then imposed on the set W, consistent with the axioms of probability theory, and with the grading mechanism, such that $w \ge w'$ if and only if $p(w) \ge p(w')$. The standard definitions of conditioning are introduced. In order to avoid the collapse of all the grading structure, the principle of preponderance is introduced, which requires that for any X, if P_iX then $p(X) > \frac{1}{2}$.

1.2. REVIEW OF KAPPA CALCULUS

As in the previous subsection, we have a set of disjoint possible worlds W and formulas can be true or false in each possible world. Again, a propositional formula ϕ is identified with the set of possible worlds in which it is true. Instead of a grading mechanism, each possible world is given a kappa value, where kappa is a function from worlds to non-negative integers: $\kappa: W \to \mathcal{N} \cup \{0\}$, such that k(w) = 0 for at least one $w \in W$. The kappa value of a possible world is the degree of surprise in encountering that possible world. Intuitively, the lower the kappa, the higher the probability, and worlds with $\kappa = 0$ are considered serious possibilities. The definition of κ is extended to sets of possible worlds (propositions and propositional formulas) via Spohn's calculus, as follows:

$$\kappa(\phi) = \min_{w \in \phi} \kappa(w),\tag{1}$$

where if ϕ is false in all possible worlds, $\kappa(\phi)$ is defined to be 1, and to conditioning:

$$\kappa(\phi|\psi) = \kappa(\phi \wedge \psi) - \kappa(\psi). \tag{2}$$

The kappa assignment is frequently (Pearl 1993) seen as an order-of-magnitude approximation of a probability function p(w) defined over W, as follows: write p(w) as a polynomial in some small quantity ϵ , and take the power of the most significant term of that polynomial to be the kappa, i.e.:

$$p(w) \approx C \epsilon^{\kappa(w)}$$

for some constant C. Treating ϵ as infinitesimally small, Spohn's calculus is consistent with (and follows from) the axioms of probability theory.

Note that seemingly the only way to state that a proposition ϕ is certain, or even very likely, is to state that the degree of surprise in encountering $\neg \phi$ is high, i.e., stating that $\kappa(\neg \phi) > 0$.

Also, if $\epsilon < \frac{1}{2}$ then clearly $\kappa(\phi) > 0$ implies $\kappa(\neg \phi) = 0$, but not vice versa (i.e., it is certainly possible that $\kappa(\phi) = \kappa(\neg \phi) = 0$).

2. Comparison of the Schemes

We begin by comparing kappa-calculus to Åqvist's grading mechanisms without considering probabilities. Later on we introduce the probability distribution and continue the comparison.

2.1. EQUIVALENCE OF KAPPA-CALCULUS TO GRADING MECHANISMS

Ignoring for the moment the probability measure on the set of possible worlds, or the value of ϵ for kappa calculus as an order-of-magnitude probability approximation, we begin defining a natural mapping between the schemata.

Given a k-level grading mechanism G over the set of possible worlds W, we define a function $G \colon W \to \mathcal{N}$, the *grade of a world*, such that G(w) = i just when $w \in C_i$ in grading mechanism G. We now define a level-to-kappa mapping function, $F_k \colon \mathcal{N} \to \mathcal{N} \cup \{0\}$ as: $F_k(i) = k - i$. The mapping from a grading mechanism to kappa calculus is simply defining $\kappa(w) = F_k(G(w))$ for each $w \in W$, and extending the definition to sets of possible worlds as for the earlier definition of κ in Equation (1) with the special case $\kappa(\emptyset) = \infty$. It is clear (by construction) that the grading mechanism G is non-vacuous (that is, C_k is non-empty) if and only if there exists a world $w \in W$ for which $\kappa(w) = 0$.

Likewise, define a kappa-to k-level mapping function F'_k : $\mathcal{N} \cup \{0\} \to \mathcal{N}$, as follows: $F'_k(i) = \max(1, k - i)$. Now, given a set of possible worlds W, and a kappa ranking, we define the k-level grading mechanism over W as the partition uniquely determined by:

$$w \in C_i$$
 if and only if $F'_k(\kappa(w))$.

The following theorem follows from the definitions:

THEOREM 1. F'_k is the left-inverse of F_k (that is, $F'_k(F_k(i)) = i$ for any $k, i \in \mathcal{N}$), and the k-level grading mechanism is equivalent to k-calculus where the κ values are limited to the range $\{0, \ldots, k-1, \infty\}$.

Obviously, since the mapping is semi-invertible, kappa-calculus subsumes Åqvist's grading mechanisms (and if we allowed grading mechanisms to have an infinite number of partitions, they would be equivalent). Åqvist could have just used kappa-calculus directly in his paper, instead of having to invent grading mechanisms. The degrees of evidential strength in Åqvist's paper have no immediate counterpart in kappa-calculus, but we could define them as follows:

DEFINITION 2. $P_i X$ if and only if $\kappa(\neg X) \ge k - i + 1$, and $R_i X$ if and only if $\kappa(X) \ge i$.

This definition is far more concise than the one made by Åqvist, yet is essentially equivalent:

THEOREM 2. P_iX in a k-level grading mechanism G, if and only if P_iX in the respective kappa-calculus representation (and likewise for R_iX).

Proof: (\rightarrow) Suppose P_iX in a k-level grading mechanism. Then for every $j \geq i$, we have $w \in C_j \to w \in X$, and thus $G(w) \geq i \to w \in X$, or alternatively, using the k-level to kappa mapping, $F_k(i) = k - i$, we have $\kappa(w) \leq k - i \to w \in X$. Thus, since possible worlds are internally consistent, the is no $w \in \neg X$ such that $\kappa(w) \leq k - i$, and since the kappa value of a set of possible worlds is the minimum kappa of the set, we have $\kappa(\neg X) \geq k - i + 1$, proving the implication.

 (\leftarrow) Suppose P_iX in the kappa-calculus representation (that is, where $\kappa(w) = F_k(G(w))$). Then $\kappa(\neg X) \geq k - i + 1$, and thus $w \in \neg X \to \kappa(w) \geq k - i + 1 \to F'_k(\kappa(w)) \leq i - 1$. Therefore, $w \in \neg X \to G(w) \leq i - 1$ (since F'_k is the left-inverse of F_k), and thus $C_i \in X$ for all $j \geq i$.

Proof of the theorem for R_iX is similar, but also follows immediately from the proof for P_iX and the following theorem.

THEOREM 3. The degrees of evidential strength defined above for kappa-calculus have the same properties as for grading mechanisms, i.e.,

- 1. $P_i X$ if and only if $R_{k-i+1} \neg X$.
- 2. $P_i X$ is inconsistent with $R_j X$ for any i, j.
- 3. $P_i X$ implies $P_{i+1} X$ for positive integer i < k.

Proof: Immediate from the definitions, as follows (item for item):

1. By Definition 2, P_iX if and only if $\kappa(\neg X) \ge k - i + 1$, and $R_{k-i+1}\neg X$ if and only if $\kappa(\neg X) \ge k - i + 1$, which is exactly the same term as for P_iX .

- 2. If P_iX for some $1 \le i \le k$ then $\kappa(\neg X) \ge k i + 1 > 0$. Suppose that R_jX for some $1 \le j \le k$. Then by Definition 2, $\kappa(X) \ge j > 0$. However, every possible world $w \in W$ must satisfy either X or $\neg X$ (using the same "excluded middle" assumption made by Åqvist), and there exists $w \in W$ such that $\kappa(w) = 0$. If this possible world w is in X, then by Equation (1), $\kappa(X) = 0$, a contradiction. Likewise if $w \in \neg X$.
- 3. By Definition 2 P_iX implies $\kappa(\neg X) \ge k i + 1 \ge k i$, and $\kappa(\neg X) \ge k i + 1 \ge k i$, and $\kappa(\neg X) \ge k i$ implies $P_{i+1}X$.

It is also possible to prove the theorem by applying Theorem 2.

2.2. RE-INTRODUCING THE PROBABILITIES

Now that we have a mapping between grading mechanisms and kappa-calculus (which was just shown to be a variant, or slight extension, of grading mechanisms), we can introduce the probability distribution over possible worlds. Note that the common trend of using kappa-calculus as an approximation of order-of-magnitude probabilities, implies that the "principle of preponderance" is obeyed.

We must point out, however, that kappa-calculus is strictly consistent with the axioms of probability only if we use an infinitesimal ϵ . Thus, probabilities of possible worlds in different sets of the partition in the grading mechanism should also be stratified by "an order-of-magnitude" probability ratio. It is not clear that this is indeed the case in the definitions in legal reasoning. Nevertheless, perhaps the mere institution of these grades in legal reasoning is an indication that at least *subjectively* (i.e., probabilities subjectively assigned to such possible worlds by a human) this is the case.

The main problem in combining probabilities with a grading-system was typified by the example given in Åqvist's paper (for k=4): a violation of the "principle of preponderance". That is, we get a case where a proposition is "presumable", yet is less likely than its negation. The suggestion (made by Åqvist) that we insist on $p(C_4) > \frac{1}{2}$ is an obvious, albeit somewhat ad-hoc solution.

A more general statement of the problem is that, if probabilities in different sets (i.e., different C_i) in the grading mechanism are not "orders of magnitude" apart, then in fact the disjunction of several propositions can "graduate" to a higher grade, if we only make such a disjunction into a primitive proposition (thereby modifying the set of possible worlds, W). It is thus the case that the grading mechanism is very sensitive to the syntax of the propositional formulas, and to the set of possible worlds which we allow to participate in the grading mechanism.

Forcing $p(C_k) > \frac{1}{2}$ may prevent unlikely facts from being accepted as "probable" or "presumable" or "certain", but would not prevent us from passing merely "probable" facts as "certain" without really changing the probability distribution, but just the syntax of the propositional formulas. This is certainly an undesirable property of the grading scheme.

For example, paraphrasing Åqvist's "cause of death" example (Åqvist 1992), with possible worlds: w_1 = poisoned by wife, w_2 = accidental liquid poisoning, w_3 = taking poison deliberately (suicide), w_4 = murdered by someone else (not wife), w_5 = accidental gas poisoning, w_0 = other cause of death. Suppose now (modifying the original example) that the evidence is such that (with a 4-level grading mechanism), we have C_4 = { w_1 }, C_3 = { w_4 }, C_2 = {}, and C_1 = { w_0 , w_2 , w_3 , w_5 }. Suppose the following probability distribution is assigned: $p(w_1)$ = 0.7 (thus obeying the "principle of preponderance"), $p(w_4)$ = 0.1, and $p(w_0)$ = $p(w_2)$ = $p(w_3)$ = $p(w_5)$ = 0.05. Total probability is 1 as required.

Now, suppose we want to decide a case where the deceased's brother is a beneficiary of a life-insurance policy taken out on the deceased. The beneficiary is claiming for double-indemnity, to be awarded in case of murder. In this example, the proposition "death by murder", entitling claimant to double indemnity payments, is "certain" (since all possible worlds in C_4 , C_3 , C_2 entail "death by murder"). In this case, the claimant would win his double-indemnity case if the required degree of evidence for such a claim were "certain", "probable" or "presumable".

However, one could claim that this is incorrect, and state: all accidental deaths are equivalent here, and thus we should *not* have both w_2 and w_5 , but instead some w_6 = accidental poisoning, with a probability 0.1 (which is the same distribution, just changing the syntax and creating a predicate that stands for a disjunction). Intuitively, this should change nothing in the certainty of "death by murder", but this is not the case in this model. We have $C_3 = \{w_4, w_6\}$ and since w_6 is not in "death by murder", then "death by murder" is only "presumable". Now, if the requirement for the evidence level is "certain" or "probable", the claimant would now lose his double-indemnity case, where he would have otherwise won it!

3. Suggested Solution

Observe that the above undesirable effect could not occur if we used kappacalculus with infinitesimal epsilon (or the equivalent probabilities in the grading mechanism). Nevertheless, this would require that we consider facts "approved not without distinction" as having infinitesimal probabilities, which is somewhat counterintuitive.

To avoid this problem, it is sufficient if we required the following constraint, in addition to $C_k \neq \emptyset$, for all $w \in W$:

$$w \in c_i \to p(w) > \sum_{j < i} \sum_{w' \in C_j} p(w').$$

This constraint implies the principle of preponderance, and subsumes Åqvist's requirement that $p(C_k) > \frac{1}{2}$. Observe that the example in the previous section, while obeying the principle of preponderance, violates our constraint, since $p(w_4) = p(w_2) + p(w_5)$ while $w_4 \in C_3$ and $w_2, w_5 \in C_1$.

4. Contextual Assessment of the Method

However mathematically sound the restatement of the Åqvist model may be, its usefulness entirely depends on compatibility with the context that has to provide the qualitative grading of the elements of evidence, that in turn the Åqvist model uses as input.

What makes the Åqvist model (and, perhaps all the more so, our probabilistic reformulation of it) quite problematic in the broad perspective of juridical proof, is that it's what lays upstream of it that perhaps defies modeling, and anyway it is not addressed at all by either Åqvist's model, or our restatement of the method. The input is a subjective grading of the elements of evidence, and is supposed to be provided by human experts. In elaborating the input they are to provide, this is to be the end product of complex mental processing (not merely cognitive: consider colouring by emotion). Such mental processing involves both perception and "higher" (cognitive or noncognitive) functions which invoke each other recursively, or, anyway, are pervasively interwoven (even the cultural vs. bio- or physiological extremes are not uncontroversially demarcated; cf. Nissan 1997a). Among the other things, a rather obvious aspect of this is that by reasoning, new goals for perception are set, in both the investigation, and, say, cross-examination. Explanation for alternative accounts of a narrative (see, e.g., Kuflik et al. 1991; Fakher-Eldeen et al. 1993) involves more evidence to be assessed. However, at the end of the day, the sad, inescapable fact remains that if anybody is to provide subjective estimates as input to an automated or otherwise formal component embodying the Aqvist model, then the virtual size of this component within the broad picture of legal evidence and proof is quite diminutive.

Our contribution in the present paper has been to show that the formalism proposed by Åqvist is entirely amenable to, and thus is fully compatible with, the statement in terms of probabilistic reasoning of the kappa calculus. Probabilistic reasoning is a well-researched, powerful tool outside as well as within artificial intelligence. It can contribute to Bayesian Law, or to its critique, or, more usefully, to overcoming the divide. For the latter purpose, however, it is other facets of the extant panoply of AI & Law or just AI techniques that could most usefully contribute: such themes as argumentation, agents' epistemic states, narrative coherence, and so forth, are all sides that deserve development. (For example, culture-laden literary concepts of narrative improbability sometimes impinge on lay perceptions of a crime narrative as reported by the media; cf. Nissan (2000), Nissan and Dragoni (2000).)

Take temporal reasoning: thus far, chronologies had been dealt with in MARSHALPLAN (see Schum 2001), but not in AI treatments of legal time (cf. Martino and Nissan 1998a). Yet, time is a necessary facet, even though on occasion (e.g., in Nissan 1995) it may be implicit in a representation. (More in general, see, e.g., Jackson (1998a) on time and law; also consider legal and pragmatic space: Nissan (1997b).) Such is the nitty-gritty of informational organization and present-

ation: these two themes, quite likely in association with argumentation research, arguably deserve the main thrust of efforts, in finding out how to enhance the treatment of the body of evidence as being handled by any of the given professional roles entrusted with it, in an investigation or litigational context. One incontroversial statement, however, is that historically, absorbing concern with probabilities paved the way for both its adepts and critics – as well as AI & Law's eventual awareness and inclusion of evidentiary research into its own agenda – for the delineation of a new horizon of inquiry.

References

- Åqvist, L. (1992). Towards a Logical Theory of Legal Evidence: Semantic Analysis of the Bolding– Ekelöf Degrees of Evidential Strength. In Martino, A. A. (ed.), Expert Systems in Law, 67–86. North-Holland: Amsterdam.
- Bolding, P. O. (1960). Aspects of the Burden of Proof. Scandinavian Studies in Law 4: 9–28.
- Charniak, E. and Shimony, S. E. (1990). Probabilistic Semantics for Cost-Based Abduction. In Proceedings of the 11th Annual National Conference on Artificial Intelligence (AAAI'90), 106–111. AAAI Press: Menlo Park, CA.
- Charniak, E. and Shimony, S. E. (1994). Cost-Based Abduction and MAP Explanation. Artificial Intelligence, 66: 345–374.
- Ekelöf, P. O. (1964). Free Evaluation of Evidence. Scandinavian Studies in Law 8.
- Fakher-Eldeen, F., Kuflik, Ts., Nissan, E. Puni, G., Salfati, R., Shaul, Y. and Spanioli, A. (1993). Interpretation of Imputed Behavior in ALIBI (1 to 3) and SKILL. Informatica e Diritto 19, 2nd series, 2(1–2): 213–242.
- Henrion, M., Provan, G., Del Favero, B. and Sanders, G. (1994). An Experimental Comparison of Numerical and Qualitative Probabilistic Reasoning. In Lopez de Mantaras, R. and Poole, D. (eds), Uncertainty in Artificial Intelligence: Proceedings of the Tenth Conference, 319–326. Morgan Kaufmann: San Mateo, CA.
- Jackson, B. (1998a). On the Atemporality of Legal Time. In Ost, F. and van Hoecke, M. (eds), Temps et Droit. Le droit a-t-il pour vocation de durer, 222–246. Bruylant: Brussels.
- Jackson, B. (1998b). Truth or Proof?: The Criminal Verdict. International Journal for the Semiotics of Law 11(33): 227–273.
- Jackson, B. (1998c). Bentham, Truth and the Semiotics of Law. (Current Legal Problems, 51.) In Freeman, M. D. A. (ed.), Legal Theory at the End of the Millennium, 493–531. Oxford University Press: Oxford.
- Kuflik, Ts., Nissan, E. and Puni, G. (1991). Finding Excuses with ALIBI: Alternative Plans that are Deontically More Defensible. Computers and Artificial Intelligence 10(4): 297–325.
- Kvart, I. (1994). Overall Positive Causal Impact. Canadian Journal of Philosophy 24(2): 205–227.
- Martino, A. A. and Nissan, E. (eds) (1998a). Formal Models of Legal Time: Law, Computers and Artificial Intelligence, special issue of Information and Communications Technology Law 7(3).
- Martino, A. A. and Nissan, E. (1998b). La prova giudiziaria: area emergente dell' "AI & Law". Oltre la statistica forense e l'algebra delle giurie, l'integrazione di metodi AI per affrontare un dominio proteiforme. Proceedings of AI, 104–108. Padua: Italy.
- Nissan, E. (1995). SEPPHORIS: An Augmented Hypergraph-Grammar Representation for Events, Stipulations, and Legal Prescriptions. Law, Computers, and Artificial Intelligence 4(1): 33–77.
- Nissan, E. (1997a). Emotion, Culture, Communication. Pragmatics and Cognition 5(2): 355–369.
- Nissan, E. (1997b). Notions of Place (2 parts). In Martino, A. A. (ed.), Logica delle norme, 256–302 + 303–361. SEU: Pisa.

- Nissan, E. (1999). Artificial Intelligence and Criminal Evidence: A Few Topics. Proceedings of the Second World Conference on New Trends in Criminal Investigation and Evidence. Amsterdam, 10–15 December 1999. Edited by C. M. Breur et al., 495–521. Antwerpen: Intersentia.
- Nissan, E. (2000). The Jama Legal Narrative. Part II: A Foray into Concepts of Improbability. Preproceedings of the AISB 2000 Symposium on AI and Legal Reasoning, 37–45 Birmingham, 17 April 2000, pp. 37–45. Now in Information and Communications Technology Law 10(1), 2001, special issue, edited by D. M. Peterson et al., pp. 39–52.
- Nissan, E. (2001). Can You Measure Circumstantial Evidence? The Background of Probative Formalisms for Law. Review article on I. Rosoni, Quae singula non prosunt collecta iuvant: La teoria della prova indiziaria nell'età medievale e moderna (Giuffrè, Milan, 1995). Information and Communications Technology Law (in press).
- Nissan, E. and Dragoni, A. F. (2000). Exoneration, and Reasoning About It: A Quick Overview of Three Perspectives. At Intelligent Decision Support for Legal Practice (IDS 2000). In Proceedings of the International ICSC Congress "Intelligent Systems & Applications" (ISA'2000), December 11–15, University of Wollongong, Australia, Vol. 1, pp. 94–100.
- Pearl, J. (1993). From Conditional Thoughts to Qualitative Decision Theory. In Uncertainty in AI, Proceedings of the Ninth Conference, 12–20.
- Santos, E., Jr. and Shimony, S. E. (1994). Belief Updating by Enumerating High-Probability Independence-Based Assignments. In Lopez de Mantaras, L. and Poole, D. (eds), Uncertainty in Artificial Intelligence: Proceedings of the Tenth Conference, 506–513. Morgan Kaufmann: San Mateo, CA.
- Shimony, S. E. (1993). The Role of Relevance in Explanation. I: Irrelevance as Statistical Independence. International Journal of Approximate Reasoning 8(4): 281–324.
- Shimony, S. E. and Charniak, E. (1990). A New Algorithm for Finding MAP Assignments to Belief Networks. In Bonissone, P. P., Henrion, M., Kanal, L. N. and Lemmer, J. F. (eds), Uncertainty in Artificial Intelligence: Proceedings of the Sixth Conference, 185–193. North-Holland: Amsterdam.
- Schum, D. (2001). Evidence Marshaling for Imaginative Fact Investigation. Artificial Intelligence and Law 9: 165–188.
- Spohn, W. (1988). A Dynamic Theory of Epistemic States. In Harper, W. L. and Skyrms, B. (eds), Causation in Decision, Belief Change, and Statistics, 105–134. D. Reidel, Dordrecht: Holland.