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Why Is There Philosophy of Mathematics at All?, by Ian Hacking. Cambridge: Cambridge University Press, 2014. Pp. xv + 290. H/b £50. P/b £17.99.

Some may be tempted to read the title of Ian Hacking's latest book as a call to dismiss the philosophy of mathematics. This would be a mistake. Hacking does not think that philosophy of mathematics will ever go away, and it is this enduring appeal that motivates Hacking's book. Hacking asks why mathematics has perennially fascinated philosophers, to the point that the philosophy of mathematics is not the philosophy of a special science – like the philosophy of physics or biology – but rather a central field of analytic philosophy.

As with much of Hacking's work, his latest book unfolds through the *history* of mathematics. Nonetheless, Hacking emphasizes that he is not giving interpretations of the texts he discusses (hence his lowercase spelling of 'leibnizian' and 'cartesian'). He does not advance a particular thesis in the philosophy of mathematics but rather asks how the central questions and enduring views in the field have come about. Hacking answers that philosophers' enthusiasm for mathematics has two sources: proof and application.

Hacking's interest in proof is primarily a concern with the experience of proof. He distinguishes between leibnizian proof – the formal line-by-line proof with which analytic philosophers have typically been concerned – and cartesian proof – the sort of proof one can get into one's mind all at once and which explains why its conclusion is true (Hacking slides freely between these two characterizations of cartesian proof, though it is not clear that they are equivalent). It is cartesian proof that Hacking claims is central to mathematicians' fascination with their subject. Demonstrating the importance of cartesian proof to mathematics may be one of the most important features of Hacking's book. At the same time, more could be done to establish that there is a clear dichotomy between these forms of proof and that focusing on cartesian proof would be important to the philosophy, rather than the history or sociology, of mathematics.

According to Hacking, cartesian proof sets the stage for the philosophy of mathematics. Mathematicians' experience of following a convincing proof, which evinces the necessity of its conclusion, is what persuades them that there is a mind-independent, non-spatiotemporal realm of mathematical facts. While following such a proof, a mathematician feels that its conclusion was 'out there' to be discovered all along, rather than invented by the author of the proof.

Hacking gives examples from mathematicians' informal writings, in which they describe their work as exploration and their results as 'out there' in the same way as the results of analytical chemistry, for example. Of course, whether an immaterial mathematical realm exists is the central concern of philosophical debates about platonism. Therefore, the experience of proof is a reason why there is philosophy of mathematics at all.

Hacking also devotes substantial attention to the applications of mathematics. The way in which mathematical results derived a priori routinely prove valuable in the a posteriori inquiry of the natural sciences is another reason why there is philosophy of mathematics. Early in his book, Hacking emphasizes that mathematics has not solely been applied to other disciplines but rather has also been applied to itself. He discusses the Langlands program, which relates geometry and number theory. Here, one wishes that Hacking would pause for more detailed discussion. He has compellingly argued that applications of mathematics to mathematics are worth greater philosophical investigation, and it would be interesting to see him undertake such investigation.

Throughout his discussion of proof and application, Hacking explores the contest between what he calls the butterfly model and the Latin model of mathematical advancement. According to the butterfly model, mathematics has evolved along a predetermined course, like a pupa's development into a butterfly. According to the Latin model, mathematics has taken its course because of historical contingencies, like a natural language. Hacking appears to favor the Latin model. Mathematics could have developed without its emphasis on rigorous proof; the work of Chinese and Indian mathematicians showed that persuasive and true conclusions can be derived without rigorous proof.

Also contingent was the emergence of the distinction between applied and pure mathematics. Indeed, what has fallen under each category has changed over time. In the time of Euclid, geometry was considered pure mathematics and arithmetic impure. Following Kant, the roles were reversed. The distinction between applied and pure itself evolved from a distinction between mixed and pure. Mixed mathematics is a form of inquiry that is partially a priori and partially empirical. Applied mathematics is developed a priori but used in empirical science. The applied-pure distinction, therefore, has not been essential to the development of mathematics.

Contrarily, Hacking argues that complex numbers give us an example of mathematics developed on the butterfly model. Some questions in number theory cannot be settled without the introduction of complex numbers. Once mathematicians began to study number theory, it was inevitable that their inquiries would lead them to the point where they could make no further progress without the complex numbers. Any route to sophisticated number theory goes through the complex numbers, just as any route from pupa to butterfly involves the same developments.

The final two chapters of Hacking's book are devoted to platonism. First, Hacking chronicles the history of 'platonism,' noting that our current understanding of the term differs vastly from what certain historical figures had in mind and even from the definition given in *Webster's Third New International Dictionary* of 1961. He then, appealingly, chooses to investigate platonism through the writings of mathematicians, turning only later to contemporary philosophy.

Hacking contrasts the views of mathematicians Alain Connes and Timothy Gowers. Connes holds that mathematics is the study of 'archaic reality,' composed of objects that exist independently of us but require us to impose organization before we can study them. Connes also has a Pythagorean streak, as he claims we will eventually find material reality to be located within mathematical reality. Against Connes, Gowers holds that mathematics does not need philosophy. Even a philosophical epiphany would not affect the work of most mathematicians. At the same time, Gowers holds that most mathematicians are and should be formalists. Being more concerned with proof than with truth, they are able to make greater progress.

The debate between Connes and Gowers is important because it makes clear how different philosophical nominalism is from mathematical anti-platonism. Mathematicians like Gowers dismiss platonism because they find it irrelevant to their practice. They are not inclined to assert that mathematical objects do not exist. Philosophical nominalists, though, are eager to make such assertions. Philosophers' nominalism has little to do with the themes of proof and application that Hacking has emphasized. Mathematical anti-platonism, contrarily, is loosely connected to proof, insofar as mathematicians tend to imagine pictures of mathematical objects in the course of their work (i.e. in their cartesian proofs). By invoking philosophers such as Field and Hellman alongside mathematicians such as Dedekind and Kronecker, Hacking effectively makes the case that philosophical debates about the ontology of mathematics have been conducted in relative isolation from the concerns of mathematicians.

I remarked above that Hacking does not enter the fray of contemporary analytic philosophy of mathematics. He does, however, hint at a preferred solution to ontological questions about mathematics: to dismiss them. Having cited Penelope Maddy and John Burgess at several points in the book, Hacking adopts a form of the naturalism that Maddy and Burgess have advanced. Philosophical questions about mathematics are to be settled by the methods of mathematicians in the course of their ordinary practice. Hacking's arrival at this conclusion via the history of mathematics and philosophy makes his book a valuable and novel contribution to the naturalistic literature.

Hacking's book unfolds in a relatively breezy style. The explanation of a concept may flow into an anecdote followed by a brief biography of a notable mathematician or philosopher. This does not mean that the book feels disorganized but rather that it makes for easier and more engaging reading than one finds in most philosophy of mathematics. At the same time, this means that Hacking devotes more attention to presenting information than to his own interpretations or analyses. Some may find this unsatisfying, as it is often unclear what information is philosophically significant and what is of mere sociological or historical interest (of course, much of Hacking's philosophy has emphasized that historical questions are closely connected to philosophical ones). However, this also means that Hacking's book gestures towards many areas for further philosophical discussion.

Though perhaps not suitable as an introduction to the philosophy of mathematics, given that Hacking avoids entering many philosophical disputes, the book would be highly useful in stimulating student interest in the philosophy of mathematics and would be a worthwhile companion to more technical readings. Moreover, it provides much fodder for philosophers duly inclined to take the history and practice of mathematics as the impetus for philosophical theorizing.

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