THE LOGIC OF ANALOGY

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Abstract

The Logic of Analogy is a study of the valid logical forms of qualitative and quantitative analogical argument, and the rules pertaining to them. It investigates equally valid conflicting arguments, statistics-based arguments and their utility in science, arguments from precedent used in law-making or law-application, and examines subsumption in analogical terms. Included for purposes of illustration is a large section on Talmudic use of analogical reasoning.

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Foreword

I analyzed in some detail the basic formalities of the argument by analogy close to ten years ago in my book *A Fortiori Logic*. I there showed in what ways it resembles and differs from a fortiori argument. However, I left the matter at that, and did not consider the inconsistencies one can easily come across in the use of analogical argument. I also did not sufficiently investigate, as I should have, the use of such argument in scientific and legal (and in particular in Talmudic) discourse. In the present essay I try to broaden and deepen my investigation. The material presented below is original; no one has, to my knowledge, surprisingly, ever investigated the formal logic of analogy in such detail.

1. Qualitative analogy

To begin with, let us review some of the main findings of my past research regarding analogical argument and see where we can improve upon them. The following text is mostly drawn from my book *A Fortiori Logic* (chapter 5.1), but with some significant editing.

Qualitative analogical argument consists of four terms, which we may label P, Q, R, S, and refer to as the *major, minor, middle and subsidiary terms*, respectively (remember the nomenclature). The major premise contains the terms P, Q, and R; the term S appears in both the minor premise and conclusion. The names major term (P) and minor term (Q), here, unlike in a fortiori argument, do not imply that P is greater in magnitude or degree than Q. For this reason, we can *conventionally* decide that the minor term will always be in the minor premise, and the major term will always be in the conclusion; meaning that all moods will have the form of so-called 'from minor to major' arguments.[1]

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[1] In my book *A Fortiori Logic*, where my treatment of analogical argument was aimed at comparison with a fortiori argument, I had to impose the same forms as in the latter to the former. That is, positive subjectal and negative predicatal moods were 'minor to major', and negative subjectal and positive predicatal moods were 'major to minor'. Here, where my treatment of analogical argument is independent, such distinctions are irrelevant; and it is wiser to make all moods 'minor to major' or all

This means that any valid ‘minor to major’ mood could, in principle, be reformulated as a valid ‘major to minor’ mood.[2]

The argument by analogy may then take the following four *copulative* forms (with a positive major premise, to start with).

a. The **positive subjectal** mood. *Given that subject P is similar to subject Q with respect to predicate R, and that Q is S, it follows that P is S*. We may analyze this argument step by step as follows:

> Major premise: P and Q are alike in that both have R. Note that this premise is fully convertible; it has no direction.

This implies both ‘P is R’ and ‘Q is R’, and is implied by them together.

> Minor premise: Q is S.

The term S may of course be any predicate; although in legalistic reasoning, it is usually a legal predicate, like ‘imperative’, ‘forbidden’, ‘permitted’, or ‘exempted’.

> Intermediate conclusion and further premise: All R are S.

This proposition is obtained from the preceding two as follows. Given that Q is S and Q is R, it follows by a

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moods ‘major to minor’, and the former choice (with the minor term always placed in the minor premise) is easier to remember.

[2] But when dealing with *quantitative* analogy (see further on) we must tread carefully, and distinguish between superior, inferior and equal terms.

substitutive third figure syllogism that there is an R which is S, i.e. that 'some R are S'. This particular conclusion is then *generalized* to 'all R are S', provided of course we have no counterevidence. If we can, from whatever source, adduce evidence that some R (other than Q) are *not* S, then of course we cannot logically claim that all R are S. Thus, this stage of the argument by analogy is partly deductive and partly inductive.

Final conclusion: P is S.

This conclusion is derived syllogistically from All R are S and P is R.

If the middle term R is known and specified, the analogy between P and Q will be characterized as 'complex'; if R is unknown, or vaguely known but unspecified, the analogy between P and Q will be characterized as 'simple'. In **complex analogy**, the middle term R is explicit and clearly present; but in **simple analogy**, it is left tacit. In complex analogy, the similarity between P and Q is indirectly established, being manifestly due to their having some known feature R in common; whereas in simple analogy, the similarity between them is effectively directly intuited, and R is merely some indefinite thing assumed to underlie it, so that in the absence of additional information we are content define it as 'whatever it is that P and Q have in common'.

Quantification of terms. Let us next consider the issue of quantity of the terms, which is not dealt with in the above prototype.

In the singular version of this argument, the major premise is 'This P is R and this Q is R', where 'this' refers to two different individuals. The minor premise is

'This Q is S', where 'this Q' refers to the same individual as 'this Q' in the major premise does. From the minor premise and part of the major premise we infer (by syllogism 3/RRI[3]) that there is an R which is S, i.e. that some R are S – and this is generalized to all R are S, assuming (unless or until evidence to the contrary is found) there is no R which is not S. From the generality thus obtained and the rest of the major premise, viz. this P is R, we infer (by syllogism 1/ARR) the conclusion 'This P is S', where 'this P' refers to the same individual as 'this P' in the major premise does.

In the corresponding general version of the argument, the major premise is 'all P are R and all Q are R' and the minor premise is 'all Q are S'. From the minor premise and part of the major premise we infer (by syllogism 3/AAI) that some R are S – and this is generalized to all R are S, assuming (unless or until evidence to the contrary is found) there is no R which is not S. From the generality thus obtained and the rest of the major

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[3] Here, the symbol R refers to a singular affiRmative proposition, as against G for a singular neGative one. I introduced these symbols in my book *Future Logic*, but singular syllogism is not something new. The Kneales (p. 67) point out that Aristotle gives an example of syllogism with a singular premise in his *Prior Analytics*, 2:27. The example they mean is supposedly: "Pittacus is generous, since ambitious men are generous,and Pittacus is ambitious" (1/ARR). Actually, there is another example in the same passage, viz.: "wise men [i.e. at least some of them] are good, since Pittacus is not only good but wise" (3/RRI). Note that the reason I did not choose the symbol F for aFfirmative was probably simply to avoid confusion with the symbol F for False. In any case, some symbols were clearly needed for singular propositions, since the traditional symbols A, E, I, O only concern plural propositions.

premise, viz. all P are R, we infer (by syllogism 1/AAA) the conclusion 'all P are S'. Note that the minor premise *must* here be general, because if only some Q are S, i.e. if some Q are not S, then, if all Q are R, it follows that some R are not S (by 3/OAO), and we cannot generalize to all R are S; and if only some Q are R, we have no valid syllogism to infer even that some R are S.

As regards the quantity of P and Q, there is much leeway. It suffices for the major premise to specify only that some Q are R; because, even if some Q are not R, we can still with all Q are S infer that some R are S (3/AII), and proceed with the same generalization and conclusion. Likewise, the major premise may be particular with respect to P, provided the conclusion follows suit; for, even if some P are not R, we can still from some P are R and all R are S conclude with some P are S (1/AII). Needless to say, we can substitute negative terms (e.g. not-S for S) throughout the argument, without affecting its validity.

It is inductive argument. Thus, more briefly put, the said analogical argument has the following form: Given that P and Q are alike in having R, and that Q is S, it follows that P is S. The validation of this argument is given in our above analysis of it. What we see there is that the argument as a whole is *not entirely deductive, but partly inductive*, since the general proposition 'All R are S' that it depends on is obtained by generalization.

Thus, it may well happen that, given the same major premise, we find (empirically or through some other reasoning process) that Q is S but P is not S. This just tells us that the generalization to 'All R are S' was in this case not appropriate – it does not put analogical

argument as such in doubt. Such cases might be characterized as ‘denials of analogy’ or ‘non-analogies’. Note also that if ‘All R are S’ is already given, so that the said generalization is not needed, then the argument as a whole is not analogical, but entirely syllogistic; i.e. it is: All R are S and P is R, therefore P is S. Thus, *analogy as such is inherently inductive*. And obviously, simple analogy is more inductive than complex analogy, since less is clearly known and sure in the former than in the latter.

Note well: inductive does not mean arbitrary. Induction is a logical process with its rules, even if it is more indulgent than deduction. One cannot just make a claim or mere speculation and give it credibility by characterizing it as ‘inductive’. Its logical possibility and consistency must first be considered, and then ways of validating it found. Any ‘analogical’ argument not here specifically formally justified may be considered as invalid, until and unless some precise formal justification for them is put forward.

Other moods. The above, prototypical mood was positive subjectal. Let us now consider the other possible forms of analogical argument.

b. The **negative subjectal** mood. *Given that subject P is similar to subject Q with respect to predicate R, and that Q is not S, it follows that P is not S*. This mood follows from the positive mood simply by obversion of the minor premise and conclusion, i.e. changing them to ‘Q is non-S’ and ‘P is non-S’ (since the negative term ‘non-S’ is included in the positive symbol S of the positive mood). This argument is of

course just as inductive as the one it is derived from; it is not deductive.

c. The **positive predicatal** mood. *Given that predicate P is similar to predicate Q in relation to subject R, and that S is Q, it follows that S is P*. We may analyze this argument step by step as follows:

> Major premise: P and Q are alike in that R has both. Note that this premise is fully convertible; it has no direction.

This implies both 'R is P' and 'R is Q', and is implied by them together.

> Minor premise: S is Q.
>
> Intermediate conclusion and further premise: S is R.

This proposition is obtained from the preceding two as follows. Given that R is Q, it follows by conversion that there is a Q which is R, i.e. that 'some Q are R', which is then *generalized* to 'all Q are R', provided of course we have no counterevidence. If we can, from whatever source, adduce evidence that some Q are *not* R, then of course we cannot logically claim that all Q are R. Next, using this generality, i.e. 'all Q are R', coupled with the minor premise 'S is Q', we infer through first figure syllogism that 'S is R'. Clearly, here again, this stage of the argument by analogy is partly deductive and partly inductive.

> Final conclusion: S is P.

This conclusion is derived syllogistically from R is P and S is R.

Note that the generalized proposition here (viz. all Q are R) concerns the minor and middle terms, whereas in positive subjectal argument it (i.e. all R are S) concerned the middle and subsidiary terms.

Let us now quantify the argument. In the singular version, the major premise is: this R is both P and Q, and in the general version it is: all R are both P and Q. The accompanying minor premise and conclusion are, in either case: and a certain S is Q (or some or all S are Q, for that matter); therefore, that S is P (or some or all S are P, as the case may be). We could also validate the argument if the major premise is all R are P and some R are Q; but if only some R are P, i.e. if some R are not P, we cannot do so for then the final syllogistic inference would be made impossible[4]. Such argument is clearly inductive, since it relies on generalization. No need for us to further belabor this topic.

d. The **negative predicatal** mood. *Given that predicate P is similar to predicate Q in relation to subject R, and that S is not Q, it follows that S is not P*. This mood follows from the positive mood by reductio ad absurdum (we cannot here use mere obversion as with subjectal argument): given the major premise, if S were P, then S would be Q (since analogical argument is non-directional, P and Q are interchangeable in it); but S is not Q is a given; therefore, S is not P may be

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[4] However, if we know that some R are P, and do *not* know that some R are not P, we can generalize the positive particular to obtain the 'all R are P' proposition needed to infer the final conclusion. In that case, the argument as a whole would be doubly inductive, since involving two generalizations.

inferred. This argument is of course just as inductive as the one it is derived from; it is not deductive.

Moods **with a negative major premise**. All the above-mentioned moods could equally well have a negative major premise (expressing non-similarity or dissimilarity, which mean the same), and yield a corresponding valid conclusion – one, as we shall now show, of opposite polarity to the preceding. We may refer to such movements of thought as **disanalogy**.

The **positive subjectal** mood would be: *Given that subject P is not similar (i.e. is dissimilar) to subject Q with respect to predicate R, and that Q is S, it follows that P is not S*. Here, the major premise means either (a) P is R but Q is not R; or (b) P is not R but Q is R. The minor premise is given as Q is S, and the conclusion is the negative P is not S. This can be validated as follows: (a) given Q is S and Q is not R, it follows that there is a S which is not R; this may (in the absence of counterevidence) be generalized to 'no S is R'; whence, given P is R, we infer that P is not S. Alternatively, (b) given Q is S and Q is R, it follows that there is a S which is R, i.e. some S are R; this may (in the absence of counterevidence) be generalized to 'all S are R'; whence, given P is not R, we infer that P is not S. The **negative subjectal** mood follows by obversion, and has as its minor premise that Q is not S and as its as its conclusion that P is S.

The **positive predicatal** mood would be: *Given that predicate P is not similar (i.e. is dissimilar) to predicate Q in relation to subject R, and that S is Q, it follows that S is not P*. Here, the major premise means either (a) R is not P but R is Q; or (b) R is P but R is not Q. The minor

premise is given as S is Q, and the conclusion is the negative S is not P. This can be validated as follows: (a) given R is Q and S is Q, it follows that there is a S which is R; and given R is not P, we may (in the absence of counterevidence) generalize to 'no R is P'; whence we infer that S is not P. Alternatively, (b) given R is not Q and S is Q, it follows that there is a S which is not R; given R is P, we may (in the absence of counterevidence) generalize to 'all P are R'; whence we infer that S is not P. The **negative predicatal** mood follows by reductio ad absurdum, and has as its minor premise that S is not Q and as its conclusion that S is P.

We can call analogical argument with a positive major premise (expressing similarity) **comparison**, and that with a negative major premise (expressing dissimilarity) **contrast**. As we shall see further on, such arguments may result in conflicting conclusions, when they are compounded with different middle terms.[5]

We can similarly develop an equal number of **implicational** moods of analogical argument, where P, Q, R, S, symbolize *theses* instead of terms and they are related through implications rather than through the copula 'is'. The positive antecedental would read: *Given that antecedent P is similar to antecedent Q with respect to consequent R, and that Q implies S, it follows that P implies S*. The negative antecedental would read: *Given*

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5 I briefly mentioned moods with a negative major premise in my past treatment of the topic; but I did not fully analyze them. I now view them as more important than I realized at the time, having lately become aware of the issue of compounding comparison and contrast.

the same major premise, and that Q does not imply S, it follows that P does not imply S. The positive consequental mood would read: *Given that consequent P is similar to consequent Q in relation to antecedent R, and that S implies Q, it follows that S implies P*. The negative consequental mood would read: *Given the same major premise, and that S does not imply Q, it follows that S does not imply P*. Moods with negative major premises can similarly be formulated; but the minor premise and conclusion will have opposite polarity, i.e. if the minor premise is positive, the conclusion will be negative, and vice versa. All implicational moods are, of course, partly inductive arguments since they involve generalizations. Validations of the implicational moods should proceed in much the same way as those of the copulative moods.

2. Quantitative analogy

Analogy may be qualitative or quantitative. The various moods of analogical argument above described are the qualitative. In special cases, *given the appropriate additional information*, they become quantitative. For quantitative analogy, as for qualitative analogy, since the major and minor terms (P and Q) are functionally interchangeable, we may conventionally consider all moods as 'minor to major'. However, in the context of quantitative analogy, where there are underlying quantities, we must nevertheless distinguish between 'inferior to superior', 'superior to inferior', and 'equal to equal' inferences.[1]

a. The **positive subjectal** moods of quantitative analogy would read:
   - Given that subject P is *greater* than subject Q with respect to predicate R, and that Q is S (Sq), it follows that P is *proportionately more* S (Sp) (argument from inferior to superior).
   - Given that subject P is *lesser* than subject Q with respect to predicate R, and that Q is S (Sq), it follows that P is *proportionately less* S (Sp) (argument from superior to inferior).

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[1] My treatment here of quantitative analogy differs somewhat from that in my book *A Fortiori Logic*. The present treatment should be regarded as more accurate.

- Given that subject P is *equal* to subject Q with respect to predicate R, and that Q is S (Sq), it follows that P is *proportionately as much* S (Sp) (argument from equal to equal).

Note that each of these quantitative major premises implies the qualitative major premise ‘subject P is similar to subject Q with respect to predicate R’; for this reason, we already know by qualitative analogy that, in conclusion, P is S; what the quantitative analogical argument does is provide an additional quantitative specification in the conclusion, telling us whether P is proportionately (compared to Q, with respect to R) more, less or as much S.

The **negative subjectal** mood of quantitative analogy is then simply:

> Whether it is given that subject P is greater or lesser or equal to subject Q with respect to predicate R, and it is given that Q is *not* S, it follows that P is *not* S.

Note that this has here been expressed as one mood, but it could equally be presented as three moods by repeating it for each of the three major premises. The proposed conclusion here is not quantitative; it does not merely deny that P is proportionately more, less or equally S – it denies that P is S to any degree, just as the minor premise denies that Q is S to any degree. This means that this mood is essentially qualitative, and not quantitative. Its operative major premise is ‘subject P is similar to subject Q with respect to predicate R’. The validity of this negative mood is thus established, as

previously, by mere obversion of the negative subsidiary term.

b. The **positive predicatal** moods of quantitative analogy would read:
   - Given that predicate P is *greater* than predicate Q in relation to subject R, and that S (Sq) is Q, it follows that *proportionately more* S (Sp) is P (argument from inferior to superior).
   - Given that predicate P is *lesser* than predicate Q in relation to subject R, and that S (Sq) is Q, it follows that *proportionately less* S (Sp) is P (argument from superior to inferior).
   - Given that predicate P is *equal* to predicate Q in relation to subject R, and that S (Sq) is Q, it follows that *proportionately as much* S (Sp) is P (argument from equal to equal).

Note that each of these quantitative major premises implies the qualitative major premise 'predicate P is similar to predicate Q in relation to subject R'; for this reason, we already know by qualitative analogy that, in conclusion, S is P; what the quantitative analogical argument does is provide an additional quantitative specification in the conclusion, telling us whether S is proportionately (in relation to R) more, less or as much P (compared to Q).

The **negative predicatal** mood of quantitative analogy is then simply:

> Whether it is given that predicate P is greater or lesser or equal to predicate Q with respect to

> subject R, and it is given that S is *not* Q, it follows that S is *not* P.

Note that this has here been expressed as one mood, but it could equally be presented as three moods by repeating it for each of the three major premises. The proposed conclusion here is not quantitative; it does not merely deny that S is proportionately more, less or equally P – it denies that S is P to any degree, just as the minor premise denies that S is Q to any degree. This means that this mood is essentially qualitative, and not quantitative. Its operative major premise is 'predicate P is similar to predicate Q in relation to subject R'. The validity of this negative mood is thus established, as previously, by reductio ad absurdum.

Obviously, for the positive moods of both subjectal and predicatal analogy, the reasoning depends (though often tacitly) on an *additional premise* that *the ratio of Sp to Sq is the same as the ratio of P to Q (relative to R)*. Very often in practice, the ratios are not exactly the same, but only roughly the same (this of course affects the argument's validity strictly speaking, though we often let it pass). Also, the reference to the ratio of P to Q (relative to R) should perhaps be more precisely expressed as the ratio of Rp to Rq. Note that this argument effectively has five terms instead of only four (since the subsidiary term S effectively splits off into two terms, Sp and Sq). Of course, the additional premise about proportionality is usually known by inductive means. It might initially be assumed, and thereafter found to be untrue or open to doubt. In such event, the argument would cease to be quantitative analogy and would revert to being merely qualitative analogy. Thus,

quantitative analogy is inherently even more inductive than qualitative analogy.

Note that the arguments here are, briefly put: (i) just as P > Q, so Sp > Sq; (ii) just as P < Q, so Sp < Sq', (iii) just as P = Q, so Sp = Sq. In other words, positive quantitative analogy may as well be from the inferior to the superior, from the superior to the inferior, or from equal to equal; it is not restrictive regarding direction. In this respect, we may note in passing, it differs radically from a fortiori argument. In the latter case, the positive subjectal mood only allows for inference from the inferior to the superior, or from equal to equal, and excludes inference from the superior to the inferior; and the positive predicatal mood only allows for inference from the superior to the inferior, or from equal to equal, and excludes inference from the inferior to the superior. All this seems obvious intuitively; having validated the qualitative analogy as already shown, all we have left to validate here is the idea of ratios, and that is a function of simple mathematics.

We can similarly develop the corresponding forms **with a negative major premise** (i.e. the 'contrast' or 'disanalogy' forms) as follows.

Regarding **subjectal** argument. (a) In cases where it is known that qualitatively 'subject P is similar to subject Q with respect to predicate R', then the quantitatively negative major premise 'P is *not* greater than Q with respect to R' can be restated positively as 'P is either lesser than or equal to Q with respect to R'; 'P is *not* lesser than Q with respect to R' can be restated positively as 'P is either greater than or equal to Q with

respect to R'; and likewise, 'P is *not* equal to Q with respect to R' can be restated positively as 'P is either greater or lesser than Q with respect to R'. The conclusions follow as already above detailed. That is, with a positive minor premise, not-greater implies a proportionately less or equal conclusion; not-lesser implies a proportionately more or equal conclusion; and not-equal implies a proportionately more or less conclusion. With a negative minor premise, the conclusion is simply negative. But (b) in cases where it is known that qualitatively 'subject P is *not* similar to subject Q with respect to predicate R', then the three quantitatively negative major premises are irrelevant, and the minor premise 'Q is S' yields the conclusion 'P is not S', or alternatively 'Q is not S' yields the conclusion 'P is S' (as earlier seen). Therefore, (c) in cases where it is not known whether the underlying relation of P and Q relative to R is positive or negative, the conclusion is moot.

Regarding **predicatal** argument. (a) In cases where it is known that qualitatively 'predicate P is similar to predicate Q in relation to subject R', then the quantitatively negative major premise 'P is *not* greater than Q in relation to R' can be restated positively as 'P is either lesser than or equal to Q with respect to R'; 'P is *not* lesser than Q with respect to R' can be restated positively as 'P is either greater than or equal to Q with respect to R'; and likewise, 'P is *not* equal to Q with respect to R' can be restated positively as 'P is either greater or lesser than Q with respect to R'. The conclusions follow as already above detailed. That is, with a positive minor premise, not-greater implies a

proportionately less or equal conclusion; not-lesser implies a proportionately more or equal conclusion; and not-equal implies a proportionately more or less conclusion. With a negative minor premise, the conclusion is simply negative. But (b) in cases where it is known that qualitatively ‘predicate P is *not* similar to predicate Q in relation to subject R’, then the three quantitatively negative major premises are irrelevant, and the minor premise ‘S is Q’ yields the conclusion ‘S is not P’, or alternatively ‘S is not Q’ yields the conclusion ‘S is P’ (as earlier seen). Therefore, (c) in cases where it is *not* known whether the underlying qualitative relation of P and Q relative to R is positive or negative, the conclusion is moot.

We can similarly develop the various corresponding **implicational** moods of quantitative analogy. Thus, all moods of qualitative analogical argument can be turned into quantitative ones, provided we add additional information attesting to ‘proportionality’.

3. Terms of unequal breadth

The issue of quantitative analogy brings to mind the issue of analogies involving terms which are not co-extensive, but one is broader than and includes the other, as a more generic term includes a more specific term or as an unconditional term includes a conditional one[1]. This is still qualitative analogy, note well. It concerns the scope of terms, not their magnitude or degree as subjects or predicates.

Consider, for a start, **positive subjectal** analogy such that the middle predicate R is not identical for the major subject P and the minor subject Q. We are given that 'P is Rp' and 'Q is Rq', but we do not yet have a comparative major premise with which to construct an analogical argument. To obtain one, we have to find the operative common property of P and Q. Clearly, it is *the more inclusive* (*or less conditional*) predicate of the two we were given (viz. Rp and Rq).

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[1] Note that in some cases, though the two terms compared are specific/conditional, they may still resemble each other sufficiently to be considered as one and the same term for the purposes of analogical argument. It is only when the terms are not so identified, but must be differentiated, that the issue of unequal scope arises.

That is to say: (a) if Rp includes Rq, so that Rq is Rp (but not vice versa), then the effective middle term is the *broader* one, Rp, and the major premise is 'subject P is similar to subject Q with respect to predicate Rp', from which, given that Q is S, it follows that P is S. Note well that we cannot in such case build an analogical argument (of minor to major form) from the narrower middle term Rq.

On the other hand: (b) if Rq includes Rp, so that Rp is Rq (but not vice versa), then the effective middle term is the broader one, Rq, and the major premise is 'subject P is similar to subject Q with respect to predicate Rq', from which, given that Q is S, it follows that P is S. Note well that we cannot in such case build an analogical argument (of minor to major form) from the narrower middle term Rp.

It might seem paradoxical to say in (a) that we can infer from Rp but cannot infer from Rq, and in (b) that we can infer from Rq but cannot infer from Rp, and yet with the same minor premise 'Q is S' obtain the same conclusion 'P is S'. But we should keep in mind that the basis of analogy, the middle term Rq or Rp used in the major premise, is different in each case, so that arguments (a) and (b) are quite distinct claims; and anyway, we are here dealing with inductive argument.[2]

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[2] Of course, we can draw a conclusion based on the narrower, less inclusive, middle term, Rq in case (a), and Rp in case (b), by proceeding from major to minor. But our standard form, conventionally, is minor to major.

The corresponding **negative subjectal** moods have the same major premises, and both infer from the minor premise ‘Q is not S’ the conclusion ‘P is not S’.

With regard to **positive predicatal** analogy, where the middle term is a subject and the major and minor terms are predicates, we begin with two propositions ‘Rp is P’ and ‘Rq is Q’, from which we need to build a comparative major premise. Here, the basis of analogy is the subject for which both P and Q can be predicated. Clearly, it is *the less inclusive* (*or more conditional*) subject of the two we were given (viz. Rp and Rq).

That is: (a) if Rp includes Rq, so that Rq is Rp (but not vice versa), then the effective middle term is the *narrower* one, Rq, and the major premise is ‘predicate P is similar to predicate Q with respect to subject Rq’, from which, given that S is Q, it follows that S is P. Note well that we cannot in such case build an analogical argument (of minor to major form) from the broader middle term Rp.

But: (b) if Rq includes Rp, so that Rp is Rq (but not vice versa), then the effective middle term is the narrower one, Rp, and the major premise is ‘predicate P is similar to predicate Q with respect to subject Rp’, from which, given that S is Q, it follows that S is P. Note well that we cannot in such case build an analogical argument (of minor to major form) from the broader middle term Rq.

Again, it might seem paradoxical to say in (a) that we can infer from Rq but cannot infer from Rp, and in (b) that we can infer from Rp but cannot infer from Rq, and yet with the same minor premise ‘S is Q’ obtain the same conclusion ‘S is P’. But we should keep in mind

that the basis of analogy, the middle term Rp or Rq used in the major premise, is different in each case, so that arguments (a) and (b) are quite distinct claims; and anyway, we are here dealing with inductive argument.[3]

The corresponding **negative predicatal** moods have the same major premises, and both infer from the minor premise 'S is not Q' the conclusion 'S is not P'.

The same principles apply to analogical arguments **with a negative major premise**, even though they involve major and minor terms that are dissimilar, rather than similar as above. This is because the contrasting major premise must be a negative mirror image of the comparative major premise, with the same middle term. Thus, all the moods here resemble those above, except that their major premises will be negative (indicating disanalogy) and their conclusions will be contradictory to the foregoing (granting that the minor premises remain the same). There is no need for us to belabor this issue further.

Likewise, **quantitative** analogies involving middle terms of unequal breadth follow the rules already established once we have determined the operative middle term in each case.

What about cases where the two middle terms Rp and Rq are not equal and neither fully overlaps the other, i.e.

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[3] Of course, we can draw a conclusion based on the broader, more inclusive, middle term, Rp in case (a), and Rq in case (b), by proceeding from major to minor. But our standard form, conventionally, is minor to major.

where they merely *intersect*. In such cases, we have the conjunction ‘Rp and Rq’ as our operative middle term, R. Given a major premise with this compound middle term, we can use it in any kind of analogical argument already established as valid. Remember that analogical argument is inductive, so there is no restriction on the scope of the middle term; any middle term which happens to be true is valid.

However, while this seems simple enough at first sight, the plot thickens when we consider the other terms in such analogical arguments and quantify them. Thus, in subjectal argument, if all P are Rp and all Q are Rq, only some P and only some Q are *both* Rp and Rq, whence the minor premise and conclusion must be formulated as concerning ‘certain Q’ and ‘certain P’ respectively; which makes it practically useless. Again, in predicatal argument, while we can say of the compound R that it is all both P and Q, we cannot in the validation process generalize from ‘some Q are R’ to ‘all Q are R’, as we need to do if we wish to infer from the minor premise ‘S is Q’ that ‘S is R’, and thence (via ‘R is P’) the conclusion ‘S is P’; so, here analogy is effectively invalid. Thus, we can say without going into more detail that argument by analogy is not applicable in cases involving intersection.

We have thus far dealt with middle terms of different scope, but what about **subsidiary terms** of different breadth?[4]

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[4] I must say, I am surprised by the results shown here for subsidiary terms, because they lack symmetry. We have here one

In positive subjectal argument, the operative subsidiary term is the predicate in the minor premise (Sq, say); the subsidiary term in the conclusion may be different (Sp, say), if and only if the primary conclusion 'P is Sq' implies the further conclusion 'P is Sp'; and this is possible only provided that 'Sq is Sp', meaning that Sp must be *broader* than Sq.[5] Likewise, in the corresponding negative mood, if 'P is not Sq' is to imply 'P is not Sp', Sp must be *narrower* than Sq.[6]

In positive predicatal argument, on the contrary, the operative subsidiary term is the subject in the minor premise (Sq, say); the subsidiary term in the conclusion may be different (Sp, say), if and only if the primary conclusion 'Sq is P' implies the further conclusion 'Sp is P', and this is possible only provided that 'Sp is Sq', meaning that Sp is *narrower* than Sq.[7] Likewise, in the corresponding negative mood, if 'Sq is not P' is to imply 'Sp is not P', Sp must be *narrower* than Sq.[8]

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mood requiring that Sp be broader than Sq, and three moods where Sp must be narrower than Sq. This, in my experience, is unusual. It seems to me that either all four moods should be the same, or two moods one way and two the other way. But try as I might I do not see any error in my treatment here; so, I must accept this finding.

[5] Syllogism: all P are Sq, all Sq are Sp, so all P are Sp.

[6] Syllogism: all P are nonSq, all nonSq are nonSp (= all Sp are Sq), so all P are nonSp.

[7] Syllogism: all Sq are P, all Sp are Sq, so all Sp are P.

[8] Syllogism: all Sq are nonP, all Sp are Sq, so all Sp are nonP.

4. Face-off with a fortiori argument

It is worth comparing and contrasting analogical argument with a fortiori argument, because these argument forms are often confused by people. For this purpose, I will simply here reproduce verbatim what my reflections on this topic in my *A Fortiori Logic*, chapter 5.1:

Clearly, while qualitative analogy is somewhat comparable to purely a fortiori argument, quantitative analogy is somewhat comparable to a crescendo argument; but they are still far from the same. Let us first compare and contrast *qualitative* analogical argument to *pure* a fortiori argument. For this purpose, let us first focus on the positive subjectal mood, viz.:

> P is more R than (or as much R as) Q,
>
> and Q is R enough to be S;
>
> therefore, P is R enough to be S.

Here, as in analogy, the major premise implies that both P and Q are R, but unlike in analogy, it additionally implies that $Rp \geq Rq$, i.e. that the quantity of R in P is greater than (or equal to) that in Q. Thus, though we can

deduce the major premise of analogical argument from that of a fortiori argument, we cannot reconstruct the major premise of a fortiori argument only from that of analogical argument. Similarly, though the minor premise of a fortiori argument implies that Q is S, and therefore implies the minor premise of analogical argument, the reverse is not true. The difference between the two minor premises is that in a fortiori argument there is the element of *sufficiency of R to be* S, which is clearly lacking in argument by analogy. For the same reason, although the conclusion of a fortiori argument implies that of analogy, the latter does not by itself enable us to reconstruct the former.

Moreover, even though each of the propositions (the major and minor premises and the conclusion) involved in a fortiori argument implies the corresponding proposition of analogical argument, this does not mean that an a fortiori argument implies an analogical one. For, the a fortiori argument is deductive, i.e. its conclusion follow necessarily from its two premises; whereas, as we have just shown, the argument by analogy, even in its complex form, is inherently inductive, i.e. it requires a generalization of its minor premise to enable us to draw its conclusion. Therefore, even if both arguments may be said to yield a common conclusion, namely 'P is S', that conclusion has a very different logical status in the one and in the other.

It follows that we can neither reduce a fortiori argument to argument by analogy, since the latter's conclusion does not imply the former's (even though the premises of the former do imply those of the latter), nor can we do the reverse, since the premises of the latter do not

imply those of the former (even though the conclusion of the former does imply that of the latter). It does happen that we know enough to form the major premise needed for a fortiori argument, but we do not know enough for its minor premise; or we know enough to form the minor premise needed for a fortiori argument, but we do not know enough for its major premise – in such cases we might have enough information to at least formulate an analogical argument. Thus, sometimes we have more information than we need for an analogy, but not enough for an a fortiori argument – in such cases we can only formulate an analogy.

Therefore, though we can say that a fortiori argument and argument by analogy have some features in common, we must admit that they are logically very distinct forms of argument. This is a formal and undeniable demonstration, once and for all. To repeat: neither argument can be *reduced* to the other. However, every valid a fortiori argument *implies* a corresponding argument by analogy involving less information and certainty. The premises of the latter, as we have just seen, lose the quantitative and/or sufficiency factors involved in the former; and the conclusion of the analogical argument is, as a result, both less informative and less sure (being now inductive instead of deductive). But of course, except for the present theoretical clarification, there is in practice no point in resorting to such implication, since the given a fortiori argument is better in all respects.

As regards the opposite direction, it cannot be said that every analogical argument implies a corresponding a fortiori argument. All we can say is that we can,

sometimes, when the facts of the case permit it, construct an a fortiori argument which implies the given analogical argument. This is possible if the latter argument has a middle term (R), or an appropriate middle term can be found for it, which can both be used as a continuum of comparison (which, I think, is always possible in practice, although we *cannot* tell a priori which term is greater than the other) and at the same time serve as the sufficient condition for the subject (Q) to access the predicate (S) in the minor premise (and this is, of course, *not* always possible in practice). Thus, the construction of a corresponding a fortiori argument from a given analogical argument is not a mechanical matter and cannot always be performed. In effect, when it is found possible, it just means that we should in the first place have resorted to the stronger a fortiori argument yet foolishly opted for the weaker analogical argument.

All that we have said here applies equally well, *mutatis mutandis*, to the negative subjectal forms of these arguments, and to positive and negative predicatal forms, and again to the four implicational forms. These jobs are left to interested readers. For reminder, the form of positive predicatal a fortiori argument is as follows:

> More R (or as much R) is required to be P than to be Q,
>
> and S is R enough to be P;
>
> therefore, S is R enough to be Q.

As regards comparison and contrast between *quantitative* analogy and *a crescendo* argument, i.e. 'proportional' a fortiori argument, the following need be said. The major premises are the same in both. But the minor premises and conclusions obviously differ, insofar as in quantitative analogy there is no idea of a threshold value of the middle term as there is in a fortiori argument. This explains why the 'proportionality' is essentially non-directional in quantitative analogical argument (inference is always possible both from minor to major and from major to minor); whereas it is clearly directional in a fortiori argument (inference is only possible from minor to major in positive subjectal and negative predicatal argument, and from major to minor in negative subjectal and positive predicatal argument).

Note in passing that although quantitative analogy and mere pro rata argument (i.e. used alone, outside of a crescendo argument) are not formally identical the two are effectively the same. Compare for example the following two formulas; clearly, the provisos in them are essentially the same (a concomitant variation between the values of S and the values of R) even if the terms are differently laid out.

Given that P is greater than Q with respect to R, and that Q is S (Sq), it follows that P is proportionately more S (Sp), *provided that* the ratio of Sp to Sq is the same as the ratio of P to Q (quantitative analogy).

Given that if R has value Rq then S has value Sq, it follows that if R has value more than Rq (say Rp), then S has value more than Sq (say Sp), *provided that* the

values of S vary in proportion to the values of R (pro rata argument).

To conclude: there is, to be sure, an element of 'analogy' in all human thinking, including in syllogism and in a fortiori argument, since all abstraction is based on mental acts of comparison and contrast; but to say this loosely is not the same as equating syllogism or a fortiori argument to argument by analogy. When we look into the exact forms of these arguments, we clearly see their significant differences.

5. Conflicting analogies

We have thus seen that analogical argument has numerous moods, which are formally expressible and capable of validation. We shall now consider the issue of *conflicting analogies*, by considering two or more middle terms, i.e. R1, R2, etc., which yield different or conflicting conclusions. One analogy may be more credible or weighty than another. This refers to compound analogical argument *comprising both comparison and contrast* (instead of each in isolation from the other). We must here focus our attention on four compounds, which combine two like forms (not just any pair of forms, note). We may call either argument (the comparison or the contrast) 'the argument' and the other 'the counterargument' (although I here place the comparison before the contrast, the opposite order would do just as well of course).

First compound: positive subjectal moods.

> Comparison: given that subject P is similar to subject Q with respect to predicate R1, and that Q is S, it follows that P is S.
>
> Contrast: given that subject P is *dis*similar to subject Q with respect to predicate R2, and that Q is S, it follows that P is not S.

Second compound: negative subjectal moods.

> Comparison: given that subject P is similar to subject Q with respect to predicate R1, and that Q is not S, it follows that P is not S.
>
> Contrast: given that subject P is *dis*similar to subject Q with respect to predicate R2, and that Q is not S, it follows that P is S.

Third compound: positive predicatal moods.

> Comparison: given that predicate P is similar to predicate Q in relation to subject R1, and that S is Q, it follows that S is P.
>
> Contrast: given that predicate P *dis*similar to predicate Q in relation to subject R2, and that S is Q, it follows that S is not P.

Fourth compound: negative predicatal moods.

> Comparison: given that predicate P is similar to predicate Q in relation to subject R1, and that S is not Q, it follows that S is not P.
>
> Contrast: given that predicate P is *dis*similar to predicate Q in relation to subject R2, and that S is not Q, it follows that S is P.

Here we see that by referring to different aspects of P and Q, namely R1 and R2, we may obtain conflicting conclusions, and therefore finally no conclusion. Note that the minor premise is made identical in both cases, and the two major premises are not formally in conflict (since their middle terms differ), and the two argument forms are equally valid. Yet the conclusions are contradictory! Such conundrum is, of course, made possible by the fact that analogical argument is not

purely deductive, but in part inductive. Its conclusions are suggestive, not decisive.

To be sure, in some cases we may be able to resolve the contradiction by refuting the analogy (i.e. the similarity or dissimilarity) claimed in the argument or the counterargument, or both; but this is of course not always possible. In some cases, even after an analogy relative to some middle term is found weak, we may still be able to posit the same analogy relative to another middle term which more strongly supports the putative conclusion; in which case, the conundrum remains.

Obviously, as when faced with any contradiction, we are called upon to carefully check our premises and ensure their credibility. And clearly, while some analogies may not resist criticism, and finally fall, or at least remain inconclusive, others may stand with relative ease, being objectively credible. So, it is inevitable for us, in the pursuit of knowledge, to be faced with such conundrums.

A special case of conflicting analogy is when R1=R2, i.e. when there is only one middle term R. In such cases, the two major premises in the four above compounds are contradictory, and the comparison and contrast arguments cannot both be valid. Also, if either of R1 and R2 implies the other, but not vice versa, then the two major premises are contrary[1], and the conflicting arguments cannot be both valid.

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[1] This is easily proven. If R1 implies R2, then the two major premises are incompatible through R2; if R2 implies R1, then they are incompatible through R1. But since either case is possible,

In any case, it should be emphasized that *no two things are the same in all respects*, or they would not be two but one; and *no two things are different in all respects*, or they would not be in the same universe. This means that the above-listed compound arguments are applicable to all things, and the problem of distinguishing significant similarities and dissimilarities from less significant ones is unavoidable. It follows that we constantly estimate by some means or other, in each context, which similarities and dissimilarities are the most significant.

This thought suggests that we should, ideally, for any two items (the subjects or predicates labeled P and Q), systematically find and list all the ways (i.e. the middle terms R1, R2, etc.) in relation to which they are similar or dissimilar. We would then verify, for each middle term considered, how the minor and major terms (P and Q) relate to the subsidiary term S. Where the relation of S to Q is known and to P is not, we would infer the latter from the former as shown above. Where the relation of S to P is known and to Q is not, we would infer the latter from the former in the same way.

Then, at the end of this systematic research process we would have some idea as to how often the conclusion is positive rather than negative, or negative rather than positive. But of course, such *complete enumeration*, though ideal and theoretically conceivable, is usually not possible in practice. There is just too much

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neither is necessary; so, the two major premises are merely contrary, not contradictory.

similarity and difference between any pair of things. In practice, we investigate and refer to the relations between things as and when they happen to come to our attention. Our knowledge evolves gradually as our experience (whether obtained by passive observation or active experiment) grows and our theoretical insights concerning it become more complex and accurate. Over time, then, our views may change regarding which conclusion is the most significant.

6. Statistics-based analogical arguments

The difficult question we need to try and answer here is: how to decide which of the two opposed arguments is the most convincing? I suspect that in everyday practice *intuition* plays a large role in most cases – our perceptions of which common factor, R1 or R2 (or others still), is the most 'significant' in the context concerned. A more formally expressible way to answer our question may, however, be to multiply the number of comparisons and contrasts (not limiting ourselves to two middle terms), and then base our final conclusion on the more numerically weighty side. This is a statistical method.

The principle would here be: If two things (P and Q) are alike in numerous ways (collectively, R1) and differ in numerous ways (collectively, R2), and they are alike more often than they differ, then we may assume that a subject or predicate (S) found to relate to the one (say, Q) probably also relates to the other (P) - the degree of probability being determined by the ratio of similarity to dissimilarity. If the major premise is that they are different more often than they are alike, then the probability is instead in favor of the conclusion being opposite to the minor premise.

The justification for such statistical argument is generalization: a relation that we found to hold in a majority of *known* cases may, by extrapolation, be assumed to hold in most *unknown* cases; inversely, if the relation holds only in a minority of known cases, there is no reason to expect it to hold in subsequent unknown cases. There is admittedly no certainty here, only probable expectation; but there is some justification: the conclusion is more likely to be thus than otherwise. The greater the probability the more trustworthy our conclusion.

We can thus propose the following four moods of what we may characterize as *statistics-based* analogical argument. Such forms of argument are clearly logically fuller than the forms initially proposed, because they consciously deal with the issue of conflicting analogies. Note that I have conventionally put the minor term in the minor premise and the major term in the conclusion in every case, although I could equally well have opted for the opposite ordering; this was done just to facilitate remembrance. In subjectal argument, the major term P is subject of the conclusion and the subsidiary term S is predicate; whereas in predicatal argument, S is subject of the conclusion and P is predicate. In positive argument, the conclusion has the same polarity as the minor premise; while in negative argument, the conclusion has the opposite polarity to the minor premise.

<u>Positive subjectal analogical argument:</u>

> Given that subject P is like subject Q with respect to *considerably many* predicates

(collectively, R1), and that Q is S (some new predicate), it follows that P is probably S too. For, given that subject P is *un*like subject Q with respect to *relatively few* predicates (collectively, R2), and that Q is S, it does *not* follow that P is probably not S. Conclusion: P is probably S.

Negative subjectal analogical argument:

Given that subject P is *un*like subject Q with respect to *considerably many* predicates (collectively, R1), and that Q is S (some new predicate), it follows that P is probably not S. For, given that subject P is like subject Q with respect to *relatively few* predicates (collectively, R2), and that Q is S, it does *not* follow that P is probably S. Conclusion: P is probably not S.

Positive predicatal analogical argument:

Given that predicate P is like predicate Q in relation to *considerably many* subjects (collectively, R1), and that (some new subject) S is Q, it follows that S is probably P too. For, given that predicate P is *un*like predicate Q in relation to *relatively few* subjects (collectively, R2), and that S is Q, it does *not* follow that S is probably not P. Conclusion: S is probably P.

Negative predicatal analogical argument:

Given that predicate P is *un*like predicate Q in relation to *considerably many* subjects (collectively, R1), and that (some new subject) S is Q, it follows that S is probably not P. For, given that predicate P is like predicate Q in

> relation to *relatively few* subjects (collectively, R2), and that S is Q, it does *not* follow that S is probably P. Conclusion: S is probably not P.

The middle terms R1 and R2 are here referred to as 'collective' with the intent that each of them represents numerous unspecified middle terms for which the stated proposition applies. In subjectal moods, the middle terms are predicates of the major premises; while in predicatal moods, they are subjects. Obviously, if the expressions "considerably many" and "relatively few", applied to the middle subjects or predicates (the Rs), can be more precisely quantified, and the bigger number grows and the smaller number shrinks, the probabilities of the conclusions increase.

Needless to say, all problematic conclusions arrived at here are inductive, meaning that they are valid only until and unless new empirical findings or deductions or stronger probabilities override them. They are not fixed, final results, but the best available results in the given context.

7. A scientific illustration

Needless to say, analogy is very often used in everyday thought, and therefore (though perhaps, ideally, more rigorously) in scientific thinking. All conceptualization (and therefore all knowledge, ultimately) is, of course, based on analogy, since we need to become aware of the apparent similarities and differences of things in order to decide whether to classify them together or apart.

I found a scientific illustration of analogical thinking in a recently published book on paleontology[1], which I happened to have purchased and started reading (with no purpose other than pleasure) just as I was developing the above thoughts on analogy. It is worth examining this illustration in some detail (without delving very deeply in the paleontological details) to see what logic can be learned from it.

There we are told that the hunting behavior of dinosaurs, for instance, is induced from other known features of dinosaurs with reference to "modern analogues" chosen, not randomly by referring to just any other predators, such as wolves or sharks, but by means of "**bracketing**." This consists in comparing dinosaurs

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[1] Michael J. Benton. *The Dinosaurs Rediscovered: How a scientific revolution is rewriting history*. London: Thames & Hudson, 2019-20.

more specifically to extant *close relatives* of theirs in the evolutionary tree, namely birds and crocodiles. The basis for analogy between 'close relatives' is, clearly, that they are already known (or even merely believed at that stage) to share many *distinctive* characteristics. The author explains:

> "If crocodiles *and* birds share some detail… then dinosaurs had it too. We can't say dinosaurs had feathers simply because birds have feathers – crocodiles do not have feathers, so dinosaurs are not bracketed as far as that character is concerned."[2]

Putting this argument in more formal terms we obtain the following:

> Subject A (dinosaurs) is known to have many characteristics (middle terms, left tacit here, e.g. genetic or morphological traits) in common with subjects B (birds) and C (crocodiles), therefore, with regard to some feature D (say, an anatomical detail or a behavior pattern): if both B and C have D, then A probably has D too, or if both B and C lack D, then A probably lacks D too; but if B has feature D whereas C lacks it, or if B lacks feature D whereas C has it, we cannot (with equal certainty) predict whether A has or lacks D.

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[2] See pp. 16-17. Reasoning by bracketing was first proposed by Larry Witmer in 1995. The resort to 'analogy with living forms' (p. 189) to interpret aspects of fossil forms was an established method long before that, of course.

This is, of course, merely probable reasoning – for it remains conceivable, and may well happen on occasion, that A differs as regards D from the indications *suggested by* B and C. It remains true that A may have some unique, novel trait D while B and C both lack it; or A may distinctively lack D while B and C both have it; or A and B may both have D while C lacks it; or A and B may both lack D while C has it; and so forth. Nevertheless, the proposed method of bracketing provides us with some direction, due to the major premise that A is already established as having many distinctive features (which are left tacit here, but together constitute the logically operative middle term) in common with both B and C.

Note that the form of this argument is positive subjectal, with A as the major term, B and C as two minor terms, the unspecified properties they all share as middle terms, and D as the subsidiary term. What is not mentioned here is the mass of differences between A on the one hand, and B & C on the other, although being non-identical they are bound to have many differences. This can be seen if we cast the argument more explicitly in the form of a standard statistics-based analogy:

> Given that subject P (A, dinosaurs) is like subject Q (comprising both B and C, birds and crocodiles) with respect to *considerably many* predicates (collectively, R1 – here unspecified), and that Q is S (some predicate D), it follows that P is probably S too. For, given that subject P is *un*like subject Q with respect to *relatively few* predicates (collectively, R2 – here unspecified), and that Q is S, it does *not* follow

> that P is probably not S. Conclusion: P is probably S (i.e., in our example, A is probably D).

Clearly, the second part of the compound shown above (i.e. the negative counterargument) was *left tacit* in the above example, it being presumed that the differences between A and B & C, with respect to another set of middle terms (unspecified), which could point us to an opposite conclusion, were *insufficiently frequent to stand out and matter*. The counterargument is, no doubt, at least subconsciously considered by scientists in practice, drawing on their vast stores of individual and collective knowledge. But to be on the safe side, in practice scientists should always consciously consider and determine the relative likelihood of the counterargument. Because in fact, both sides of the full argument are logically relevant.

It should be obvious that the use of two minor terms (B and C), in preference to only one (either B or C alone), is that this increases the probability of the conclusion about A, which effectively is impressed on us convergently, twice instead of only once. Moreover, if the analogues B and C point to divergent conclusions (both D and not D), we are left with doubts concerning A. As already suggested, the terms A, B, and C should preferably be closely related, as this increases the probability of the result. If they have some characteristic(s) in common, that is good; but if they have some *distinctive* characteristic(s) in common, that is much better, for that fact ties them more closely together, and increases the chances (though of course,

still does not ensure) that they will also share the concluded characteristic (D).

Obviously, too, this kind of compound reasoning can be pushed further, by involving more than two modern analogues. *The more analogues the merrier*, since this (to repeat) increases the probability of the conclusion. That is, if subject A is correlated with several more analogues (instead of just B and C) and they are also found to have D, the probability grows that A is also D. This, then, is one important lesson we can learn from the technique of bracketing – viz. that the probability of the conclusion can be increased by referring, not just to more numerous middle terms (as earlier remarked), but also to more numerous minor terms.

As regards probability ratings, that is not just talk here. It is true that in ordinary discourse, probabilities are very roughly 'estimated' based on personal experience and memory, and even bias, and people may well disagree as to their directions and magnitudes. But in scientific discourse, the issue is taken much more seriously, and great effort and expense are invested to determine probabilities as accurately as possible. Contemporary scientists[3] use a wide array of more and more sophisticated observational and experimental techniques, marvelous technological tools and measuring instruments, ingenious mathematical and computational methods, and extremely powerful computers, to obtain the data they seek. Their professional credibility and reputation depend on their

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[3] Such as the paleontologists in the referenced book.

rigor. The consequence is certainty increasing over time, sometimes at an exponential rate.

Modern researchers are admirable in the amount of care and effort they put in to arrive at their conclusions. This is well illustrated in the book on dinosaurs we have here mentioned[4]. By the year 2000, some 500 species of dinosaur had been discovered and named in the world. Scientists wished to classify them relative to each other, in a complete evolutionary tree, as accurately as possible. They collected, merged, and tabulated all known information from hundreds of published papers; and using complex software and powerful computers managed to find the statistically most likely classifications for hundreds of known species. More recently, they have started to reexamine specimens stored in museums and universities across the world, looking for the presence or absence of 457 anatomical characters in each case, to obtain a still more complete and more accurate tree.

Obviously, such a tree facilitates bracketing, among other things. It is a brief, visual repository of large numbers of comparisons and contrasts.

I should also mention, as illustrations of the use of analogy in scientific contexts, **medical applications of analogical argument**. When we visit a doctor for a checkup, he examines us in a variety of ways, and may diagnose some unhealthy condition, and prescribe some

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[4] Pp. 76-77, 82-83.

possibly preventive or curative measures, and venture a prognosis. All this involves reasoning of various kinds, especially causal and analogical. The following is the kind of thinking the doctor may engage in:

Diagnosis: The symptoms a, b, c, observed in this patient resemble those of other people who were found to have disease so-and-so; therefore, my patient is likely to get probably or has the same disease.

Prevention/cure: Clinical tests and widespread practice have shown that such-and-such medical treatment is efficacious in preempting or combating disease so-and-so; therefore, my patient will likely benefit from similar treatment.

Prognosis: If the patient does not follow such treatment, his condition will likely deteriorate in this way and that; but if he takes proper care of his medical needs, as I prescribe, his condition will likely improve in various respects.

Diagnostic reasoning largely consists of causal logic; but it also manifestly involves some analogical logic, insofar as one compares and contrasts symptoms observed in the patient to a list of symptoms known to medical professionals to date through extensive research and experience. Symptoms may be observed with the naked eye by the doctor, or felt and reported by the patient, or identified by various medical tests using simple tools or complex technologies.

Obviously, different diseases may display some similar symptoms. One further confirms the identification of the disease by referring to more and more symptoms,

ideally to an exhaustive list of symptoms. To be able to zero-in to and pinpoint the applicable disease with certainty, one needs to find an exclusive set of symptoms. If some essential (i.e. necessary, *sine qua non*) symptom(s) of a disease is/are lacking, then that disease can be eliminated from the list of possible diseases for the observed symptom or set of symptoms.

Similar reasoning is used in deciding preventive and curative measures, which may include not only medicines and surgical operations, but lifestyle and environmental recommendations, nutrition, exercise, physiotherapy, and so forth; and to prognosis, which may be positive or negative, and have various degrees of probability.

8. Use of analogy in making and applying law

Analogical argument is common not only in everyday thought and discourse by everyone, and in more scientific contexts, but it is also quite widespread in legal contexts. It is an instrument of law development and application used in all legal systems. Examples are easily found in ancient systems (like the Greek, the Roman or the Talmudic), in medieval systems (like the Christian, the Islamic or the Rabbinic), and in modern systems (like the British, the American or the French). It does not matter whether the political system involved is essentially dictatorial (as, say, in Russia or China today) or essentially free and democratic (as in Western countries today) – reasoning by analogy by legislators or judges is widespread.

Legislators aim to enact new laws, producing ‘statutory law’, while judges aim in principle to apply the laws the latter hand down to them, although, by establishing binding precedents, courts effectively amplify the law, producing ‘case law’, and moreover some supreme courts take this interpretative power far beyond the manifest original intent of legislation and get quite ‘creative’.

Analogical argument helps maintain some degree of consistency and uniformity in the law. If analogies and disanalogies were ignored, a law system might include a smorgasbord of relatively contradictory laws, which could be used to arbitrarily form lenient or stern judgments, as judges please, depending on their political or other personal prejudices or even just their current moods. Such *à la carte* legislation is obviously contrary to justice.

The argument by analogy may be used in legal contexts in several ways: (a) we may formulate new laws on the basis of general ethical or political principles[1]; (b) we can derive specific laws from constitutional guidelines; (c) we can make new laws by imitation of existing laws for comparable situations; (d) we can argue for the application of an existing law to a particular case under consideration; (e) we can make use of legal precedents, examining past cases resembling the present case, and proposing a like judgment for it; or (f) we can resort to some combination of these ways. For each of these ways, or a combination of them, an argument by analogy can be constructed, provided we perceive (and preferably make explicit) some significant commonality between the source and target situations. The argument

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[1] For instance, arguing that since a man has a natural right to life and liberty, he cannot be executed or imprisoned at will (but only eventually under specific conditions, i.e. following demonstrated criminal behavior punishable by law, and after due process). The legislation is intended to give concrete, practical expression to the abstract, philosophical principle.

would look something like the following (positive subjectal, comparing):

> Since [major premise] the situation under consideration (= major term, P) *resembles* the situation envisioned by such and such general ethical or political principles (a), or constitutional guidelines (b), or existing laws (c, d), or legal precedents (e) (= minor term, Q), with respect to this and that (= middle term, R),
>
> and [minor premise] this source (Q) prescribes some legal course of action[2] (= subsidiary term, S),
>
> it follows by analogy that [conclusion] for the target situation (P) we ought to establish or apply a like legal course of action (S).

Needless to say, while the analogy may be *prima facie* quite convincing, it might eventually be credibly contested; because such argument is never logically decisive, but at best indicative. It might be argued that P does not resemble Q sufficiently or in significant respects R, or that while it is comparable with respect to R, it is rather different with respect to certain other factors (another middle term), and therefore that the formulation for P of a law or judgment S similar to that previously settled for Q is not wise. Such counterargument can also be formulated in standard form, as follows (negative subjectal, contrasting):

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[2] Such as an appropriate verdict or penalty.

> Since the situation under consideration (P) *does not* resemble the situation envisioned by such and such general ethical principles (a), or constitutional guidelines (b), or existing laws (c, d), or legal precedents (e) (Q), with respect to this and that (R), or with respect to certain other factors, and this source (Q) prescribes some legal course of action (S), it follows by disanalogy that for the target situation (P) we ought *not* establish or apply a like legal course of action (S).

Analogical argument should not be confused with a fortiori argument, which is more complex (see my work *A Fortiori Logic* for a thorough treatment of such argument[3]). At this point, we should of course propose numerous examples from various historically and geographically different legal systems[4]. I shall, however, be content with the presentation of one Talmudic example, which I find intellectually interesting and challenging because of the convoluted thinking it involves. The reader would do well to read it carefully, even if indifferent to Talmudic content, as there is much to gain in logical acuity and skill from this demanding exercise.

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[3] See section 4 of the present work for comparisons and contrasts between the two forms of argument.

[4] The reader can, I assume, readily find many such examples through legal websites or in libraries.

9. A Talmudic illustration

We shall now examine a Talmudic illustration of the sort of more complex analogical reasoning we introduced earlier, with reference to a discussion found in the Babylonian Talmud, tractate *Baba Kama*, pp. 20a-21a. My attention was drawn to this long *sugya* (pericope) by R. Louis Jacobs, who presents a detailed literary analysis of it in one of his works[1]. I here only present a small of part of the discussion, and that as briefly as possible, because I am not really interested in the specific legal issue under discussion, but merely wish to illustrate and evaluate the use of analogy in the halachic discourse of the Talmud. My account is based on the Soncino English translation of the Talmud[2] as

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[1] R. Louis Jacobs. *Structure and Form in the Babylonian Talmud.* Cambridge: Cambridge UP, 1991. See chapter 5 (pp. 56-64). Indeed, it is through reading that essay that I realized that my presentation of analogical argument in *A Fortiori Logic* was far from complete, and I was moved to write the present more thorough essay. The aim of Jacobs' analysis is to show how the Talmud collects and orders information and arguments from different sources and times to form an instructive literary unit; it does not randomly or chronologically report discussions but organizes them purposely in a seemingly logical progression. My aim here is very different: it is to study the logical discourse used.

[2] The full text can be found in Halakhah.com. The explanatory comments in square brackets are given there, too.

well as on Jacobs' reading; but all logical analyses and eventual critical comments are entirely my own.

It is evident from this lengthy example that analogical argument plays a large role in Talmudic (and later, rabbinic) reasoning. We learn from it that when the rabbis wish to establish a new legal ruling, they resort to various analogies found in Mishnaic (or, in other contexts, in Biblical or otherwise traditional proof-texts, or even as a last resort in authoritative statements by rabbinic deciders[3]), as the possible basis of that proposition – and this is where the issue of differing or even conflicting analogies comes into play. The issue being: which of a set of proposed analogies is the most apt, the one to prefer? The problem here, as against in more scientific contexts, is the difficulty in evaluating the relative relevance of conflicting analogies.

The central question posed by our *sygya* is the following. A certain rabbi, R. Hisda, wonders whether "*one who occupied his neighbour's premises unbeknown to him would have to pay rent or not*." I shall here call, for the sake brevity and clarity, the occupier 'the squatter' and the owner of the premises 'the

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[3] In some cases, even within this *sugya*, they just seem to rely on the greater authority of some exponent. This is, of course, *ad hominem* argument, although its intent is positive. The authorities referred to are so considered because they are viewed as bearers of the oral traditions handed down since the time of Moses. However, there is no denying that they are in fact often at odds. Traditional commentary on this fact asserts that they are nevertheless (somehow) all right.

landlord'. The Gemara[4] offers the following clarification of the issue:

> "But under what circumstances? It could hardly be supposed that the premises were not for hire [and would in any case have remained vacant], and he [the one who occupied them] was similarly a man who was not in the habit of hiring any [as he had friends who were willing to accommodate him without any pay], for [what liability could there be attached to a case where] the defendant derived no benefit and the plaintiff sustained no loss? If on the other hand the premises were for hire and he was a man whose wont it was to hire premises, [why should no liability be attached since] the defendant derived a benefit and the plaintiff sustained a loss? — No; the problem arises in a case where the premises were not for hire, but his wont was to hire premises."

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[4] The Talmud includes Mishna and Gemara. Each Mishna passage is presented verbatim, then discussed by the Gemara, though other topics might also be treated in passing. The term 'Gemara' refers to the anonymous editor(s) who compiled discussions, associated somehow with the stated Mishna, by various named rabbis in various periods, putting those discussions in some purposeful order, usually with a commentary binding them together. Other commentators, such as Rashi or Tosafot, may come into play long after the Gemara, asking questions or clarifying points not found explicitly treated by the Gemara. The Mishna is dated at c. 200 CE and the Gemara at about 500 CE.

From which we know that in the case under consideration the squatter benefits (since he lacked somewhere to stay free of rent), but the landlord does not suffer a loss (since he allowed the place to remain empty at that time, even if he usually sought to rent it) – in Hebrew this case is referred to as *zeh neheneh ve-zeh lo-haser* (= this one benefits and that one does not suffer loss). After the fact, the landlord might say to the squatter "Since you have derived a benefit [as otherwise you would have had to hire premises], you must pay rent accordingly;" while the squatter might refuse to pay rent to the landlord, arguing "What loss have I caused to you [since your premises were in any case not for hire]?"

The answer to the question is sought through consideration of the legal rulings made in other contexts involving a protagonist/defendant (like the squatter) who benefits from something and an antagonist/plaintiff (like the landlord) who does not suffer a loss, i.e. having the same *zeh neheneh ve-zeh lo-haser* scenario. If in such comparable situation the ruling was that the protagonist is liable to pay something to the antagonist, it is assumed that the same ruling of liability can be applied to 'our' case (i.e. the above-mentioned case of landlord versus squatter). If in such comparable situation the ruling was non-liability, then in our case that will be assumed to be the applicable ruling. The analogical argument pursued here is thus the following:

> Just as, in the proof-text, where the protagonist benefits and the antagonist does not suffer loss, the law was that the former is obligated (or not obligated, as the case may be) to pay some compensation to the latter;

> likewise, in our case, where the protagonist benefits and the antagonist does not suffer loss, the law must be that the former is obligated (or not obligated, as the case may be) to pay some compensation to the latter.

Call these two sentences the source of analogy and the target of analogy. Note well that both cases involve the scenario *zeh neheneh ve-zeh lo-haser*; this is what binds them together, their common ground. The first paragraph provides a hypothetical proposition (the source) that in a previous case involving this scenario (the antecedent) the ruling was so and so (the consequent); the second paragraph formulates a like if-then statement (the target) for the new case, arguing that since it has the same antecedent, it may be assumed to have the same consequent. In this way, a ruling is proposed for the new case. It must be stressed, however, that this inference is inductive, not deductive; it is not logically inconceivable that the ruling might turn out to be different in the two cases on other, more plausible, grounds.

We can rephrase this argument in the standard format for (positive subjectal) analogical argument as follows:

> Given that our case (= major term, P) is similar to the proof-text case (= minor term, Q) in involving the scenario *zeh neheneh ve-zeh lo-haser* (= middle term, R), and that in the proof-text case (Q) the law was so-and-so (obligation to pay, or not, as the case may be) (= subsidiary term, S), it follows that in our case (P) the law

> should likewise be so-and-so (obligation to pay, or not, as the case may be) (S).

A putative example in our *sugya* of such analogical argument-form is the following. Another rabbi, Rami bar Hama, claims that the solution to the problem posed by R. Hisda is to be found in Mishna *Baba Kama* 2:2, which reads[5]:

> "In what case is this statement applied, that one pays the full value of the food eaten by the animal? It is a case where the animal ate the food on the property of the injured party; but if the animal ate food in the public domain, the owner of the animal is exempt from liability. And even if the animal ate food in the public domain, if the animal derives benefit from eating another's produce in the public domain, the owner pays for the benefit that it derives, just not for the full cost of the food."

This passage of the Mishna comprises three sentences. The first is a reference to a law given in Exodus 22:4. This Torah passage states that "If a man cause a field or vineyard to be eaten, and shall let his beast loose, and it feed in another man's field; of the best of his own field, and of the best of his own vineyard, shall he make restitution."[6] The second sentence in our Mishna is

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[5] I here quote the three sentences in the Mishna of interest to us using the translation in Sefaria.org because it is clearer than the one given in the Soncino ed.

[6] Translation taken from Mechon-mamre.org.

derived from the first by a *davka* (just so) reading, taking it to mean that the liability exists *only if* the loose beast feeds illicitly in a private domain; whence it is inferred that if the problem arose in the public domain, there is no liability (although, logically, partial liability is also a possibility). Note that this is a Mishna ruling based on inference; it is not an explicit Torah given.

The pattern of *davka* inference is always like this: if the proof-text specifically mentions case X ("in another man's field," in the present context), and does not explicitly mention cases other than X (i.e. non-X), then it is assumed that the intent of the omission must have been *to exclude* non-X (namely, here, the public domain). This is a common form of reasoning in Talmudic and rabbinic logic. It should be clear that *davka* inference is inductive, not deductive, since it is logically conceivable (though in fact not the case here) that another text might have been found that included non-X without this implying contradiction (i.e. there could well have been another Torah passage specifying that in the public domain, too, there is liability).

Indeed, even if no Torah passage is found that explicitly provides the missing information, it does not follow that *davka* inference is inevitable and sure. An opposite form of reasoning is possible, and indeed is sometimes practiced; it is called *lav davka* (not just so). One could have in the present context, for example[7], argued that

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[7] Needless to say, I am not here advocating the use of *lav davka* reasoning in the present context. I am merely illustrating the form that a *lav davka* reading would have taken in the present

the reason the Torah did not mention an animal eating food in the public domain was because it considered it obvious enough that in such case the animal's owner is liable to pay the food owner full compensation. That is, the argument goes, the Torah only mentioned the case of an animal eating food in the private domain requiring full compensation because it considered that it was not so obvious. In this perspective, anything left unattended in the public domain is 'obviously' protected by law, whereas in the private domain the property owner might well be expected to protect all objects therein, say by fencing or a guard dog; and the Torah comes forth to say: "No, even in the private domain the law must protect unattended objects." Such thinking is quite conceivable; so, *davka* reasoning is not deductive, but merely inductive. Likewise, of course, for *lav davka* reasoning.

The second sentence in our Mishna, then, informs us that if a domestic animal illicitly eats food left unattended in the public domain, the animal's owner is not liable to pay the food's owner for his loss. The third sentence informs us that the protagonist (the animal owner) is nonetheless obligated disburse to the antagonist (the food owner) what feeding his animal *would have* cost him, i.e. the amount of money he saved due to his animal feeding illicitly (presumably, a much lesser amount).[8]

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context. I have no interest in contesting the *davka* reading implied in the Mishna.

[8] The exact basis of this additional ruling by the Mishna is not, as far as I can see, explicitly stated or immediately apparent. It

We thus have two Mishna rulings that seem contradictory at first blush: the first states that there is no liability (but it means: not the full liability occurring in the private domain); the second states that there is some liability (but it means: a minimal liability equal to the usual cost of ordinary feed). Rami focuses on the last sentence to build his argument. The analogy, as he sees it, is as follows:

> Just as, in Mishna *Baba Kama* 2:2, where the animal owner benefits and the food owner does not suffer loss (in a *de jure* viewpoint, because what he did in fact lose was lost in the public domain), the law was that the former is obligated to pay the latter the minimal cost of feeding (even though he is not liable to pay full compensation);
>
> likewise, in the R. Hisda case, where the squatter benefits (since he disposed of no other place) and the landlord does not suffer loss (since he was content to leave the place empty), the law should be that the former is obligated to pay the latter a minimal rent (even though he is not liable to pay full compensation).

Or putting it in standard form (positive subjectal analogy):

> Given that our case (P) is similar to the Mishna *Baba Kama* 2:2 case (Q) in involving the scenario *zeh neheneh ve-zeh lo-haser* (R), and

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could simply be rabbinical fiat. Maybe its basis is obvious to cognoscenti, but I don't know what it is.

> that in the Mishna case (Q) the law was that the protagonist (animal owner) is obligated to pay the antagonist (food owner) the amount of his benefit (S), it follows that in our case (P) the law should likewise be that the protagonist (squatter) is obligated to pay the antagonist (landlord) the amount of his benefit (S).

As we shall see, such argument can be opposed in various ways. The most obvious counterargument to it would be as follows (positive subjectal analogy with a negative major premise):

> Given that our case (P) is *not* similar to the proof-text case (Q) in involving the scenario *zeh neheneh ve-zeh lo-haser* (R), and that in the proof-text case (Q) the law was so-and-so (obligation to pay, or not, as the case may be) (S), it follows that in our case (P) the law should on the contrary *not*-be so-and-so (obligation to pay, or not, as the case may be) (i.e. *not*-S).

Indeed, in the Talmudic narrative under consideration, a third rabbi, Rava, rejects the analogy proposed by Rami, arguing that "in the case of the Mishnah the defendant derived a benefit and the plaintiff sustained a loss, whereas in the problem before us the defendant derived a benefit but the plaintiff sustained no loss." Thus Rava argues (in a more *de facto* spirit than Rami) that in the Mishna the food owner has, objectively, suffered a financial loss (the real value of the food eaten minus the smaller compensation due from the animal owner), whereas in the case at hand the landlord has not

done so (since he would not, in fact, have received rent at that time if his place had not been squatted).

This means that Rava does not agree with Rami that the landlord is due compensation from the squatter. Rava thus proposed the following counterargument, put in standard form:

> Given that the present case (P) is *not* similar to the Mishna *Baba Kama* 2:2 case (Q) in involving the scenario *zeh neheneh ve-zeh lo-haser* (R), and that in the Mishna case (Q) the law was that the protagonist (animal owner) is obligated to pay something to the antagonist (food owner) (S), it follows that in our case (P) the law should on the contrary be that the protagonist (squatter) is *not* obligated to pay anything to the antagonist (landlord) (*not*-S).

According to Rava, then, the scenario of the Mishna referred to is that of *zeh neheneh ve-zeh ken-haser* (= this one benefits and that one *does* suffer loss); and this does not correspond to the putative scenario of the case at hand, which is *zeh neheneh ve-zeh lo-haser*. As we have seen earlier, the Gemara explicitly states that in such case, i.e. where the protagonist benefits and the antagonist suffers loss, the former must indeed pay compensation to the latter. For it is obvious, in its view, that if the squatter had no other premises to occupy and the landlord wished to rent the place at that time, there is indeed need to pay rent[9].

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[9] The Gemara also considers that in the event of 'no benefit for the one and no loss for the other', the former is not liable to pay

Notice that we have come across, here, examples of both a positive argument (similarity between cases) and a negative counterargument (dissimilarity between cases). We thus apparently have, in this *sugya*, examples of two related moods of the argument (analogy and disanalogy of positive subjectal form). Since they involve the same middle term, their major premises are contradictory and they cannot both be valid. Note that Rami and Rava were contemporaries; they were third generation Amoraim (fl. c. 300 CE).

At his point, it should be noted that the Talmud comes to the defense of Rami by means of the following remark: "Rami b. Hama was, however, of the opinion that generally speaking fruits left on public ground have been [more or less] abandoned by their owner [who could thus not regard the animal that consumed them there as having exclusively caused him the loss he sustained, and the analogy therefore was good]." (Note that the explanations given in square brackets in the Soncino edition water down somewhat the position of the Gemara.)

The Gemara is here trying to 'rescue' Rami's argument from Rava's objection by claiming that the food left in the public domain was effectively *hefker*, i.e. mentally given up on by its owner, so that the latter could not blame the animal for its loss; whence, when the Mishna

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the latter. The scenario of 'no benefit for the one and loss for the other' is not addressed in the Gemara, but (I gather from Jacobs' account, n.3) there is a Tosafot commentary about it. Such a scenario is conceivable; one could for instance refer it to a vandal damaging vacant premises.

ruled that the animal owner had to pay a small amount, it was not as compensation for a loss sustained by the food owner (as Rava claimed) so much as payment for the benefit received by the animal owner. In this perspective, then, the Mishna precedent was indeed a case of *zeh neheneh ve-zeh lo-haser* (as Rami claimed) and not a case of *zeh neheneh ve-zeh ken-haser* (as Rava claimed).

The Gemara is here projecting (maybe a couple of centuries later) a thought into Rami's mind that he did not openly express, so as to make him seem to have anticipated Rava's objection and taken it into account. However, the Gemara's intervention turns out to be weak. Jacobs, in an endnote (n.6), informs us of an interesting objection to it by a Tosafist that, in Jacobs' words, "the Talmud cannot mean that the owner has automatically and totally abandoned the food since, if that were the case, there would be no payment at all, the food no longer being his." This observation effectively neutralizes the Gemara's attempted refutation of Rava's counterargument.[10]

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[10] There would be no reason for the animal owner to pay anything to the food owner if the latter did not own the food any longer at the time the animal ate it. Jacobs suggests that perhaps the meaning is "not that he [the food owner] has abandoned the food, but that the Torah has abandoned it in declaring that there is no *shen* [i.e. no liability] in the public domain." However, I do not see any significant difference between the Torah abandoning and the food owner abandoning, since the latter would naturally follow from the former. If the food owner abandoned, it was surely because he knew that the Torah abandoned; if he did not know the Torah (or more precisely, the *davka* inference from it), he would have no

So, this additional discussion turns out to be something of a useless digression. We are left with an argument by Rami and a counterargument by Rava, and we need to know which of the two to prefer. Both seem convincing, at least superficially, and it is hard to choose between them. The Talmud is evidently not wholly satisfied with the arguments of Rami and Rava, or even with its own defense of Rami against Rava, since it goes off looking for other arguments that might more convincingly answer the question put by R. Hisda; but it does not make clear why it does that.

For our part, the following **critical remarks** seem relevant. Please note well that I have no halakhic axe to grind. I am not trying to prove the Talmud, or any rabbi mentioned in it, right or wrong. I do not care what the legal outcome of the discussion might be, though I am of course concerned with the logical propriety or inadequacy of the arguments encountered. My ultimate interest in examining this Talmudic passage is to see what lessons can be learned from it for formal logic (and, as will be seen, I did indeed learn some lessons).

As already shown in our theoretical treatment of conflicting analogy, there is no formal way to resolve the conflict between a comparison and a contrast; formally, either thesis might be right. One has to dig deeper into the problem at hand and try to find reasons

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reason to regard his property as being as good as lost the moment he left it unattended – he would naturally assume or at least hope he would readily recover it upon his return (or else would not leave it unattended). The resulting neutralization of the Gemara's argument is therefore unaffected.

to prefer one thesis or the other. In the discourse under scrutiny, we can certainly point out that one possible flaw is the variable (or ambiguous or equivocal) use of terms. Each of the predicates 'benefits', 'suffers loss', 'is liable', and their negations, although on the surface seemingly uniform in meaning, is in the course of this discussion (and again as it is extended later on in post-Talmudic commentaries) used in selected restrictive ways, which can be characterized as conventional (or even as subjective or as arbitrary).

Thus, the squatter in R. Hisda's narrative is regarded by the Gemara as having 'benefited' only if, when he occupied the premises, he had no alternative place to stay at his disposal; i.e. only if he needed the place he squatted. (Needless to say an invited guest is not a squatter.) But objectively, one could argue that *the mere fact* that the squatter voluntarily occupied that place implies that he considered doing so as of some value to himself (else he would not have done it). In which case, *all* squatting is benefiting somewhat, and *no* scenario involving squatting could be truly said to involve no benefit to the protagonist. The same can be said for the animal owner (in the Mishna referred to): as of the moment his animal has fed, whether in the private or public domain, he has objectively (albeit fortuitously) benefited somewhat.

Again, the landlord is regarded by the Gemara as having 'suffered loss' only if he was actively seeking or at least mentally desired to rent the place out; otherwise, if he was apparently content to leave the place vacant, he is viewed as not having suffered loss. But one could reasonably argue that he has suffered loss by *the mere*

fact that his property was used without his knowledge or permission, even if he was not actively seeking or even desiring to find a tenant (he might perhaps have been keeping the place vacant in case his mother-in-law came to visit). In which case, *all* squatting causes loss, and *no* scenario involving squatting could be truly said to involve no loss for the antagonist. The same thinking applies to the food owner: as of the moment his food has been eaten, whether in the private or public domain, he has objectively suffered loss (even if the law, whether Torah or Mishna, conventionally denies it).

On this basis, i.e. when we insist on *uniform terminology*, both the Mishna case and the R. Hisda case necessarily involve the scenario 'this one benefits, and that one suffers loss' – and Rami is wrong to view them as both *zeh neheneh ve-zeh lo-haser*; while Rava, though partly right in viewing the Mishna case as *zeh neheneh ve-zeh ken-haser*, is partly wrong in viewing the R. Hisda case as *zeh neheneh ve-zeh lo-haser*. Note that it is the Gemara which interprets the squatter as benefiting *restrictively*, only if he had no other premises to occupy, and the landlord as losing *restrictively*, only if he was hoping or trying to rent the place at the time. But since the Gemara's interpretations are restrictive, and it allows for other possible scenarios (notably, 'no benefit for the one and no loss for the other', and eventually 'no benefit for the one and loss for the other'), it is not arguing (as I am here doing) in favor of uniform terminology.

So much for the antecedent scenario (serving as the basis of analogy). As regards the consequent legal obligation (or not), here too we can observe variety in

meaning. In the Mishna, following a *davka* (just so) reading of Ex. 22:4, the animal owner is declared exempt from compensating the food owner for the food lost, although the latter is nonetheless, by additional Mishnaic ruling, required to pay the former the (presumably relatively small) amount he would have had to disburse to feed his animal (had not that animal illicitly satisfied its hunger with the more expensive food it found unattended). Here, then, the protagonist is considered as being strictly-speaking 'not liable', even while he is legally obliged to pay the antagonist something; the smaller amount he is required to pay is not considered as falling under the term 'liable'. This is a conventionally *restricted* use of the term 'liable'[11]. Objectively, of course, any obligation to pay any amount is a liability. In that event, the Mishna's verdict is effectively that there is liability, even if one smaller than it might have been. Whence, in the case brought forward by R. Hisda, the verdict ought to be that the squatter must pay the landlord a minimal amount of rent (the minimum market rate for such a property at that time and place).

Granting all these considerations, it appears that the correct application of the Mishna precedent (taken as a whole) to the case at hand would be that the scenario involved is 'benefit for the one and loss for the other',

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[11] The fiction being that the antagonist cannot, for his loss, make a financial *claim* (on the protagonist); but the protagonist nevertheless has a *duty* to pay the money he saved (to the antagonist). This is a fanciful distinction because, surely, given the latter legal duty, a legal claim could be made in court.

and the resulting legal ruling should be partial compensation[12]. The food owner does objectively suffer loss, and the animal owner is objectively liable to pay something; and the landlord can also be viewed as suffering loss, and on that basis the squatter can be regarded as liable to pay something. In that event, *neither* Rami's argument by analogy *nor* Rava's counterargument by disanalogy can be claimed to be as accurate as they initially seem. Putting our novel thesis in standard form, we obtain:

> Given that our case (P) is similar to the Mishna *Baba Kama* 2:2 case (Q) in involving the scenario *zeh neheneh ve-zeh ken-haser* (R), and that in the Mishna case (Q) the law was that the protagonist (animal owner) is obligated to pay the antagonist (food owner) the amount of his benefit (S), it follows that in our case (P) the law should likewise be that the protagonist (squatter) is obligated to pay the antagonist (landlord) the amount of his benefit (S).

The Talmud does not take into consideration this simple alternative interpretation, based on uniform terminology. From the start of its reflection, it binds

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[12] *Some* compensation is at least implied. The compensation is not *full* because the Mishna has ruled that it cannot be, on the basis of a *davka* reading of Ex. 22:4. But had this Torah passage been read *lav davka*, compensation could well have been full, note. So, the compensation is necessarily *partial*. An additional rabbinical judgment makes it equal to the *minimal* amount the protagonist would have had to disburse had not the events described occurred.

itself to a more complicated approach, from which various logical possibilities arise. Perhaps it opts for this tortuous path because it is not really looking for a solution to the problem (determining a particular legal principle or law) but using the narrative as a convenient occasion to explore different situations and opinions. In that event, it has to keep the issue open and unresolved, even if somewhat artificially, so as to keep the conversation going. (We have seen a clear example of this in the Gemara's gauche attempt to rescue Rami from Rava.) The Talmud's motive is evidently primarily academic and didactic rather than exclusively focused on law-making.

But, so doing, the Talmud misses out on the said additional logical possibility! It never conceives it, let alone propose some credible reason to eliminate it. As we have seen above, the Gemara defines the problem needing solution from the get-go as a search for a precedent in which the protagonist benefits and the antagonist does not suffer loss. It arrives at that putative definition by claiming outright that the two scenarios, in which the former does not benefit and the latter does not suffer loss (for which there would be no liability) or the former does benefit and the former suffers loss (for which there would be liability), are not applicable to the case at hand. And it does not mention or eliminate the third possible scenario (which a Tosafist noticed), viz. that wherein the protagonist does not benefit and the antagonist suffers loss.

The Gemara does not tell us on what basis it has eliminated the said two alternative scenarios it mentions, nor explain why it does not mention the third

possible scenario. Yet it adheres with impressive certainty to the fourth scenario (viz. 'this one benefits and that one does not suffer loss'). Most readers allow such offhand (or sleight of hand) claims to pass uncritically because they believe the Gemara has total knowledge and therefore absolute authority. But surely, if the Gemara resorts to reason at all, it must do so consistently and explain all its positions. It must convincingly justify the certainties it displays.

One can readily agree with the Gemara that a squatter who usually pays rent elsewhere would be liable to pay rent to this landlord too, assuming the latter was looking for or wishing for a paying tenant; but *why* would this liability of the squatter disappear if the landlord was not looking to rent his place out and had not given permission for free occupation of his premises? And *why* would a squatter who could have stayed in a friend's place free of charge not be nonetheless liable to pay rent for staying in this landlord's place uninvited, even if the latter was not looking for or wishing for a paying tenant? The Gemara does not justify its fancy fine distinctions, even though they are far from axiomatic.

Step back a moment and consider the absurdity of the Gemara's claim here in the light of common moral standards. Can it be supposed that a homeless vagrant can freely enter and live in (or otherwise use) premises belonging to a homeowner without the latter's knowledge and permission? Surely that would constitute *theft* of private property, even if temporary and subject to certain conditions (namely, that the squatter could have stayed at other places free of charge

and the landlord's place was currently not up for rent). It would be as surely theft as if a stranger cheerfully 'borrowed' someone's automobile for a while without the owner's okay, arguing that his pals usually let him do that and the car was standing idle! Clearly, the Gemara's claim here is effectively a denial of property rights, and a sanction of gross dishonesty. Maybe in those days social norms were that different, but I doubt it.

The only credible statement, I'd say, is that someone squatting a place without permission is always liable to pay some compensation to the landlord, *irrespective of* any conditions relating to either the one or the other. Indeed, he should additionally be prosecuted for trespass! The Gemara nowhere considers or refutes (as it should have) this obvious proposition. Of course, one can imagine a *force majeure* situation – say someone lost in a snowstorm who comes across an empty, potentially lifesaving, cabin – certainly in such an exceptional situation squatting would be morally acceptable. But the Gemara does not refer its permissiveness to mortal danger.

We could go still further in our critique. So far, we have moved away from the parameters imposed on us by the Talmud by proposing a switch from the given middle term 'this one benefits and that one *does not* suffer loss' to the more accurate middle term 'this one benefits and that one *does* suffer loss', and we consequently accepted – through analogy – 'partial compensation' as the subsidiary term.

But we can go deeper and ask how a squatter, who *intentionally* occupies a vacant home, can be compared to an animal-owner, who allowed his animal to roam freely in the public domain *probably inadvertently*[13]. If we say, by analogy to the given case of the stray animal, that the squatter need only pay a minimum rent for his illicit occupation of the vacant home, are we not enticing him to squat? Are we not telling him: 'you can stay in a five-star apartment and need only pay one-star rent for it'? Clearly, this is quite unfair to the landlord. It means that anyone who does not want to pay the full rent can resort to squatting and get away with a much lesser rent-payment.

The way we got to this absurd conclusion, remember, was by reference to a middle term relating to benefit by the protagonist and loss by the antagonist. But we are not logically forced to restrict our search to this sort of middle term! A completely different parameter or set of parameters might be found for use as middle term, with the aim of obtaining a more equitable conclusion. This is assuming we insist on law-making on this subject (i.e. that of squatting) by means of an argument by analogy from precedent. But this is, of course, not the only way to enact a new law. New laws can be established by majority voting in a legislature, for instance.

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[13] He could, of course, have done so intentionally in the hope that it would find something to feed on out there. In that case, comparison to the squatter is indeed possible. But in the more common case of accidental straying, such comparison is not appropriate.

In short, there are many possible ways for us to 'think out of the box' handed down to us by the Talmud in this matter. We do not need to get mentally bogged-down in the received framework. We can project the goal, then look for the means to it with an open mind. The means, in this context, being one in accord with core Torah principles. And surely, fairness is one of these principles.

It is admittedly very unorthodox to criticize a Talmudic argument without leaving it an escape hatch. Normally, students of the Talmud take for granted whatever it says; and if some 'difficulty' in what it says is found, some convoluted 'resolution' is quickly suggested so as to maintain its overall credibility. But my interest here is not to defend, or even to attack, this document. I am not engaged in 'virtue signaling'. I am just concerned with the logic of the discourse, whatever its purpose or result. My sole intent here is to show that arguing by analogy from a judicial precedent to establish some new legal principle or law is a complex process involving much thought and discussion.

As regards my proposed alternative thesis, viz. that the case under scrutiny (landlord *vs.* squatter) can be derived by analogy from the Mishna case (food owner *vs.* animal owner) through the middle term *zeh neheneh ve-zeh ken-haser* (the subsidiary term then being partial liability), it should be emphasized that I consider this still an *inductive* conclusion. I am not suggesting that it is not open to eventual challenge. There might be some other proof-text or some other inference that belies it or at least surpasses it in credibility. There might, for instance, be analogical argument(s) from some other

Mishna(s), arguing though some other middle term(s) and yielding some contrary conclusion(s). We must then somehow weigh the alternatives and decide which is the most convincing. For example, we might find numerically more reasons that support this conclusion rather than that one. Analogy is inductive, not deductive, argument. It involves trial and error.

The above observations have significance for the formal logic of analogy. An important question they raise is: is an analogy valid if the terms used are analogous only conditionally or in specific instances? There is surely a formal difference between the general term 'benefits' and the narrower term 'benefits under such and such conditions' (for example, 'the squatter benefits provided that he has no alternative lodgings at his disposal'). Likewise, the terms 'suffers loss' and 'suffers loss under conditions so and so' (e.g. 'the landlord suffers loss provided he looked for or at least wished for a tenant') are not equivalent but differ in breadth. Again, the terms 'liable' and 'liable conditionally' (e.g. 'the squatter is liable to the landlord only if the law is that he has to pay as rent the full value of the place, not if he only has to pay a lesser rent) – these are not identical terms.

As we have seen in our earlier theoretical treatment of analogical logic, the mere claim that there is an analogy is not necessarily true, even if made sincerely. There may be ambiguity or equivocation in the terminology (whether done innocently or with intent to deceive) which invalidates the attempted inference. The apparent middle term may not be identical for the major and minor terms, and likewise the subsidiary term may lack

uniformity. Such problems of scope can be overcome under certain precise conditions, but not always.

Let us try and draw a lesson in analogical logic from the Talmudic example. That is, let us determine under precisely what terminological conditions analogy can be claimed and an argument involving it be declared formally valid. We must first determine whether we truly have a major premise with a middle term (R) true of the whole extensions of the major and minor terms (P and Q); and we must also make sure that the subsidiary term (S) is the same in the conclusion (concerning the major term) as it is in the minor premise (concerning the minor term), or if not, determine what the justification for a difference might be. I deal with the purely theoretical aspects of this issue in detail earlier on in the present essay (in section 3) under the heading of 'Terms of unequal breadth'[14].

In the above Talmudic arguments, the putative middle term is the conjunction '*zeh neheneh ve-zeh lo-haser*'. However, as we have just seen the terms 'benefits' (say, K) and 'does not suffer loss' (say, L) may not be used uniformly. The question is: what happens when the putative middle term is a compound (K + L) composed of more specific or conditional elements? And more to the point, what happens if instead of the generic and

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[14] Note for the record that I got involved in the theoretical study of this issue in response to the quandaries posed by the present Talmudic *sugya*. I placed my abstract analysis earlier on in the text to stress its formal significance for all analogical logic, not just for analogy in the Talmud.

unconditional pair of elements K and L, we are faced with more specific or conditional pairs of elements, say K1 and L1 (for the source), or K2 and L2 (for the target). Likewise, what if the subsidiary term, call it M in generic/unconditional form, has different specific/conditional values, say M1 and M2, in the source and target propositions? In such events, our analogical argument would look as follows:

> Source: just as, in the proof-text, where the protagonist has K1 and the antagonist has L1, the law was so and so (say, M1).
>
> Target: likewise, in our case, where the protagonist has K2 and the antagonist has L2, the law must be that so and so (say, M2).

We can reformulate these sentences as if-then propositions, i.e. the source as 'if (K1 + L1), then M1'; and the target as 'if (K2 + L2), then M2'. As we saw earlier on, in our theoretical investigation of positive subjectal analogical argument, these two if-then propositions cannot give rise to a valid analogy if the terms they involve are truly unequal. Precise logical rules are applicable in such event, and they cannot be ignored. Putting the argument in standard (positive subjectal) form, we obtain with the generic terms (K, L, M) the following analogy:

> Given that our case (P) is similar to the proof-text case (Q) in involving the scenario (K + L) (R), and that in the proof-text case (Q) the law was M (S), it follows that in our case (P) the law should likewise be M (S).

But as we shall see, the only valid specific form for this argument is the following ‘from minor to major’ mood:

> Given that our case (P) is similar to the proof-text case (Q) in involving the scenario (K1 + L1) (R), and that in the proof-text case (Q) the law was M1 (S), it follows that in our case (P) the law should likewise be M1 (S).

The proof of this statement is as follows. Here, the operative middle term must be (K1 + L1), because the minor premise has (and must have) the proof-text case (Q) as its subject. As we have learned earlier, in our theoretical investigation of terms of unequal breadth, the effective middle term (R) must be the broader (more generic, less conditional) one. Therefore, the above argument is valid only in cases where (K1 + L1) is broader than or equal to, and includes, (K2 + L2). In which event, of course, (K2 + L2) must imply (K1 + L1), and the specific compound (K1 + L1) is effectively the generic compound (K + L). If these conditions are met, the argument indeed has a functioning middle term and a working major premise. But if on the contrary (K2 + L2) is broader than and includes (K1 + L1), or if those two terms intersect but do not overlap, or do not even intersect, then the argument is invalid, because it lacks a functioning middle term and a working major premise.

As regards the subsidiary term, since the predicate of the precedent Q in the minor premise has to be M1, the predicate of our case P must also be at least M1. It can however also be M2, provided M2 is broader than or equal to, and includes, M1. In which case, M1 implies M2, and the specific term M2 is effectively the generic

term M. In such case, note, we are merely following up the above analogical argument with a syllogistic argument; the analogical argument *per se* is not changed. However, if on the contrary M1 is broader than and includes M2, or if those two terms intersect but do not overlap, or do not even intersect, then the argument cannot conclude with M2 for case P.

It is possible and even likely, given the stringency of these rules of formal logic, that some of the arguments found in the Talmudic *sugya* under consideration, and other narratives, do not constitute logically valid analogies, because they are contrived by means of ambiguities or equivocations, and wrongly treat some specific/conditional (middle and/or subsidiary) terms as generic/unconditional ones. Analogical argument is not arbitrary rhetoric, but reasoning subject to strict law. Wherever these logical laws are disobeyed, the argument is fallacious.

Let us now apply the above formal tests on Rami's analogical argument as explicated by the Gemara. Rami apparently reasoned as follows.

> Just as, in Mishna *Baba Kama* 2:2, where the animal owner benefits (in that his animal has been fed, and he saved the price of feed) (K1) and the food owner does not suffer loss (he doesn't *de jure* by *davka* inference from the Torah, although he does *de facto* as the Mishna admits) (L1), the law was that the former must pay the minimal cost of feeding (by Mishnaic ruling, albeit not obligated to pay the full price

> of food consumed to the latter *de jure* by *davka* inference from the Torah) (M1);
>
> likewise, in our case, where the squatter benefits (but only, according to the Gemara, if he was a habitual tenant and had no other place to go for free) (K2) and the landlord does not suffer loss (provided, according to the Gemara, he was content to leave the place vacant) (L2), the law should be that the former must pay the minimal market value of rent (but is not obligated to pay full rent to the latter) (M2).

Notice that Rami ignores (or puts in brackets) a number of things (specified on his behalf by the Gemara), so as to increase impressions of resemblance. Examining this, it appears as if K1 is broadly intended, while L1 is narrower in scope than it is made to seem (since the word 'loss' is not applied to all loss); as for K2 and L2, they are both clearly conditional (since the words 'benefit' and 'loss' are not applied to all events of squatting). K1 could perhaps be viewed as englobing K2, but L1 certainly cannot do the same for L2 (since the limiting conditions are not similar). Thus, the conjunction (K1 + L1) cannot, as formally required, be implied by (K2 + L2). So, I would say that there is an illicit process in this inference; that is, Rami's argument by analogy (as the Gemara presents it) is formally invalid since it lacks an inclusive middle term. As regards the subsidiary term, M1 is more restrictive than it looks, but its restriction could be passed on to M2 *mutatis mutandis*, so there is no problem there.

We have thus shown, by means of one example, that the Talmud can include invalid reasoning by analogy. This is not surprising for, as already said, analogical argument does have complex theoretical rules not always easy to apply in practice. Anyone might well make errors with it, unless very prepared and very careful. As we have seen, Rava rejects Rami's argument; but he does not do so for the reasons of scope here pointed out. Nor does the Gemara show awareness of these problems, although it tries to shore up Rami's argument in reply to Rava's criticism.

Even so, the Talmud evidently senses, if only vaguely, that there is some inadequacy in the arguments by analogy and disanalogy formulated by Rami, Rava, and even the Gemara itself, with reference to Mishna *Baba Kama* 2:2. This is evident, as already pointed out, from the fact that it goes searching for other possible precedents.

The Talmud next attempts to solve the problem posed by R. Hisda with reference to another case, discussed in Mishna *Baba Batra* 1:3, in which the protagonist is the owner of a field, surrounded on all four sides by fields owned by the antagonist; here again, after a long back and forth discussion, the conclusion is moot. The Talmud then refers to yet another discussion, found in Mishna *Baba Metzia* 10:3, in which the protagonist owns the upper storey of a house, while the antagonist owns the ground floor; and again, the analogies proposed are open to debate and inconclusive. Many more stories, authoritative opinions, and arguments are

brought to bear with apparently no indisputable final conclusion.[15]

This results (as often in the Talmud) in unfortunate prolixity. The central issue posed (viz. whether the apparent scenario of *zeh neheneh ve-zeh lo-haser* implies liability or nonliability) is almost lost in a sea of superfluous detail and the reader's mind easily may lose the thread. We have already suggested that the reason for the Talmud's digressions from the primary issue at hand may be that it sees the discussion as an opportunity to communicate in passing other (loosely associated) information it considers worthy of interest in a wider perspective. It is not trying to get to the point, so much as trying to intellectually scan the area around it.

Another important observation is that the discussion (again, as often in the Talmud) does not always result in clear intermediate conclusions, let alone in a practical terminal result that can be posited as halakha. Some statements end effectively with an ellipsis… their finality is left open. (The effective inconclusiveness of

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[15] R. Louis Jacobs comments (p. 64) that "After the whole *sugya* has eventually arrived at the conclusion that A [the squatter] is not liable, R. Nahman's case is presented for discussion in that, on the surface, it seems to contradict the conclusion towards which the rest of the *sugya* has been leading." But my own impression is that, in view of the mixed *chronology* of the discussion, no definitive final conclusion can really be claimed; if such had been achieved at some point in time, all discussion would have ceased thereafter. Assuming the historicity of the account, it must be ordered chronologically (rather than in a logical or literary progression) to see more clearly and objectively its direction and result.

the Rami-Rava debate is a case in point.) The writer(s) of the Talmud may have thought the unstated conclusions obvious; but it obviously was not so since subsequent commentators (i.e. Rashi, Tosafot, and many others) are forced to try and elucidate the missing information, and often disagree as to what it might be.

In truth, looking at the above example, albeit armed with a formal analysis of analogical argument in general (which the Talmud authors lacked), I do not offhand see any way to definitively solve the particular problem at hand. The various arguments given in this long Talmudic debate all seem, more or less, reasonably credible to me at first reading. But as shown above with reference to the first set of analogical arguments (Rami's and Rava's), and the Gemara's take on them, closer scrutiny may reveal certain flaws in the reasoning. I must therefore regard the different points of view as all having an element of arbitrariness or subjectivity. The contestants put forward interesting arguments in support of their respective viewpoints, but none apparently settles the matter decisively. My guess is that finally, in this kind of situation, a halakhic ruling is imposed by majority or by authority or by traditional practice, rather than by pure logic.

There is in this Talmudic discourse, then, a lot of obscurity, ambiguity, equivocation, and uncertainty, which makes difficult a finite and definite reading, even if it does have considerable value as thought-provoking and educational material. But such deficiencies need not concern us here, since we were not really interested in solving the specific legal problem at hand, but rather sought to observe the use of analogical logic in the

Talmud, and evaluate it by formal means, and perhaps learn lessons from it.

I have here written many pages discussing only the first debate in the present *sugya*. There are many more debates in it, and it would take very many more pages (possibly a whole book) to fully analyze them in equal detail. However, to repeat, my goal here is not to thoroughly analyze the whole *sugya*, but merely to demonstrate through at least one example in it that the Talmud, like many other legal traditions, ancient and modern, near and far, resorts to analogical argument from precedents to derive new legal principles or laws. Having already achieved this goal, I can in good conscience stop the analysis here; indeed, must do so since I would otherwise be ranging too far off topic (namely, analogical logic in general).

10. More about analogy in the Talmud

Based on the above example, and other readings of Talmudic discourse over the years, I think it is safe to say offhand that the Talmud (including both Mishna and Gemara, in both the BT and JT) makes widespread use of such reasoning. But of course, this proposition still needs to be demonstrated by an exhaustive listing and competent detailed analysis of each and every instance of logical discourse in this massive work[1].

I have already pointed out, in my book *Judaic Logic* (2004), the undercurrents of analogical reasoning in some of the 13 *Midot* (hermeneutic principles) of Rabbi Ishmael, notably in the rules called *kal vachomer* (a fortiori argument), *gezerah shavah* (analogy based on homonymy or synonymy), *binyan av* (causal reasoning), and *heqesh*, *semuchim*, *meinyano*, *misofo* (analogies based textual proximity).

Additionally, in my later book *A Fortiori Logic* (2013), I have listed some Torah passages which can be interpreted as analogical arguments, notably Ex. 2:11-

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[1] I hope, and expect, some scholars will eventually dare attempt such an ambitious project; for my part I am already too old to take up the challenge, although I have tried to do a small share of the work.

14 (which suggests *gezerah shavah*) and Lev. 10:9-11 (which resembles *binyan av*). The Nakh (the rest of the Jewish Bible) can be expected to contain many examples, too.

I have written extensively about *kal vachomer*, the first rule of R. Ishmael, in my past works and will not repeat myself here.

The second rule of R. Ishmael, the principle of *gezerah shavah*, which is based on the terms having some Biblical wording or intent in common, may be said to constitute simple analogy. This is because (evident) same wording, or (assumed) same 'intent' of different wordings, do not provide a sufficiently substantive explicit predicate (R) in common to the subjects compared (P and Q). Words are explicit, but they are incidental to what they verbalize; therefore, the assumption that the Torah intends them as significant enough to justify an inference is open to debate.

In other words, the traditional Judaic belief (for some people, a dogma) that names are part of the nature of the things they name, if not their very essence, is – as far as formal logic is concerned – only a theory. There is nothing obvious or axiomatic about it. It is a hypothesis that must remain open to scrutiny and testing like any other. Modern linguistics would deny this hypothesis in view of the demonstrable fact that all languages, including Hebrew, have evolved over time. Things do not change in nature just because we change their names.

In any case, *gezerah shavah* inference suggests an argument by analogy of roughly the following (positive

subjectal) form: since text P and text Q, found in the Torah, are similar in literal wording or in verbal intent (R), then given that Q implies some information S, it follows that P implies the same information S.

This brings to mind *gematria* and other systems of 'numerology' found in Judaism, which compare the 'numerical value' (variously calculated) of two words, phrases or sentences, and regard their equality (or sometimes, near-equality) as a basis of analogical inference. These exegetic techniques seem to date from Talmudic times (some claim earlier), though they were greatly developed later. They are used in haggadic (non-legal) contexts, rather than halakhic ones[2]. I have personally no faith in them[3], and have argued in the

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[2] They are often used as homiletic tools in the synagogue. As they make possible surprising connections between narratives or ideas, they grab the attention of auditors in the way a magical trick would. This seems quite acceptable to my mind, provided it is intended playfully, not seriously.

[3] I have no faith in them as realistic systems of inference. However, I do personally, like many fellow Jews, use a couple of traditional numbers as *merely conventional symbols*. When I see the number 26 (the primary numerical value of the Tetragrammaton), say on a clock, I am by choice habitually reminded of God and of His mercy. Or again, when I give charity I tend to do so in multiples of 26, or alternatively of 18 (the numerical value of *chai*, the Hebrew word for life), with the intent to benevolently wish mercy or life to the recipients. In my mind, these numbers are mere words, constructed arbitrarily out of numerals instead of letters; I do not imagine a real connection between them and their putative objects, nor refer them to a general system of numerology. Therefore, I do not use them (or any others)

past[4] that their probable absurdity could be demonstrated systematically by drawing up a list of the numerical values (under each of the different systems) of every word in the Hebrew dictionary and then grouping all words with the same numerical value together (which should reveal enough contradictory equations, I wager, to dissuade believers).

The third rule of R. Ishmael, the principle of *binyan av*, falls squarely under the heading of complex (positive subjectal) analogy. In fact, our description of complex analogy is an exact description of *binyan av* reasoning. When the rabbis want to extend the scope of a Torah law (S), they show that some new subject (P) has some feature (R) in common with the Torah-given subject (Q), and assuming that this feature is the reason for the law (this assumption constitutes a generalization, even if it superficially may seem to be a direct insight), they carry the law over from the given case to the unspecified case. However, note, sometimes the common ground is not identified explicitly; in which case, of course, the analogy is simple.

As regards the twelfth rule of R. Ishmael, which refers to contextual inferences (*meinyano* and *misofo*, *heqesh* and *semuchim*, and the like), here is how I describe such

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superstitiously as lucky numbers, nor use them for any sort of serious inference.

[4] Over 25 years ago, in *Judaic Logic*, Appendix 3. No one, to my knowledge, has in the meantime followed up on the idea of a systematic study for the purpose of scientific verification or falsification. Not knowing Hebrew well enough, I cannot do the job.

reasoning in my book *Judaic Logic* (chapter 10.2): "All these take into account *the textual closeness* of an expression or sentence to certain other(s), and on this basis assume that there exists a conceptual relation between the passages under scrutiny, which makes possible an inference of certain attributes from the context to the expression or sentence." Inference based on context is simple analogy, since it is without explicit explication.

Contextual inference can be cast as positive subjectal analogy, roughly as follows: since text P and text Q are similar in their being placed close together in the Torah narrative (R), then given that Q implies some information S, it follows that P implies the same information S.

Here, the analogy is based on the incidental fact of location of text relative to other text within a Biblical document, not on any substantive motive. Granting that the Torah is Divinely given or inspired, adjacence of texts is not in itself an incredible basis of analogy; it is a formally acceptable basis. However, there is a problem with it, insofar as contextual analogy is not considered throughout the document, but only evoked selectively, in cases where it is convenient for the justification of some legal principle or ruling. This objection would no doubt be rejected by the rabbis, through an argument that there are surely reasons for the close location of all verses in the document even if we humans are not aware of them all. But this is, of course, an appeal to faith, not a proof.

In some cases, of course, analogy is explicitly proposed in the Torah. For instance, in Deut. 22:26, which compares rape and murder, saying "for as when... even so..." (*ki kaasher... ken hadavar hazeh*). Such analogy is evidently more substantive, and the common ground it suggests might readily be made explicit. It would be interesting to make a listing of all such cases in the whole Jewish Bible (the Torah and the Nakh), as I and others have done for a fortiori argument.

Clearly, analogical argument plays a considerable role in Judaic logic (see my past works for more details and examples). And no doubt similarly in other religious logics, Christian, Islamic, Hindu, Buddhist, and so forth. A lot of work is needed to find all its instances and examine the skill and credibility of each instance. It is also important to know not just the practice of analogy in different traditions, but also just how consciously it is done, i.e. how far each tradition has gone in theoretical reflection on and understanding of what it was doing. In Judaism, we have (as above shown) some theoretical exposition of analogical reasoning in the hermeneutic principles expounded in different lists, although these lists are not as thorough and formal as they could and should have been.

11. Subsumption in analogical terms

Subsumption is the inclusion of a particular instance in a class, or of a narrower class in a wider one. All concept-formation is based on subsumption, and proceeds by identifying the similarities and differences between things, and then grouping together those with certain characteristics in common, and distinguishing them from things without such characteristics. We normally think of subsumption in a positive syllogistic thought process, following Aristotle, and quite rightly, as follows:

> Anything that has certain characteristics B is C;
>
> this item A1 has characteristics B;
>
> therefore, A1 is C.

This informs us that all Bs are C, so that having property B is a *sufficient* condition for the subsumption of an item like A1 under C. The relation of subsumption is stronger in cases where the characteristic B is exclusive to C; that is, where it is additionally given that no non-B are C, so that *only* Bs are C. This implies a negative syllogism, for items like A2 that do not have property B, as follows:

> Anything that lacks certain characteristics B is not C;

this item A2 lacks characteristics B;

therefore, A2 is not C.

In such cases, having the property B is a *necessary* condition, a *sine qua non*, for belonging (as an instance or subclass) to concept C. Where B is both a sufficient and necessary condition for classification as a C, B can be used (if need be) as a defining characteristic of C.

Clearly, all conceptual knowledge is based on this thought process that we call subsumption. Clearly, too, subsumption involves analogical thinking. We can restate the above syllogisms as analogical arguments, forcing things a bit, as follows:

> Given (as above) that A1 has certain characteristics B, it is similar to certain other things which also have B (so far unnamed, call them D); and given (as above) that all B are C, it follows (by syllogism, through all D are B) that all D are C, and thence by analogy that A1 is also C (although we could have obtained the same conclusion syllogistically directly through all B are C, bypassing D). Here, the commonality (viz. that A1 and D have B in common) is the driving force of the (positive) inference.
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> Given (as above) that A2 *lacks* certain characteristics B, it is *dis*similar to certain other things which *do* have B (so far unnamed, call them D); and given (as above) that all B are C and no non-B are C, it follows (by syllogism, through all D are B) that all D are C, and thence

> by *dis*analogy that A2 is distinctively not C (although we could have obtained the same conclusion syllogistically directly through no non-B are C, bypassing D). Here, the distinctiveness (viz. that D has but A2 distinctly lacks B) is the driving force of the (negative) inference.

Note that our casting the two arguments, here, in the form of analogy or disanalogy is somewhat artificial, since the syllogistic path is shorter and clearer (and we are still resorting to syllogism to arrive at the desired results). Not only that, but the two analogical arguments are logically weaker than the two preceding syllogisms, since the syllogisms yield deductive conclusions whereas the analogies yield merely inductive conclusions. However, my purpose here is merely to show the analogical undercurrent of the syllogistic thought; it is not to suggest preference of the analogical statements over the syllogistic ones.

Perhaps if we distinguish between complex and simple analogy the role of such argument in subsumption may be enhanced. There are two kinds of definition in forming concepts. In 'deductive' definition, we form the concept C by defining it from the start through some specified property B; and in such case, the syllogisms shown above are obviously the most natural instruments of subsumption of instances like A1. However, in the case of 'inductive' definition, we do not clearly know at the outset how precisely to define the putative concept C; we sense that there is some property in common and exclusive to certain instances, but we cannot yet say just what it is; so, we coin a term 'C' for a start and then

proceed to gradually look for a definition ‘B’ of it, one capable of subsuming instances like A1 and excluding instances like A2. In the latter case, it is obvious that analogical argument plays a more prominent role, because simple analogy can proceed without explicit specification of the middle term.

My point is just that putting individuals in a class, or subclasses is a wider class, depends on ‘seeing’ the similarities and differences between the items under consideration, and that underlying thought process is manifestly analogical, whether explicitly or tacitly. How this ‘seeing’, or direct insight, occurs is something of a mystery. Indeed, it is a big epistemological mystery; and perhaps, precisely because it is such a fundamental power of human consciousness, it is an unsolvable mystery. But what is sure is that if we could not tell the similarities and differences between things, we could not form any concepts.

Man’s power of abstraction depends on this faculty of insight into similarities and differences; and subsequent conceptualization depends on consciously differentiating, grouping and naming things on its basis. Animals (at least the higher ones) no doubt can likewise tell similarities and differences between things, since they can recognize edible foods or dangerous predators. But in their case, this faculty seemingly does not proceed on to concept-formation, but remains at a relatively concrete level of sensory memories (sights, sounds, smells, tastes, touch-sensations).

12. Analogy and causality

It would be fallacious reasoning to infer from the fact of *similarity* between two phenomena that the former necessarily *caused* the latter. It can and does happen that analogy implies causality, but it is not always the case. A hypothesis to that effect can of course be proposed in any given case; but such hypothesis must then be tested empirically with reference to the applicable rules of causal logic. In other words, the logical status of such inference is at best inductive, not deductive.

To give an example, Wilhelm Windelband, in his *History of Ancient Philosophy*[1], addressing the claim by some historians that Greek thought owed a great deal to Oriental thought in view of observed similarities between some of their doctrines, points out that such commentators "fell into the error of transmuting analogies into causal relations," and justifies his criticism by pointing out that "equally notable disparities[2] might also have been found" (p. 21). In his view, while admitting that some influence no doubt

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[1] New York, N.Y.: Dover, 1956. Trans. By H. E. Cushman, from the 2nd German ed. (1893).

[2] Disparities means disanalogies. What he is saying is that when considering analogies, we should also consider disanalogies, and weigh the resulting impact of both these factors.

occurred, the Greeks significantly improved upon any such external inputs.

We might add that individuals in different cultures, or even the same one, can have similar ideas without having influenced each other or been subjects to a common influence. What occurs in such cases is human intelligence responding in like ways to similar data inputs. There is a common cause, but it is not necessarily the simple cause of mutual or common influence; it is a deeper cause. The same reasoning can be applied in other domains, of course. For instance, in biology if two individuals have a similar physical trait (e.g. eye color), it does not necessarily mean that they are siblings or that they have a common parent, or even a remote distinctive ancestor; there may be some other genetic explanation.

Annex: Conflicting a fortiori arguments

It is interesting to compare and contrast the behavior of conflicting analogies to that of conflicting a fortiori arguments. This concerns a fortiori arguments with different middle terms (R1, R2), and a single subsidiary term (S), with the same extreme terms (P, Q), except that their roles as major and minor terms are reversed.

But first let us briefly compare and contrast the structures of analogical and a fortiori arguments.

- In qualitative argument by analogy, the major premise informs us only of the similarity or dissimilarity of the major and minor terms (P, Q) in relation to the middle term (R); and the minor premise and conclusion tell us of the respective relations of P and Q to the subsidiary term (S), with no mention of R. In quantitative analogical argument, the major premise informs us of the quantitative relations of P and Q to R; but the minor premise and conclusion still make no mention of the middle term R. This is inductive inference: given the said premises, the conclusion is only probable (dependent on certain reasonable assumptions).
- Whereas in a fortiori argument, the major premise informs us of the quantitative relations of P and Q to R; and the minor premise and conclusion tell us of

the respective relations of the terms P and Q to the term S *through the sufficiency or insufficiency of the middle term R*. This is deductive inference: given the said premises, the conclusion follows necessarily (unconditionally).

Thus, the former kind of argument is simpler than the latter. In the former, the major premise may carry less or as much information as in the latter; but in the former, the minor premise and conclusion make no mention of the middle term (R), whereas in the latter they distinctively do. Keep these differences in mind.

Let us now examine conflicting a fortiori arguments.

In the case of subjectal a fortiori argument, we need to focus on the following two arguments (one positive and one negative), whose major premises go in opposite directions, and whose minor premises have opposite polarities, and whose conclusions have contradictory implications:

P is more R1 than (or as much R1 as) Q,
and Q is R1 enough to be S;
therefore, P is R1 enough to be S,
whence P is S.

P is less R2 than (or as much R2 as) Q,
(= Q is more R2 than (or as much R2 as) P,)
and Q is R2 not enough to be S;
therefore, P is R2 not enough to be S,
whence P is not S.

Obviously these two arguments are in conflict, because they yield contradictory final conclusions, viz. P is S and P is not S. Their major premises are not formally contradictory – i.e. two terms P and Q may well be comparatively both greater and smaller in relation to different common features (R1, R2). Therefore, the problem lies in the minor premises of these two arguments – and indeed, one implies that Q is S and the other that Q is not S. So, one or the other (or both) of the minor premises of these two arguments must be false.

Keep in mind that a fortiori argument is essentially unidirectional (unlike analogical argument); this is why the above two subjectal inferences go from Q to P. In the special case where the two above a fortiori major premises are egalitarian (i.e. involve 'as much R'), the minor premise and conclusion are interchangeable (as they are in argument by analogy) in each of the two arguments (i.e. the arguments could equally well go from P to Q as from Q to P). But of course, the minor premises are not arbitrary – they must be proposed as true.

The two arguments would yield the same positive or negative conclusion if the two minor premises have the same polarity. If the two minor premises are positive ('Q is R1 enough to be S' and 'Q is R2 enough to be S'), then the conclusions would both be positive ('P is R1 enough to be S' and 'P is R2 enough to be S'), so that 'P is S' would be implied true *by both*. Alternatively, if the two minor premises are negative ('Q is R1 not enough to be S' and 'Q is R2 not enough to be S'), then the conclusions would both be negative ('P is R1 not

enough to be S' and 'P is R2 not enough to be S'), so that 'P is not S' would be implied true *by both*. In such cases, there is no conflict between the arguments.

However, if one of the egalitarian a fortiori arguments has a positive minor premise and the other a negative one, as may well occur, their implied conclusions ('P is S' and 'P is not S') would be contradictory, and the arguments would be in conflict.

In the case of predicatal a fortiori argument, we need to focus on the following two arguments (one positive and one negative), whose major premises go in opposite directions, and whose minor premises have opposite polarities, and whose conclusions have contradictory implications::

> More R1 (or as much R1) is required to be P than to be Q,
> and S is R1 enough to be P;
> therefore, S is R1 enough to be Q,
> whence S is Q.

> Less R2 (or as much R2) is required to be P than to be Q,
> (= More R2 (or as much R2) is required to be Q than to be P,)
> and S is R2 not enough to be P;
> therefore, S is R2 not enough to be Q,
> whence S is not Q.

Obviously these two arguments are in conflict, because they yield contradictory final conclusions, viz. S is Q and S is not Q. Their major premises are not formally contradictory – i.e. two terms P and Q may well be comparatively both greater and smaller in relation to different common features (R1, R2). Therefore, the problem lies in the minor premises of these two arguments – and indeed, one implies that S is P and the other that S is not P. So, one or the other (or both) of the minor premises of these two arguments must be false.

Here again, keep in mind that a fortiori argument is essentially unidirectional; this is why the above two predicatal inferences go from P to Q. In the special case where the two above a fortiori major premises are egalitarian, the minor premise and conclusion are interchangeable in each of the two arguments (i.e. they could as well go from Q to P as from P to Q). But of course, the minor premises are not arbitrary – they must be proposed as true.

The two arguments would yield the same positive or negative conclusion if the two minor premises have the same polarity. If the two minor premises are positive ('S is R1 enough to be P' and 'S is R2 enough to be P'), then the conclusions would both be positive ('S is R1 enough to be Q' and 'S is R2 enough to be Q'), so that 'S is Q' would be implied true *by both*. Alternatively, if the two minor premises are negative ('S is R1 not enough to be P' and 'S is R2 not enough to be P'), then the conclusions would both be negative ('S is R1 not enough to be Q' and 'S is R2 not enough to be Q'), so that 'S is not Q' would be implied true *by both*. In such cases, there is no conflict between the arguments.

However, if one of the egalitarian arguments has a positive minor premise and the other a negative one, as may well occur, their implied conclusions (‘S is Q’ and ‘S is not Q’) would be contradictory, and the arguments would be in conflict.

Clearly, whereas in the case of conflicting analogical arguments the conclusions disagree even though the major premises are compatible and the minor premises are identical – in the case of conflicting a fortiori arguments the conclusions disagree albeit compatibility of the major premises, because of the contrariety of the minor premises – which, though they mention different middle terms (R1, R2), have opposite implications irrespective of middle term. In subjectal argument these implications are ‘Q is S’ and ‘Q is not S’: while in predicatal argument they are ‘S is P’ and ‘S is not P’. Needless to say, such inconsistency means that one (or both) of the given minor premises must be false.

Works by Avi Sion

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