*Peter M. Simons*

# Gestalt and Functional Dependence

## §1 Preamble: The Grelling/Oppenheim Gestalt Papers

The following piece is designed to accompany the ensuing essays on the logic of Gestalt concepts by Kurt Grelling and Paul Oppenheim. All of these essays are written with admirable clarity, so it is quite unnecessary to summarize what they say. I shall be more concerned with general scene-setting, both conceptual and historical, and with evaluations and comparisons.

Of the three essays, one, "The Concept of Gestalt in the Light of Modern Logic", has been previously published in German, in the 1938 issue of *Erkenntnis*. The following English version is a translation of that paper, to which are appended lightly edited "Supplementary Remarks on the Concept of Gestalt" from the same volume, which appeared in English. (The cultural, geographical and linguistic shift in the centre of gravity of scientific philosophy is here marked in miniature.) This paper is the central one of those published here, and represents a watershed in the attempts to give the concept(s) of Gestalt a firm logical basis. The other two papers are related to it and to some extent presuppose it. The joint paper "Logical Analysis of 'Gestalt' as 'Functional Whole'" is a development of the ideas in the earlier paper. Like its companion piece, "A Logical Theory of Dependence", which bears the name of Grelling alone, it was sent in for the fifth International Congress for the Unity of Science in Cambridge, Massachusetts, in 1939, and both pieces were scheduled to appear in Volume 9 of *Erkenntnis*, whose distribution was prevented by the German invasion of the Low Countries in 1940. Both pieces are here made generally available for the first time.

These essays represent the most sustained contribution by Grelling and Oppenheim to the problem of Gestalt concepts, but they do not exhaust their work in this area. For the sake of completeness, the remaining relevant pieces, all of which have been published (some in English translation) elsewhere, may be mentioned (for complete bibliographical information, see the references at the end of this article and the bibliography at the end of the volume). The following are all by Grelling:

(1) "Melody as Gestalt" of 1975,
(2) "On Definitions by Equivalence Classes and by Group Invariants" (1969),
(3) "On the Logical Relations between Groups and Equivalence Classes" (1970).

By both authors is a very brief discussion note

(4) "Concerning the Structure of Wholes" of 1939.

In addition, Oppenheim cooperated after the war with Rescher in a joint paper which may be looked on as an update of the 1938 Grelling/Oppenheim paper:

(5) "The Logical Analysis of Gestalt Concepts".

None of these papers has been reprinted here. The Rescher/Oppenheim piece, in the *British Journal for the Philosophy of Science* for 1955, is very readily available, and the pieces (1)–(3) by Grelling are also obtainable. Papers (2) and (3) are of rather marginal interest for the logic of Gestalten, being concerned with alternative mathematical approaches to the objects Grelling and Oppenheim decide to adopt as Gestalten (certain equivalence classes). The discussion piece (4), a commentary to a rather inexact article by a certain Andras Angyal, is too minor to be worth reprinting; everything it says which is worth saying is found in greater detail in the 1938 article. The essay on melodies is only a fragment, and its positive part is again better done in the 1938 joint article.

## §2 What's in a Name?

That which we call a Gestalt by any other name would not, it seems clear, sound quite so interesting. Some theories have serendipitous names, which make them sound interesting whatever their scientific merits. Preface 'theory' by any of the following, and you have examples: 'catastrophe', 'game', 'invisible hand', 'quantum' and 'systems' (the order is alphabetical). 'Gestalt' is just such a lucky find. Meinong and Husserl both coined their own terms for what Ehrenfels called 'Gestalt qualities': Meinong called them 'founded contents' and Husserl called them 'figural moments'. Neither term stuck, and one needs no second hearing (even in the original) to know why not. 'Gestalt' purveys an intriguing mixture of science and mystery, and with a Teutonic tinge which tickles the palate of German-speakers and non-German-speakers alike. This has ensured that 'Gestalt' has entered other languages as a loan-word, a fact even recognized by lexica. How the world might have been had this not been so was inadvertently shown recently when the Austrian Post Office honoured the 125th birthday of Christian von Ehrenfels with a special stamp. On the reverse of the first-day cover was a brief biography in German, with translations into English and French. The English translation began, 'Christian of Ehrenfels, founder of the shape psychology...' Who would have taken Shape Psychology or Shape Theory or even Shape Therapy seriously?

Although 'shape' would always have been a bad translation of 'Gestalt' in psychological contexts, there is an English near-equivalent which has about the right generality and philosophical loading, namely 'form'. Teutonic mystery apart (which was of course very dear to the fanatical Wagnerian Ehrenfels), 'form psychology' would just about have done. What is more, the philosophical pedigree of 'form' and its many cognates more than does justice to the aspirations to generality of Gestalt theory. The German words '*Gestalt*' and '*Form*' are almost synonymous in everyday use. Taken as a verb, *gestalten* describes precisely the activity of the potter, sculptor or Demiurge in forming, shaping, moulding, and there is indeed more than a hint of Plato's *Timaeus* in Ehrenfels' *Kosmogonie*. That we cannot say that 'Gestalt', even in its technical use, is just a synonym of 'form', is due principally to the greater variety of meanings of the latter. At best, a Gestalt is one variety of form. Getting clear what the term 'Gestalt' may be reasonably taken to mean is the point of the essays by Grelling and Oppenheim. In my view they are remarkably successful in being precise about something which has too often been the victim of sloppy usage. Grelling was indeed murdered by practitioners of the ideology which perverted, among many others, precisely the term on which he here attempted to shed light. The fate of the word was shared by others of related content, in particular *Ganzheit*, whose morphological impropriety alone should have kept it from being coined, and *Gemeinschaft*, the social *Ganzheit* (*das Volk*), as contrasted with *Gesellschaft*, the social sum. The obfuscations surrounding '*Ganzheit*' were the subject of a moderately famous attack by Schlick.[1] While Schlick's principal target was bad philosophy, he was certainly politically at variance with the propounders of *ganzheitliche* politics. And it is of course inherently easier to (mis-)use holistic concepts in the service of totalitarian and authoritarian ideologies than to do the same with atomistic concepts. The great totalitarian theoreticians, Plato, Hegel and Marx, can be used to underpin dictatorships in a way which Locke or Hume cannot. But the user of holistic concepts in political philosophy cannot be condemned simply by the bad company he is forced to keep. Schlick's recommendation that holism or atomism are purely methodological points of view without ontological significance certainly goes too far, whatever the merit of his attack on woolly uses of holistic terms in the interwar years. The direction opened up by Grelling and Oppenheim shows that carefully managed uses of '*Gestalt*' and related terms may indeed cut ontological ice.

The *terms* 'Gestalt' and so on are nowadays for the most part not fashionable enough for it to be worth pointing out again, as Schlick did, the known dangers of sliding into profound-sounding trivialities and nonsense. We shall here pass over the targets of Schlick's attack in silence. It must be noted though that a similar danger to that offered in days of yore by '*Ganzheit*' is posed nowadays by the term 'system'. In much of its present-day usage, this borders perilously on the vacuous. To examine the credentials of this term in the way Grelling and Oppenheim did for 'Gestalt' would be a large task, and one for another time. But the relative success of their account of terms a couple of generations older is encouraging; by separating the wheat from the chaff it *might* be possible to offer a precise and tidy concept of system. Whether this would then have the characteristics requisite for broad popularity is another matter. But of course any science needs and uses concepts with anti-atomistic import. So it is a natural impulse to attempt to generalize and eventually even perhaps formalize such concepts, so that they are universally applicable, and foster interdisciplinary conceptual community ('Unity of Science!', as Grelling and Oppenheim say at one point). The methodological problem is to steer between the Scylla of overspecificity on the one hand and the Charibdis of emptiness on the other ('*Everything* is a Gestalt/Ganzheit/system...!') In my view, the term 'Gestalt' is not limited in its applicability to real things, nor was such a limitation contemplated in early uses of the term. Ehrenfels himself, though not always clear about the different kinds of Gestalt, did employ Gestalt-theoretic concepts in his study of prime numbers, in my view perfectly congruously. The Grelling-Oppenheim reconstruction makes it clear that *all* Gestalten are abstract entities, whether or not they are realized in concrete entities. Among the applications of their concepts they instance geometry and logic. If this is right, and there is no *a priori* restriction in the field of application of the concept of Gestalt, it is indeed a *formal* concept, to be ranked alongside such concepts as *individual*, *property*, *relation*, *complex*, and *state of affairs*.[2] The fact that the concept first came to prominence in psychological investigations is then accidental from the point of view of the formal concept itself.

## §3 On Defining 'Gestalt'

To Ehrenfels goes the credit of making it crystal clear that there *are* Gestalten. This was done essentially with the help of a single example: that of a melody, which remains the same in all its transpositions. The relationship of Ehrenfels' views to those of Mach is dealt with by Smith and Mulligan in their essay in this volume, so we may pass over this aspect of background to the development of the Gestalt concept. One remark only is worth making. Ehrenfels, like most thinkers on matters philosophical in his generation, was initially none too clear about when he was talking about an object and when he was talking about one's experience of an object. His initial approach to the problem of how we can apprehend a melody allows him to tilt in either direction: towards an account of the psychology of apprehension and towards an account of the ontology of the item apprehended. His use of the term 'Gestalt quality' if anything encourages the ambiguity, in view of the long tradition stemming from the British empiricists (in whose footsteps all students of Brentano trod) of using 'quality' for a sensible idea which may (primary qualities) or may not (secondary qualities) be an accurate representation of the qualitative properties of the object of presentation. A number of Brentano's students, in particular Twardowski, Meinong and Husserl, advanced to the point of making a clear distinction between objective qualities of objects of presentation and the subjective presentations or moments of presentations by means of which such objective qualities are intended. Such a distinction was a necessary prerequisite for overcoming the prevailing psychologism in logic and in other disciplines such as aesthetics and value theory in general. While it would be wrong to accuse Ehrenfels of psychologism, he was not intimately involved in the struggle against pychologism, and expended less care in differentiating act, content and object than some of his colleagues. So in his last published words on Gestalt theory, we find him dividing Gestalt qualities into '*apprehensions* of processes and of momentary states'[3], though almost immediately afterwards he claims that a certain objective formation (a word), as contrasted with the apprehension of the word, is itself a Gestalt-quality. Ehrenfels characteristically approached objects of whatever kind through the way in which we have access to them, and was less careful than he might have been in keeping the constitution of the object distinct from the way in which we apprehend it. This tendency may have been underscored by his subjectivistic voluntaristic theory of values, which explicitly denied the existence of what Meinong called supra-personal values.

We just found Ehrenfels dividing Gestalt qualities into two mutually disjoint classes: dynamic and static. A melody is a dynamic Gestalt: it unfolds in time and cannot be taken in at a stroke. A geometrical shape, e.g. in a painting, is a static Gestalt, and can be apprehended in one, at least in principle. Indeed Ehrenfels' interest in providing a classification of Gestalten persisted from the time of his famous article right up till the end of his life. However before Gestalten can be classified, we must first have a clear idea what a Gestalt is. I say 'have a clear idea' rather than 'define', since it is not clear at first sight whether such a general concept as that of Gestalt can be defined at all; indeed, what we mean by 'define' in this connection is not obvious.

Ehrenfels did supply a brief definition of 'Gestalt quality' (p. 93 above), but his attempts to get clear what Gestalten are principally take the two different routes of giving examples and offering characterizations of Gestalten. Grelling and Oppenheim, too, offer a definition, and therefore it may be worth pondering briefly what a putative definition of 'Gestalt' must be like. Is Gestalt an indefinable concept? Ehrenfels' sparse remarks on how the concept might be reduced to others suggests that he may have thought so, and this would explain why he set so much store by his two alternative procedures for getting the sympathetic reader to understand what he is talking about. Examples can have a great demonstrative and convincing effect, as Ehrenfels' melody case shows, but examples alone can only give the reader an idea which is rough: they may rule out some inappropriate cases, they may provide some central ones for comparison. But they cannot draw a boundary round what counts and keep out just what does not. For this we need more. Ehrenfels does indeed offer more, but whether his characterizations suffice fully to determine the concept is far from clear. They might perhaps be considered as a set of *necessary* conditions for something to be a Gestalt. Grelling and Oppenheim effectively claim for their concept that it also provides *sufficient* conditions, but more on this anon.

The so-called 'Ehrenfels conditions' will be discussed in the next section. First, however, a minor difficulty must be removed. Suppose we are right that *Gestalt* is a formal concept. Does this not rule out its being defined? After all, such formal concepts as we have mentioned, such as *individual*, *relation* etc. are indefinable. However, a concept may be formal and yet definable, provided only that it is defined solely in terms of formal concepts (logical and ontological). So, for instance, the concept *fifth power of a relation* is a definable formal concept, as is *object without proper parts* (i.e. *atom*), on the warrantable assumption that *proper part* is a formal concept. Hence nothing in principle stands in the way of *Gestalt* being both a formal concept and definable.

## §4 Ehrenfels' Conditions on Gestalten

The first characteristic property of Gestalten is their one-sided dependence on their basis or fundament. This means that the existence of the object or objects which form the basis of a Gestalt is a necessary prerequisite for the existence of the Gestalt, but not vice versa. Gestalten are *moments*, in the sense we have attempted to make clear elsewhere.[4] The term 'basis' is simply the correlate to 'Gestalt' in this case. If we term the relation between a dependent object and the objects on which it is immediately dependent *foundation*, then a Gestalt is a *founded* object, and the *founding* objects whose existence its existence presupposes are its fundament or basis. In Ehrenfels, in contrast to some later Gestalt theorists, in particular of the Berlin School, a Gestalt need not exist if its fundament does, so the foundation or dependence relation is in this sense one-sided rather than mutual. Consider a flight acrobatics air display team consisting of four aircraft, which get themselves into a tight diamond formation with the rear aircraft slightly below the leader: this is known as Box Formation. The particular instance of Box Formation flown by this team just now could not exist without the four planes, but they can well exist without it: they need not fly in Box at all, but some other formation, e.g. in wing abreast.

If this last example is a good case for Ehrenfels, there are others which seem to back up the Berlin view better. A melody is a case in point. The counterfactual conditions on identification of events such as the sounding of a C sharp by Player X here and now are much less flexible than those of physical objects such as aircraft. It is perfectly obvious that it is no part of the essence of a particular aircraft that it fly at a certain distance and angle from three other particular aircraft at a certain time. In another possible world, as they say, it could have been sitting in its hangar being repaired, or be winging its defecting pilot across the Iron Curtain. By contrast it is by no means clear that if Player X had been transported to Kuala Lumpur rather than the Royal Albert Hall before playing his C sharp, that he would have been playing the very same C sharp. Nor is it clear that if, instead of playing CCEGG,GG,EE, I had played ECGCG,EG,EG, that we can assert cross-world identities for *any* of the tones sounded. If that is so, then the very tones of the first series could not have all existed without playing the Blue Danube melody, and the dependence is mutual. (The qualification 'all' is necessary because we may without difficulty assert cross-world identities for tones in melodies which might have been interrupted.) However, in either case we can in truth say that the Gestalt is dependent on its basis, irrespective of whether this dependence is one-sided or mutual.

The examples and the point show up an ambiguity in the term 'Gestalt' or its near-equivalent 'Gestalt quality' which seems to have gone unnoticed in Grelling and Oppenheim's analysis. Taken as a universal, the Blue Danube melody is clearly *not* dependent on a particular sequence of tones tapped out by Simons on his son's toy xylophone. The melody *an sich* is perfectly impervious to such gratuitous exemplifications. Yet what Grelling and Oppenheim call a Gestalt is most naturally taken as a logically reconstructed universal. In fact, since they construe Gestalten extensionally as certain equivalence classes, and since the identity of a class is determined by its members, it turns out on their analysis that the xylophone-tapping *is* after all essential to the Blue Danube! This result alone is enough to show that taking Gestalten or other universals as equivalence classes is mistaken, as will be argued again below. But suppose we overlook this peculiarity of their analysis. In everyday parlance, it is one and the same melody which may be played any number of times, and none of these individual performances of the melody is essential to it as such; it would still have been the same melody had this particular performance never occurred. We must therefore distinguish the melody *an sich*, the universal, repeatable, single melody, from its many instances in concrete sequences of tones. Only an individual instance of the Blue Danube Melody is directly dependent on the particular tones on which it is founded. The universal is at best *generically* dependent on its realizations in concrete sequences of tones, on a moderate realist theory of universals at any rate. On an extreme Platonist theory, even this generic dependence is lacking:[5] the melody does not ever have to be discovered or played to exist as a universal. This ambiguity of 'Gestalt' as between type and token is relatively harmless, as long as we recognize the possibility of both senses. Ehrenfels, like many of his fellow students of Brentano, was perfectly at home with the idea of dependent particulars standing to their non-substantial kinds as individual horses stand to the substantial kind *horse*. On the other hand this distinction between universals and particulars in categories other than substance, which is a cornerstone of Aristotelian and Scholastic ontology, is largely lost in modern philosophy, in no small part because the standard logical means of expression (predicate logic and its set-theoretic extensions) standardly do not contain singular terms designating dependent particulars.[6] Grelling and Oppenheim belong to this modern tradition, and therefore understand 'Gestalt' to denote a universal. This makes it impossible to give a clear sense in their terms to Ehrenfels' notion of one-sided dependence of particular occurrences of the Blue Danube melody on particular tone sequences in which such occurrences are found. However since the predominant interest both of Ehrenfels and of Grelling and Oppenheim is in that which is suitably invariant in varying realizations, i.e. the universal, this neglect of Gestalt-

*instances* is not detrimental to their study in any irreparable way. We accordingly leave consideration of these instances likewise aside, although a full ontological account of Gestalten would have to bring them in again.

Whether (instances of) Gestalten are taken to be one-sidedly dependent on their fundament (as Ehrenfels assumes) or mutually dependent (as Wertheimer and Köhler assume), or sometimes one, sometimes the other (our position), makes no difference to the assessment of Ehrenfels' two further criteria for something's being a Gestalt: (1) *Supersummativity* and (2) *Transposability*. (Actually only the first of these was formulated by Ehrenfels as a criterion of Gestalt.) Supersummativity is the oft-quoted condition that a Gestalt be something other than the sum of its parts. This condition has sometimes been ridiculed on the grounds that it is completely trivially satisfied, there being no object which is a mere sum of parts.[7] But the criticism is misplaced. It is possible to find pure sums. For something to be a sum of objects, it is not necessary that the different summands have no relation to one another, nor that they have no effect on one another. The vague idea of a sum may be made precise in mereology, and this is indeed the concept of sum which Grelling and Oppenheim, making use of their knowledge of the work of Woodger and Tarski, presuppose.[8] For the sum of a number of objects to exist, it is both necessary *and sufficient* that all of these objects exist. If the existence of an object depends not merely on the existence of certain parts forming a division of the object (in the sense used by our authors in their §2.1) but also on there being some particular relations holding among these parts, then the resulting object is *not* the sum of these parts. To take their example, a brick wall is not the sum of its bricks and mortar, because its being a brick wall is dependent on the bricks' being in a certain loosely circumscribed configuration. At any rate, if the bricks and mortar are in two separate heaps side by side, we still have the sum, but no brick wall. But no-one will deny that the bricks lying in a heap have specific relations, including causal ones, to one another.

The condition of supersummativity is nevertheless very easy to satisfy: almost all the objects we are interested in (including ourselves) satisfy it for all possible divisions. Exceptions include mere pluralities of objects and amounts of stuff. But they are the exceptions. Grelling and Oppenheim point out that the *term* 'Gestalt' as used i.a. by Köhler means something other than Ehrenfels' 'Gestalt quality', since Köhler's Gestalten satisfy *only* the supersummativity criterion, and not the second, more important transposability criterion. The word 'Gestalt' is therefore reserved by them for the Ehrenfelsian concept, and the Köhler concept is termed first 'determinational system' [*Wirkungssystem*], later redubbed 'functional whole', following Koffka, for their second joint paper. Functional wholes are of necessity real, since the parts have reciprocal effects on one another. On the other hand, a Gestalt is itself something abstract, and may indeed have an abstract fundament, as Grelling and Oppenheim attest with a geometrical example. Our interest here is with the concept of Gestalt, not that of functional whole. Nevertheless, it was clearly important for the authors to make the distinction, elementary though it now seems, since even proponents of Gestalt theory were obviously not clear about the different things they were calling by one name.[9]

Gestalten in the original sense must be transposable. This thought is indeed the basic insight behind Ehrenfels' original study, though even he is prepared to allow certain exceptions. The musical example is particularly felicitous, because 'transposition' in its usual musical sense describes precisely what happens when the same piece of music is played in different keys. As a special and obvious case, we may take a simple melody. But of course the case is only a special one, and Ehrenfels' second insight was to see that the notion of transposition may be generalized to all kinds of modifications in which some aspect of form remains constant. To remain with music, the inversion, retrograde and retrograde inversion of a canonic or dodecaphonic theme do not give us the same melody as the theme, but something else, other than a melody, remains constant or invariant throughout these variations, for which we have no special name. Again, a rhythmic motive, such as the fate motive in Beethoven's 5th Symphony, is transformable in various ways, realizable in different melodic versions, while remaining recognizable. It may even have been Ehrenfels' admiration for Wagner which helped him to generalize the melody example,[10] since Wagner's use of recurring motives other than melodies goes recognizably beyond mere transposition. In modern music the abstractness of structures which may be preserved under transformation may be such that the same structure cannot be recognized again by ear.[11] Perhaps the most striking achievement of Grelling and Oppenheim's analysis of Gestalt concepts is the way in which they attempt to give an account of what is meant by 'transposition' which is both general enough to satisfy the aspirations of Gestalt theorists and at the same time not empty of content.

## §5 Determinables and Determinates

Consider a number of concrete things, e.g. a number of schoolchildren in a class. They are describable and classifiable in innumerable ways, for instance according to height, sex, age, hair colour, intelligence, musical ability, parental income, and so on. Take the simple binary classification into boys and girls. All the children fall into one or other of these, and none into both. When we say of a child that it is a boy or a girl, we are giving its *sex*. Similarly when we say it is 3 feet or 4 feet tall etc., we are giving its *height*. The two answers 'boy' and 'girl' answer the question 'What sex?', the various heights the question 'What height?' and so on for all the other dimensions of classification. The concepts *male*, *female*, *3 feet tall* etc. can all be seen as specific properties which the children have or may have. The properties form *families* of contraries: nothing can be both 3 feet and 4 feet tall at the same time, or both male and female at the same time. If we choose the dimensions of classification of our objects carefully, then each object will have precisely one property from each family. The concepts *height, sex* etc. do not on the other hand describe properties of the children, but rather are the kinds to which the properties of the children belong: *male* and *female* to *sex* and so on. The sexes are just (the properties of being) male and female, or on some usages also the extensions of these properties. Following Johnson[12] we call the properties of the objects 'determinates', and the higher-order kinds to which these belong 'determinables'. Each determinable has a family of contrary determinates.[13] It is determinables that Grelling and Oppenheim call *Klassifikatoren*; the English translation of this is 'classifiers'.[14] However it is useful to have a correlative term for the possible values of classifiers, so we have adopted Johnson's terminology, which is definitely meant to cover the same phenomenon. When Grelling and Oppenheim call classifiers 'functions' this may mislead one into supposing the values are objects, but they are not. E.g. the value of the determinables *sex*, *height*, *hair colour* for Priscilla are three abstract properties, say *(being) female*, *3 feet* and *blonde* respectively. The function terminology is rather a Procrustean bed for such a commonplace affair; rather than say that the value of the function *sex* for the argument Priscilla is *female*, it is easier and more natural to say that Priscilla's sex is female. It is worth remarking that determinables and determinates may, like functions, be of more than one place. For instance the determinable *distance* is two-placed, like its determinate *35 miles from*. This illustrates an obvious law: if a determinable is *n*-placed, then so are all its determinates. A three-place determinable is given by *angle (formed by ... and —— with respect to –:–)*. As the examples show, determinables often form a *magnitude* of which the determinates are possible *values*. There is then usually some kind of ordering relation or topology on the values (e.g. the real half-line structure among values of the scalar *temperature*, an *n*-dimensional Euclidean structure for example among values of the vector magnitude *direction*). There are all sorts of deep and interesting questions which arise here concerning the relationships between concrete objects and their properties and relations, which are determinates to various determinables, and various abstract mathematical structures which are naturally used to model the magnitudes. There are obviously strict constraints on what kind of structure is suitable to model certain magnitudes, and these cannot be purely accidental or conventional. For instance, the natural numbers suit themselves to counting discrete things, but the positive and negative reals suit themselves to measuring distance on a line. It is plausible to think that the need for negative numbers emerges only with the introduction of linear vector magnitudes like this, which suggests that the usual mathematical derivation of all number systems from the natural numbers by means of various constructions is quite untrue to the application of such numbers in measurement. One of the few philosophers to have taken this point seriously is Frege, in his unfinished theory of real numbers.[15]

One reason why Frege's better-known theory of natural numbers stands head and shoulders above most of its rivals is that he troubles to explain the link between the abstract mathematical objects the pure mathematician studies and their application in humdrum daily practice like counting barrels of beer. The key to Frege's theory is the idea that when a range of objects have an equivalence relation defined on them, then it is possible to find a function from these objects to certain abstract objects such that two objects yield the same value of the function if they stand to one another in the equivalence relation. Now the exact nature of the transition from equivalences to identities is a ticklish affair, as Frege was perhaps the first to recognize,[16] and it is doubtful whether there is yet a satisfactory theory of it, by which I mean one which avoids what Angelelli has called the 'looking-around method'.[17] What this means is the following. Since there is no unique function taking equivalences into identities, and since it is natural to want the convenience of just one abstract object which we can call *the* number, *the* direction etc., most logicians settle for a conventional stipulation of which particular abstract object is to be the value of the function for each argument. With this step, one leaves facts behind and opts for convenience. This was the step Frege took when he decided to make the number belonging to the concept *F* the extension of the concept *concept equinumerous with F*.[18] The general method of taking the abstract objects associated with an equivalence relation to be the set of equivalence classes quickly established itself as canonical,[19] and it is this method which Grelling and Oppenheim employ when they come to define *Gestalt*: the relation '... is transposable into—— ' (for each particular kind of transposition) is an equivalence, and they therefore take the corresponding Gestalt to be the equivalence class for any given argument. But the method of equivalence classes is purely extensional, and leads to absurd results if we take it seriously as telling us what certain abstract objects *are*, or even how they are to be conventionally represented. Consider for example the school class of our previous example, and the equivalence relation *is as tall as*. This partitions the class into subclasses of schoolchildren, each of which consists of children of the same height. Apart from its being literally absurd to call e.g. the class {Tom, Dick, Priscilla} *the height 3 feet* (restricted to this class), since the class has three members, whereas a height is not the sort of thing that can have members, the results are wrong even if we take the class as merely a conventional representative of the height. For the height is such that it is not necessarily the case that Tom, Dick and Priscilla have it (now, in this class), whereas the class *is* necessarily such that it contains Tom, Dick and Priscilla.[20] It will not do to say that the class can be designated by a definite description, "the class of children who are 3 feet tall", for we are attempting to *define* the expression "3 feet tall" and so are unable to make use of it without circularity. So although the description has the right kind of modal flexibility (designating different classes of children in different possible circumstances, as they say), while the direct designation of the class by naming its members is modally rigid, we are forced to use the inappropriate tag for the class, whose modal properties make it therefore unfit to represent the height (in another world Tom, Dick and Priscilla might all be 4 feet, and all the children who are actually 4 feet might be 3 feet instead, which means that the height of 3 feet could have been the height of 4 feet and vice versa according to the usual theory).

This is an argument which applies to *any* use of equivalence classes to represent determinate characteristics of objects for which these characteristics might have had different values (within mathematics itself matters are much simpler). *A fortiori* the Grelling-Oppenheim definition of *Gestalt* is modally inappropriate, in particular as applied to objects with accidental properties.

How this defect should best be remedied is a topic which would require a sizeable monograph. It is perhaps most in spirit with the extensionalized taming of modality embodied in possible-worlds semantics to regard a Gestalt as not an equivalence class within the actual world but as an equivalence class across all worlds, or as a function from possible worlds into equivalence classes. To make sense of this one has to make sense of cross-world equivalence relations.[21] Some such trick will certainly circumvent the modal inappropriateness argument. Whether we are any closer to understanding what a Gestalt is as a result is doubtful. In any case, we have simply replaced a world-bound convention for a cross-world one. We are still "looking around", only our perspectives have widened. My own preferred starting point for a solution is to strengthen the transition constraints on the abstraction step: an equivalence '$a \underset{X}{\sim} b$' and its associated identity 'the $X$ of $a$ = the $X$ of $b$' have to be not merely materially equivalent but *synonymous*.[22] This makes the identity of the $X$ of $a$ as determinate (hence as indeterminate) as the meaning of '$\ldots \underset{X}{\sim} ---$'. This is hard to reconcile with Platonism, for which there ought to be a determinate answer to the question whether the $X$ of $a = q$ for an arbitrary singular term $q$, it being assumed that a Platonic object is an entity with (determinate) identity. Tying the identity of abstracta down to the possibly indeterminate meaning of equivalence relational predicates promises rather a form of conceptualism-nominalism, which does not take abstract objects seriously. This raises the general question of the viability of a nominalist reduction of all abstracta, Gestalten included. While such a question is in my view open,[23] and of course no attempt can be made here to answer it, I should be happier if nominalism were, as I believe, correct.

However we may – perhaps surprisingly for such a fundamental ontological question – leave the issue on one side for the purposes of our discussion of the analysis of *Gestalt*. For even the most die-hard nominalist does not deny the *abbreviatory convenience* of terms purporting to designate abstracta. He contends only that these – and perhaps more problematically, bound variables taking their place – are not ontologically committing. So a Platonist and a nominalist may both speak the same *prima facie* Platonistic language and discuss the appropriateness of the definition of *Gestalt* while differing as to the ultimate ontological significance of their discussion.

The Frege transition from an equivalence to an identity between abstracta is clearly insufficient to yield an interesting theory in itself. For the abstracta thereby introduced to be interesting they must have other attributes, and these may be considered to be derived from those attributes of their underlying concreta which are *invariant* under the equivalence.[24] Another generalization which must be made is to take account of determinables with more than one place. With a 2-placed determinable the underlying equivalence among concreta is 4-placed, when the determinable is 3-placed, the equivalence is 6-placed, and so on. For example, the relation among concreta which corresponds to the determinable *distance* is that which holds among four objects $a$, $b$, $c$, $d$ when the distance of $a$ from $b$ is the same as that from $c$ to $d$. In general, if $E$ is a $2n$-placed equivalence, i.e. a relation such that the following always hold:

(Refl) $a_1a_2\ldots a_n\, E\, a_1a_2\ldots a_n$,
(Eucl) $a_1a_2\ldots a_n\, E\, c_1c_2\ldots c_n \wedge b_1b_2\ldots b_n\, E\, c_1c_2\ldots c_n \rightarrow a_1a_2\ldots a_n\, E\, b_1b_2\ldots b_n$,
then for all $a_1b_1a_2b_2\ldots a_nb_n$:
$$a_1a_2\ldots a_n\, E\, b_1b_2\ldots b_n \leftrightarrow X_E(a_1a_2\ldots a_n)=X_E(b_1b_2\ldots b_n).$$

The value of $X_E$ for given arguments is then an abstract $n$-place determinate relation. Under such multi-placed equivalences we may also weed out the properties and relations which are invariant, thereby carrying over from the concrete objects to their associated abstracta just those for which it does not matter which concrete ones we take in an equivalence class. It is this characteristic of the procedure which merits calling it *abstraction*. We trade *quantification* over concreta for *designation* of abstracta: therein lies the convenience.[25]

Once determinables with more than one place are admitted, we tap a rich source of new determinables. We can plug one or more places and consider the determinable resulting: for instance by plugging either place in the (symmetrical) determinable *distance* we obtain such determinables as *distance from Rome*, *distance from Athens* and so on.

Determinables correspond to wh-questions in English. To the question "How far is Rome from Athens?" we have in fact *three* determinables: *distance, distance from Rome, distance from Athens,* according to how we carve up the question.

## §6 Functional Dependence

One of the central methodological tools behind the Grelling-Oppenheim investigations is the notion of functional dependence, examined for its own sake in Grelling's "A Logical Theory of Dependence". This notion is interesting in its own right, quite apart from its use in their joint papers, since it raises a number of fundamental issues concerning the nature of measurement, of causality, and of functions. It swiftly emerges in Grelling's paper that there is not just one concept of dependence, but a swarm of them. If anything, there are rather more different concepts of dependence than even Grelling recognizes, so it is wrong to talk of *the* concept of functional dependence. We use the epithet 'functional' where Grelling talks simply of dependence, because there are many different kinds of dependence, of which the kind discussed is only one. In particular the kind of dependence has nothing to do with *existential* or *ontological* dependence, the dependence of one object for its existence on another. Nor is it directly connected with *logical* dependence, the relationship between propositions and sets of propositions (there *is* a connection, but it is not as simple as might appear at first sight). Finally, there is the notion of *causal* dependence. To the extent that the authors' notion of functional dependence captures the idea of one quantity's depending on another (or several others), it might be claimed that causal dependence turns out to be a special case. But functional dependence is in my view far too weak a relation to capture the causal aspect of causal dependence, which has to do with some things' being in a certain way or acting in a certain way *making* something come about. There is a moderately respectable tradition running through empiricist thought from Hume, through Mill and Mach to Russell, which claims in effect that functional dependence among suitable objects (usually some kind of phenomena) is to *replace* the notion of causality. That this is unconvincing is shown above all by the indirectness of the connection between functional dependences and causal connections. Anyone in doubt of this is invited to look at Mill's Canons of Induction, which effectively say only that if certain magnitudes are found to vary together or stay constant together then they are *in some way* causally connected.

Even this is not guaranteed to be so: correlations may come about by accident. In any case, if the order of explanation is meant to be from functional dependence to causation, it is clearly wrong: it is the causal connections among things which explain the functional dependences among their determinables, not vice versa.

Grelling's account of functional dependence uses a somewhat compressed notation, so it is worth running briefly through some of the main ideas in a more expanded version. We then consider possible criticisms which may be brought against the approach.

Like Grelling, we confine attention primarily to the simplest case of one-placed determinables, which we symbolize by $f$, $g$, $h$, etc., denoting classes of these by $\varphi$, $\psi$ etc. We assume for the sake of simplicity that all the determinables we are dealing with at a given time can be meaningfully applied to the same objects. This assumption is substantial, since the arguments of a multi-place determinable may come from different ontological categories, e.g. *height of* [PERSON] *in* [UNIT OF LENGTH], whose value is not a length but a positive real number. However for introductory purposes it is too complicated to relativize quantification to each relevant domain. Another assumption which Grelling makes – and again it is a substantial one – is that the usual laws of identity apply to the values or outputs of determinables, i.e. to determinates. This is substantial because these values will often be abstract entities, and it is not uncontroversial that these can be said to have a determinate identity in the way that concrete objects do, as was indicated in the previous section. However the values of a determinable need not in all cases be abstract, for example the six values of the determinable *wife of Henry VIII* – an example which however pushes to its limits the terminology of determinable/determinate, since determinates are classically attributes rather than individuals. One of the nice aspects about the Grelling analysis is that the only relations we need to consider among determinates are identity and difference, which are precisely the ones most intimately linked to underlying equivalences. So the terminology and notation of determinables or functions promises to be dispensable. Its merit consists in allowing generalization across superficially dissimilar cases which is very cumbersome if carried out at the level of concreta using only equivalences. We symbolize the arguments of determinables by $x$, $y$, $z$, etc. The first basic idea concerning a determinable is whether it only takes one determinate value on its domain (in which case it is *constant*) or whether it is *variable*:

$$\text{D1} \quad \text{const}(f) \equiv \forall xy(f(x) = f(y)) \qquad \text{(cf. Grelling's D4)}.$$

Note that whereas Grelling defines a class of functions, we define a predicate of functions (determinables).

At the other extreme is a determinable which *never* has the same value for different arguments: Grelling calls such a determinable *monotone*:

$$\text{D2} \quad \text{mon}(f) \equiv \forall xy(f(x) = f(y) \supset x = y) \qquad \text{(cf. Grelling's D5)}.$$

A constant determinable is perforce not monotone.

Simple as these definitions are, their generalization to classes of monadic determinables brings complication. First consider the following abbreviatory definitions:

$$\text{D3} \quad \varphi(x) = \varphi(y) \equiv \forall f(f \, \varepsilon \, \varphi \supset f(x) = f(y)),$$
$$\text{D4} \quad \varphi(x) \neq \varphi(y) \equiv \exists f(f \, \varepsilon \, \varphi \wedge f(x) \neq f(y)),$$
$$\text{D5} \quad \varphi(x) \#\, \varphi(y) \equiv \forall f(f \, \varepsilon \, \varphi \supset f(x) \neq f(y)).$$

(the equivocation on '=' and '≠' is obviously harmless).

D3 generalizes the idea of a determinable taking the same value for two arguments to a class of determinables. D4 simply negates this. But D5 is a stronger notion contrary to D3 – all the determinables of a class taking different values for two arguments. Now if we define the constancy of a class of determinables by replacing '$f$' in D1 by '$\varphi$', we have two different concepts of variability, a weak one and a strong one:

$$\text{D6} \quad \text{wvar}(\varphi) \equiv \exists xy(\varphi(x) \neq \varphi(y)),$$
$$\text{D7} \quad \text{svar}(\varphi) \equiv \exists xy(\varphi(x) \#\, \varphi(y)).$$

The full implications of these complications need not be followed up here, but it should be noted that when a number of determinables are considered together in a definition, and some idea of their variability is involved, then one must consider different concepts according as the variability is strong or weak. For instance it is sometimes important to know that two determinables simultaneously take different values at the same two arguments (are *codifferent*). When generalizing this concept to two classes of determinables, we have in fact three different cases to consider, according as the classes simultaneously vary strongly, or weakly, or one strongly and one weakly. Should such points seem rather abstract and irrelevant to the issue at hand, it is to be noted that the ideas of varying and staying the same are the bread-and-butter of Grelling's definitions, so that ambiguities and complications here affect his whole scheme by way of complicating the cases to be considered. This is shown most clearly by one of Grelling's central definitions, that of *equi-dependence*:

D8 $f \text{ equidep } g \equiv \forall xy(g(x) = g(y) \supset f(x) = f(y))$,
D8′ $f \text{ equidep } \varphi \equiv \forall xy(\varphi(x) = \varphi(y) \supset f(x) = f(y))$.

Equality (identity) of values of $g(\varphi)$ brings with it equality of values of $f$: hence the name. Grelling considers equidependence to capture one sense of (functional) dependence, which we may call *total* functional dependence. For if, no matter how the determinables outside $\varphi$ vary, if we keep $f$ constant by simply keeping all of $\varphi$ constant, then the value of $f$ in some sense depends on those of $\varphi$, or, we may say, $f$ depends totally on $\varphi$. For instance, on what does the airspeed of a flying aircraft depend? We may list the factors: altitude, loading, engine power, throttle setting, control surface setting, attitude, angle of flight to the horizontal, turbulence. The list then begins to wear thin. Some factors, such as the plane's price, registration number, or colour, are quite irrelevant, others perhaps only marginally relevant. Putting all these relevant factors together, we have a class on which the airspeed is totally functionally dependent. If these all stay the same, so does the speed. And conversely, if the speed varies, it is because one of these varies.

While the concept of total functional dependence as here illustrated by equidependence is in some sense an important part of the concept of dependence, it does not exactly capture what is meant by 'dependence'. For one thing, we have the theorem

T1 $(f \text{ equidep } \varphi \wedge \varphi \subset \psi) \supset f \text{ equidep } \psi$.

but it is clear that the difference between $\varphi$ and $\psi$ can consist merely of "junk" determinables, irrelevant to $f$, suggesting that we need to somehow pare $\varphi$ down to a minimal set. Another consequence of D8′ is

T2 $\text{const}(f) \supset f \text{ equidep } \varphi$.

A constant determinable is equidependent with respect to all classes of determinables. But if a determinable remains constant in complete imperviousness to the variations among others, this is a sign that it is precisely *independent* of them. This result applies even when we consider not completely constant determinables but ones which are held constant by (are equidependent on) others but show lack of variation when further factors vary. For instance, holding other things (such as mass, density, element) constant, the rate of radioactive decay of an element does not vary when we vary the element's temperature. Thus we conclude precisely that rate of decay is *independent* of temperature. This applies even within mathematics. The constant real function $f(x) = 1$ for all $x$ has a value independent of its argument. Constant functions may form an admissible limiting case of the concept *function*, but they offer no case at all of functional *dependence*. This is perhaps the reason why they always present a learning difficulty – not just because one tends to confuse the function with its value, but because they do not appear to present functions *of* anything.

Another difficulty is provided by monotone determinables. If $\varphi$ is a monotone class of determinables, then every determinable is trivially equidependent with respect to it. Assuming that the determinable *fingerprint pattern* is monotone on human beings, then if equidependence meant dependence, it would follow that a person's sex, birthdate, height at any time, etc., are all dependent on his or her fingerprint pattern, which is absurd. There is no way of knowing what determinables, if any, are dependent on a monotone determinable or class of such, since this cannot be held constant for different arguments to investigate what happens to another test determinable. In such a case we have no use for the functional dependence concept, though again this case is admitted for mathematical functions – we readily allow that the square of a number is a function, the function $\xi^{2/3}$, of the cube of a number.

To rule out these irritating but trivial cases we may consider the following definition of total functional dependence:

D9 $\quad f \operatorname{tfd} g \equiv \sim\operatorname{const}(f) \wedge f \operatorname{equidep} g \wedge \sim\operatorname{mon}(g).$

But while this rules out these problem cases, it brings problems of its own. Consider the one-place determinable *mean distance of the planet ... from the sun*.[26] This is, as it happens, monotone, there being no two distinct planets at the same mean distance from the sun. It would then follow that *no* determinable of a planet is totally functionally dependent on its mean distance from the sun, which is also absurd – gravitational attraction of a given planet, its rate of irradiation, and more *do* depend on its distance from the sun, and (other things – the mass of the sun, its radiation output – being equal) nothing else.

One way round such problems is to distinguish, as Grelling does, several different concepts of dependence, and suit them to the cases. Grelling defines *variational dependence* as follows:

D10 $f \operatorname{vardep} \varphi \equiv \forall xy(\exists! g \, \varepsilon \, \varphi(g(x) \neq g(y)) \supset f(x) \neq f(y))$.

Variational dependence is just a converse to the idea of a *ceteris paribus* equidependence: for all $g$ in $\varphi$, if everything in $\varphi$ but $g$ is held equal, then $g$ varies only if $f$ varies. When $\varphi$ is a singleton, variational dependence reduces to the converse of equidependence, and this shows that it, too, is unfitted to define functional dependence, since every monotone determinable is trivially variationally dependent on any class of determinables, and every determinable is trivially variationally dependent on any class of constant determinables.

Grelling therefore experiments with various combinations of his major definitions, and there are indeed various more or less interesting concepts which can be got in this way. There are also related notions, such as the determinables in a class being variously interdependent, the concepts of *ceteris paribus* constancy and variability, and various concepts of *partial* functional dependence, as e.g. the pressure of a fixed mass of ideal gas is partially (not totally) dependent on its temperature. While this area is still almost completely unexplored by logicians, it must be recognized that no amount of manipulation of the sort envisaged will itself give us a satisfactory solution to the problem of defining dependence, simply because the concept is inherently modal, and therefore requires an intensional language for its formulation. Grelling was in this respect a child of his time, in that he uses an extensional language. His work can therefore have heuristic or preparatory value, but cannot be the definitive solution. To suppose it could be would be to suppose, as Russell did, that an acceptable theory of necessity and possibility can be given using the usual quantifiers within predicate logic.

While different concepts of dependence may be modal in slightly different ways, to see how dependence concepts in general are modal, it suffices to consider examples. For instance, it was mooted above that *fingerprint pattern* is monotone over human beings – therein lies the forensic use of fingerprints. But there is no contradiction in supposing that two people might share their ten fingerprint patterns down to the last whorl, unlikely though this is. The monotonicity of fingerprint patterns is contingent. On the other hand that a person's sex is equidependent on his or her sex chromosome type is not contingent, but at least naturally necessary. Likewise, the planet example can be taken care of by considering counterfactual cases: if two planets *were* at the same mean distance from the sun, then they *would* (other things, notably the laws of nature, being equal) share a good number of values of various determinables. Of course the example is somewhat strained by the restriction to planets of our sun, whereas the laws of motion apply to other bodies besides. But again, it just *might* be *de facto* the case that a determinable with unrestricted range is monotone, yet we should not want therefore to rule out that some other determinable be dependent on this one. It is a difference between a law and an accidental generalization precisely that laws support counterfactuals in a way bald generalizations do not. It is *only* if we have a modal theory of dependence that we could hope to explain causality via dependence. The notion of *compulsion* which is an inseparable part of that of causation is ineluctably modal: given the causes, the effect *could not* (*ceteris paribus*) have failed to ensue. Even if Hume was right that we do not *perceive* causation, any more than we perceive what might have been, this would show only the inadequacy of empiricism, not that causation is mythical. Of course it is still unlikely that causation can be reduced to dependence, even in a modal theory of the latter.

It is similarly not clear whether the kinds of functional dependence with which we are concerned can be defined in terms of a modally enriched language of determinables and their values, of the sort we have used up to now. That is something that can only be established by trying it out.

Another extension of the concepts here mentioned would take in probabilistic connections between determinables: while the fact that two sets of fingerprints show the same pattern is very good evidence that they were made by the same person, it is not 100% conclusive, only 99.99..%. The relaxation of deterministic conditions in favour of probabilistic ones would also make the theory more applicable, e.g. to social sciences, perhaps even in statistical physics.

The conclusion of this section is that Grelling's approach to functional dependence will not wash because of the logical limitations of his extensional framework. The same limitation was pointed out above in connection with the Grelling-Oppenheim definition of *Gestalt* as an equivalence class. However there is no reason for smugness in this conclusion: the work remains to be done, and Grelling remains the high-water mark to date. Modal dependence theory is at present barely more than a glint in the eye of this or that logician.[27]

## §7 Is the Concept of Gestalt Really Necessary?

A concept can have a nice name and much exposure and still be no more than old wine in a new bottle. For instance, if one distils the essence of the concept *system* from a recent definition,[28] one gets something like the following:

> A system is an object with properties and parts such that it, its properties, its parts, and the properties of its parts stand in relations to one another and to objects which are not parts of it (if there are any), and their parts and their properties.

Aside from the fact that this appears to make all objects systems, we might ask what it is good for, since all the concepts employed in the definition are rather more accessible than the defined concept. Not that this is without its uses even so, but if we can indeed use 'object' without needing 'system', and simply draw attention to the fact that objects have parts and properties and stand in relations to one another etc., surely the term 'system' is an idle wheel. At the very least, its employment ought to bring advantages of some kind, such as clarity, recognition of similarities, opening up of genuinely new fields of research, overcoming old prejudices. Now in the case of 'system' it is probably true that at least one of these secondary aims, namely recognition of similarities, even across disciplines, has been *de facto* achieved to some extent. Would these similarities have been recognized anyway? Perhaps. That's speculative sociology of knowledge. But that does not show that the introduction of all the system terminology is not counterproductive. It seems that systems theory is awaiting its Grelling and Oppenheim.

It is a good heuristic strategy in philosophy (not elsewhere) to assume until the contrary is demonstrated that a putatively new philosophical concept is either bogus or has been largely anticipated under another name (usually by Aristotle). Right from the start, Ehrenfels faced this accusation regarding the Gestalt concept, and indeed we have seen that Gestalten have much in common with *forms*. The fact that Ehrenfels' discovery or rediscovery of a concept had a liberating effect in science, especially on psychology, might be plausibly accounted for by the restricted theoretical framework of the psychology of his day, with its atomistic, sensationalistic prejudices. That does not suffice to make the Gestalt concept really new; since a similar liberating effect might have been achieved by a renaissance of older ideas. Nor does it suffice to show the Gestalt concept is indispensable, since the liberating effect might have been due to some constituent of the concept which is more fundamental, and which had likewise been lacking hitherto. I think there *is* something in the Gestalt concept which, if not wholly new, is at least serendipitously highlighted by Ehrenfels' formulation and also by the Grelling-Oppenheim reconstruction. I shall come back to it after considering some early attempts to show that *Gestalt* is dispensable in favour of other items.

Historically in pole position (in relation to Ehrenfels' work) is Mach's attempt to account for Gestalt phenomena in terms of muscular sensations. This anticipation, its effect on Ehrenfels and its shortcomings are dealt with at length in the essay by Smith and Mulligan. It is not exactly an attempt to eliminate the Gestalt concept by showing that it reduces to more familiar or more accessible concepts, but rather an attempt to show that the phenomena which persuaded Ehrenfels that the idea of a Gestalt quality was required can be explained without this concept.

Anton Marty considered Ehrenfels' Gestalt concept and came to the conclusion that Gestalten, psychologically speaking, are nothing other than certain groups of sensations between which certain relations subsist, and that a Gestalt in general is simply a sum of relations between the objects making up the fundament of the Gestalt.[29] Exactly what this sum is meant to be is not made clear, though the best reconstruction is to take it as a logical *product* in the sense of *Principia Mathematica*. Ehrenfels was not unhappy about Marty's characterization: he describes it as 'not so bad, and compatible with all essential consequences of Gestalt theory, as long as one is not misled into thinking that any sum as such represents another whole in its own right.'[30] Unfortunately Ehrenfels does not trouble to get Marty's account exactly right. He describes a Gestalt as '*the* sum of relations', whereas Marty's text does not contain this implication of totality: the *total* sum is only a special case, that associated with what

Grelling and Oppenheim call a *simple* correspondence between complexes. Consider the case Ehrenfels takes: two notes, C and G, sounding together to form a perfect fifth chord. Within certain limits, it is possible to vary the relative loudnesses of the two component tones without losing the chord – the C may be louder, the G may be louder, or they may be equally loud. So it is possible to have the same chord although some of the relations among the fundaments change. Indeed it is precisely the flexibility of the Gestalt concept that it allows just such partial variation of factors, and looks for what remains invariant. Ehrenfels' oversight apart, there is much to be said for Marty's idea (providing we understand 'relation' as referring to universals, not particulars). What two instances of a given Gestalt have in common is largely (though not exclusively) a matter of the interrelations of the members and parts of their fundaments. In the case of a perfect fifth, it is the frequency ratio of 3:2 between the two tones. This fact is adequately captured in the Grelling-Oppenheim definition. It must be stressed however that the qualitative nature of the fundament is not usually irrelevant. The crucial relation for a perfect fifth is a ratio of 3:2, but not just any ratio of 3:2 will do – e.g. the ratio that obtains between Rupert's height and Rebecca's when he is half as tall again as she. We require a *frequency* ratio, and one of molecular vibrations at that – not of e.g. electromagnetic or water waves. This constrains the elements of the fundament to be sounds. The only Gestalten for which the nature of the elements is completely immaterial are purely *formal* or mathematical Gestalten.[31] The mistake of thinking that all Gestalten must be formal is a kind of Pythagoreanism.

In their first joint article, Grelling and Oppenheim mention another attempt to dispense with the Gestalt concept in favour of something simpler: Ajdukiewicz's suggestion, apparently made in 1934, that 'Gestalt' be taken to mean simply 'relation'. Grelling and Oppenheim object to this on the grounds that this would make the Gestalt concept completely dispensable, every relation then being a Gestalt and vice versa. As an objection, this is quite beside the point, since Ajdukiewicz *is* simply claiming that the concept is dispensable. If he is right, he has done everyone a service by paring away unnecessary terminology. To put some meat on their objection, they need to show that there are relations which are not, in their terms, Gestalten, and furthermore that it is fruitful to employ the restricted concept *Gestalt* in addition to the broader concept *relation*.

As their reply to Ajdukiewicz indicates, Grelling and Oppenheim take it that every Gestalt is a relation, but not every relation is a Gestalt. This is in my view exactly the wrong way round, at least according to their theory. We can demonstrate how a relation is a Gestalt very easily. Let $P$ be an arbitrary $n$-placed relation. As domain of positions take the first $n$ natural numbers, and as domain of values take the objects in the field of $P$.[32] Then a complex is an assignment of objects in the field of $P$ to the first $n$ natural numbers, i.e. it is an $n$-membered *sequence* $\langle a_1,\ldots,a_n\rangle$. Now take the correspondence $R(P)$ defined by

$$\langle a_1,\ldots,a_n\rangle R(P) \langle b_1,\ldots,b_n\rangle \equiv Pa_1\ldots a \wedge Pb_1\ldots b_n$$

It is obvious that $R(P)$ is an equivalence relation. That which all such $n$-placed sequences have in common is that they all fulfil the relation $P$. The $n$-fold Cartesian product of the field of $P$ is thus partitioned into two equivalence classes, of those $n$-tuples which satisfy $P$ and those which do not. They are thus Gestalten in the sense of Grelling and Oppenheim, and the class of those satisfying $P$ just *is* the relation $P$ *in extension*. Conversely, any finite Gestalt may be considered a complex relation. But unless we accept relations with infinitely many places, there are Gestalten (e.g. in mathematics) which are not relations. While this reduction makes use of certain special features of Grelling and Oppenheim's extensionalist approach, it can be applied to relations and Gestalten considered as abstracta arising under abstraction from equivalence relations in the manner considered above.

We instanced Gestalten with infinitely many fundamenta as a counterexample to the thesis that all Gestalten are relations. (It would be overstretching the concept of relation to allow relations to have arbitrarily many terms, including transfinitely many.) Another reason for denying the reduction is that there are Gestalten which may have different numbers of fundaments in different realizations, for instance the Gestalt *brass band* or the Gestalt *cyclic group*.

Once again we see the flexibility of the Gestalt concept. This is what I want to take as the virtue of the concept, and it is based on the flexibility of the concept of *transposition*. Grelling and Oppenheim in effect make this concept the core of their interpretation: the remaining definitions are in a sense all preparation for its introduction. The transposition concept is a formal one: a transposition is a structure-preserving isomorphism between complexes, and a Gestalt is the invariant under these transpositions. This characterization invites comparison with the more general mathematical notion of a morphism employed in category theory, a comparison which we do not go into here. It is important that the Gestalt concept did not arise in mathematics, but rather in the very concrete area of perceptual psychology. Its applicability is thus secured from the start, whereas much of category theory is only mathematically interesting when applied to mathematical examples of extremely large cardinality, which therefore have little or no application in real life. The fact that Gestalten can constrain the material region of their underlying fundamenta is also something they do not share with the mathematical concept of structural isomorphism.

The really fruitful aspect of the Gestalt concept is thus, in my view, its being built on the idea of trans*form*ation. The idea of transformational invariants was not new with Ehrenfels – it is at the basis of Felix Klein's *Erlanger Programm* for geometry, which itself was revolutionary, perhaps the first truly structuralist approach to mathematics. It is doubtful whether Ehrenfels knew of Klein's work. In any case, Ehrenfels' Gestalt concept, despite its similarities with the mathematical approach, was more down-to-earth. It is only with hindsight that we discern the parallel. But precisely because of its nearness to empirical application, Ehrenfels' concept could have an immediate impact in empirical science, whereas it took much longer for abstract mathematical structuralism to filter down to empirical science, which had by that time in any case been infiltrated by approaches deriving from Gestalt psychology.

The other fruitful aspect of approaching form via transformations is that the latter are often relatively easier to recognize, characterize, vary and generalize. We may thus gain access to Gestalten of various degrees of generality and formality. This brings me to a final virtue. While Ehrenfels' discovery was liberating because it freed psychology of atomistic shackles, the Gestalt concept is not in any absolute sense holistic, and certainly by no means mystical. It is true that there is a genetic connection between early Gestalt theory and later, excessively holistic *Ganzheitstheorie*, a development which Ehrenfels himself greeted.[33] But this development is not a reflection of something inherent in the Gestalt concept itself. One can employ this concept with success without believing in an all-inclusive absolute whole with its own supreme Gestalt. A post-Cantorian Platonist mathematician is forced, on pain of contradiction, to deny that there is in any meaningful sense a whole which contains all the objects of mathematics. A believer in absolutely simple monads can still consider the Gestalten embodied in their static and dynamic configurations. It is possible to derive new Gestalten by ignoring internal structure (treating complexes as simples) or, equally well, by considering how to combine given elements in manifold ways.

If the Gestalt *concept* (or something very like it) appears indispensable, it is another question whether the *term* 'Gestalt' is the best we can have. Many uses of this term are simply historical, referring to Gestalt theory – especially Gestalt psychology – in its heyday, and while some of the tenable results of this research have passed into their disciplines, the term 'Gestalt' is not often met with in modern literature as marking a working concept. This is almost certainly nothing more than wasteful terminological proliferation of the kind mentioned in 1938 by Naess in his comment on Grelling and Oppenheim's paper. The various authors are too busy doing their own positive science to take the trouble to strive for enhanced understanding via standardizing terminology. This appears to be particularly marked in psychology and the social sciences, to some extent also in linguistics. A new terminology can also screen lack of originality in a competitive but superficial academic culture (it needn't but it can). As our authors remark, it depends whether you want to stress historical continuity or filter out unwanted connotations. Teutonic mystery apart, I have nothing particular to set against 'Gestalt', and it is nice to honour the giants on whose shoulders we stand. To finish, I should like to quote a programmatic statement about the importance of the Gestalt concept by a philosopher who was far from superficial, the recently deceased D. C.Williams. In the Preface to his *Principles of Empirical Realism*, Williams apologizes for not having dealt with Gestalten to the extent they deserve and adds:[34]

> The topics of analysis, analytic truth and implication, of definition, of primary and secondary qualities, of temporal extension and passage, of personal identity, of valuation, of consciousness as such, and even of whole and part generally, or the one and the many, converge on this notion [of '*Gestalt* qualities'] and promise to be explicated only so far as it is explicated.

## Notes

[1] Schlick (1935). All references in this form are to the list on pp. 189ff. References without parentheses are to the Bibliography on pp. 231–478.

[2] On formal concepts in general, and in particular the distinction between formal logical and formal ontological concepts, cf. Husserl 1900/1901, §67. *Gestalt* is ontological, not logical.

[3] Ehrenfels 1937, p. 140.

[4] Cf. Smith, ed. 1982.

[5] On generic dependence, cf. Smith, ed. 1982, pp. 122ff.

[6] Of course there is nothing to stop us introducing such terms. This is done, not for predicate logic, but for Leśniewski's Ontology, in Simons (1983).

[7] Cf. Popper's remarks on the concept of a Gestalt as being more than the sum of its elements: 'I think that everything is a Gestalt in this sense', Medawar (1980), p. 75. A few lines later, however, Popper says the Gestalt concept is '*almost* all-embracing' (ibid., my emphasis).

[8] Cf. §4.5 of "The concept of Gestalt", referring to Tarski (1937). Tarski of course took the concept from Leśniewski and adapted it to the different logical setting.

[9] Of course there are multifarious connections between the two concepts, as the authors point out. A third aspect, the concept of emergence, is dealt with in the article Rescher and Oppenheim 1955.

[10] Ehrenfels calls certain sequences of 0s and 1s in his theory of prime numbers *Leitreihen*, and almost automatically relates the mathematical concepts to musical ones: cf. Ehrenfels 1922, p. 37, pp. 53ff.

[11] Some of the consequences are discussed in Simons (1985).

[12] Johnson (1921), pp. 173ff.

[13] The assumption that the determinates of a determinable are contraries is in fact an oversimplification, though for most purposes it is warrantable to make it. But where we are faced with incomplete information (temperature: between 5000° and 6000°C) or with concepts whose extensions overlap (colour: blue/turquoise) we do not have mutual exclusivity.

[14] In the original version of their "Supplementary Remarks" the authors use 'classificator', but this is an unnecessary coinage, and has been corrected in the version printed in this volume. Rescher and Oppenheim 1955 use 'attribute', which has much going for it as a term. One would then talk about values of the attribute. This terminology has some currency in the theory of information systems. However both there and in Rescher and Oppenheim it tends to be confined to the one-place case. In his dependence paper, Grelling prefers 'function'.

[15] Frege (1903), pp. 155ff.

[16] Frege (1884), pp. 73ff.

[17] Angelelli (1979), (1984). The term 'looking-around method' derives from the following passage in Carnap (1947), p. 1:

If two designators are equivalent, we say also that they have the same *extension*. If they are, moreover, L-equivalent, we say that they have also the same *intension*... Then we look around for entities which might be taken as extensions or as intensions for the various kinds of designators.

[18] Frege (1884), p. 79f. As p. 80n.1 suggests, even Frege was aware that this is to some extent arbitrary. A second layer of arbitrariness was uncovered by Frege in the concept *extension*, which led to the more extended looking-around of his (1893), pp. 16ff.

[19] By the time of Scholz and Schweitzer (1935), it was then customary to call the representation of abstracta by equivalence classes 'definition by abstraction', which, as Angelelli (1984) rightly remarks, is not a matter of abstraction at all, but of looking around.

[20] On the necessity of set-membership cf. Sharvy (1968), Fine (1981), Forbes (1985), Ch.5.

[21] On the problems involved, cf. Salmon (1982), Ch.13.

[22] As argued in Simons (1981). In Frege (1884), p. 75 it seems that synonymy (sameness of content) is required; by p. 79 the requirement has been lost. Even in Frege (1893), p. 16n.1 the case of synonymy is recognized (and distinguished, as 'having the same sense', from the more general 'having the same reference'), but sense plays no part *within* Frege's logic.

[23] But cf. the good work done in trying to close the case in favour of nominalism by Field, Bonavec and Horgan.

[24] Cf. Weyl (1949), p. 9, Simons (1981).

[25] This was for instance Zermelo's approach to finite arithmetic: every statement about finite numbers is a statement about finite sets. Cf. Hallett (1984), pp. 245ff.

[26] I owe the example to an anonymous referee for Oxford University Press.

[27] I have hopes that the glint I have seen in Kit Fine's eye will bear fruit. It should be pointed out that extensional concepts of dependence *do* have application in a perhaps unexpected quarter, namely the theory of information systems. If objects in an information system are characterized by the values they take of certain determinables (attributes) – where inexact information may also be allowed – then objects which take the same values of all determinables are *indiscernible* in the system. By adding new determinables we can usually improve our discerning power. But if we add a determinable which is equidependent on those we have already we cannot discern more. This thought is behind the following definition of dependence: a set of determinables is dependent if it has a proper subset which discerns as much as it does. Cf. the works by Orłowska in the bibliography. The convergence with Grelling's work (which is of course in effect being published here for the first time) is remarkable, although the initial angles of approach are very different.

[28] Ropohl (1978). The set-theoretic coating which Ropohl gives his description has been stripped away. Also overlooked was his insistence that systems are models, not objects, advice which he does not himself adhere to in the article. Unfortunately we do not get the quintessence of a system, because a system is only an ordered quadruple. We must be content with a quartessence.

[29] Marty 1908, p. 110.

[30] Ehrenfels 1922, p. 105.

[31] Cf. Simons 1985, p. 131f. The concept *formal* here used is that of Husserl (cf. note 2). An ontological concept is formal if it is by nature applicable to all

possible regions of entities, whatever their material or qualitative determinations. Cf. also Wittgenstein (1922), 4.126ff.

[32] For the explanation of these terms, see Grelling and Oppenheim's "The Concept of Gestalt", §1.2f.

[33] Cf. Ehrenfels 1922, p. 102f. for enthusiastic remarks on Driesch's *ganzheitliche* biology.

[34] Williams (1966), p. xi.

## References


Angelelli, I. (1979) "Abstraction, Looking-Around and Semantics", *Studia Leibnitiana, Sonderheft* 8, 108–123.

– (1984) "Frege and Abstraction", *Philosophia Naturalis*, 21, 453–71.

Bonavec, D. (1982) *Reduction in the Abstract Sciences*, Indianapolis: Hackett.

Carnap, R. (1947) *Meaning and Necessity. A Study in Semantics and Modal Logic*, Chicago: University of Chicago Press.

Field, H. (1980) *Science Without Numbers*, Oxford: Blackwell.

Fine, K. (1981) "First-Order Modal Theories I – Sets", *Noûs*, 15, 177–205.

Forbes, G. (1985) *The Metaphysics of Modality*, Oxford: Clarendon.

Frege, G. (1884) *Die Grundlagen der Arithmetik*, Breslau: Koebner. Reprint Hildesheim: Olms, 1977. English translation *The Foundations of Arithmetic*, Oxford: Blackwell, 2nd ed. 1953.

– (1893) *Grundgesetze der Arithmetik*, Vol. I, Jena: Pohle. Reprint Darmstadt: Wissenschaftliche Buchgesellschaft, 1962. Partial English translation *The Basic Laws of Arithmetic*, Berkeley: University of California Press, 1964.

– (1903) *Grundgesetze der Arithmetik*, Vol. II, Jena: Pohle. Reprint Darmstadt: Wissenschaftliche Buchgesellschaft, 1962.

Grelling, K. (1969) "On Definitions by Equivalence Classes and by Group Invariants", translated by Kurt Bing and Edith H. Luchins, *Methodology and Science*, 2, 116–122.

– (1970) "On the Logical Relations between Groups and Equivalence Relations", translated by Kurt Bing and Edith H. Luchins, *Methodology and Science*, 3, 5–17.

Hallett, M. (1984) *Cantorian Set Theory and Limitation of Size*, Oxford: Clarendon.

Horgan, T. (1984) "Science Nominalized", *Philosophy of Science*, 51, 529–49.

Johnson, W. E. (1921) *Logic*, vol. 1. Cambridge: Cambridge University Press.

Medawar, P., and Shelley, J. (eds.) (1980) *Structure in Science and Art*, Amsterdam: Excerpta Medica.

Orłowska, E. (1980) "Dependency of Attributes in Information Systems", *ICS PAS Reports*, 425.

– (1983) "Dependencies of Attributes in Pawlak's Information Systems", *Fundamenta Informaticae*, 6, 247–56.

Ropohl, G. (1978) "Einführung in die allgemeine Systemtheorie", in H. Lenk and G. Ropohl (eds.), *Systemtheorie als Wissenschaftsprogramm*, Königstein: Athenäum, 1978, 9–49.

Salmon, N. (1982) *Reference and Essence*. Oxford: Blackwell.

Schlick, M. (1935) "Über den Begriff der Ganzheit". *Wissenschaftlicher Jahresbericht der Philosophischen Gesellschaft an der Universität zu Wien – Ortsgruppe Wien der Kantgesellschaft für das Vereinsjahr 1933/34 und 1934/35*, 23–37. Reprinted in *Gesammelte Aufsätze 1926–1936*, Vienna: Gerold, 1938, 251–66. English translation, "On the Concept of Wholeness", in *Philosophical Papers. Vol. 2 (1925–1936)*, Dordrecht: Reidel, 1979, 388–99.

Scholz, H. and Schweitzer, H. (1935) *Die sogenannten Definitionen durch Abstraktion*, Leipzig: Meiner. (*Forschungen zur Logistik und zur Grundlegung der exakten Wissenschaften*, 3.)

Sharvy, R. (1968) "Why a Class Can't Change Its Members", *Noûs*, 2, 303–14.

Simons, P. M. (1981) "Abstraction and Abstract Objects", in E. Morscher, O. Neumaier & G. Zecha, eds., *Philosophie als Wissenschaft/Essays in Scientific Philosophy*, Bad Reichenhall: Comes, 355–70.

– (1983) "A Leśniewskian Language for the Nominalistic Theory of Substance and Accident", *Topoi*, 2, 99–109.

– (1985) "Computermusik und die Identität des Musikwerks". *Conceptus*, 19, 69–75.

Tarski, A. (1937) Appendix E to J. H. Woodger, *The Axiomatic Method in Biology*, Cambridge: Cambridge University Press, 161–72.

Weyl, H. (1949) *Philosophy of Mathematics and Natural Science*, Princeton: Princeton University Press.

Williams, D. C. (1966) *Principles of Empirical Realism. Philosophical Essays*, Springfield: Thomas.

Wittgenstein, L. (1922) *Tractatus Logico-Philosophicus*, London: Routledge & Kegan Paul.