Naturalness and Arbitrariness*

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Peter Forrest and D.M. Armstrong have given an argument against a theory of naturalness proposed by David Lewis based on the fact that ordered pairs can be constructed from sets in any of a number of different ways.[1] I think the argument is good, but requires a more thorough defense. Moreover, the argument has important consequences that have not been noticed.

I introduce a version of Lewis's proposal in section one, and then in section two I present and defend my version of the argument. After addressing a worry about my argument in section three, in section four I argue that a similar "argument from arbitrariness" jeopardizes Lewis's solution to the Kripke/Wittgenstein puzzle of the content of thought.

1. Primitivist Class Nominalism

Lewis's notion of a natural property or relation is one that is maximally "fundamental" and "carves nature at the joints". If we take an "abundant" construal of properties and relations, on which we have a property for every set of objects and a relation for every set of 'tuples, no matter how arbitrary or miscellaneous the set, then only an elite minority of the properties and relations turn out natural. Examples might include the properties of fundamental physics and spatiotemporal relations. Negations and disjunctions of natural properties, and other "gerrymanders" such as grue and bleen, **being ten feet from Bill Clinton**, etc., are not natural because they are less fundamental and don't carve nature at the joints.[2]

Lewis distinguishes, and remains agnostic between, two ways to account for the distinction between natural and unnatural abundant properties and relations.[3] One is to admit, in addition to abundant properties and relations,

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[1]See Armstrong (1986, 86–87) and Armstrong (1989, 29–32), and Forrest (1986, 90–91).

[2]See Lewis (1983, 1984, 1986*b*, 59–63). In this paper I ignore the complication of degrees of naturalness.

[3]See Lewis (1986*b*, 59–69).

"sparse" properties and relations as a separate class of entities, *sparse* because there simply are no unnatural sparse properties or relations, only natural ones. One can then pick out the natural abundant properties and relations as those that correspond to sparse properties and relations.[4] Two leading conceptions of sparse properties and relations take them to be "immanent universals" on one hand (this is Armstrong's view[5]), or "particularized properties", or "tropes", on the other. The second way to account for naturalness is to postulate only the abundant properties and relations, but admit a primitive distinction among these entities between those that are natural and those that are not.

In this paper I will argue against a certain proposal for taking naturalness as a primitive. But taking naturalness as a primitive is not a *claim*; it is rather a certain kind of theoretical activity: appealing to the term 'natural' without definition. It cannot, therefore, be objectionable in the most straightforward way, because it is not the sort of thing that can be true or false. Of course, one might have various *reasons* for taking naturalness as a primitive, and those reasons might be false. What I will call "Primitivism" is a certain reason for taking naturalness as a primitive: the belief that naturalness is ontologically basic, incapable of reductive analysis. I also understand Primitivism as implying that naturalness is in no way conventional—it is an objective fact about the world that some properties and relations are natural, while others are not.[6]

A theorist might have other reasons to take naturalness as primitive.[7] He might simply know of no analysis; he might be agnostic among several proposed analyses; he might not want to embroil his theory in controversy over the analysis of its basic terms, etc. These reasons don't involve claims about naturalness, but rather various sorts of agnosticism. Perhaps perfectly appropriate agnosticism, but I want to discuss a certain claim about the nature of naturalness, not the attitudes of theorists of naturalness; I therefore focus on Primitivism. (My target, therefore, is not a view to which Lewis is explicitly committed; Lewis has not specified in print his reasons for taking naturalness as a primitive, and has denied in correspondence a commitment to Primitivism.) As I see it, the version of the Forrest/Armstrong objection that I want to defend needn't apply to all theorists who take naturalness as a primitive, for such theorists

[4]I have reservations about this project—see my (1995). Lewis (1986*a*) has argued against the version of this project that appeals to immanent universals.

[5]See Armstrong (1978*a*,*b*). Note that Armstrong does not accept abundant properties and relations.

[6]See Lewis (1986*b*, 63–64) and Lewis (1983, 347).

[7]Here I am indebted to correspondence with Lewis.

may simply be agnostic among various conceptions of naturalness to which the argument does not apply.

I do not argue against Primitivism *per se*, only Primitivism in conjunction with a certain view of the nature of abundant properties and relations (which Lewis accepts), namely “Class Nominalism”.[8] On this view properties and relations are constructed set-theoretic items. The basic entities countenanced by the Class Nominalist are sets and concrete particulars; the properties are then identified with the sets, and the n-place relations are identified with the sets of ordered n-tuples, where these n-tuples are constructed from sets according to any one of the various common devices.[9] The conjunction of Class Nominalism and Primitivism I call “Primitivist Class Nominalism”, or “PCN” for short.

2. Against Primitivist Class Nominalism

2.1 The Armstrong/Forrest arbitrariness objection

The objection begins with the observation that the notion of an ordered pair is not an undefined notion of typical set theories; rather, ordered pairs are constructed from (unordered) sets by any one of a number of methods, each of which preserves the following identity condition:

$$\langle x, y\rangle = \langle z, w\rangle \text{ if and only if } x = z \text{ and } y = w.$$

The import of the identity condition is to insure that ordered pairs, under any method of construction, are individuated by their membership and the relative order of those members (this is in contrast to sets, which are individuated by membership alone). The two most common methods for this construction are Kuratowski’s and Wiener’s, according to which $\langle x, y\rangle$ is identified with $\{\{x\},\{x,y\}\}$ and $\{\{x,\varnothing\},\{y\}\}$, respectively, but any way of associating sets with objects that yields an ordered pair for any two objects and obeys the stated identity condition will do.

It would appear that PCN is in tension with the fact that ordered pairs can be constructed in different ways. Consider the application of naturalness to relations. If relations are reduced to ordered pairs and thus ultimately to sets,

[8] See Lewis (1986*b*, section 1.5).

[9] Class Nominalism is most plausible when coupled with the acceptance of non-actual possible objects (otherwise we identify actually coextensive properties and relations).

then it would seem that sets must be the things that, fundamentally, are natural or fail to be natural. But the set to which a given relation is reduced depends on the method for constructing pairs. Since the choice of a method is surely arbitrary and conventional, the worry is that naturalness of relations would turn out conventional, which would contradict the "objectivity" component of Primitivism.

This worry is roughly that expressed by Armstrong as follows:[10]

> Can we use the Wiener-Kuratowski device, and substitute for the ordered pairs an unordered set of sets? For ⟨a, b⟩ we substitute, perhaps, {{a}, {a, b}}, and for ⟨b, a⟩ {{b}, {a, b}}. However, as a piece of serious metaphysics, this seems quite unacceptable… the correlation between ordered pairs and unordered sets of sets is quite arbitrary. The substitution just given could as well have been reversed.

and by Forrest:[11]

> …as is well known, Kuratowski's identification suffers from a grave defect if it is treated as anything more than a model-theoretic device. For it is a convention, not a discovery, that $\langle a, b\rangle$ is to be identified with $\{a, \{a, b\}\}$ rather than, say, $\{\{a, \varnothing\}, \{a, b\}\}$, and serious ontology is not done by convention.

But the argument as stated is problematic, specifically in its appeal to "conventionality" and "arbitrariness", for Lewis might grant that the choice of a method for constructing pairs is arbitrary and conventional, but that given any such choice, there are nonconventional, objective answers to questions of naturalness.[12] The arbitrary choice of a method might be likened to arbitrary linguistic choices: we can choose to let terms mean anything we like; it is only after such choices that "the world does its thing", picking out which of our sentences are true and which are false.

As I see it, this response can only be adequately addressed by formulating a more careful and detailed version of the argument. As a start, we must fix some ideas about what the semantics for the term 'relation' will look like if class nominalism is true. The idea I will work with is that each method for constructing pairs corresponds to a different meaning for the term 'relation'—

[10]Armstrong (1986, 87).
[11]Forrest (1986, 91).
[12]I thank an anonymous referee for helpful comments here.

if X has in mind Kuratowski's method for constructing pairs whereas Y has some other method in mind, then they express numerically distinct propositions with the same sentences whenever those sentences contain the term 'relation'. This is not the only possible semantics for 'relation', but my argument goes through under other assumptions.[13]

The various meanings for 'relation' share a certain common functional or structural feature; after all, in the example above, X and Y are closer in what they mean by 'relation' than either is to someone who uses 'relation' as a term for a kind of tree. This common structural feature is due to the fact that in each case, a "relation" is defined as a set of "ordered pairs", where the term 'ordered pair' obeys the identity condition mentioned above. Thus, certain sentences will retain their truth values through a shift from one definition of 'relation' to another. For example, the sentence:

(S) The **being ten feet from** relation is symmetric

will have the same truth value regardless of what method for making pairs is involved in the definition of 'relation'. This may easily be shown to follow from the identity condition on ordered pairs, and the fact that 'symmetric' itself is defined in terms of ordered pairs (R is symmetric iff for any x and y, if $\langle x, y\rangle$ is in R then so is $\langle y, x\rangle$). We might think of (S) as a "structural" predication, since the predicate 'is symmetric' is defined in terms of ordered pairs in such a way as to only depend on structural features of sets of pairs, and not on the particular method of construction of those pairs.

Of course, not all sentences involving 'relation' will have their truth values preserved through shifts in methods of making pairs. The sentence

($\varnothing$) The **being ten feet from** relation contains $\varnothing$ (in its transitive closure)

[13] An alternate semantics for 'relation' would have that term be "partially interpreted". Think of an interpretation of English as a model; on this view, there are many models for English that differ only over what method for constructing pairs is used in interpreting talk of relations. Relation talk is "partially interpreted" because there is no one distinguished model; a claim of English would then be taken to be true simpliciter iff true on all acceptable models. My arguments require slight revision if this view is correct. In essence, the truth of principle (AF) (which I employ in sections 2.2 and 2.3) is built into this semantics for 'relation'. As John Collins pointed out, the argument is much faster this way; I stick with the assumptions in the text to avoid begging any questions.

is true if ‘relation’ means ‘set of Wiener pairs’ (because the null set gets “built into” every Wiener pair), but not if ‘relation’ means ‘set of Kuratowski pairs’. Thus, ‘contains the null set’ is not a structural predicate of relations, but rather one that depends on the particular method of construction.

What about sentences concerning naturalness? Is the sentence:

(N) The **being ten feet from** relation is natural

like (∅) or like (S)? Is the predicate ‘is natural’ structural? According to PCN, ‘natural’ cannot be defined in terms of ordered pairs the way ‘symmetric’ was defined, so it seems likely that if PCN is true, a sentence like (N) would be like (∅) rather than (S), and would turn out true on some methods for making pairs, but false on others. It is this idea, I believe, that Forrest and Armstrong find unacceptable, on the grounds that the choice of a method is arbitrary and conventional, whereas naturalness is supposed to be objective. This connection between objectivity and claims of naturalness may be summarized as follows:[14]

(AF) If naturalness is to be an “objective primitive”, then sentences of the form ‘Relation R is natural’ cannot vary in truth value when we vary the method for constructing ordered pairs

I think that (AF) is a correct principle, and would argue for it as follows. Variation in the truth value of a sentence of the form (N) could be explained in three ways. **First**, we could assimilate this variation to the case of the variation that would occur if we suddenly started using ‘relation’ to refer to something altogether different from sets of ordered pairs; certain kinds of trees, for example. This seems inadequate simply because the shift in the meaning of ‘relation’ isn’t *that* severe; if it were, then we ought to count class nominalists who use different methods for pair-making as talking about entirely different subjects.

Secondly, we could compare the variation to that of (∅); questions of naturalness have no “objective” truth values; their answers are mere artifacts of our methods for making ordered pairs. This is the sort of thing a class

[14] A more sophisticated version of the principle would focus only on sentences where relations are named by abstracts, such as: “The relation **being an *x* and *y* such that** ϕ is natural”, where ϕ is some open sentence containing no more than ‘*x*’ and ‘*y*’ free, and which makes no reference to relations, ordered pairs, etc. All of my uses of (AF) should be taken as uses of this more sophisticated principle, although I will usually simply coin names for relations (e.g., ‘R’), rather than using the abstract notation.

nominalist would say about the question of whether relations always contain the null set (in their transitive closures). But this claim about naturalness could not provide a counterexample to (AF), because of the claim in the antecedent of (AF) that naturalness is "objective".

Thirdly, we could say that the variation is due to the existence of an ontological distinction between, say, Kuratowski pairs and Wiener pairs. If Kuratowski pairs really were special, then it would be no surprise to find that claims of naturalness depend on the method of pair making. The rejection of this option is based on rejecting the existence of any such "joint in reality" dividing some kinds of pairs from others. Granted, the claim of PCN is that facts about naturalness are brute, but the objection is that these *particular* claimed brute facts are implausible. It is hard to say what our evidence is for rejecting the existence of such a joint in reality, but its rejection does indeed seem compelling. Perhaps one line of our thought here is that we don't see what could possibly count as a reason for believing which of the methods is the distinguished one. In his famous argument that numbers are not objects, Paul Benacceraf rejects the idea that there is an unknowable fact as to which of the familiar reductions of numbers to sets is *the* correct one:[15]

> In awaiting enlightenment on the true identity of 3 we are not awaiting a proof of some deep theorem. Having gotten as far as we have without settling the identity of 3, we can go no further. We do not know what a proof of that *could* look like. The notion of "correct account" is breaking loose from its moorings if we admit of the possible existence of unjustifiable but correct answers to questions such as this.

While I would not reject unknowable facts in general, in the case of a distinguished reduction of numbers, unknowable facts seem implausible. Ordered pairs seem analogous. The only features that distinguish one method from another involve mathematical convenience, and so seem irrelevant to the existence of an ontologically distinguished method.

2.2 PCN-1

Having rejected the only three ways that principle (AF) could fail to be true, I will now use it to argue against various versions of PCN that I will distinguish. (AF) gives us a direct objection to the simplest version of PCN. Recall the

[15] Benacerraf (1965, 58).

response to the original Armstrong/Forrest arbitrariness objection: once a method for making ordered pairs and thus a meaning for 'relation' is selected, "objective" answers to questions about naturalness are thereby determined. Versions of PCN divide over how these objective answers would be determined. To state any version of PCN it is necessary to answer two questions:

i) What is the nature of the fundamental facts about naturalness?

ii) How do these facts determine truth values of sentences like (N)?

PCN-1 answers i) in part by postulating a certain *property* of sets of sets of possibilia, which we may call "**naturalness**". Facts about which sets have this property exhaust the fundamental facts about naturalness. PCN-1's answer to ii) is that once we decide on a meaning for the term 'relation' by choosing a method, M, for making pairs, the sentence 'R is natural' is true iff the set with which R is identified under M has the property of naturalness. For example, once we decide that 'relation' will be interpreted in terms of Kuratowski's method for making pairs, then (N) is true iff the set of sets $\{\{x\},\{x,y\}\}$, where x is ten feet from y, has the property of naturalness.

What further distinguishes PCN-1 is a further answer to i), a claim about which sets have the property of naturalness: only sets that should, intuitively, count as natural relations under *one* distinguished method.[16] Suppose, for example, that Kuratowski's method is the distinguished method; then the set of Kuratowski pairs of objects separated by ten feet would be a natural set, whereas the set of Wiener pairs of objects separated by ten feet would not. Thus, (N) would be true if 'relation' were defined in terms of Kuratowski pairs, but false were 'relation' defined in Wiener's way.

PCN-1 is obviously inconsistent with (AF), and so may be rejected. It is my guess that Forrest and Armstrong were thinking of PCN along the lines of PCN-1, since it is so directly refuted by the considerations of conventionality and arbitrariness that went into the defense of (AF). But what Armstrong and Forrest do not consider is the possibility of other versions of PCN that escape this argument.

2.3 PCN-2

PCN-1 had the objectionable feature that only sets of (say) Kuratowski pairs have the property of naturalness, but what about a version of PCN according

[16] I ignore naturalness of properties and non-binary relations throughout.

to which sets of Kuratowski pairs *and* sets of Wiener pairs have the property of naturalness? This new version of PCN may be stated with more care by focusing on our two questions of formulation for PCN:

i) What is the nature of the fundamental facts about naturalness?

ii) How do these facts determine truth values of sentences like (N)?

PCN-2 retains the first part of PCN-1's answer to i): the fundamental facts involving naturalness consist of the instantiation of the **naturalness** property of sets. It also retains PCN-1's answer to ii): once the meaning of 'relation' is fixed by selecting a method, M, of making ordered pairs, the truth value of a sentence such as (N) may be determined simply by asking whether the set of M-pairs $\langle x, y\rangle$, where x is ten feet from y, has the property of naturalness. PCN-2 is distinguished by its admission of far more sets that have the property of naturalness. If the set of Kuratowski pairs of objects separated by ten feet is a natural set, then so will be the set of Wiener pairs of objects separated by ten feet. In general, we have:

(P2) if a set of M-pairs has the property of naturalness, where M is some method for making pairs that obeys the identity condition for pairs, then so will the corresponding set of M'-pairs, for every other such method M'.

Thus, there will be no one distinguished method; every method is on a par. In virtue of (P2), sentences like (N) will retain their truth values regardless of how we change the meaning of 'relation' (so long as it is defined as a set of ordered pairs under some acceptable method of construction).

But there is a further problem that confronts any view that, like PCN-1 and PCN-2, postulates a single property of naturalness, and evaluates the truth of (N) relative to method M by considering the naturalness of the set of M-pairs of objects that are separated by ten feet. This problem derives from the possibility of a set that corresponds to different relations under different methods, only some of which are natural. Let u and v be two objects that are ten feet apart, and let u' and v' be two other objects that are not ten feet apart. Define a new method X for constructing the ordered pairs, just like Kuratowski's method save that the sets identified with the pairs $\langle u, v\rangle$ and $\langle u', v'\rangle$ are "swapped"[17]; and let

[17] Method X may be defined more carefully as follows:

S_X be the set of X-pairs of objects that are separated by ten feet. Under method X, S_X counts as the **being ten feet from** relation; but under Kuratowski's method S_X corresponds to a highly unnatural relation: a relation, R, which we could tortuously name as follows:

> **being an x and y such that a) x and y are ten feet apart and it is neither the case that ($x = u$ and $y = v$) nor ($x = u'$ and $y = v'$), or b) $x = u'$ and $y = v'$**

Does S_X have the property of naturalness? Either answer is trouble. If S_X is not natural then (N) turns out false in a language where 'relation' means 'set of X-pairs', and thus, by (AF), false in any language with an alternate method of constructing pairs. But (N), intuitively, is true. On the other hand, if S_X is natural then the sentence 'R is natural' turns out true in a language where 'relation' means 'set of Kuratowski pairs'. This too is wrong; R does not carve the world at the joints. PCN-2 must be rejected.

Phillip Bricker suggested a hybrid of PCN-1 and PCN-2 that appeals to a primitive distinction between natural and unnatural methods of constructing pairs. The idea is that in order to "get the right answers" for questions of naturalness, one must construct the ordered pairs using a natural method. Set S_X may simply be taken to be unnatural; the fact that (N) then turns out false in a language where 'relation' means 'set of X-pairs' is allegedly unobjectionable because that language employs an unnatural method of construction; moreover, the further consequence that (N) turns out false in all other languages is resisted by rejecting (AF) in favor of a version restricted to natural methods. And a similarly restricted version of (P2) allows the hybrid theory to retain the reply to the objection to PCN-1: *both* the set of Kuratowski pairs and the set of Wiener pairs of objects separated by ten feet are natural. But I am inclined to stubbornly insist that the *un*restricted version of (AF) ought to hold. While a class of distinguished methods may be less implausible than the single distinguished method postulated by PCN-1, it is still implausible. The hybrid theory's distinction between natural and unnatural methods seems to mistake a pragmatic distinction (Kuratowski's method is more mathematically convenient

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$$\langle x, y\rangle = \begin{cases} \{\{u\},\{u,v\}\} & \text{if } x = u' \text{ and } y = v' \\ \{\{u'\},\{u',v'\}\} & \text{if } x = u \text{ and } y = v \\ \{\{x\},\{x,y\}\} & \text{otherwise} \end{cases}$$

than method X) for an ontological one. Thus, I continue to reject the idea that any method for constructing ordered pairs is ontologically distinguished from any other.

2.4 PCN-3

The main difficulty at this point is that a single set should sometimes count as a natural relation with respect to one method, but unnatural with respect to another. One response to this difficulty is to defend a version of PCN which postulates, not a single property of naturalness, but a plurality of properties of naturalness: one for each method of constructing ordered pairs. We thus have Kuratowski-naturalness, Wiener-naturalness, X-naturalness, etc.[18] Question i) for formulating versions of PCN is thus answered: facts about this plurality of naturalness properties exhaust the facts of naturalness. Question ii) is answered as follows: relative to a language which employs method of construction M, (N) is true iff the set of M-pairs of objects separated by ten feet has the property of M-naturalness.

Compare the case of the property **being a transitive relation**. Given the plurality of methods, we have not a single property of transitivity, but rather a plurality of different properties of transitivity: Kuratowski-transitivity, Wiener-transitivity, etc. In general, M-transitivity is the property of being a set, S, of M-pairs such that if M-pairs $\langle x,y\rangle$ and $\langle y,z\rangle$ are in S then so is the M-pair $\langle x,z\rangle$. Thus we may give a general characterization of the plurality of transitivity properties, thanks to the existence of a definition of transitivity in terms of the schematic term 'ordered pair'.

If naturalness were not a primitive notion, but rather defined in some way, then we could give an analogous general characterization of the plurality of naturalness properties. For example, if we accepted a definition of naturalness (for binary relations) in terms of sparse universals, we could say that a set, S, is M-natural iff there is some dyadic universal, U, such that S is the set of M-pairs of objects that stand in U. But since according to PCN there is no such definition of naturalness, no general characterization of the plurality of naturalness properties seems possible. And that, I say, is the difficulty with PCN-3. Not a knockdown objection, but a forceful one nonetheless: it would require an infinitude of primitive properties, with no hope of subsumption under a single formula or explanation. If PCN requires such an unlovely

[18]This idea of this response is due to David Lewis (private correspondence).

menagerie to do its work, we'd do better to look elsewhere for a theory of naturalness.

2.5 PCN-4

Perhaps there is a way to subsume the plurality of naturalness properties under a single undefined primitive notion of naturalness. Define the various naturalness properties, not via universals, but rather in terms of a single naturalness *relation*: a set, S, is M-natural iff it bears this naturalness relation to method M. Set S_X, the set of X-pairs of objects separated by ten feet, will bear the naturalness relation to method X, but not to Kuratowski's method. Recall our two questions for distinguishing versions of PCN:

i) What is the nature of the fundamental facts about naturalness?

ii) How do these facts determine truth values of sentences like (N)?

As for i), according to PCN-4, the fundamental facts involving naturalness are exhausted by the facts concerning a certain relation, **naturalness**, which holds between sets and methods. We answer ii) by claiming that (N) is true in a language where 'relation' is interpreted according to method M iff the set of M-pairs of objects separated by ten feet bears the naturalness relation to M.

According to PCN-4, the **naturalness** relation holds between sets and *methods*, but if so then there must actually be such things as methods. Trouble arises when we ask what these methods are supposed to be. There are certainly various sorts of construction we could employ here. The Kuratowski method, for example, might be identified with a two-place function f, where $f(x,y) = \{\{x\},\{x,y\}\}$. But the Kuratowski method could just as well be taken to be a different function $g : g(x,y) =$ the characteristic function of $f(x,y)$. And in either case, the functions will surely be construed as sets of ordered pairs. Since the ordered pairs are capable of multiple constructions, the functions will be as well.

The fact is that there is no one entity in the ontology of PCN that can legitimately be called *the* method of Kuratowski. For any method there are a number of constructions, each as legitimate a candidate for "being that method" as the rest, and this reintroduces our difficulty at the level of "meta-methods". There are four possibilities for formulating PCN-4, which correspond to our four possibilities for formulating PCN; none is acceptable. We cannot say that our naturalness relation relates sets and methods under one distinguished meta-method of construction, for the existence of such a distinguished meta-method is implausible, and would run afoul of a principle analogous to (AF). The analog of PCN-2 would be the claim that if the naturalness relation holds between a set and one construction of a given method, then it holds between that set and every construction of that method; but as with PCN-2, some entity might be interpretable as more than one method, only one of which ought to bear the naturalness relation to a given set. As with PCN-3, a plurality of naturalness relations, one for each meta-method, would be objectionable on grounds of theoretical economy. As a last resort we might, taking our cue from PCN-4, replace our two-place naturalness relation with a three-place relation between sets, method-constructions, and meta-methods: the relation holds between S, *M*, and *MM* iff S counts as a natural set, when interpreted as a set of ordered pairs constructed according to *M*, which in turn is interpreted as a method as constructed according to meta-method *MM*. This of course just postpones the evil day: meta-methods are just as arbitrarily constructable as methods, and the difficulty reappears at the level of meta-meta-methods. The regress is vicious because our goal is to find a stable, acceptable statement of PCN, and at each level the attempt fails.

I can think of no other plausible ways to formulate PCN, so I conclude that we should reject that theory. Notice that other views about naturalness are unaffected by my argument. A Primitivist could give up on Class Nominalism and take relations to be *sui generis* entities. And as Armstrong and Forrest note, a Primitivist could retain a kind of class nominalism by continuing to identify relations with classes of ordered pairs, but taking ordered pairs themselves as *sui generis* (as Forrest and Armstrong also point out, however, Lewis has independent reason *not* to take this way out.[19]) If on the other hand we retain the class nominalist reduction of relations to unordered sets, my argument does not apply to a view that analyzes naturalness in some way in terms of facts that don't involve ordered pairs (such as the analysis in terms of universals I mentioned earlier[20]), for given any method for constructing pairs, we may

[19]See Armstrong (1986, 86–87), and Forrest (1986, 90–91) for this point. Lewis defends the principle that "the 'generating' relation of a system should never generate two different things out of the very same material. There should be no difference without a difference in content." See Lewis (1991, 38). Lewis wields this principle against structural universals in Lewis (1986*a*) and discusses it at length in Lewis (1991, section 2.3).

[20]Another analysis of naturalness that would be unaffected by my argument is an alternate proposal of Lewis's: naturalness is to be analyzed in terms of a complex similarity relation over

simply apply the analysis of naturalness to pick out the natural sets under that method of construction.

It may be worth mentioning briefly that Lewis's recent proposal about the nature of sets exacerbates the difficulty of this section. In the appendix of *Parts of Classes* Lewis considers the prospects of eliminating the set-theoretic primitive relation of membership in favor of the part-whole relation. On his "structuralist" conception of set theory, for any objects, there is no one entity that is, once and for all, the set containing just those objects. Rather, different items count as the set of those objects relative to different *singleton functions*, which assign singletons to objects. (Relative to a given singleton function, multi-membered sets are taken to be the mereological fusions of their singleton subsets.) But if there is no distinguished singleton function, there is no single object that is, for example, the set of all negatively charged objects, and thus no one object that the class nominalist can identify with the property of being negatively charged. Thus, the argument of this section will apply, only now it applies to natural properties as well as natural relations.[21]

3. Does the Argument Prove Too Much?

Surely, the fact that there is no uniquely distinguished method for constructing the pairs does not mean that all uses of pairs by philosophers and mathematicians are faulty! But couldn't my argument be extended to show these uses illegitimate? And if not, then what is so special about the use of pairs made by PCN?

I agree that most uses of pairs are legitimate. The case of PCN is special, for it grants significance to the particular objects to which pairs are reduced, by applying primitive naturalness to sets of pairs. (In fact, it seems to me that my argument applies whenever a Class Nominalist postulates a non-conventional primitive property or relation of relations.[22]) Other uses of pairs are legitimate

possibilia. See Lewis (1983, 347, 348).

[21]The problem arises in another way as well: singleton functions are construed mereologically by Lewis, and as he and others show, can be constructed in different ways. See Lewis (1991, Appendix).

[22]And still more generally, whenever an unanalyzable property or relation is applied to "arbitrarily constructable entities". I would argue, for example, that a similar argument refutes Quine's proposal in "Whither Physical Objects?" to reduce physical objects to 'tuples of reals and thus to pure sets. The reduction is arbitrary in two ways that Quine mentions (units of measure and frame of reference—p. 501) and also in that the reduction of reals to sets can take

because on those uses no such significance is granted— rather, the pairs are only used to get a job done.

Consider the use of ordered pairs in semantics as the meanings of dyadic natural language predicates. The predicate 'taller than' will be assigned a different semantic value depending on how we construct pairs. If we are loath to accept a distinguished method for making pairs, must we accept that there is no objective fact as to what a given word means? In a sense, yes: if we reduce meanings to sets of ordered pairs then we should admit that it is a matter of the pair-making convention which particular entity is the meaning of 'taller than'. But we can accept this conclusion. The goal of semantic theory is merely to provide a systematic account of certain intuitions speakers have about their language, intuitions for example about the truth conditions of sentences or about the validity of inferences. To accomplish this task, semantic theory is free to employ whatever "internal" apparatus it chooses. What particular entities play a role in this internal apparatus is not part of the ultimate output of the theory, but is only an "artifact of the model".

There is, then, a sense in which what a word means is a matter of the pair-making convention: a semanticist who utilized Wiener pairs would assign numerically distinct entities as the meanings of predicates than a semanticist partial to Kuratowski's method would, and neither would be giving a better description of reality. Put another way: a principle along the lines of (AF) would be implausible for sentences like "The meaning of 'taller than' contains $\varnothing$ (in its transitive closure)." There is, however, another sense in which meaning isn't conventional in this way, but my argument is consistent with this sense. This can be seen by observing that the truth of the following sentences does not depend on the method for pair making:

(7a) 'Ted is taller than Mike' is true iff 'Mike is shorter than Ted' is true

(7b) 'is taller than' expresses a transitive relation

(7c) 'Ted is taller than Mike' is true iff Ted is taller than Mike

(7d) 'is taller than' applies to Ted and Mike (in that order)

Wiener- and Kuratowski- semanticists alike would agree on these sentences. I suggest that it is claims like these that we have in mind when we think that

place in various ways. The crucial fact is that Quine needs to apply physical primitives (e.g., those involving fields) to entities that are capable of multiple reductions.

"what words mean is not a matter of the pairs convention". The phrase 'what a word means' is ambiguous. Are we discussing the entity that is assigned to be that word's semantic value? Then we are discussing something that is (partly) a matter of the convention of making pairs. Or are we discussing meaning relations between words as in (7a), formal properties of a word's meaning as in (7b), or relations between words and the world as in (7c) and (7c)? Then what we are discussing is not a matter of the pairs convention; indeed, a principle like (AF) for such sentences would be both plausible and consistent with my argument.

4. Naturalness and "Kripkenstein"

I close by briefly applying the argument from arbitrariness of the previous section to Lewis's solution to Saul Kripke's version of a puzzle of Wittgenstein's.[23] Surely, by using the word 'plus', I express the plus function, which assigns to any two numbers their sum. But my behavior seems to be consistent with my meaning instead the *quus* function, which assigns to *x* and y their sum if each is less than some suitably chosen number; seventeen otherwise. If the chosen number is high enough, it seems that nothing about my behavior could constitute my meaning plus and not quus when I say 'plus'. Surely my behavior determines what I mean, so it seems we ought to conclude that I do not mean plus any more than quus. But I *do* mean plus and not quus.

Lewis proposes we solve the puzzle using naturalness.[24] Plus, but not quus, is a natural function; since natural functions are prima facie more eligible to serve as meanings for our words, I mean plus rather than quus. Nothing about my behavior determines this— I may never have heard of naturalness. It is simply a fact about reference that reference goes to the most natural candidate.

This solution applies primitive naturalness to number-theoretic functions, but many believe that numbers are not primitive entities, but are rather constructions from sets.[25] Since there are various different ways of doing the construction, the argument from arbitrariness applies. As Paul Benacceraf has pointed out, we cannot take the number two to *be*, once and for all, any of the

[23]Kripke (1982).

[24]Lewis (1983, 375–77).

[25]There is no problem with Lewis's solution of the Kripkenstein puzzle if numbers are taken as primitive entities. See Wetzel (1989, part one) and Resnik (1980, 231) for analogous responses to Benacceraf's argument that numbers "are not objects".

sets with which it is identified in these various constructions, for i) the sets in the various cases are different— 2 is identified with $\{\{\varnothing\}\}$ by Zermelo, and with $\{\varnothing,\{\varnothing\}\}$ by von Neumann — and ii) as mentioned earlier, the notion of a distinguished construction of the numbers seems implausible.[26] Given this parallel with the case of ordered pairs, we have analogs to our four versions of PCN: naturalness as applied to numbers might be taken as a property of set-theoretic functions relative to one distinguished method for constructing numbers, a property of set theoretic functions had by functions for all methods of construction, a plurality of properties, or a relation between set-theoretic functions and methods. The first falls with PCN-1 to considerations of conventionality, the second with PCN-2 because of set theoretic functions that count as different number-theoretic functions under different methods of construction, and the third like PCN-3 because of its prohibitive cost in complication of theory; the fourth, like PCN-4, reintroduces the arbitrariness difficulty with its reification of "methods".

It should be clear that this argument is not particular to numbers. It generalizes whenever our talk of "entities", like our talk of numbers, is capable of multiple, equally good, reconstructions. Talk of primitive naturalness in these areas must be abandoned.

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[26]See Benacerraf (1965).

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