Discrete Configuration of Probability of Occurrence of Events in Wave Spaces

George P. Shpenkov and Leonid G. Kreidik
Institute of Mathematics & Physics
UTA, Bydgoszcz, Poland
gshp@atr.bydgoszcz.pl

This paper is devoted to a wave theory of probability of wave spaces, including atomic and molecular spaces. It reveals kinematics of wave probabilistic processes (the form of the process), *i.e.*, the spatial distribution of singularities – nodal points, where amplitude of probabilistic potential achieves extreme and zero values. Results obtained are applicable to problems of condensed matter physics. In particular, they reveal morphology of crystals and the law of constancy of angles between edges, and elucidate the "strictly forbidden" symmetry found in quasicrystals, *etc*.

Keywords: density of probability, wave equation solutions

1. Introduction

In this paper we describe the *wave approach* to the problem on the *spatial* (not energetic) *structure* of material spaces at molecular, atomic, and subatomic levels, unsolvable *directly* in the framework of quantum mechanics (QM). We mean the *interatomic* and *intratomic*

spatial geometry of disposition of the corresponding constituents (*atoms* and *nucleons*). This geometry (structure) is directly related with the symmetry that is *inherent in material spaces at all levels*. The new notions of probability, different from Born's probability, and of the wave function, different from Schrödinger's wave Ψ-function [1, 2], are in the base of our approach.

Because the *wave exchange* of matter-space and motion-rest (*matter-space-time* for brevity) is in the nature of all physical phenomena, the probability of possible states at wave exchange must also have the *wave character* and reflect the states of *rest* and *motion*. The possibility of rest and motion gives rise to the potential-kinetic field of *reality*, where rest (*potential* field) and motion (*kinetic* field) are inseparably linked between themselves in the unit *potential-kinetic field*. The wider notion *exchange* (introduced first in the work [3]) instead of *interaction* reflects *wave behavior* of micro-objects [4] in their dynamic equilibrium with the ambient field, at rest and motion, and interactions with other objects.

The new concept of probability, *phase probability*, presented here, was set forth first in [5]. It reveals the geometry of distribution of specific points in wave spaces, including atomic, where amplitude of probabilistic potential achieves extreme and zero values. The phase probability and its density introduced describe any wave events, including *probability of concentration of substance* in such points of space. Because of this, it is especially important for an analysis and prediction of molecular and crystalline structures. Actually, the wave theory of probability reveals the structure (morphology) of crystals, including the five-fold symmetry observed in quasicrystals [6], "*strictly forbidden by the mathematical laws of crystallography*" [7], *etc*. The *law of constancy of angles between edges* (and facets) for all crystals of the same substance finds thus a new physical and logical

justification. The other important effects of the results obtained are in [1].

2. Phase and energetic probabilities; Probability potential

Speaking about the *wave field of probability*, we bear in mind the *mathematical image of the wave field of possibility*. We call such probability the *phase probability* and denote it by the symbol $\hat{p}$. Two opposite phase probabilities, the *kinetic p_k* and *potential p_p* express, respectively, the probability of states of motion and rest. They define the *potential-kinetic phase probability*

$$\hat{p} = p_p + ip_k . \tag{2.1}$$

The *density of phase probability* $\hat{\Psi}$ describes the distribution of phase probability $\hat{p}$:

$$\hat{\Psi} = d\hat{p}/dV = d\hat{p}_p/dV + id\hat{p}_k/dV = \Psi_p + i\Psi_k , \tag{2.2}$$

where $d\hat{p}$ is the elementary phase probability, dV is the elementary volume of space, Ψ_p and Ψ_k are, respectively, the potential and kinetic densities of phase probability.

We further assume that the phase probability $\hat{p}$ (2.1) and its density $\hat{\Psi}$ (2.2) satisfy the *wave probabilistic equations*:

$$\Delta\hat{p} - \frac{1}{c^2}\frac{\partial^2 \hat{p}}{\partial t^2} = 0 \qquad \Delta\hat{\Psi} - \frac{1}{c^2}\frac{\partial^2 \hat{\Psi}}{\partial t^2} = 0 , \tag{2.3}$$

The phase probability $\hat{p}$ and its density $\hat{\Psi}$ must describe any wave events. In every concrete case, the character of studying objects and the concrete chosen parameters-measures of the description are determined by these events. If the density of energy of the field is

proportional to the wave amplitude of density of phase probability squared, then

$$dE_p / dV = \mathbf{z}_p \Psi_p^2, \qquad dE_k / dV = \mathbf{z}_k \Psi_k^2,$$
$$dE / dV = \mathbf{z}_p \Psi_p^2 + \mathbf{z}_k \Psi_k^2 \qquad (2.4)$$

where dE_p, dE_k, and dE are differentials of the potential, kinetic, and total energy; $\mathbf{z}_p$ and $\mathbf{z}_k$ are some coefficients of proportionality depending on the selection of phase probability and on the character of the field. For the class of fields satisfying the condition $\mathbf{z}_p = \mathbf{z}_k = \mathbf{z}$, we have

$$dE / dV = \mathbf{z}\,(\Psi_p^2 + \Psi_k^2) = \mathbf{z}\left|\hat{\Psi}\right|^2. \qquad (2.5)$$

Along with the phase probability, we operate with the notion of *energetic probability*. It is needed due to the simple reason that the distributions of total energy and masses are different (although they are related between themselves in the wave field-space of exchange). We must distinguish them. The differential of energetic probability *dw*, by the definition, should be assumed to be proportional to the differential of energy *dE*, *i.e.*, $dw = \mathbf{h}dE$, where *?* is the coefficient of proportionality. In such a case the densities of potential, kinetic, and total energetic probabilities are determined as

$$dw_p / dV = \mathbf{x}\,\Psi_p^2, \qquad dw_k / dV = \mathbf{x}\,\Psi_k^2,$$
$$dw / dV = \mathbf{x}\left|\hat{\Psi}\right|^2 \qquad (2.6)$$

where $\mathbf{x} = \mathbf{z}\mathbf{h}$ is the coefficient of proportionality depending on the character of the field and the choice of the wave function $\hat{\Psi}$. The *characteristic elements* of the wave probabilistic geometry − *extremes and zeroes* of the functions Ψ_k and Ψ_p − *define its discrete structure*.

Potential and kinetic extremes are mutually conjugated because the conjugated functions $\hat{\Psi} = \Psi_p + i\Psi_k$ (2.2) and

$$\hat{\Psi} = i(\Psi_p + i\Psi_k) = (-\Psi_k)_p + i(\Psi_p)_k \qquad (2.7)$$

satisfy the wave equation. Moreover, these extremes are also "conjugated" to zeroes of the wave function, because the kinetic extremes spatially coincide with the potential zeroes and the potential extremes are spatially imposed upon the kinetic zeroes.

The extremes and zeroes of Ψ_k and Ψ_p coincide with the extremes and zeroes of their squares, Ψ_k^2 and Ψ_p^2, in three-dimensional space of reality. Therefore, they define the same probabilistic geometry of density of states and the energies related to the extremes and zeroes. Since $\hat{p}$ and $\hat{\Psi}$ satisfy the same wave equation (2.3), the extremes and zeroes of phase probability $\hat{p}$ and its density $\hat{\Psi}$ coincide; in this sense, the functions $\hat{p}$ and $\hat{\Psi}$ are equivalent.

The value of the constant coefficient (the normalizing factor) of the $\hat{\Psi}$-function does not matter because *only its extremes and zeros* define the discrete structure of a studying object. Therefore, it makes sense to introduce the notion the *probabilistic potential* (or the *probability potential*) proportional to the wave function, which we also designate by the symbol $\hat{\Psi}$.

The wave *probabilistic potential* $\hat{\Psi}$ in the spherical polar coordinates (with the physical polar Z-axis) is represented in the form of the product of the four multiplicative components-functions of probability: $\hat{R}(\boldsymbol{r})$ (where $\boldsymbol{r} = kr$), $\Theta(\boldsymbol{q})$, $\hat{\Phi}(\boldsymbol{j})$, and $\hat{\Xi}(t)$, which represent by themselves the multiplicative components of probability potential. The radial, polar and azimuth components of the potential of probability form the *spatial amplitude of the potential of*

probability $\hat{y}(r,q,j) = \hat{R}(r)\Theta(q)\hat{\Phi}(j)$. Thus, the potential of probability $\hat{\Psi}$,

$$\hat{\Psi} = \hat{R}(r)\Theta(q)\hat{\Phi}(j)\hat{\Xi}(t) = \hat{y}(r,q,j)\hat{\Xi}(t), \qquad (2.8)$$

is determined by the product of spatial and time potentials of probability. Their amplitudes are described, in accordance with (2.3), by the following equations:

$$\Delta\hat{y} + k^2\hat{y} = 0, \quad \text{and} \quad d^2\hat{\Xi}/dt^2 = -w^2\hat{\Xi}, \qquad (2.9)$$

where $k = 2p/l = w/c$ is the wave number, the constant quantity (*w* is the *fundamental*, carrier, *frequency of the wave field of exchange at the subatomic level* equal to $w = 1.86916197 \cdot 10^{18} \, s^{-1}$ [1]). After the conventional separation of variables, Eq. (2.9) falls into the equations of radial $\hat{R}_l(r)$, polar $\Theta_{l,m}(q)$, and azimuth $\hat{\Phi}(j)$ components.

3. Space configuration of probability; Solutions

For any model of an object of study, the radial solutions define the characteristic spheres of extremes (domains of more intensive radial displacements) and zeroes (where radial displacements are absent) of the radial function. For a variety of problems, it is sufficient to know that such characteristic spheres exist. It is very important for determination of the spatial geometry of a studying object.

   Polar components $\Theta_{l,m}(q)$ of spatial density of probability $\hat{\Psi}$ define characteristic parallels of extremes (primary and secondary) and zeroes on radial spheres (shells). Azimuth components $\Phi_m(\varphi)$ define characteristic meridians of extremes and zeroes. Potential and kinetic polar-azimuth probabilities select together the distinctive coordinates (points) of extremes and zeroes on the radial shells. Graphs of these functions (see Fig. 3.1 and Table 3.1)

show that there are *primary* and *secondary* extremes (we will also call them *nodes*), which determine, correspondingly, stable and metastable states of probabilistic events.
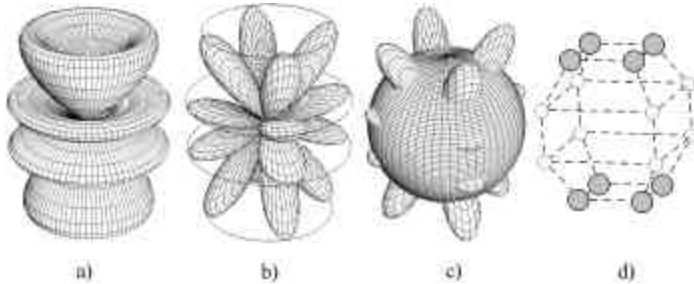

Fig. 3.1. The graphs of (a) $Q_{5,2}(q)$, (b) $Y_{5,2}(q, j)$, (c) $Y_{5,2}(q, j)$ together with $R_5(r)$, indicating (d) the disposition of *primary* and *secondary* potential extremes (designated conditionally by shaded and blank spheres) of $y_{5,2}(r, q, j)_p$ in the spherical field of probability.

The completely realized polar-azimuth *n*-th shell of the *potential* nodes is defined, in accordance with the wave equation of probability (2.3), by the function

$$\Psi_{l,m}(r_{l,n}, q\,j)_p = C_\Psi R_l(r_{l,n})\Theta_{l,m}(q)\cos(mj + a), \qquad (3.1)$$

where $r_{l,n}$ is the radius of *n*-th extremal radial shell of the function $R_l(r)$. We will call such shells the *whole shells*. The geometry of shells is determined by the polar-azimuth functions.

The "*fractional*" (uncompleted) *shells* are defined by the half-integer solutions of the form

$$\Psi_{l,l}(r_{l,n}, q\,j)_p = C_\Psi R_l(r_{l,n})\sin^l q \cos(lj + a), \qquad (3.2)$$

where *l* is a real number, with extremes lying in the equatorial plane.

Table 3.1. Solutions of Eq. (2.3) presented in the form of the spatial distribution of potential extremes-nodes; numbers 1, 2, 3, … , 110 are the ordinal numbers Z of the primary polar-azimuth nodes and, simultaneously, they are the ordinal numbers of the last primary node of a probabilistic object.
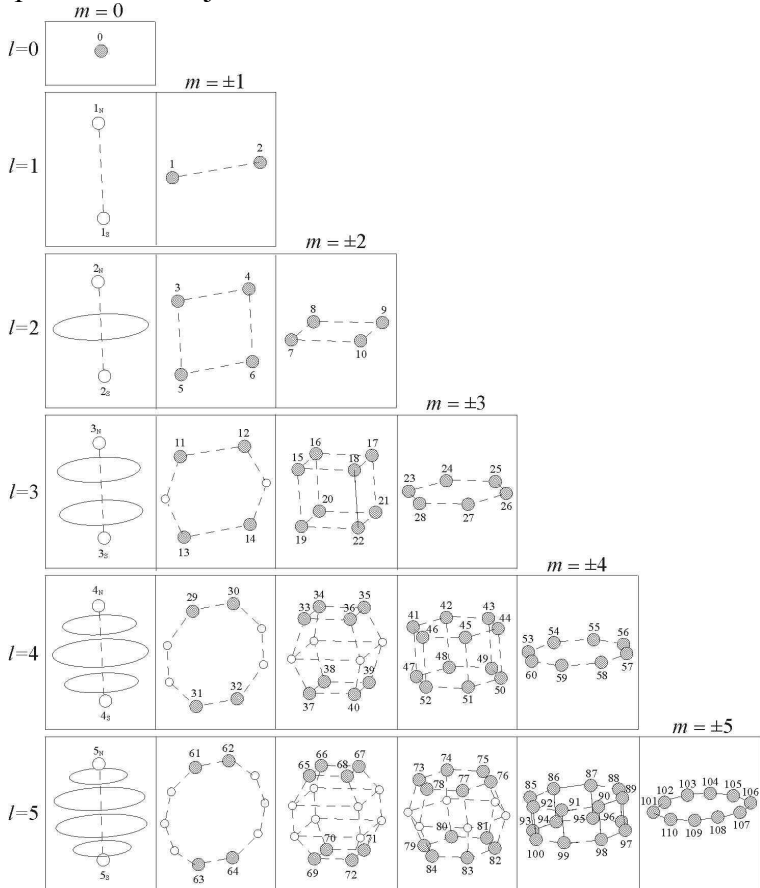
"N" and "S" are subscripts designating the "north" and "south" polar nodes (at $m = 0$).

In a general case, the complete structure of *any probabilistic object* (we call it an *abstract atom* [1]) with the ordinal number Z is defined by the two sums:

$$\Psi_Z = \sum C_y R_l(\boldsymbol{r}_{l,n})\Theta_{l,m}(\boldsymbol{q})\cos(m\boldsymbol{j} + \boldsymbol{a}_m) \oplus$$
$$\oplus \sum C_s R_s(\boldsymbol{r}_{s,j})\sin^{s/2}\boldsymbol{q}\cos\left(\tfrac{s}{2}\boldsymbol{j} + \boldsymbol{b}_s\right), \quad (3.3)$$

where the subscript Z indicates the number of primary nodes and, simultaneously, the ordinal number of the last primary node of a probabilistic object; $s = 0, 1, 2, 3, ...$; $\boldsymbol{a}_m$ and $\boldsymbol{b}_s$ are the initial phases. The first sum in (3.3) consists of embedded whole shells; the second sum consists of embedded half-integer subshells.

The extremes and zeros of the phase probability are significant in an equal degree. Zero values of the wave spherical field of probability define the radial shells of zero probability of radial displacements (oscillations). Naturally, they are the shells of stationary states. On the contrary, shells of extremal values of the wave field of probability define domains of more intensive radial displacements and, accordingly, these shells describe nonstationary (unstable) states.

Thus, the extremes of the wave field of probability do not quite mean that they are domains of the most probable localization of microparticles. (The QM formalism, accentuating the attention to *extremes* of the wave function squared, is unable to describe the qualitative peculiarities of probabilistic processes).

As was mentioned above, there is the difference between distributions of two kinds of extremes: (2.2) the *density of phase probability* $\hat{\Psi}$ and (2.5) the *density of energy of wave fields* proportional to $\left|\hat{\Psi}\right|^2$. This difference is demonstrated in Fig. 3.2.
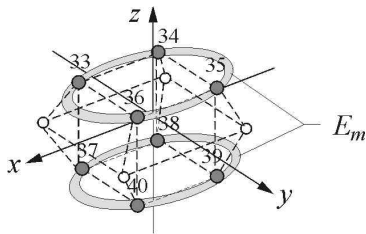
Fig. 3.2. Distribution of extremes of probabilistic states (small black and blank spheres) and extremes of the total energy E_m (two shaded rings) for the shell with $l=4, m=\pm2$.

4. Conclusion

The data presented reveals the kinematics (form) of wave probabilistic processes, *i.e.*, the spatial distribution of nodal points, where amplitude of probabilistic potential achieves extreme and zero values. By definition *reality* is the realized *possibility*, and *probability* is their measure. Hence, discrete structure of probability obtained, and presented in Table 3.1, reflects at the same time the structures really existed in nature.

Actually, the theory of wave fields-spaces of possibility and reality has the general discrete-wave feature because it is applicable to an analysis of any discrete-wave material spaces. In particular, characteristic angles of the functions (3.1) and (3.2), as it is shown in [1, 5], are materialized in characteristic angles of crystal forms of minerals. They repeat at the macro-level the angles of disposition of corresponding nodes of shells, presented in Table 3.1, and define the shape (morphology) of crystals [8]. This testifies thus in favor of the wave nature of the *law of constancy of angles between edges* (and facets) for all crystals of the same substance.

Half-integer solutions in the equatorial domain (3.2) have *any-fold symmetry*. Accordingly, they reveal the nature of the five-fold symmetry, "*strictly forbidden by the mathematical laws of*

crystallography" [7], found in 1984 [6], which attracts at present the world-wide attention.

The comprehensive analysis, conducted in [1], confirms also that the directions of chemical bonds in ordered structures are determined by the superposition of elementary solutions of the equation for the wave probabilistic field (2.3) [9]. These solutions can be used for the prediction of the molecular and crystalline structures. Moreover, they reveal the nature of Mendeleev's Periodic Law not discussed here (see [1]), *etc*.

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