WHY EXTENSIONAL EVIDENCE MATTERS

Draft of August 22, 2023

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ABSTRACT

Intensional evidence is any reason to accept a proposition that is not the truth values of the proposition accepted or, if it is a complex proposition, is not the truth values of its propositional contents. Extensional evidence is non-intensional evidence. Someone can accept a complex proposition, but deny its logical consequences when her acceptance is based on intensional evidence, while the logical consequences of the proposition presuppose the acceptance of extensional evidence, e.g., she can refuse the logical consequence of a proposition she accepts because she doesn't know what are the truth-values of its propositional contents. This tension motivates counterexamples to the negation of conditionals, the propositional analysis of conditionals, hypothetical syllogism, contraposition and or-to-if. It is argued that these counterexamples are non-starters because they rely on a mix of intensionally based premises and extensionally based conclusions. Instead, a genuine counterexample to classical argumentative forms should present circumstances where an intuitively true and extensionally based premise leads to an intuitively false conclusion that is also extensionally based. The other point is that evidentiary concerns about intensionally based beliefs should be constrained by the truth conditions of propositions presented by classical logic, which are nothing more than requirements of coherence in distributions of truth value. It is argued that this restriction also dissolves some known puzzles such as conditional stand-offs, Adams pair, the opt-out property, and the burglar's puzzle.

1. INTRODUCTION

The material account of conditionals states that a conditional statement $A \rightarrow B$ is only true if $\neg(A \& \neg \underline{B})$. Therefore, if we have the negation of 'If God exists then the prayers of evil men will be answered', then we can conclude that 'God exists and the prayers of evil men will not be answered' (Stevenson, 1970: 28). This paper argues that this and other peculiar puzzles involving conditionals can be explained by a tension between the use of extensional and intensional evidence. Intensional evidence is any reason to accept a proposition that is not the truth values of the proposition accepted or, if it is a complex proposition, is not the truth values of its propositional contents. Extensional evidence, on the other hand, is non-intensional evidence. Sometimes a person can accept a complex proposition, but deny its logical consequences when her acceptance is based on intensional evidence, while the logical consequences of the proposition presuppose the acceptance of extensional evidence. Consequently, she can refuse the logical consequence of a proposition she accepts because she does not know the truth-values of its propositional contents.

This article will be organized as follows. In Section 2, the distinction between intensional and extensional evidence will be introduced and clarified. Section 3 argues that despite intensional evidence being more salient than extensional evidence in our daily epistemic concerns, it must always come to terms with extensional evidence in logical matters. Specific contexts where extensional evidence is preferred will also be introduced. Sections 4-6 will utilize the distinction between intensional and extensional evidence to disarm counter-examples against the classical negation of conditionals, the propositional analysis of conditions, hypothetical syllogism, or-to-if, and contraposition. It will be argued that these counterexamples are non-starters because they rely on premises that are intensionally based. For the counterexamples against classic logic to be successful, they need to present a circumstance where a premise that is intuitively true on extensional grounds leads to a conclusion that is apparently false on extensional grounds. Ultimately, evidentiary concerns should be constrained by the truth conditions of propositions, which, in classical logic, are nothing more than coherent distributions of truth-values. Therefore, if our intensionally based beliefs are incompatible with the possible combinations of truth values presented by classical logic, they are ultimately incoherent and require revision. These considerations are also applied in sections 7-10, which address the following conditional puzzles: the opt-out property, Adams pair, conditional stand-offs, and the burglar's puzzle. Section 11 concludes.

2. TWO TYPES OF EVIDENCE

Intensional evidence involves any reasons to accept a proposition that are not the truth-values of the proposition or, if it's a complex proposition, its propositional contents. The fact that there is a known connection between red spots and measles is intensional evidence to accept the conditional 'If John has red spots, he has measles'. Intensional evidence requires a defeasible reasoning that supports the proposition, but can be defeated by additional information. The presence of red spots is an indicator of measles, but is possible that you do not have measles after all. It was just a rash. Intensional evidence only suffices for the acceptability of a conditional. It is inconclusive evidence.

Extensional evidence plays a significant role in conditional logic. For instance, if we know that John had red spots and measles, this extensional evidence is sufficient to accept the conditional 'If John has red spots, he has measles.' Extensional evidence is a crucial component of deductively valid reasoning as the truth of both the antecedent and consequent are not only compelling, but also indefeasible. Hence, it is impossible to have red spots and not have measles if it is not the case that if John has red spots, he has measles. Therefore, extensional evidence serves as conclusive evidence for the truth of a conditional[1].

It is worth noting that extensional evidence does not necessarily imply classical logic. Although most conditional logics assume that the truth of both A and B are sufficient evidence to accept $A \rightarrow B$[2] and that A and $\neg B$ are sufficient evidence to deny $A \rightarrow B$, the assumption that

[1] The concept of intensional and extensional evidence, originally introduced by Stevenson (1970), has been adopted and expanded in this paper. Stevenson (1970:31) defines intensional evidence as a 'body of evidence that confirms p ⊃ q' without confirming the stronger propositions of ¬p or q. Extensional evidence, on the other hand, is considered as nonintensional evidence. The current paper employs a broader interpretation of this distinction by applying it not only to the material conditional but also to any simple or complex proposition. Additionally, this paper introduces further concepts related to this distinction, such as defeasible and conclusive evidence, acceptability and truth conditions, and criteria of truth and truth conditions. It is important to note that Stevenson does not support or present arguments for these associated concepts in his original work.

[2] I will use '→' for indicative conditionals, '⊃' for the material implication and the capital letters *A*, *B*, *C*…. for propositional variables. The symbols and variables quoted will be modified to ensure that the notation remains uniform.

$\neg A$ or B are enough to accept $A \rightarrow B$ is exclusive to classical logic. Nonetheless, only one of the issues concerning negated conditionals involves classical logic, as we shall discuss in more detail later.

The distinction between intensional and extensional evidence is not limited to complex propositions, but also applies to simple propositions. For example, the weather forecast predicting heavy rain for tomorrow is intensional evidence supporting the belief that there will be heavy rain on August 2nd, whereas the actual occurrence of heavy rain on August 2nd is extensional evidence for the same belief. Similarly, being told by a trustworthy individual that a match was cancelled is intensional evidence supporting the belief that the match was cancelled, while the cancellation of the match itself is extensional evidence supporting this belief. It is important to note that a true statement may serve as intensional evidence for a false statement, but never as extensional evidence. Additionally, any intensional evidence that is a true proposition may also be considered extensional evidence of itself.

The difference between intensional and extensional evidence highlights the greater epistemic relevance of the former over the latter. Relying on extensional evidence can be problematic as it goes against our epistemic practices, which often involve uncertainty regarding the truth values of the propositions being evaluated. When assessing whether to accept a proposition A, it is typically not known whether A is true or not, requiring the use of intensional evidence.

In the case of complex propositions such as conditionals, intensional evidence is strongly preferred. This is due to several reasons. Firstly, we often do not know the truth values of the propositional constituents of $A \rightarrow B$. Secondly, conditionals are used to express connections between things, which require intensional evidence to determine whether these connections hold. Thirdly, for the inferential use of conditionals, intensional evidence is required to confirm the premises of a *modus ponens* or a *modus tollens* without begging the question or making the argument unsound [3]. Fourthly, while acceptance of intensional evidence for a simple proposition implies acceptance of extensional evidence, the acceptance of intensional evidence for a complex proposition does not necessarily imply the acceptance of extensional evidence. If one has intensional evidence to accept A, then she will think that A is true; but if one has intensional evidence to accept $A \rightarrow B$, she can think that this proposition is true without compromising the individual truth-values of A and B.

Intensional evidence is ubiquitous; however, systems of logic invariably treat conditionals as a type of function. Classical logic, for instance, treats connectives as truth functions and requires knowledge of truth-values to establish the validity of inferential forms. The system assumes a type of evidence - the extensional kind - which conflicts with our epistemic practices. It is not surprising, therefore, that applying the fundamentals of logic to everyday examples of conditional reasoning leads to a range of puzzles and counter-intuitive examples.

3. THE RELEVANCE OF EXTENSIONAL EVIDENCE

It could be argued that extensional evidence always outweighs intensional evidence, but we tend to hold the opposite view because intensional evidence is more relevant to our daily epistemic concerns. First, let us consider the relevance of extensional evidence. Possible extensional reasons to accept $A \rightarrow B$ are $A \& B$, $\neg A \& B$ and $\neg A \& \neg B$. But is it true that these combinations of truth-values are never used to establish a conditional's truth value? Not quite.

[3] See Johnson (1921) and Stevenson (1970: 30).

$A \& B$ is sufficient to accept puzzle conditionals (such as ‘I know where the prize is, but all I will tell you is that if it is not in the garden, it is in the attic’), Kennedy shooter conditionals (such as ‘If Oswald did not kill Kennedy, someone else did’), or incidental conditionals where *A* and *B* coincidentally happen to be true (such as ‘If he leaves at ten, a car accident will happen’).

How about the other circumstances, when $\neg A \& B$ is true or $\neg A \& \neg B$ is true? $\neg A \& B$ can be a reason to accept even-if conditionals (‘Even if he felt embarrassed, he showed no signs of it’), since they are accepted when *B* is assumed as true regardless of the truth value of *A*. $\neg A \& \neg B$ is enough to accept puzzle conditionals and sportscast play-by-play commentary conditionals (‘If Messi waits just a second longer, he scores on that play’)[4]. $\neg A \& \neg B$ can also be a reason to accept Dutchman conditionals (‘If John’s speaking the truth, I’m a Dutchman’).

Grice also presented a variety of contexts where it is implicitly acknowledged that the reasons employed to assert conditionals are extensional. According to Grice (1989a: 59), the conditional ‘If Smith is in the library, he is working’ would normally carry the implication that the speaker has intensional grounds to back his claim—what Grice called Indirectness Condition. But the speaker could opt out from this implication adding: ‘I know just where Smith is and what he is doing, but all I will tell you is that if he is in the library he is working’. The speaker asserted this conditional because he had just looked and found him in the library, but wants to play a game with his interlocutor. Grice (1989a: 60) also presented the example of a guessing game:

> You may know the kind of logical puzzle in which you are given the names of a number of persons in a room, their professions, and their current occupations, without being told directly which person belongs to which profession or is engaged in which occupation. You are then given a number of pieces of information, from which you have to assign each profession and each occupation to a named individual. Suppose that I am propounding such a puzzle … about real people whom I can see but my hearer cannot. I could perfectly properly say, at some point, “If Jones has black (pieces) then Mrs. Jones has black too.” … indeed, the total content of this utterance would be just what would be asserted (according to truth-table definition) by saying “Jones has black ⊃ J Mrs. Jones has black.” Thus one undertaking of the previous action has been fulfilled.

In this game, the use of information is explicitly extensional. The hearer asserts the conditional because they know what the truth values of the conditional constituents are, and they want their interlocutor to make an educated guess using this conditional as a piece of information. Finally, Grice (1989a: 60) asks us to consider a game of bridge with special conventions in which a bid of five no trumps is announced to one's opponents as meaning ‘If I have a red king, I also have a black king.’ This conditional is extensional through and through.

In denying the logical significance of intensional evidence, one can contrast it with the conclusive aspect of extensional evidence. Intensional evidence supports a proposition through defeasible reasoning, which can be defeated by additional information. For instance, red spots on a person may indicate measles, but it is possible that the person does not have measles, and it is just a rash. On the other hand, extensional evidence is involved in a deductively valid reasoning, where it is impossible for both the antecedent and the consequent to be true, and the conditional to be false. It is impossible for Socrates to have red spots and measles, and still be false that if Socrates has red spots, he has measles. The truth of both the antecedent and the

[4] von Fintel (2012: 467).

consequent provides conclusive evidence of the truth of the conditional. Extensional evidence is sufficient to establish the truth of a conditional, whereas intensional evidence can only establish the acceptability of a conditional since it is not conclusive evidence.

It is reasonable to assume that extensional evidence always prevails over intensional evidence. For example, suppose I assert, 'If you flip that coin, it will come up heads.' However, since a fair coin has at least a 50% chance of resulting in tails, there is no intensional evidence to accept the conditional. Consequently, my assertion is unjustified, and you promptly deny the conditional. But if, after I make the assertion, I flip the coin and it comes up heads, the result provides extensional evidence that the conditional is not only acceptable, but true. Your negation was a mistake.

Now, imagine a modified circumstance in which I know that the coin toss is rigged to ensure that the result of the toss will always be heads. Knowing this, I assert, 'If you flip the coin, it will come up heads.' In this case, the same conditional would be acceptable before the toss since I have intensional evidence to accept it. But suppose that despite my excellent intensional evidence, the result of the toss turns out to be tails (perhaps the rigged mechanism failed, etc.). Again, extensional evidence has the last word on the issue. Ultimately, the truth-value of the conditional is determined by the truth-values of its propositional constituents.

The predominance of extensional evidence over intensional evidence happens because intensional evidence can vary with time and is based on imperfect information. However, if an epistemic agent were to correct her beliefs given the opportunity, optimal information would always be extensional. Our intensional-based beliefs will ultimately be grounded in facts that determine the truth-values of the relevant propositions, i.e., extensional evidence. Thus, the tension between intensional and extensional evidence will always be resolved in favour of the latter since intensional evidence inevitably has to come to terms with extensional evidence.

Note that just as our epistemic biases may cause us to prefer intensional evidence over extensional evidence, they may also lead us to favor acceptability conditions, which are the conditions under which a proposition is deemed acceptable, over truth conditions, which are the conditions under which a proposition is actually true or false. While the negation of a conditional may not necessarily imply a conjunction if we rely solely on acceptability conditions, we must remember that acceptability conditions are not a proper substitute for truth conditions, just as intensional evidence is not a proper substitute for extensional evidence. It is important not to confuse claims about what is considered acceptable or unacceptable with claims about objective truth or falsehood. A proposition may be considered acceptable by an epistemic agent based on the intensional evidence available, but still turn out to be false, or it may be deemed unacceptable due to a lack of intensional evidence and yet still be true. The considerations associated with acceptability conditions cannot serve as a metric for determining which logic we should use because they are reliant on the vagaries of our epistemic constraints, while truth conditions are determined by factual matters that are independent of epistemic agents and their epistemic situation.

4. THE NEGATION OF CONDITIONALS

If indicative conditionals are material, then $\neg(A \rightarrow B)$ implies $A \& \neg B$. However, this assumption leads to counter-intuitive instances where the conclusion is a conjunction that the person ignores, despite accepting the premise due to intensional evidence. For instance, if I deny the conditional 'If God exists then the prayers of evil men will be answered,' I must admit that 'God exists and the prayers of evil men will not be answered' (Stevenson, 1970: 28). Therefore, from the negation of a simple conditional, one could prove that God exists. This is implausible since someone could refuse the conditional based on assumptions about the moral dispositions of God, even if they do not believe in the existence of God.

Edgington (1986: 16)[5] presented another version of the trivial proof of God's existence that relies on a different conditional: 'If God doesn't exist, then it is not the case that if I pray, my prayers will be answered (by Him).' Intuitively, this conditional is true. However, if I do not pray, the antecedent of the conditional in the consequent is false, which implies that the negation of the conditional is false. Thus, the only way to maintain the assumption that the whole conditional is true is by admitting that the antecedent of the whole conditional is false, and therefore, we must admit that God exists.

Klinger (1971: 191) provided yet another counterexample. Imagine a lawyer attempting to use classical logic to defend their client. We can suppose that the judge has a basic understanding of Logic I, just enough to follow the argument. The lawyer admits that their client was found at the crime scene, but argues that this fact alone is not a sufficient condition for guilt. They represent this argument with the conditional statement: 'It is not the case that if the accused was found on the crime scene, he is guilty'. From this, we can infer the surprising conclusion: 'The accused was found at the crime scene and is not guilty'. However, to avoid this unexpected conclusion, we cannot reinterpret the negated conditional as 'If the accused was found at the crime scene, he is not guilty'. Doing so implies that being found at the crime scene is a sufficient condition for innocence, which is not the case.

The root of these counterexamples lies in the tension between the use of extensional evidence and our common epistemic practices. The material account relies on extensional calculus, which works under the assumption of omniscient logic. In other words, it assumes that the evaluator of the conditional knows the truth-values of its propositional constituents. However, in practice, the evidence we have available when evaluating a conditional is often intensional. When evaluating a conditional, $A \rightarrow B$, we usually do not know whether A and B are true or not. For instance, to determine whether John was late for work if he left his home late, we need to consider the traffic conditions. Our ignorance of the truth values of A and B is ignored by the extensional calculus. This explains why it is intuitive to think that A and $\neg B$ entails $\neg(A \rightarrow B)$. However, the converse is not intuitively true. The extensional evidence is sufficient to reject the conditional, but the rejection of the conditional can be motivated by intensional evidence that depends on the epistemic situation of the evaluator.

The counterexamples all share the same structure: they interpret a premise based on intensional grounds (e.g., the negation of a conditional), which forces a conclusion based on extensional grounds (e.g., a conjunction). This seems too strong, since one can accept the premise without committing to the extensional conclusion. However, one could object that the negation of conditionals assumes an extensional basis for the premise, i.e., that accepting $\neg(A \rightarrow B)$ on extensional grounds entails accepting $A\&\neg\underline{B}$ on extensional grounds. Thus, the counterexamples are non-starters, as they rely on premises that are intensionally based. For counterexamples against classical logic to be successful, they must present a possible

[5] The counter-example is attributed to W.D. Hart (Edgington, 1986: 37, footnote 6).

circumstance where the premise of a valid argument form is extensionally based, but the conclusion seems false even on extensional grounds. Classical logic deals with extensional grounds and their possible combinations, so a valid argument form preserves not only truth, but also grounds for belief. The only difference is that they preserve extensional grounds for believing.

One objection to this insistence on extensional thinking is that it conflicts with our common practices that rely on intensional evidence and imperfect information. This suggests that we need new logical systems that are better suited for these practices. However, logical systems are attempts to identify and classify patterns, such as argumentative forms, that preserve truth. To establish whether an argumentative form preserves truth or not, we need to ascertain all possible combinations of truth-values in the premises and conclusion. The study of logical consequence thus involves identifying truth-value distributions and our commitments to truth-values in any given circumstance. Logic consists of coherent truth-value allocation in contexts of optimal information. Therefore, alternative logical systems would not only require commitments to truth-values absent in counterexamples that rely on intensional evidence, but also provide truth-value distributions that improve upon classical logic. This is a tall order.

The underlying rationale of the counterexamples assumes that a logical system should track inferential dispositions that start with the acceptance of complex propositions whose constituent truth-values are unknown. However, the truth-value of a conditional is dependent on the truth-values of its constituent propositions in any logic system, classical or not. Therefore, the counterexamples are motivated by a requirement that no logical system satisfies. As a result, they cannot be decisive against a classical logic system.

Ultimately, evidentiary concerns, that is, how someone decides whether to believe in a proposition, should be constrained by the truth conditions of propositions and their possible combinations stipulated by logical systems. Intensional evidence have epistemic significance because they are standards used in contexts of imperfect information to distinguish whether a given proposition is true or false, that is, in contexts where the only evidence available to assess the relevant proposition is intensional. However, truth conditions have logical significance because they determine the conditions in which a proposition is true or false and all their possible combinations, in contexts of truth-value omniscience. Our evidentiary concerns should adhere to the constraints of truth-value combinations presented by classical logic, and not the other way around.

In addition, classical logic provides further intensional evidence to support the truth of $A\&\neg B$ when $\neg(A \rightarrow B)$ is true. This evidence is independent of any prior knowledge of the truth values of A and B. Acceptance of conjunctions, like any other proposition, can be based on intensional grounds. For instance, I may accept the proposition 'The weather tomorrow will be rainy and cold' based on trust in the weather forecast prediction. In such a case, the evidence used to accept the conjunction is intensional. Once we accept that the conjunction is true, we also make commitments to the truth-values of its conjuncts; namely, we accept that both conjuncts are true. The same holds for the negation of a conditional. Knowing the logical consequences of negating a conditional allows us to identify the available extensional evidence for this proposition.

If truth value combinations cannot be satisfied in a conclusion, it is because of an incoherence in the interpretation of the premise. For example, what appears to be a negation in Stevenson's counterexample, 'If God exists then the prayers of evil men will be answered', is actually another conditional with a negated consequent, 'If God exists then the prayers of evil men will not be answered'[6]. Similarly, the conditional presented by Edgington, 'If God doesn't exist, then it is not the case that if I pray my prayers will be answered (by Him)', can be reinterpreted as 'If God does not exist, then if I pray my prayers will be ignored by Him' (Ortiz, 2010: 2). Finally, the Klinger counter-example can be disarmed by recognizing that the consequent has a modal operator of possibility implicit in it. When this modal operator is specified, the conditional is more reasonably interpreted as 'It is not the case that if the accused was found on the crime scene, he cannot be innocent'. With this interpretation, we can do justice to the lawyer's argument while eliminating the counter-intuitive aspect of the correspondent conjunction, which should be interpreted as 'The accused was found on the crime scene and he can be innocent'. It is also plausible to reinterpret the negation of the conditional as internal: 'If the accused was found on the crime scene, he could be innocent'.

5. THE PROPOSITIONAL ANALYSIS OF CONDITIONS

Akman (2017) argued that propositional analysis of necessary and sufficient conditions presented in most logic textbooks should be discarded because it leads to a contradiction. This analysis translates 'A is sufficient for B' as the conditional 'if A, then B', symbolized as $A \supset B$. Similarly, 'A is necessary for B' is interpreted as 'if not A, then not B', which can be symbolized as $\neg A \supset \neg B$, equivalent to $B \supset A$. These two assumptions lead to the symbolic representation of 'A is necessary and sufficient for B' as $(B \supset A)\&(A \supset B)$. Now, if we assert that 'A is neither necessary nor sufficient for B', Akman claimed that this proposition is equivalent to the acceptance of both 'A is not necessary for B' and 'A is not sufficient for B'. This, according to the propositional analysis, is represented as $\neg(B \supset A)\&\neg(A \supset B)$, which is a contradiction in classical logic. But since it is obvious that one could deny that A is either necessary or sufficient for B without implying a contradiction, the propositional analysis of conditions is surely false (Akman, 2017: 378).

However, this reasoning is flawed since 'A is neither necessary nor sufficient for B' is the negation of 'A is necessary and sufficient for B', which is symbolized as $(B \supset A)\&(A \supset B)$. The negation of this proposition is not $\neg(B \supset A)\&\neg(A \supset B)$, but $\neg(B \supset A) \vee \neg(A \supset B)$, which is not a contradiction in classical logic. It is still possible to derive a contradiction from the propositional analysis if we accept both 'A is not a necessary condition for B' and 'A is not a sufficient condition for B'. This joint acceptance is represented symbolically as $\neg(B \supset A)\&\neg(A \supset B)$, which is again a contradiction in classical logic, since the negation of $A \supset B$ and $A\&\neg B$, as well as $B \supset A$ and $B\&\neg A$, leads to the acceptance of both $A\&\neg B$ and $B\&\neg A$, which is equivalent to accepting both $A\&\neg A$ and $B\&\neg B$.

Neglecting this counter-intuitive aspect of the material conditional, Akman attempts to prevent the contradiction by advancing a first-order analysis, interpreting conditions as one-place predicates. Akman's solution interprets a statement such as 'A is a sufficient condition for B' as 'everything that possesses property A possesses property B,' symbolically represented as $\forall x(Ax \supset Bx)$. Similarly, the statement 'A is a necessary condition for B' is interpreted as 'nothing possesses property B if it does not possess property A,' represented symbolically as

[6] See, for example, Richards (1969: 421), Fulda (2005: 1421), and Lycan (2005: 91).

$\forall x(Bx \supset Ax)$. This approach aims to prevent the generation of a contradiction because the negation of both claims is logically equivalent to $\neg(\forall x(Ax \supset Bx) \vee \forall x(Bx \supset Ax))$, which is not a contradiction in classical logic (Akman, 2017: 379).

The first-order analysis of conditions is a promising step in the right direction. It offers a more nuanced analysis of conditions through an elegant use of predicate logic. By interpreting conditions as properties and explaining sufficiency and necessity in conditionality statements as inference relations, this analysis clarifies our intuitions. However, Akman's use of predicate logic does not accurately represent most attributions of conditions. Akman assumes that every conditionality statement involves universal quantifiers, but most attributions of conditions do not work that way.

Consider the statement 'Socrates being a philosopher is a sufficient condition for Socrates being Greek'. Following Akman's solution, this statement must be interpreted as 'Everything that possesses the property of being a philosopher possesses the property of being Greek'. However, this interpretation is too strong, as it is clear that I am making an attribution of condition specific to Socrates. A more sensible formulation of the first-order analysis interprets this statement as 'If Socrates possesses the property of being a philosopher, he possesses the property of being Greek,' represented without a universal quantifier as $Aa \supset Ba$.

This qualification is also significant because it shows that the first-order analysis makes use of the negation of the material conditional and thus remains unsuccessful in preventing contradictions. Suppose that I claim both 'Socrates being a philosopher is not a sufficient condition for being Greek' and 'Socrates being a philosopher is not a necessary condition for being Greek.' Taken together, these statements are equivalent to $\neg(Aa \supset Ba) \supset \neg(Ba \supset Aa)$, leading us to $(Aa\&\neg Ba)\&(Ba\&\neg Aa)$, which is a contradiction.

It comes as no surprise that the contradiction did not result from the propositional analysis of conditionals, but rather from the truth conditions of negated material conditionals. To understand the relationship between conditionals and conditionality statements, consider that if $A \supset B$ is true, then based on the truth conditions of the material conditional, if A is true, then B must also be true, meaning that A is a sufficient condition for B. Similarly, if $B \supset A$ is true, then if A is false, then B must also be false, meaning that A is a necessary condition for B. Now, let us represent the natural language conditional as $A \rightarrow B$. By replacing the material conditional with the natural language conditional, we can still maintain the rationale that motivates the propositional analysis of conditions. If $A \rightarrow B$ is true, then A is a sufficient condition for B, and if $B \rightarrow A$ is true, then A is a necessary condition for B.

In our propositional analysis of conditions, we interpret 'A is not a sufficient condition for B' and 'A is not a necessary condition for B' as $\neg(A \rightarrow B)$ and $\neg(B \rightarrow A)$, respectively. This implies $A \rightarrow \neg B$ and $B \rightarrow \neg A$. However, notice that their conjunction does not generate a contradiction. If we use $A \rightarrow \neg B$ in a *modus ponens*, we can infer $\neg B$ from A, but then $B \rightarrow \neg A$ only allows us to infer $\neg B$ from A through *modus tollens*. Similarly, if we use $B \rightarrow \neg A$ in a *modus ponens*, we can infer $\neg A$ from B, but then we can only use $A \rightarrow \neg B$ in a *modus tollens* to infer $\neg A$ from B. There is no situation where we can infer both A and $\neg A$ or B and $\neg B$.

The same reasoning applies to the first-order analysis, with the only difference being that instead of interpreting $A \rightarrow \neg B$ as the consequence of $\neg(A \rightarrow B)$ and $B \rightarrow \neg A$ as the consequence of $\neg(B \rightarrow A)$, we interpret $Aa \rightarrow \neg Ba$ as the consequence of $\neg(Aa \rightarrow Ba)$ and $Ba \rightarrow \neg Aa$ as the consequence of $\neg(Ba \rightarrow Aa)$.

The conclusion reached is surprising. Although rejecting the truth of $\neg(A \rightarrow B)$ based on intensional grounds does not provide grounds for accepting $A\&\neg B$ on intensional grounds, as discussed earlier, this disagreement is not a genuine counterexample. Therefore, to accept that a proposition *A* is neither necessary nor sufficient for *B* is to accept a contradiction, despite its counterintuitive nature. If we believe otherwise, it is because we are accustomed to epistemic constraints that favor intensional evidence, acceptability conditions, and criteria of truth. Our epistemic practices bias us. In this sense, classical logic is no different from many scientific findings in physics and biology that conflict with our expectations of reality. Why should this bother us? It's just business as usual. Let us keep our textbooks safe from the bonfire.

6. HYPOTHETICAL SYLLOGISM, OR-TO-IF AND CONTRAPOSITION

The hypothetical syllogism purported counterexamples illustrate the conflict between intensional and extensional evidence. Dale (1972: 439-440) offers the following example:

(1) If I knock this typewriter off the desk then it will fall.

(2) If it falls then it is heavier than air.

(3) If I knock this typewriter off the desk then it is heavier than air.

(4) If the typewriter is heavier than air then an elephant is heavier than air.

(5) If I knock this typewriter off the desk then an elephant is heavier than air. From (1)-(4) hypothetical syllogism.

The issue in this case is that while there is intensional evidence to support (1)-(4), the only evidence supporting (5) is extensional, which means that we can only be sure that both the antecedent and the consequent are true given the inference by hypothetical syllogism from previous propositions. This intuition can be criticized for assuming, without argument, that the only evidence to accept a conditional is of the intensional kind. Once this misunderstanding is clarified, it becomes perfectly natural to accept the conclusion on extensional grounds. In this case, the acceptance of previous premises leads to further commitments of truth values, whether we are aware of this or not. Additionally, hypothetical syllogism can only be refuted by an example where the premises are accepted on extensional grounds, but the conclusion is unacceptable on extensional grounds.

Next, we have the inferential form $A \vee B \vDash \neg A \rightarrow B$, commonly known as ‘or-to-if’. Let us imagine a context where two balls are placed in a bag labeled as ‘*a*’ and ‘*b*’. The only thing we know is that one of these balls is red, but we do not know which one. In this case, we accept that ‘either *a* is red, or *b* is red’, and we feel entitled to infer from this that ‘if *a* is not red, *b* is red’. The context can be modified slightly so that we know that ball a comes from a collection in which 99% of the balls are red, but we do not have any reason to think that b is red. Perhaps b comes from a collection in which only 1% of the balls are red. My confidence that *a* is red justifies my belief that ‘either *a* is red, or *b* is red’, but it does not justify the conclusion that ‘if *a* is not red, *b* is red’ (Edgington, 1987: 55–56).

The difference in intuitions between two contexts can be easily explained. In the first context, inferring the conditional from the disjunction is not unusual because both are accepted based on intensional grounds. Therefore, there is no evidentiary tension because the same type of evidence is used in both cases.

In the second context, the evidence to accept the disjunction, ‘either *a* is red, or *b* is red,’ is extensional, based on the assumption that *a* is red. However, this evidence does not appear to be sufficient to justify the conclusion that ‘if *a* is not red, *b* is red.’ In other words, although extensional evidence seems adequate to accept a disjunction, it is not intuitively adequate to accept a conditional. It is assumed that intensional evidence is needed to establish a connection between the antecedent and the consequent.

The reason for this mistake is that the conditional seems to take us to a context where the antecedent is assumed to be true. As the extensional evidence in this case involves the falsity of the antecedent, it is automatically discarded as irrelevant. However, the assumption that conditionals cannot be justified by extensional evidence is at least controversial.

This dynamic also explains why some instances of 'or-to-if' do not attract criticism. For example, consider the statement 'Either the butler or the gardener did it. Therefore, if the butler didn't do it, the gardener did.' This example is intuitively valid because the reasons for accepting the disjunction (facts about the crime, main suspects, etc.) are the same as the reasons for accepting the conclusion.

This also holds true when the reasons involved are extensional. For instance, imagine a parent saying to their children during a treasure hunt, 'The prize is either in the garden or in the attic. I know that because I know where I put it, but I'm not going to tell you.' In this context, it is clear to the children that the speaker knows a particular disjunct to be true (Grice, 1989b: 44–45). Interestingly, this disjunction is intuitively equivalent to the following conditional: 'If the prize is not in the garden, it is in the attic.' The conditional can also be accepted in the same situation due to extensional reasons alone.

The reason why 'or-to-if' seems valid in both cases is that it preserves the grounds for believing in the premise. This is precisely why 'or-to-if' seems invalid in the counterexample: the premise is accepted on intensional grounds, but not the conclusion. However, 'or-to-if' is nothing more than a requirement of coherent distribution of truth-values. In other words, given the acceptance of the premise on extensional grounds, it follows that a certain conclusion is also accepted on extensional grounds. Therefore, the fact that it is implausible to think that an intensionally based premise leads to an extensionally based conclusion *is not a problem for the material account*.

Let us now turn our attention to the principle of 'contraposition': $A \to B \vDash \neg B \to \neg A$. Consider the following inference made while waiting for the judges' decision: 'Well, if he didn't win, he certainly tried his hardest. Therefore, if he didn't try his hardest, he won.' Skyrms (1978, p. 178) suggests that one might accept the premise while rejecting the conclusion, because the premise can be acceptable on intensional grounds while the conclusion cannot. However, if we evaluate both the premise and the conclusion from an extensional perspective, the premise 'if he didn't win, he certainly tried his hardest' can be accepted given that its consequent is true, even if the truth value of the antecedent is still uncertain. Nonetheless, the conclusion seems false in a circumstance where the antecedent is true, as it implies that he won despite not trying his hardest, which is unlikely. This alteration of the background facts assumed in the premise changes the condition upon which it depends, which is that he tried his hardest. Therefore, there are no coherent distributions of truth-values where the premise is true and the conclusion is false. What matters is that 'contraposition' preserves extensional grounds for believing in the premise.

7. THE OPT-OUT PROPERTY

It is intuitive to think that $A \rightarrow B$ is acceptable when $A \supset B$ is robust with respect to A, i.e., when $\Pr(A \supset B)$ is high and would remain high after learning that A (Jackson, 1987: 28). This suggests that $A \rightarrow B$ is acceptable when used in a *modus ponens* inference. However, this assumption is challenged by the following counter-example: Imagine I am certain that my wife is deceiving me, but I would never know because she is too smart to get caught. Despite my trust in her, I do not believe she is deceiving me. In this scenario, the conditional probability that I do not know she is deceiving me, given that she is deceiving me, is high. Nevertheless, I would not infer that I do not know she is deceiving me if I found out that she is indeed deceiving me (Van Fraassen, 1980: 503). In this case, the conditional 'If my wife is deceiving me, I would never know' is acceptable, but it is not employable in a *modus ponens*.

Bennett (2003: 55) attempts to explain this counter-example by arguing that the speaker would not be willing to use the conditional in a *modus ponens* but believes that any other person accepting the conditional would be willing to employ it in a *modus ponens*. However, this explanation is ad hoc and only confuses the issue.

The reason why conditionals with the form 'If *A*, I will never know *A*' cannot be used in a *modus ponens* by the speakers who assert them is that doing so would require extensional evidence that falsifies the conditional. The speaker accepts the conditional based on intensional evidence, but the conditional can only be employed in a *modus ponens* with the admission of falsifying extensional evidence.

The phenomenon of resistance to the robustness requirement and employability on *modus ponens* is referred to by Bennett (1995: 340) as the 'Opt-out Property'. According to him, most subjunctive conditionals exhibit this property. He provides the following example to illustrate this phenomenon:

> In 1970 I went to the University of British Columbia, where I worked for nine years; I am sure that if I had not gone to UBC I would have left Canada. However, I am not even slightly disposed to infer, upon learning that I did not go to UBC, that I left Canada. On the contrary, if "I did not go to UBC" is added to my belief system with its multitude of seeming memories of life there, the resulting system implies that I have gone mad and cannot tell what I did in 1970.

Bennett (1995: 340) describes the 'Opt-out Property' as a resistance to the robustness requirement and employability of *modus ponens* in most subjunctive conditionals. He provides an example to illustrate this phenomenon: 'If I had not gone to UBC, I would have left Canada.' Someone may accept this conditional if they assume the antecedent is false but would reject it if they learned the antecedent is true. The speaker would opt out of the conditional, implying that if a conditional satisfies the robustness requirement, it doesn't have the Opt-out Property. However, one problem with this explanation is that subjunctive conditionals are sometimes asserted to reinforce the belief in the truth of the antecedent. For instance, 'I think she took arsenic, for she has symptoms X, Y, and Z, and these are just the symptoms she would have if she had taken arsenic' (Anderson, 1951: 37). Bennett admits that a conditional with the Opt-out Property may be accepted by someone who believes in its antecedent and obsesses that 'I am not denying that. I say merely that a conditional which has the Property can be comfortably accepted by someone who is entirely confident that the antecedent is false; that is an aspect of the meaning of such a conditional' (Bennett, 1995: 341). However, this answer is unsatisfactory because if subjunctives had an Opt-out Property that is characteristic of their meaning, they couldn't be turned off wherever the speaker sees fit.

Bennett's acceptance of the conditional is based on abundant evidence about what is actually the case, including extensional evidence that he went to UBC and intensional evidence about what would be the case if his choices were different in the past. Realizing that the antecedent is actually false would undermine the extensional evidence that led him to accept the conditional in the first place, creating an incoherence.

The relationship between evidence and inferential employability also affects our inferential dispositions. For example, some conditionals are accepted only when we are willing to employ the conditional in a modus tollens inference instead of a modus ponens. When I accept ‘If John's speaking the truth, I'm a Dutchman,’ I am not willing to infer that I am a Dutchman if it turns out that John was telling the truth. The conditional was asserted under the assumption that the antecedent is false. In the already mentioned cheating partner example, when I accept the conditional ‘If my wife is deceiving me, I will never know,’ I am not willing to infer that I will never know that she is deceiving me if I find out that she is deceiving me after all.

8. ADAMS’ PAIR

According to the Apartheid thesis, indicative and subjunctive conditionals have distinct truth conditions. One of the primary arguments presented to substantiate this claim is the Adams’ pair. Let us examine the following set of conditionals:

(1) If Oswald did not kill Kennedy, someone else did.

(2) If Oswald had not killed Kennedy, someone else would have.

Intuitively, these conditionals have different truth conditions. Accepting (1) only requires knowledge that Kennedy was killed by someone, whereas accepting (2) requires assuming a conspiracy theory regarding its murder (Lewis, 1973: 3) [7].

The discrepancy between the two conditionals is due to a difference in available intensional evidence for each conditional. The intensional evidence that someone killed Kennedy and Oswald is the main suspect is sufficient to accept (1), but it is not enough to accept (2). This happens because asserting (2) implies that the speaker is already committed to the extensional evidence that Oswald is the killer, and thus requires stronger intensional evidence that someone else would have killed Kennedy if necessary.

One might argue against this reasoning by saying that the evidence supporting (1) not only is not intensional, but also entails both (1) and (2). The fact that Kennedy was killed by someone appears to be intensional evidence, but is actually extensional evidence. The conditional ‘If Oswald did not kill Kennedy, someone else did’ depends on whether Kennedy was killed, and thus on whether Kennedy was killed by someone. If we represent the logical form of the proposition ‘Someone killed Kennedy’ as $(\exists x)Fx$ and the logical form of the proposition ‘Oswald did not kill Kennedy’ as $\neg Fa$, then applying the existential instantiation rule to the first propositional form gives us Fb. Together with $\neg Fa$, this gives us $(a \neq b)$ by indiscernibility of identicals. The conjunction gives us $Fb \,\&\, (a \neq b)$, and by applying the existential generalisation, we have $(\exists x)Fx \,\&\, (a \neq x)$, which is the logical form of the consequent of the conditional. Thus, the conditional is entailed by its consequent. Now suppose

[7] This example is a modification of the original example presented by Adams (1970: 90).

that the antecedent of the conditional is false. Therefore, it is true that Oswald killed Kennedy and someone else killed Kennedy. Thus, the conditional will be true again[8].

The next step is to show that the evidence supports both (1) and (2). Since (∃x)Fx & (*a* ≠ x) entails (1), and since (2) has the same logical form as (1) - namely, ¬F*a* → (∃x)Fx & (*a* ≠ x) - it too is entailed by the evidence. The reason we may perceive a difference between the two propositions is likely due to our linguistic habits of interpreting subjunctive conditionals as being asserted under the assumption that the antecedent is false. However, such habits should have no bearing on logical matters.

9. CONDITIONAL STAND-OFFS

Conditional stand-offs are also instances that demonstrates the tension between intensional and extensional evidence. Stand-offs occur when one person has reasons to accept '*A* → *B*', while another has equally compelling reasons to accept what seems to be the opposite conditional, '*A* → ¬*B*'. If conditionals have truth conditions, '*A* → *B*' and '*A* → ¬*B*' cannot both be true since they appear contradictory. The reasoning is that for one of the conditionals to be false, someone must have made an error in the facts of the case. However, both individuals have valid reasons to accept each conditional. If neither is mistaken, then neither is stating something false. Therefore, conditionals do not have truth conditions. Gibbard (1981: 226–32) presented the following example to illustrate this puzzle:

> Sly Pete and Mr. Stone are playing poker on a Mississippi riverboat. It is now up to Pete to call or fold. My henchman Zack sees Stone's hand, which is quite good, and signals its content to Pete. My henchman Jack sees both hands, and sees that Pete's hand is rather low, so that Stone's is the winning hand. At this point, the room is cleared. (…) Zack knows that Pete knew Stone's hand. He can thus appropriately assert "If Pete called, he won." Jack knows that Pete held the losing hand, and thus can appropriately assert "If Pete called, he lost." From this, we can see that neither is asserting anything false.

It is important to note, however, that the example provided has a caveat. It can be argued that the example is not truly symmetrical because Jack has stronger reasons to justify his belief than Zack. This has led to attempts to provide new standoff examples that guarantee complete symmetry (Edgington, 1995: 294):

> In a game, (1) all red square cards are worth 10 points, and (2) all large square cards are worth nothing. X caught a glimpse as Z picked a card and saw that it was red. Knowing (1), he believes "If Z picked a square card, it's worth 10 points". Y, seeing it bulging under Z's jacket, where Z is keeping it out of view, knows it's large. Knowing (2), he believes "If Z picked a square card, it's worth nothing".

We need to consider the following example carefully. X and Y's beliefs are justified based on the available intensional evidence. However, this evidence is inconclusive and heavily dependent on their individual epistemic situations. The extensional evidence, on the other hand, is determined by the facts of the case. For instance, it depends on whether Z has chosen a square

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[8] Mellor (1993: 238–239). In fact, it could be said that the premise 'Someone killed Kennedy' not only entails, but is logically equivalent to the conclusion, 'If Oswald did not kill Kennedy, someone else did'; since there are no circumstances in which the conditional is true and the negation of the premise, namely, 'No one killed Kennedy', is true (Lowe, 1979: 139–140). See also Johnston (1996: 99–100).

card that is worth 10 points or not. These facts will ultimately resolve the issue and determine the truth or falsity of each conditional. Once the truth-values become relevant, the symmetry disappears, rendering it a non-issue. These conditionals are either objectively true or false, and their truth-values depend on asymmetrical fact

10. THE BURGLAR'S PUZZLE

Consider the following sentences:

(1) If Alf was the burglar, we'll find his fingerprints in the room.

(2) If Sid was the mastermind, we won't find any fingerprints in the room.

Someone could accept both (1) and (2) without knowing (3):

(3) Alf was the robber but Sid was the mastermind.

According to Ramachandran (2016: 29), it would be inappropriate for a person to deny the conjunction of the antecedents if they are unsure whether (3) is false or not. However, accepting both (1) and (2) simultaneously entails the denial of (3). This is because if it is true that Alf was the robber and Sid was the mastermind, then Alf's fingerprints would be found in the room and no other fingerprints would be found.

The problem arises from a conflict between our use of intensional evidence and the actual truth-values of the components that have logical significance. A detective could endorse both (1) and (2) based on intensional evidence associated with the behavior of Alf and Sid, without making any inferences based on the truth-values of the antecedent and the consequent. However, once the truth-values are established, such as when Sid confesses to being the mastermind, the detective will have to abandon (3) and the antecedent of (1), concluding that there are no fingerprints in the room.

When dealing with evidence, one typically ignores the attribution of truth-values. However, this attitude cannot be maintained once the truth-values are revealed. While (1) and (2) are co-tenable when considering the evidence to accept the connection between the antecedent and the consequent of each conditional, they are not co-tenable if the antecedent of one of them is true. The evidence supports the idea that if the antecedent of each pair is true, then the consequent will also be true, but it does not support accepting both antecedents initially.

We could say that there are two levels of evidentially. The first level involves accepting a conditional based on intensional reasons, which means that there is a connection between the antecedent and the consequent. The second level involves the actual truth-values, or a combination of truth-values and intensional evidence. For instance, if the antecedent is true and there are good reasons to believe that there is a connection between the antecedent and the consequent, you must accept one of the conditionals and reject the other. What matters most are the truth-values of the antecedent and the consequent, rather than the intensional evidence. In other words, the second level of evidentiality always takes precedence over the first.

11. FINAL THOUGHTS

In classical logic, the truth conditions of connectives are simplified and devoid of psychological and epistemic factors, including the role of intensional evidence and epistemic states of imperfect information. However, this simplification has generated several counter-intuitive aspects. It may be tempting to conclude that the material conditional is an inadequate representation of the logical properties of conditionals in natural language if we assume that its logical properties must include our epistemic practices.

However, these contrary intuitions are epistemically biased and should be criticized for that. Logic deals with the truth-conditions of propositions, which are determined by the metaphysical substrate that is responsible for the truth-values of its propositional components. This substrate and, therefore, the truth-values of its propositional components are largely independent of epistemic agents, their epistemic situation, degrees of confidence, and so on. Belief conditions, intensional evidence, and preservation of grounds for believing are epistemic phenomena that are affected by the epistemic agent's ignorance. Truth preservation is a semantic phenomenon that is independent of the epistemic agent's ignorance. Semantics always takes precedence over epistemic ignorance. If intensional evidence and grounds for believing preservation clash against extensional evidence and truth preservation, the latter should be prioritized.

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