Revista Brasileira de
Filosofia da Religião
ISSN 2358-8284
Associação Brasileira de Filosofia da Religião ABFR
Dossiê Fenomenologia e Filosofia Analítica da Religião

On the logical formalization of Anselm's ontological argument[1]

Sobre a formalização lógica do argumento ontológico de Anselmo

RICARDO SOUSA SILVESTRE(*)

Abstract

The general theme of this paper is the issue of formalization in philosophy; in a more specific way, it deals with the issue of formalization of arguments in analytic philosophy of religion. One argument in particular – Anselm's Proslogion II ontological argument – and one specific attempt to formalize it – Robert Adams' formalization found in his paper "The Logical Structure of Anselm's Arguments", published in *The Philosophical Review* in 1971 – are taken as study cases. The purpose of the paper is to critically analyze Adams' formalization with the intent to shed some light on the following questions: What are the virtues of formally analyzing arguments and the contributions, if any, of such an enterprise to the debate on Anselm's argument? Which lessons can Adam's work teach us about the dangers and limitations of formalization? Do these virtues and dangers teach us something about analysis of arguments in general?

Key words: Formalization in philosophy. Theist arguments. Ontological argument. Anselm's argument. Adams.

Resumo

O tema geral deste artigo é a questão da formalização em filosofia; de uma maneira mais específica, ele trata da questão da formalização de argumentos em filosofia analítica da religião. Um argumento em particular – o argumento ontológico de Anselmo encontrado no capítulo II do seu Proslógio – e uma tentativa específica de formaliza-lo – a formalização de Robert Adams encontrada em seu artigo "The Logical Structure of Anselm's Arguments", publicado no *The Philosophical Review* em 1971 – são tomados como estudos de caso. O objetivo do artigo é analisar criticamente a formalização de Adams com o propósito de lançar alguma luz nas seguintes questões: Quais são as virtudes de se analisar formalmente argumentos e as contribuições de tal empreitada para o debate acerca do argumento de Anselmo? Que lições o trabalho de Adams pode nos dar sobre os perigos e limitações da formalização em filosofia? Essas virtudes e perigos nos ensinam algo sobre análise de argumentos em geral?

Palavras-chave: Formalização em filosofia. Argumentos teístas. Argumento ontológico. Argumento de Anselmo. Adams.

(*) Doutorado em Filosofia pela University of Montreal, Canadá. Mestrado e graduação em Ciência da Computação pela UFC. Professor Associado da Universidade Federal de Campina Grande. Atua nas áreas de Lógica Filosófica, Epistemologia, Filosofia da Religião e Filosofia Indiana.
E-mail: ricardoss@ufcg.edu.br

[1] Work partially supported by CNPq (National Counsel of Technological and Scientific Development of Brazil), public notice MCT/CNPq N◦ 14/2013.

Introduction

There has been recently a great deal of interest in the debate on the role of formalization in philosophy. Authors such as Pascal Engel (2010), Sven Hansson (2000) and Leon Horsten and Igor Douven (2008) have made interesting contributions to this topic. Questions usually asked here are (1) "What are the virtues and dangers of formalization?" (2) "Which kind of work should precede the formalization proper?" and (3) "Which formal tool is most suitable for a given philosophical problem?

It is not coincidence that this debate takes place in a moment where it is safe to say that the age of logical influence on analytic philosophy is gone. Except for isolated cases, the use that contemporary analytic philosophers make of logic is, in the absence of a better word, a 'diluted' one: by mostly using elementary logical tools, they neglect what mostly characterize modern logic, namely, its proof and model theoretical sides and meta-logical considerations. There are many reasons for this state of affairs. Surely one of them has to do with a criticism very often made against the use of formal tools in philosophy: in general the formal philosopher gets so stuck in technicalities that he misses what really matters to the problem; at the end of the day it is hard to say what are his real contributions to the philosophical problem at hand.

However and despite of this, construction and evaluation of arguments keeps being an important part of the work of any analytic philosopher. This seems to be particularly true for analytic philosophy of religion, where the construction and analysis of theistic and atheistic arguments has been for decades a flourishing field of inquire. Among other things, we find here – especially in the study of one of the most famous arguments in the philosophy of religion, and in fact in the whole history of philosophy: the ontological argument – some of those isolated cases I have mentioned before.

It is not new that some very well-known contemporary versions of the ontological argument make use of a considerable amount of formal notation. I am here thinking of Kurt Gödel's (1995) and Alvin Planting's (1974) versions of the ontological argument. However, it is in the analysis of some traditional versions of the argument, particularly Anselm's version, that we find some of the most interesting instances of logical formalization in philosophy of religion. The works of Robert Adams (1971), Jonathan Barnes (1972), Paul Oppenheimer and Edward Zalta (1991), Gyula Klima (2000) and Jordan Sobel (2004) are examples of this.

I will focus here on Robert Adams' (1971) formalization of Anselm's Proslogion II argument found in his paper "The Logical Structure of Anselm's Arguments", published in *The Philosophical Review* in 1971. More specifically, my goal is to critically analyze this work with the intent to shed some light on the following questions:

1) What are the virtues of formally analyzing arguments and the contributions, if any, of such an enterprise to the debate on Anselm's argument?
2) Which lessons can Adam's work teach us about the dangers and limitations of formalization?
3) Do these virtues and dangers teach us something about analysis of arguments in general?

Before getting into the analysis proper, I will say some few words about the use of logic in the analysis of philosophical arguments. I will also present a very rough analysis of Anselm's argument that will help in the evaluation of Adams' work.

1 Some considerations on Logic-Formal Analysis in philosophy

Talk about the role of logical formal analysis of arguments presupposes an answer to the question of what a formal analysis of an existing argument is. Like many issues in formalization in philosophy, this is a disputable one. However, we might say there are some features common to all formal analysis of arguments. First, there is always some sort of previous, informal analysis of the argument; it is meant to say, for example, what the premises and conclusion of the argument are, whether or not there are subsidiary arguments and hidden premises, etc. Second, there is a formal language in which premises and conclusion are represented. Third, there is some attempt to reconstruct the inferential steps of the original argument, that is to say, some sort of formal derivation from premises to conclusion usually is there. Finally, and this might be taken as an optional feature, there is an inference theory, be it proof theoretical or semantical, or both, inside of which the derivation is evaluated.

This last point, although not always present in formalizations of arguments found in contemporary philosophy (that is why I have said it is optional), is a very important one. One of the main purposes of formalizing an existing argument is to better evaluate its soundness. Logic seems to be the ideal tool for this exactly because it is a theory of sound argumentation; one of its main purposes, we can say, is to provide a rigorous, more trustful framework in which arguments can be evaluated. Although a formal analysis that stops, say, at the level of representing premises and conclusion into a formal language might shed some light on the structure and presuppositions of the argument, it misses the most important contribution that formal logic can give, which is a rigorous way to appraise the soundness of arguments.

In a sense, the whole thing can be seen from the viewpoint of Carnap's (1950) project of conceptual explanation. On one side, we have an argument, in general, presented in a prose text, whose relevant aspects – premises and conclusion with their exact meaning, presuppositions, structure, etc. – are obscure and ambiguous. This would correspond to Carnap's notion of *explicandum*. On the other hand we have the outcome of the analysis: a derivation, represented inside a formal framework, which is supposed to be a reconstruction, or to use Carnap's terminology, an explanation of the original argument. This is the *explicatum*.

Due to its formal feature, the *explicatum* is supposed not to have those obscure features of the *explicandum*. In particular, it must be evident in the *explicatum* the exact meaning of premises, conclusion and hidden presuppositions, the structure of the argument, and whether or not it is a sound argument. The *explicatum* is also supposed to help in the evaluation of the reasonableness of the premises. Naturally, in order to be an explanation or, as we shall prefer, a reconstruction of the original argument, the *explicatum* should be minimally similar to the *explicandum*. Due to the very nature of a formal reconstruction and to the obscurity and incompleteness of informal arguments, the *explicatum* will have many elements not present in the original argument. However, this shall not make the *explicatum* to depart too much from the original argument, otherwise it cannot any more be said to be an explanation of it.

2 Anselm's Argument

Anselm's first and most famous ontological argument is found in the second chapter of his Proslogion (Anselm, 1965). Here we have the extract where the argument appears:

(i) Well then, Lord, You who give understanding to faith, grant me that I may understand, as much as You see fit, that You exist as we believe You to exist, and that You are what we believe You to be.

(ii) Now we believe that You are something than which nothing greater can be thought.

(iii) Or can it be that a thing of such a nature does not exist, since "the Fool has said in his heart, there is no God?" (Psalms14, l.1, and 53, l. 1.)
(iv) But surely, when this same Fool hears what I am talking about, namely, "something-than-which-nothing-greater-can-be-thought", he understands what he hears, and what he understands is in his mind (intellect, understanding), even if he does not understand that it actually exists.
(v) For it is one thing for an object to exist in the mind, and another thing to understand that an object actually exists.
(vi) Thus, when a painter plans before hand what he is going to execute, he has (it) in his mind, but does not yet think that it actually exists because he has not yet executed it.
(vii) However, when he has actually painted it, then he both has it in his mind and understands that it exists because he has now made it.
(viii) Even the Fool, then, is forced to agree that something-than-which-nothing--greater-can-be-thought exists in the mind, since he understands this when he hears it, and whatever is understood is in the mind.
(ix) And surely that-than-which-a-greater-cannot-be-thought cannot exist in the mind alone.
(x) For if it exists solely in the mind even, it can be thought to exist in reality also, which is greater.
(xi) If then that-than-which-a-greater-cannot-be-thought exists in the mind alone, this same that-than-which-a-greater-cannot-be-thought is that-than-which-a--greater-can-be-thought.
(xii) But this is obviously impossible.
(xiii) Therefore there is absolutely no doubt that something-than-which-a-greater--cannot-be-thought exists both in the mind and in reality[2].

Sentences (i) and (ii) might be seen as an introduction to the argument. While (i) is a sort of opening statement, (ii) is Anselm's famous definition of God: God is something than which nothing greater can be thought. (iii) marks the proof style Anselm adopted: the *reductio ad absurdum* method; it states what we might call the *reductio ad absurdum* hypothesis, that is, the negation of what is supposed to be proved. Sentences (iv) to (viii) can be taken as a preliminary argument meant to prove a key premise of the argument: that something-than-which-nothing-greater-can-be-thought exists in the Fool's mind. (ix) is an anticipation of the argument's conclusion: that God exists both in reality and in the understanding. (x) is the basic step of the argument: if this thing exists only in the mind, it can be thought to exist in reality also, and to exist in reality is greater. Sentence (xi) states a consequence of what has been said so far, in special a consequence of the hypothesis

[2] Translation by M. J. Charlesworth (ANSELM, 1965).

that this thing exists only in the mind: if it exists only in the mind, it will be at the same time that-than-which-a-greater-cannot-be-thought and that-than-which-a-greater-can-be--thought. But this, as sentence (xii) says, is impossible. Therefore the conclusion of the argument (xiii): that something-than-which-a-greater-cannot-be-thought exists both in the mind and in reality.

3 Adams' Formalization

Robert Adams' (1971) article "The Logical Structure of Anselm's Arguments" offers a formal analysis of Anselm's arguments for the existence of God. After presenting the argument of Proslogion II in a formal fashion, Adams lays down a couple of assumptions about existence and predication on which the argument seems to him to depend. He then tries to formally show how Gaunilo's famous lost island counterexample proves that these assumptions must be modified. Besides, he analyzes one of the arguments for the existence of God found in Anselm's reply to Gaunilo, which, according to him, does not depend on those assumptions; he also analyzes Anselm's ontological argument found in chapter III of Proslogion.

Here I will be concerned only with Adams' formalization of Anselm's Proslogion II argument, his assumptions about existence and predication and his formalization of Gaunilo's counter example.

Before presenting his formalization, Adams does a short informal analysis of the original argument; according to him, Anselm's argument has the following three sentences as premises:

(I) There is, in the understanding at least, something than which nothing greater can be thought;
(II) If it is even in the understanding alone, it can be thought to be in reality also;
(III) which is greater;

and the sentence below as conclusion:

(IV) There exists, therefore, . . . both in the understanding and in reality, something than which a greater cannot be thought

As far as our numeration of Ansem's statements is concerned, (I) is sentence (viii), (II) and (III) are (x), and (IV) is sentence (xiii).

The first observation to be made about Adams's analysis concerns his choice of taking (viii) as premise. As we have said, sentences from (iv) to (viii) can be taken very reasonably as a preliminary argument: while (iv) is an anticipation of the conclusion and (viii) is the conclusion, sentences (v) to (vii) seem to be meant to support (viii). That Adams skips this and takes (viii) instead as premise is significant for a couple of reasons. First, although one could try to justify this move, the fact that Adams does not even mention it and simply ignores a good part of Anselm's original argument makes his analysis less faithful to it. Second, neglecting that Anselm himself tried to justify (viii) has important consequences for evaluating the reasonableness of (Adam's reconstruction of) Anselm's argument. As we know, many people, such as William Rowe (2006, p. 37-52), have argued that this premise is a very key one in the argument, and unless it is well justified, the argument might be accused of question-begging.

The second thing is that there is in Adams analysis a fourth premise which he mentions just at the time of formally reconstructing Anselm's argument:

(V) There is no God.

It corresponds to sentence (iii). The reason why he had to wait that long has to do with the poof method Anselm uses: the *reductio ad absurdum* method.

Adams uses a first-order language to represent premises and conclusion. However, he interprets the definition of God in a modal fashion, using a modal operator and going, in this way, beyond the walls of classical first-order logic. He uses two unary predicates – U and R –, and two binary ones: G and Q, meaning as follows:

U(x) *x* exists in the understanding
R(x) *x* exists in reality
G(x,y) *x* is greater than *y*
Q(x,y) *x* is the magnitude of *y*

Besides, as we said, he also uses a modal operator, M, which means "it can be thought that" or "it is possible that", which he takes as equivalent. From now on I will read M as "it is possible that"; later I shall comment on Adam's taking "it can be thought that" and "it is possible that" as equivalent.

In order to formally represent the premises, Adams uses the following abbreviation:

$\phi(x,m) =_{def} Q(m,x) \wedge \neg M(\exists y \exists n(G(n,m) \wedge Q(n,y))$

$\phi(x,m)$ is a representation of Anselm's concept of a thing than which nothing greater can be thought. Besides using variable *x* to mean that thing, Adams uses one more free variable, *m*, to represent the magnitude of *x*. What $\phi(x,m)$ says is that *m* is the magnitude of *x*, and it is not possible that there is another thing, say *y*, whose magnitude *n* is greater than *m*.

The formal representation of the three premises and conclusion of the argument is as follows:

(I) $\exists x \exists m(U(x) \wedge \phi(x,m))$
(II) $\forall x \forall m(U(x) \wedge \phi(x,m) \rightarrow MR(x))$
(III) $\forall x \forall m(\phi(x,m) \wedge \neg R(x) \rightarrow \neg M \neg (R(x) \rightarrow \exists n(G(n,m) \wedge Q(n,x))))$
(IV) $\exists x \exists m(U(x) \wedge R(x) \wedge \phi(x,m))$

(I), (II) and (IV) are of easy understanding. (III) says that to every *x* and *m*, if *x* is God and *m* is his magnitude, but he does not exist in reality, then it is not possible that the following proposition is false (that is to say, such a proposition is necessary): if *x* exists in reality, then his new magnitude, *n*, is greater than *m*. Adopting a counterfactual reading, (III) would mean the following: if God, whose magnitude is *m*, does not exist in reality, then would he exist in reality, his new magnitude, *n*, would be greater than *m*.

(III) incorporates one of the most controversial issues in Anselm's argument: the doctrine that existence is a perfection. The doctrine appears in sentence (x) of Anselm's argument:

(x) For if it exists solely in the mind even, it can be thought to exist in reality also, which is greater.

, which Adams represents as his (incomplete) informal premise (III). Now, this principle, which can be described as the presupposition that

(G) It is greater to exist in reality as well than to exist merely in the understanding

, might be understood in at least three different ways (Matthews, 2005, pp. 90-91):

(G1) Anything that exists both in reality and in the understanding is greater than anything that exists in the understanding alone.

(G2) Anything that exists both in reality and in the understanding is greater than the otherwise same kind of thing that exists in the understanding alone.

(G3) Anything that exists both in the understanding and in reality is greater than the otherwise exact same thing, if that thing exists merely in the understanding.

Adams, who is partially aware of this ambiguity –

> All Anselm said was "which is greater." This could be taken to mean that anything which exists in reality is greater than anything which does not; this is a claim to which Anselm would probably have assented. But it could also mean just that the being under discussion (that than which nothing greater can be thought) would be greater if it existed than if not; this is all the argument requires, and I am assuming this minimal interpretation in symbolizing the argument. (Adams, 1974, p. 30)

– picks (G3) as the correct or more suitable interpretation of (G). However, even considering its attempt to be as precise as possible, G3 is still ambiguous with respect to one thing: are these two things we are comparing exactly the same object, or two objects which differ in one aspect only (existence)? Adams representation leaves no doubt: we are comparing the very and same object, the one referred to by variable *x*.

A second point I should mention regarding (III) is Adams' use of the operator M. Sure it is an interesting way to incorporate in (III) the doctrine that existence is a perfection. However, it is significant that neither Anselm's original statement of it in (x) nor (G) use any kind of modal construction. It is a theoretical choice Adams does with no trivial support in Anselm's text. And, similarly to his use of (viii) as premise, there is no attempt to justify this important theoretical movement. Besides the philosophical implications of writing (G) in this way, it is crucial for one of the main purposes of Adams' work; as well shall see below, the validity of the formal reconstruction of Anselm's argument he presents is formally linked with the use of M in (III).

Adams then proceeds to show that Anselm's argument, or to be more precise, his reconstruction of Anselm's argument, is sound. He constructs a derivation using a couple of well-known classical first-order inference rules of Quine's book *Methods of Logic*. He also uses two modal inferences:

M1. $\neg M\neg(\alpha\rightarrow\beta), M\alpha \vdash M\beta$

M2. $\exists x M\alpha(x) \vdash M\exists x\alpha(x)$

Differently however from his use of classical first-order rules, he does not state which modal formal system he is using. Despite of this, his reconstruction of Anselm's argument seems to be sound. Besides these two inferences, he also uses the following first order inferences rules (here we are using a more standard notation than Quine's):

C1. $\exists x\alpha(x) \vdash \alpha(x/t)$
C2. $\forall x\alpha(x) \vdash \alpha(x/t)$
C3. $\alpha\wedge\beta,\ \beta\wedge\varphi\rightarrow\lambda \vdash \varphi\rightarrow\lambda$
C4. $\alpha(t) \vdash \exists x\alpha(t/x)$
C5. $\alpha\wedge\beta,\ \varphi \vdash \beta\wedge\varphi$
C6. If $\Gamma, \alpha \vdash \beta$ then $\Gamma \vdash \alpha\rightarrow\beta$
C7. $\neg\alpha\rightarrow\beta\wedge\neg\beta \vdash \alpha$
C8. $\alpha\wedge\beta,\ \varphi \vdash \alpha\wedge\varphi\wedge\beta$
MP. $\alpha,\ \alpha\rightarrow\beta \vdash \beta$

Here is the derivation:

1.	$\exists x\exists m(U(x)\wedge\phi(x,m))$	Pr. (I)
2.	$\forall x\forall m(U(x)\wedge\phi(x,m)\rightarrow MR(x))$	Pr. (II)
3.	$\forall x\forall m(\phi(x,m)\wedge\neg R(x)\rightarrow\neg M\neg(R(x)\rightarrow\exists n(G(n,m)\wedge Q(n,x))))$	Pr. (III)
4.	$U(a)\wedge\phi(a,b)$	C1 1
5.	$U(a)\wedge\phi(a,b)\rightarrow MR(a)$	C2 2
6.	$MR(a)$	MP 5,4
7.	$\phi(a,b)\wedge\neg R(a)\rightarrow\neg M\neg(R(a)\rightarrow\exists n(G(n,b)\wedge Q(n,a)))$	C2 3
8.	$\neg R(a)\rightarrow\neg M\neg(R(a)\rightarrow\exists n(G(n,b)\wedge Q(n,a)))$	C3 4,7
*9.	$\neg R(a)$	Pr. (V)
*10.	$\neg M\neg(R(a)\rightarrow\exists n(G(n,b)\wedge Q(n,a)))$	MP 8,9
*11.	$M\exists n(G(n,b)\wedge Q(n,a))$	M1 6,10
*12.	$\exists yM\exists n(G(n,b)\wedge Q(n,y))$	C4 11
*13.	$M\exists y\exists n(G(n,b)\wedge Q(n,y))$	M2 12
*14.	$U(a)\wedge Q(b,a)\wedge\neg M\exists y\exists n(G(n,b)\wedge Q(n,y))$	4
*15.	$M\exists y\exists n(G(n,b)\wedge Q(n,y))\wedge\neg M\exists y\exists x(G(n,b)\wedge Q(n,y))$	C5 13,14
16.	$\neg R(a)\rightarrow M\exists y\exists n(G(n,b)\wedge Q(n,y))\wedge\neg M\exists y\exists x(G(n,b)\wedge Q(n,y))$	C6
17.	$R(a)$	C7 16
18.	$U(a)\wedge R(a)\wedge\phi(a,b)$	C8 4,17
19.	$\exists x\exists m(U(x)\wedge R(x)\wedge\phi(x,m))$	C4 18

A couple of things have to be mentioned about this reconstruction of Anselm's argument. First, it is exactly this: a reconstruction. Trivially Anselm's argument does not have this structure; at no point of his text we find evidence for most of the steps and inference rules that Adams uses. It is a reconstruction in the sense of revealing the logic beyond Anselm's argument or unclosing all otherwise hidden logical steps needed to turn Anselm's argument into a sound one.

Despite of this, and this is the second point, Adams correctly represents two important structural features of Anselm's argument. First, starting from step 9, it uses *the reductio ad absurdum* method we found in the original argument (it ends at 15). Second, Anselm's original argument switches back and forth from a universal discourse to talk about particulars. From (iv) to (viii) he speaks about *something* than which nothing greater can be thought; however, from (ix) to (xii) he changes his discourse and starts speaking about *that* than which a greater cannot be thought; then, in (xiii), he goes back to talk about *something* than which nothing greater can be thought. Adams correctly represents this movement. Using C1 and C2, he switches, in steps 4, 5 and 7, from a universal discourse to discourse about particulars (in the case, individuals *a* and *b*). Similarly to Anselm's original argument, all crucial *reductio ad absurdum* steps are done inside this particular discourse framework. Then, when he has proved that *a* exists in reality, he goes back in step 19, thought C4, to the universal type of discourse.

Third, Adams uses a somehow incomplete calculus: the first-order part is ok, but he does not say from which calculus he draws his modal inferences. Consequently, we do not know which semantics might be associated with it (since there are many modal systems), neither if the resulting system with a specific semantics would be sound and complete.

Finally, we can here appreciate better the consequences of taking "it can be thought that" as equivalent to "it is possible that". Trivially, the soundness of this reconstruction depends on the soundness of the modal inferences he uses. That they are sound when interpreting M as "it is possible that" is not a big issue. But how about Adams' interpretation? Is the soundness of these modal inferences automatically transferred when one interprets M as "it can be thought that"? It is somehow *ad hoc* to arbitrarily assume that this question can be answered with a "yes".

After this, Adams states what he takes as one of the main goals of his article: "What I do want to discuss are certain general principles about existence and predication which are presupposed in the formulation and assertion of the premises of the Proslogion 2 argument." (Adams, 1971, p. 32). They are five in number:

(1) That predication does not presuppose real existence;
(2) That (contrary to Meinong's ontology) the universe of discourse does not include objects with contradictory predicates;

(3) That a thing which exists in the understanding truly possesses all the properties which are contained or implied in its concept or definition;
(4) That one and the same thing can exist both in the understanding and in reality;
(5) That existence and nonexistence in reality and existence in the understanding are predicates or properties.

Since Adam's other stated goal is to "offer a formal analysis of Anselm's arguments for the existence of God in the Proslogion […]" (Adams, 1971, p. 28), we might wonder about the relations that are between those five principles and the formal work done so far. Well, most of these principles are well-known and do not seem at all to depend on any kind of formalization. However, some of them bear very interesting relations with the whole idea of formally analyzing Anselm's argument.

First, that predication does not presuppose real existence (1) implies that, in a first-order semantic framework, the objects belonging to the domain D must include, beyond real objects, also unreal objects. Adams is clear about that:

> In terms of the predicate calculus used in my formalization of the argument, this means that the universe of discourse over which the variables range is not restricted to things that exist in reality. Obviously, if the universe of discourse were assumed to include only real things, the first step of the argument could not without circularity be asserted as a premise. (Adams, 1929, p. 33)

What we have here is an interpretation of a particular aspect of the semantics forced by the peculiarly of the problem at hand. On the other way round, we can say that this feature of the domain D helps in clarifying (1), making it somehow more precise. Trivially, such a use of first-order semantics implies (1).

Second, it not hard to see that (2) is an intrinsic feature of classical first-order semantics. Therefore, similarly to what we have said about (1) above, the intolerance to contradictions of classical first-order logic implies the truth of (2). So, we can say that, in this respect, Adams picked up the right representational tool[3].

Third, more than any other of Adams' five principles, (5) is closely connected with his formulation. Since he formalizes the two kinds of existence (in the understanding and in reality) as predicate symbols of the logical language, it is natural that he takes this principle as one which is presupposed by the argument. But to what extent this applies to

[3] It must be said, however, that in other logics, such as paraconsistent logics, there might be objects with contradictory properties. So, this is a reinforcement of the point that Anselm's logic is classical in this sense.

Anselm's formulation? Naturally, Anselm uses the notions of existence in the understanding and existence in reality. But, does he see them as predicates or properties, as Adams seems to claim? In order to appreciate this better, let us take a look at something very closely connected with this issue: Kant's famous "being is not a property" critique to the ontological argument.

Although originally directed towards Descartes' ontological argument, Kant's objection has been often seen as an objection against Anselm's version. Trivially enough, it is an objection against Adam's formalization of Anselm's argument. But is it a genuine objection against Anselm's formulation? Some would say that it is not. Here it is Kant's relevant statement:

> By whatever and by however many predicates we may think a thing – even if we completely determine it – we do not make the least addition to the thing when we further declare that this thing is. Otherwise, it would not be exactly the same thing that exists, but something more than we had thought in the concept; and we could not, therefore, say that the exact object of my concept exists. (Kant, 1929, p. 505)

This criticism does not exactly fit Anselm's statement of his argument. As Gareth Matthews has put it:

> He does [Anselm] not speak of adding the concept of existence, or even the concept of existence in reality, to the concept of God, or the concept of something than which nothing greater can be thought. What he does instead is to ask us to compare something existing merely in the understanding with something existing in reality as well. And the second, he says, is greater. (Matthews, 2005, p. 90)

So, it is not clear at all that, for Anselm's formulation, existence is a predicate.

At this point we can better appreciate the impact of one's formulation in the logical analysis of an argument. Since Adams represents the concepts of existence in reality and existence in the understanding with the help of logical predicates, *his formulation* naturally presupposes (5). But the same cannot indisputably be said about Anselm's formulation. In fact, while Anselm's formulation is at least defensible against Kant's critique, Adam's is not.

It is important to keep in mind that the choice of representing the two existence concepts as logical predicates is exactly this: a technical choice. Many formalizations of Anselm's argument represent at least one of the concepts with the help of the existential quantifier. And in fact, it is not difficult to conceive an alternative version of Adam's formalization which represents none of the two existence concepts as properties. In order to illustrate this point, let us give a rough sketch of what this version would look like.

First we have to build an expanded first-order logic with two existential quantifiers, say, $\exists$ and $\ni$. While $\exists$ is a broad quantifier ranging over a large domain D, $\ni$ is a more restricted one ranging over domain $D' \subseteq D$. As far as Anselm's argument is concerned, D contains all objects, be them located in reality or in the understanding (it does not matter here who's understanding); D' contains only objects located in reality. Therefore, while $\exists xP(x)$ means that *x* exists in reality or in the understanding and has property *x,* $\ni xP(x)$ means that *x* exists in reality and has property P. Given this, we have two abbreviations:

$$\phi(x,m) =_{def} Q(m,x) \wedge \neg M(\exists y \exists n(G(n,m) \wedge Q(n,y))$$

$$\varepsilon(x,m) =_{def} \ni y \exists n(\phi(y,n) \wedge (y=x) \wedge (n=m))$$

ϕ is the same as Adam's abbreviation. $\varepsilon(x,m)$ means that x is God and he exists in reality. The premises and conclusion are then represented as follows:

(I) $\exists x \exists y(\phi(x,m))$
(II) $\forall x \forall m(\phi(x,m) \rightarrow M(\varepsilon(x,m))$
(III) $\forall x \forall m(\phi(x,m) \wedge \neg\varepsilon(x,m) \rightarrow \neg M \neg(\varepsilon(x,m) \rightarrow \exists n(G(n,m) \wedge Q(n,x))))$
(IV) $\ni x \exists m(\phi(x,m))$

This simple exercise is meant just to show the impact of one's technical choices in the formalization of an argument. By using a different representational tool, we not only showed that (5) is not a presupposition of Anselm's argument *per se*, but of Adams' formulation of it, but also got rid of Kant's famous criticism. The same can be said about Frege's related criticism that existence is a property of concepts, not of the objects which fall under the concept.

After stating those five principles, Adams goes on to formalize Gaunilo's famous lost island counter-example. His stated purpose with this was, first, to analyze the soundness of such an argument, and second, show that its premises also depend on the five principles on existence and predication which according to him Anselm's depend on. According to Adams, "The counterexample of the lost island shows quite clearly that assumptions (i-v) must be rejected or at least modified" (Adams, 1971, p. 34)

In order to formalize Gaunilo's counter-argument, Adams uses the following predicates:

I(x)	*x* is an island
L(x)	*x* is a land or country
P(x)	*x* has the profitable and delightful features attributed by legend to the lost island

The premises of the argument, already formalized, are as follows:

P1. $\exists x(U(x) \wedge I(x) \wedge P(x) \wedge \neg \exists y(L(y) \wedge G(y,x)))$

P2. $\exists x(L(x) \wedge R(x))$

P3. $\forall x \forall y(L(x) \wedge R(x) \wedge I(y) \wedge \neg R(y) \rightarrow G(x,y))$

The conclusion is

C. $\exists x(U(x) \wedge R(x) \wedge I(x) \wedge P(x) \wedge \neg \exists y(L(y) \wedge G(y,x)))$

P1 means that there is an individual *x* which exists in the understanding, is an island, has the profitable and delightful features attributed by legend to the lost island and, besides, there is no land greater than it. P2 simply says that there exists a real land. P3 says that any real land is greater than any island which does not exist in reality. The conclusion says that there exist such an island, both in the understanding and in reality, and that there is no greater land.

As we can see, the formalization of Gaunilo's premises and conclusion is simpler than the formalization of Anselm's argument: the predicate Q is not used, nor is the modal operator M. Adams' reconstruction of Gaunilo's argument is also simpler, although it uses the following extra inference rules:

C9. $\sigma_1 \wedge \beta \wedge \sigma_2, \alpha, \alpha \wedge \beta \wedge \lambda \rightarrow \varphi \vdash \lambda \rightarrow \varphi$

Here is the derivation:

1.	$\exists x(U(x) \wedge I(x) \wedge P(x) \wedge \neg \exists y(Ly \wedge G(y,x)))$	P1
2.	$\exists x(L(x) \wedge R(x))$	P2
3.	$\forall x \forall y(L(x) \wedge R(x) \wedge I(y) \wedge \neg R(y) \rightarrow G(x,y))$	P3
4.	$U(b) \wedge I(b) \wedge P(b) \wedge \neg \exists y(L(y) \wedge G(y,b)$	C1 1
5.	$L(a) \wedge R(a)$	C1 2
6.	$L(a) \wedge R(a) \wedge I(b) \wedge \neg R(b) \rightarrow G(a,b)$	C2 3
7.	$\neg R(b) \rightarrow G(a,b)$	C9 4,5,6
*8.	$\neg R(b)$	Pr.
*9.	$G(a,b)$	MP 8,7
*10.	$L(a) \wedge G(a,b)$	C5 5,9
*11.	$\exists y(L(y) \wedge G(y,b))$	C4 10

*12.	$\exists y(L(y) \wedge G(y,b)) \wedge \neg \exists y(L(y) \wedge G(y,b)$	C5 4,11
13.	$\neg R(b) \rightarrow \exists y(L(y) \wedge G(y,b)) \wedge \neg \exists y(L(y) \wedge G(y,b)$	C6 8,12
14.	$R(b)$	C7 13
15.	$U(b) \wedge R(b) \wedge I(b) \wedge P(b) \wedge \neg \exists y(L(y) \wedge G(y,b))$	C8 4,14
16.	$\exists x(U(x) \wedge R(x) \wedge I(x) \wedge P(x) \wedge \neg \exists y(L(y) \wedge G(y,x)))$	C4 15

It is a sound argument. And despite the similarities (both proofs use the *reductio ad absurdum* method and the universal-to-particular-to-universal movement), it is pretty clear that both arguments have a quite different structure. In fact, the structure departure starts from the logical form of the premises: whereas Anselm spoke of a being whose greatness could not possibly be surpassed, Gaunilo speaks only of an island to which no country is in fact superior.

Given this, it is clear that Gaunilo's argument fails as a counter-argument to Anselm's Proslogion II argument. Traditionally, a counter-argument in this sense is an argument that shares the same logical structure of another argument, has true or reasonable premises, but an absurd or patently false conclusion. In this way, since we cannot accept the conclusion of the counter-argument, we cannot accept the conclusion of the original argument either: despite its apparent soundness, there must be something wrong with the argument (although this method of refutation does not say what is wrong). It seems that Gaunilo intended his counter argument to function as a refutation of Anselm's ontological argument in this sense. But since according to Adams' reconstructions both arguments have a quite different structure, Gaunilo does not succeed in refuting Anselm's argument this sense[4].

4 Concluding Remarks

Let us see now what we have got from this brief analysis of Adams' formalization of Anselm' argument.

About the pros of formalization and its contributions to the debate on Anselm's argument, I would first point out that the use of a formal logical language such as first-order predicate language has a very interesting impact in the analysis of an argument. Besides anything else, this has to do with the level of detail shown in the representation of sentences offered by a formal language. Due to this, it was possible to incorporate in (III) a very precise version of (G) and (G3). I order to see the importance of this disambiguation

[4] Despite of this, Adams insists that it is still a powerful criticism, for the reasonableness of its premises depend, like Anselm's argument, on the truthfulness of the five presuppositions.

one must just recall that Gaunilo attributed to Anselm the assumption that whatever exists in reality is greater than anything that does not. Something very alike seems to be behind Norman Malcolm's criticism of the doctrine that existence is a perfection:

> The doctrine that existence is a perfection is remarkably queer. It makes sense and is true to say that my future house will be a better one if it is insulated than if it is not insulated; but what could it mean to say that it will be a better house if it exists than if it does not? (Malcolm, 1960, p. 43)

Malcolm took himself to be restating the criticism of Anselm's argument found in Kant which we mentioned earlier. But what the argument, according to Adams' reconstruction, needs is not a principle comparing concepts, but one which compares objects existing in the understanding.

Second, by using a proof theory, it is possible to reveal crucial aspects of the argument at hand, including its logical structure and soundness. As an instance of this, we have Adams' reconstruction incorporating two important aspects of the proof method Anselm used: the *reductio ad absurdum* method and the movement he does, back and forth, from a universal discourse to discourse about particulars. Thanks also to this and to his reconstruction of Gaunilo's argument, it was possible to show that Gaunilo's argument is not a counter-argument of Anselm's argument, at least not in the logical sense of the expression "counter-argument", although Adams does not mention this.

Third, the use of a formal framework allows one to better state in, a more precise way, his analysis as a whole. For instance, Adams principles (1) that predication does not presuppose real existence and that (2) the universe of discourse does not include objects with contradictory predicates find a match in first-order semantics. Since Adams represents God with the help of variables, semantically it will correspond to an object belonging to the semantic domain. And since there can be objects not satisfying the property corresponding to the predicate R, (1) automatically follows. Also, that there cannot be an object $a \in D$ having and not having the same property is a feature of first-order semantics; therefore (2).

About the dangers of formalization and Adams' drawbacks, it should first be mentioned that Adam's reconstruction is not quite faithful to Anselm's original formulation. Although it incorporates many important elements of the original formulation, it departs from the original argument in some important points. Adams simply ignores that Anselm did give an argument for premise (viii): as we saw, there is a pretty clear preliminary argument in Anselm's text meant to support (viii). This negligence, as we saw, has important consequences for the rest of Adams' formal analysis. Although any formalization of an argument has in some degree to depart from the original formulation, if the departure is too much, and mainly, if the departure points are not well justified, it is hard to see how the formal argument at hand might be taken as a formalization of the original argument. As such, it simply misses the whole purpose of logical formalization.

A second but related point is that Adams makes a couple of important theoretical choices without justification. He equates "it can be thought that" with "it is possible that" with no attempt to justify it. He also represents premise (x) with the help of a modal operator which does appear at all in Anselm's text. As we have mentioned, these choices are crucial for the rest of this formal work. Although these choices also make Adams' formalization to depart from the original formulation, they are not as serious as the point mentioned above. However, the lack of justification for these movements is a serious issue. For example, by representing "it is possible that" with the help of modal operator M and making reference to modal inferences characteristic of modal systems, he is unjustifiably assuming that all the formal explanation given to the concept of possibility can be trivially transferred to the concept of conceivability.

As in any formal reconstruction, Adams had to add elements not present in the original argument. The several steps of his formal argument clearly show that. In order to show the validity of an argument, proof theory requires a level of detail that no argument written in ordinary language could provide. Therefore, inevitably there will be elements foreign to the original formulation. The point which Adams seems to miss is that, since there are quite a number of directions to follow in the choice of these elements, whatever direction one picks has to be well justified.

Third, the fact that Adams does not use a full formal logical machinery is troublesome. As we have said, one of the main purposes of formally analyzing an argument is to be able to critically evaluate the argument inside a somehow more trustful framework. However, Adams does not do so. Since he relies on an incomplete logical system, some of the grounds on which we will rely to appraise the soundness of his reconstruction will be pretty much alike to any informal analysis.

Finally, about the relation between formal analysis and analysis in general, through this brief examination of Adam's work we could see that there is a strong link between the features of the formalization and the evaluation and analysis we make of the formalized argument. This is illustrated by Adam's principle (5) and his use of two predicates to represent the concepts of existence in reality and existence in the understanding. Surely this is a serious drawback of Adam's formalization: by introducing an element which was absent from Anselm's original formulation, he made the argument susceptible to Kant's and Frege's criticism. However, our sketch of a version of Adams' formalization which is free from such criticisms shows the dependence of one's evaluation and analysis of Anselm' argument on the formal choices he or she makes in the process of formalizing it. And this point applies to any deep analysis of an argument, be it formal or not. As soon as we start an analysis of an argument which involves representing premises and conclusion in a more detailed way than is found in the original formulation, we have to make choices that most probably will influence the evaluation of the (reconstructed) argument.

References

ADAMS, R. The Logical Structure of Anselm's Arguments. *The Philosophical Review,* 1971, 80:28-54.

ANSELM. Tr. M. Charlesworth. Notre Dame: University of Notre Dame Press, 1965.

CARNAP, R. *Logical Foundations of Probability.* Chicago: University of Chicago Press, 1950.

ENGEL, P. Formal Methods in Philosophy. In: *Proceedings of the 6th European Congress of Analytic Philosophy*, (eds.) T. Czarnecki et al. London: College Publications, 2010.

ENGEL, P. Formal Methods in Philosophy: Shooting Right without Collateral Damage. In: *The Analytic Way. Proceedings of the 6th European Congress of Analytic Philosophy,* (eds.) T. Czarnecki, K. Kijania-Placek, O. Poller e J. Wolenski. London: College Publications, 2010.

GÖDEL, K. (1995) Kurt Gödel: *Collected Works,* vol. 3. Oxford: Oxford University Press, 1995.

HANSSON, S. Formalization in Philosophy. *The Bulletin of Symbolic Logic 6:* 162-175, 2000.

HORSTEN, L.; DOUVEN, I. Formal Methods in the Philosophy of Science. *StudiaLogica* 89: 151-162, 2008.

KLIMA, Gyula (2000) Saint Anselm's Proof: A Problem of Reference, Intentional Identity and Mutual Understanding. In: Ghita Holmström-Hintikka (ed.), *Medieval Philosophy and Modern Times*,Dordrecht: Kluwer, 2000, p. 69-87.

MATTHEWS, G. "The Ontological Argument." In: *The Blackwell Guide to the Philosophy of Religion,* (ed.) William Mann,Oxford: Blackwell Publishing, 2005, p. 81-102.

OPPENHEIMER, P. and ZALTA, E. On the Logic of the Ontological Argument. In: *Philosophical Perspectives 5***:** *The Philosophy of Religion*, (ed.) James Tomblin. Atascadero: Ridgview Press, 1991.

PLANTINGA, A. *The Nature of Necessity*. Oxford: Oxford University Press, 1974.

ROWE, W. *Philosophy of Religion: An Introduction*. 4th edition. Belmont: Wadsworth, 2006.

SOBEL, J. *Logic and Theism.* New York: Cambridge University Press, 2004.

Recebido em agosto de 2015
Aceito em setembro de 2015