

Frege's Infinite Hierarchy of Senses

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'Anna believes that Bob Dylan won a Nobel Prize' and 'Anna believes that Robert Zimmerman won a Nobel Prize' may differ in truth-value. How's that possible given that 'Bob Dylan' and 'Robert Zimmerman' refer to just the same person? This is (a version of) Frege's puzzle, one of the most important problems in the philosophy of language. Frege's own solution crucially involves ascribing to an expression not only a *reference* but also a *sense*, a way in which the entity referred to is being presented. In the belief reports above, we are, according to Frege, not so much asserting a relationship between Anna and the unique musician who goes by the names 'Bob Dylan' and 'Robert Zimmerman', but between Anna and the two distinct senses these names express. This solution, when generalized, has been taken to lead to an infinite hierarchy of senses. This infinite hierarchy of senses, in turn, has been taken to render a Fregean language unlearnable. Learning just a single expression, so the objection goes, would require a thinker to apply infinite cognitive resources. Luckily, for Frege, this objection can be resisted. His account, when properly developed, does not require an infinite hierarchy of senses.

Where exactly is the infinite hierarchy supposed to come from? Here's the standard story, endorsed by most commentators from Carnap to Kripke. It has three steps. First, Frege postulates a reference shift for expressions occurring in the 'that'-clause following an attitude verb. This is what allows him to treat such contexts as extensional, a major selling point of his account. In the example above, 'Bob Dylan' no longer refers to Bob Dylan, but to the sense it ordinarily expresses. Accordingly, since 'Bob Dylan' and 'Robert Zimmerman' differ in sense when occurring in ordinary ('direct' as Frege puts it) contexts, they differ in reference when occurring in attitude ('indirect') contexts. That they cannot be substituted for one another *salva veritate* in indirect contexts then no longer counts against the extensionality of these contexts. Second, if expressions shift their *reference* in indirect contexts, they must also shift their *sense*. After all, sense determines reference. So, in the above example sentence, 'Bob Dylan' not only assumes an indirect referent, distinct from its direct referent, but also an indirect sense, distinct from its direct sense. Third, note that we can iterate attitude operators, as in 'Berta believes that Anna believes that Bob Dylan won a Nobel Prize'. If the business of such operators is to induce a shift in sense and

reference, the reasoning continues, then each additional operator must lead to an additional such shift. In a doubly indirect context, then, ‘Bob Dylan’ must refer to the sense it expresses in a singly indirect context. Accordingly, it must express yet another sense (its doubly indirect sense). Further attitude operators will lead to further shifts. Since we can always form a new sentence by prefixing yet another operator, ‘Bob Dylan’ (and, of course, any other expression) ends up associated with infinitely many indirect senses.

Is this a fair price to pay for a simple, extensional account of attitude ascriptions? Some have thought it prohibitively high. If each expression comes with infinitely many senses, then fully mastering even a single expression would take up infinite cognitive resources, so that a language that works along the lines described would be unlearnable (see Davidson (1965, “Theories of Meaning and Learnable Languages”, in *Inquiries into Truth and Interpretation*, 1984, Clarendon Press, 3-15)). However, this argument assumes that the infinitely many indirect senses associated with, e.g., ‘Bob Dylan’ are entirely independent of one another and need to be learned piece by piece. But why should this be the case? Burge (2005, “Truth, Thought, Reason”, Clarendon Press, Ch. 4) and Kripke (2008, “Frege’s Theory of Sense and Reference: Some Exegetical Notes”, *Theoria*, 181-218) have suggested ways of rendering the hierarchy learnable. What their approaches share is the assumption that the n -ly indirect reference of an expression determines its $n + 1$ -ly indirect reference: each direct sense (= singly indirect referent) is presented by exactly one singly indirect sense (= doubly indirect referent), which is presented by exactly one doubly indirect sense (= triply indirect referent), and so on. The hierarchy of indirect senses associated with, e.g., ‘Bob Dylan’ thus doesn’t branch out as we move upwards but forms a single, straight column. On the Burge-Kripke view, there is then only two things we need to grasp in order to understand ‘Bob Dylan’ in any context it may occur in, no matter how indirect. First, we need to grasp the expression’s direct sense, the sense at the foot of the infinite column. Second, we need a (perhaps implicit) grasp of a function or rule that gets us from a given sense in the column to the unique sense one level further up.

I think this approach succeeds in taming the infinite hierarchy. In doing so, it also renders the hierarchy entirely useless though. For recall the theoretical pay-off of the original reference shift. The original shift allows us to say that two expressions which co-refer in direct contexts (‘Bob Dylan’, ‘Robert Zimmerman’) no longer co-refer in (singly) indirect contexts. This works because the direct reference of an expression does *not* determine its (singly) indirect reference. The two names have the same direct reference, but *not* the same direct sense and thus, given the shift, *not* the same (singly) indirect reference. Now, in assuming that the n -ly indirect reference of an expression determines its $n + 1$ -ly indirect reference, the Burge-Kripke approach ensures, by its very design, that no further theoretical pay-off is to be had by any of the further shifts. On their approach, any two expressions which co-refer in singly indirect contexts also co-refer in doubly indirect contexts, and in triply indirect contexts, and so on. While the initial shift from direct to singly indirect reference allows us to draw more fine-grained semantic distinctions, the additional shifts, e.g.

from singly indirect to doubly indirect reference have no such effect. But then why even postulate these shifts at all? Why not simply say that each expression is subject to a one-off reference shift when placed in the scope of an attitude operator while prefixing further such operators has no additional semantic effect on the expression? (See Skiba (2015, “On Indirect Sense and Reference”, *Theoria*, 48-81) for an extended defence of this proposal).

If there is no use for the infinite hierarchy, we should only accept it if, for some reason, we must. Some have thought there to be such reasons. Burge (*ibid.*) takes a rejection of the hierarchy to conflict both with certain principles governing sense composition as well as with the possibility of providing a recursive truth theory with certain desirable features for a Fregean language. But both conflicts can be resolved (see Skiba (2015, 63-75)). The most convincing rational reconstruction of Frege’s theory of attitude ascriptions thus only requires one or two layers of sense, depending on whether one takes the single, one-off reference shift which actually pays a theoretical dividend to be accompanied by a single, one-off sense shift. Either way, the infinite hierarchy of indirect senses can and should be avoided.