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Local & Global Relativity Principles

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1. Three Relativity Principles

What is the special principle of relativity? One rough statement of the principle is

(1) Inertial, or unaccelerated, states of motion are observationally indistinguishable.

We can make the principle clearer by expanding on what "observationally indistinguishable" means. On one way of understanding this term, (1) is equivalent to

(2) If the same experiment is conducted in isolated laboratories that are both moving inertially, the outcomes of those experiments must be the same.

But don't take this talk of "experiments" in isolated "laboratories" too seriously. It is just a way of talking about isolated *physical processes*. Two particles orbiting their common center of mass far away from other matter may constitute an experiment conducted in an isolated laboratory, even though there is no physical laboratory housing them and no experimenter making measurements. Similarly, the outcomes of two experiments are the same just in case the physical systems that constitute the experiments are in the same final state, whether or not anyone could figure this out by making measurements. (By "final state" I mean "final state, intrinsically characterized"; two processes can be in the same final state even if, for example, they are moving at different velocities, or occurring in different countries.)

Even with this precisification of "observationally indistinguishable," there are two ways in which (2) is unclear. First, I haven't said anything about which kinds of experiments we're talking about. According to some histories of the principle of relativity, Galileo accepted something like (2) but limited it to "mechanical" experiments, while Einstein accepted something like (2) but extended it to all experiments, including experiments involving electromagnetic phenomena.[1] I am not

1. See [Brown and Sypel 1995: 238] and [Brown 2005: 33–36] for dissent.

interested in this difference between principles of relativity, and so in this paper I will focus on unrestricted principles that apply to all kinds of experiments.

There is a second way in which (2) is unclear. It contains "must," a modal term; but there are different ways the quantifier over experiments can interact with the modality. On one reading (maybe the most natural reading) of (2), we have

> (3) It is physically necessary that if the same experiment is conducted in isolated laboratories that are both moving inertially, then the outcomes of those experiments are the same.

This is an intra-world principle. It says that *within* any one physically possible world, outcomes of the same experiment type are the same. There is also an inter-world principle. This principle is most easily stated using possible-worlds talk:

> (4) If the same experiment is conducted in isolated laboratories that are both moving inertially, in the same or different physically possible worlds, then the outcomes of those experiments are the same.

Obviously, (4) entails (3).

(3) and (4) are principles that presuppose determinism. They presuppose that the laws and the state of the world at an instant (or on an appropriate spacelike hypersurface) fix a unique future. Stochastic laws that do not guarantee that (3) is true, or that (4) is true, can still respect the principle of relativity. It is enough for the laws to assign the same probabilities to outcomes in the two experiments. But throughout this paper I will limit my attention to deterministic laws.

(3) and (4) are also local principles. They say things about particular experiments, particular subsystems of the universe. (That is just what I mean by "local principle." That one of these local principles is true has nothing to do with locality, in another sense; it has nothing to do with whether there is any action at a distance. But the relationship between local principles and action at a distance will come up in section 3.) There are also global relativity principles that apply only to the universe as a whole. Roughly speaking, according to the global version of the special principle of relativity, if you take the entire material content of a physically possible world, and speed it up by the same amount in the same direction at each time, you end up with another physically possible world.

For a more formal statement we need some definitions. An *inertial frame* is a congruence of parallel timelike geodesics in spacetime. Think of an inertial frame as a set of worldlines of inertial observers at relative rest who fill all of space. Now suppose for simplicity that we're working with theories set in Minkowski spacetime. Then the Lorentz transformations are (some of the) symmetries of spacetime. The Lorentz transformations preserve inertial frames: they map inertial frames to inertial frames, and for any two inertial frames there is a Lorentz transformation that maps one to the other.

We need one more definition: a definition of "dynamical symmetry of T," where T is a physical theory. This term is usually defined in terms of the models of a spacetime theory. For theories set in Minkowski spacetime the models have the form $\mathrm{M} = (M, \mathrm{h}_{ab}, O_1, O_2 \ldots)$ where M is a differentiable manifold, h_{ab} is a flat Lorentz metric on M, and the O_i are geometrical objects that represent the material contents of spacetime. Then a diffeomorphism φ that maps M to M induces a map φ^* from geometrical objects on M to geometrical objects on M, which in turn allows us to define a map from models to models: given M, M_φ is $(M, \mathrm{h}_{ab}, \varphi^*O_1, \varphi^*O_2, \ldots)$. Then φ is a dynamical symmetry of T if for every model M of T, M_φ is also a model of T.[2] With this machinery in place, our global principle of relativity is

> (5) If φ is a Lorentz transformation, then it is a dynamical symmetry of all true physical theories.

2. I follow the notation in [Earman 1989: 45–46].

Now, the principle of relativity is a modal principle. With the definitions I have given, though, (5) is merely a model closure principle. Facts about what the set of models of a theory looks like do not automatically entail anything about what the theory says is possible. For (5) to have modal content we need to make some connection between facts about the theory's models and what the theory says is possible. How you make this connection can depend on your metaphysical commitments. Substantivalists, who believe that spacetime exists, may want to say that distinct models represent distinct possibilities. Relationalists, who deny that spacetime exists, may want to say that pairs of models like M and M_φ (where φ is a Lorentz transformation) represent the same possibility. (In general, relationalists may want to say that two models that are related by a diffeomorphism represent the same possibility. Some substantivalists — "sophisticated substantivalists" — say the same thing.)[3] I am a substantivalist, and not a sophisticated one, so I will assume in this paper that M and M_φ represent distinct possibilities. With this assumption in place, (5) becomes a modal principle. The possibilities that M and M_φ represent differ in the way I mentioned when I gave an informal statement of the global principle of relativity: according to M_φ the material contents of the universe are moving faster by some fixed amount in some fixed direction (relative to a given frame) than they are according to M, while everything else is the same.

(5) does not purport to capture the whole content of the special principle of relativity. It only covers the special case where spacetime has a Minkowski geometry. (The special principle of relativity is still true in Newtonian mechanics!) In Newtonian and Galilean spacetime it is the Galilean transformations that preserve inertial frames. In those spacetimes the special principle requires the Galilean transformations to be dynamical symmetries. So roughly speaking, the general version of (5) would say that whatever spacetime is like, the transformations that preserve inertial frames are dynamical symmetries.[4]

(4) entails (5) as a special case. The state of the entire universe at an instant (or on a hyperplane of simultaneity)[5] is an experiment in an isolated laboratory, the outcome of which is the state of the universe at a later instant (that is, on a later hyperplane of simultaneity that belongs to the same foliation of spacetime by hyperplanes of simultaneity as the original one). Then the proof goes as follows. Suppose that f is a Lorentz transformation and that w is a physically possible world.[6] I need to show that w_f is also physically possible. Choose any instant H_0. (So H_0 is a region of spacetime.) Then $f(H_0)$ is also an instant. Let w' be the unique physically possible world in which the state on $f(H_0)$ is the same as the state on H_0 in w. (So, for example, there is a particle at point p on H_0 in w iff there is a particle at point $f(p)$ on $f(H_0)$ in w'; the state of some field on H_0 in w is given by the tensor T iff the state of that field on $f(H_0)$ in w' is given by f^*T; and so on. That w' is unique follows from the fact that the laws are deterministic.) By (4), for each *later* instant H in the same foliation as H_0, the state on H in w matches the state on $f(H)$ in w'. That establishes that H in w matches $f(H)$ in w' for H later than H_0. But the difference between the past and the future is not intrinsic to spacetime, so we can run this argument in both

3. Earman and Norton [1987] connect the substantivalism debate to these different ways of interpreting models. On sophisticated substantivalism see [Belot and Earman 2001] and [Pooley 2006].

4. See [Budden 1997: 485–489] for a more detailed statement of the general version of (5). This approach to stating the principle of relativity goes back to [Anderson 1967]; see also [Earman 1974], [Friedman 1983], and [Norton 1993: 831–840]. Note that some of these authors require (as I do not) that transformations between frames be symmetries of spacetime. On their view, no theory set in Newtonian spacetime satisfies the global version of the special principle of relativity.

5. When I am discussing Minkowski spacetime I will always use "instant" to mean "hyperplane of simultaneity."

6. Given the assumptions I made in the text, the models of a theory just are possible worlds, on one respectable way of using "possible world": they are abstract representations that specify ways the world might have been. Sometimes it is more convenient to use possible worlds talk, so I will freely switch between the two modes of speech.

temporal directions. It follows that $w' = w_f$, and so that f is a dynamical symmetry.

I have now stated the versions of the principle of relativity that I want to talk about in this paper.[7] Among them, (5) is regarded as *the* correct modern formulation of the special principle of relativity.[8] It is regarded this way because of its connection with another common way of stating the special principle of relativity: as the claim that the laws of nature take the same form in all inertial frames. (This is how Einstein states it in his famous 1905 paper [1952].) As is well known, this way of stating the principle gets into trouble: even theories that violate the spirit of relativity can be given a generally covariant formulation, so that the equations of motion take the same form in any coordinate system, including any coordinate system adapted to an inertial frame. It was proposed that (5) captures what was intended by the talk of laws taking the same form.[9]

Let me repeat that all the principles I have stated are versions of the special principle of relativity. The general principle of relativity is, roughly, the claim that all states of motion, not just inertial states of motion, are observationally indistinguishable. I will not be interested in the general principle of relativity.[10]

As I said, (5) is regarded as the correct modern formulation of the principle, but you might worry that it does not capture the entire content of the principle of relativity, as intuitively understood. These worries are driven by the suspicion that (5) does not entail either (3) or (4).[11] Galileo's statements about relativity are best understood as affirmations of (3). A statement of the principle of relativity that did not capture what Galileo was saying would surely be leaving something out. In addition, since our direct empirical evidence is evidence about what goes on in the actual world, that evidence directly supports (3), not (5). So if (5) does not entail (3), we are not doing enough to make our theories relativistic if we merely ensure that they are compatible with (5). Finally, if (5) does not entail (3), then it is not clear what empirical content (5) really has. It merely says that if a certain world is possible, then so is a world that is qualitatively indiscernible from it.[12] But the principle of relativity certainly has some empirical content.

If (5) does not entail (3) then these complaints about (5) may have some force. My aim is to defend (5) against those who would demote it. To dispel the doubts I will argue that (5) does, in fact, entail (3). I will also explain why a counterexample to this entailment offered by Budden [1997] is not genuine.

2. The Proof

The proof of (3) from (5) begins like this. Suppose that *w1* is a physically possible world with the following features (see figure 1). There are two experiments of the same type conducted in isolated, inertially moving laboratories. (A laboratory may be isolated at some times and not at others; I mean that these laboratories are isolated while the experiments are being conducted.) But the labs are not at rest relative to each other. The experiments *E1* and *E2* have outcomes *O1* and *O2*, respectively. Maybe in both labs the experimenters time how long it takes a marble of unit mass to fall one meter. Maybe the experiments are much more complicated.

7. The fact that there were different versions of the principle of relativity was discussed in the early 20th century; see [Treder 1970] and the references therein. Treder mentions principles much like (4) and (5) and notes that (4) entails (5), but seems to deny the reverse entailment.

8. See the references in footnote 4.

9. See [Norton 1993] for the history.

10. Leibniz seemed to accept the general principle; see his letter to Huygens dated 12/22 June, 1694 (printed in [Stein 1977: 39]). As the history of relativity theory is usually told, Einstein thought that the general principle of relativity was true in the general theory of relativity, but he was wrong; Norton [1993] tells the story.

11. See [Brown and Sypel 1995] and [Budden 1997] for expression of the worries that follow.

12. Assuming that spacetime is Minkowskian. If the worlds contain Newtonian spacetime, and facts about absolute velocities are qualitative facts, then the worlds are not qualitatively indiscernible.

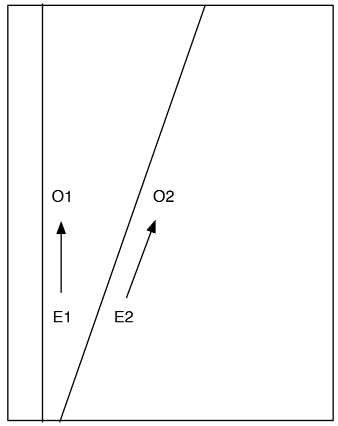

Figure 1: World 1

We need to show that *O1* and *O2* are the same type of outcome (that, for example, each experimenter recorded the same amount of time for his marble's fall), assuming that (5) is true. Since the frames and experiments chosen were arbitrary, this will establish (3).

Now, if (5) is true, then *w2* is also a physically possible world (see figure 2). It differs from *w1* by a boost.

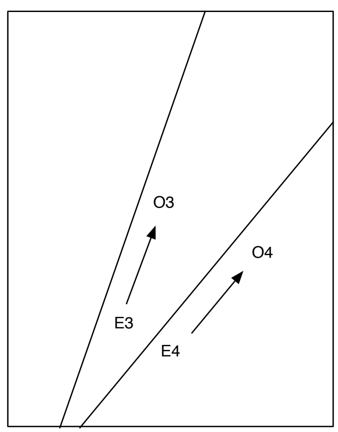

Figure 2: World 2

In *w2* two experiments, *E3* and *E4*, are conducted in isolated, inertially moving laboratories. These experiments are the same type as the experiments conducted in *w1* — *E3* matches *E1* and *E4* matches *E2*. Also, the outcomes of the experiments in *w2* match the outcomes of the corresponding experiments in *w1*. In particular, *O3* is the same type of outcome as *O1*. (By using the labels "*E3*," "*E4*," and so on, I wish to leave it open whether the experiments and outcomes in *w2* are numerically identical to the experiments and outcomes in *w1*, or just of the same type. For my purposes, it does not matter which of these is true.) Finally, *E3* takes place, in *w2*, in the same region of spacetime (and so the same state of inertial motion) that *E2* takes place in, in *w1*. (That is, the laboratories for *E2* and *E3* occupy the same region of spacetime, and the experiments are conducted in corresponding parts of the laboratories; it is left open, so far, whether *E2* and *E3* have the same outcome.)

At this point you might worry: these three facts — that *E3* is an experiment of the same kind as *E2*, that *E3* happens in the same region of spacetime in which *E2* happens (though in a different possible world), and that *O1* and *O3* are the same type of outcome — do not entail that *O1* and *O2* are the same type of outcome. But we have already made use of (5) in our proof. So there doesn't seem to be anywhere else to go from this point.

There also seems to be a principled reason why we cannot go on. (5) is a conditional principle: it says that if a world is permitted by the laws, then some other world is also permitted by the laws. But it appears to place no unconditional constraints on which worlds are physically possible. It places no constraints on what goes on within any one physically possible world. So how can it entail (3), which does place constraints on what goes on within each world?[13]

It is true that we have not yet proved that *O1* and *O2* are the same type of outcome. But the proof can be completed. We have overlooked the fact that these experiments occur in isolated laboratories. The fol-

13. Budden [1997: 491] offers the apparent inability to go on from here as a reason to think that (5) does not entail (3).

lowing is a necessary truth about isolated laboratories in deterministic worlds:[14]

(6) If the same type of experiment is conducted in the same region of spacetime in isolated laboratories in two physically possible worlds, then the outcomes of those experiments are the same.

After all, part of what it means for a laboratory to be isolated is for the outcome of an experiment in that laboratory not to depend on what is (or is not) going on outside of the laboratory. Note that (6) is not a relativity principle. It is compatible with (6) that some experiment can distinguish between different states of inertial motion. Of course, (6) does *follow from* a relativity principle: it follows from (4). But my support for (6) comes from the meaning of "isolated laboratory," not from (4).

I want to postpone a more complete justification of (6) until section 3 and see how the proof proceeds once we accept (6). It is (6) that allows us to get local results from a global principle. *E2* and *E3* are the same type of experiment conducted in isolated laboratories; they occur in the same region of spacetime; so using (6) we can conclude that *O3* and *O2* are the same type of outcome. Since I've already shown that *O3* and *O1* are the same type of outcome, it follows that *O1* and *O2* are the same type of outcome — which is what was to be shown. So (3) follows from (5). A similar argument using (6) shows that (4) also follows from (5). (Since (4) also entails (5), this means that (4) and (5) are equivalent.)[15]

14. Or, at least, it is a necessary truth about isolated laboratories in flat spacetimes, which is the kind of case I am interested in.

15. This argument was inspired by Sider and Paull's [1992] argument that global supervenience is equivalent to strong supervenience for intrinsic properties. They use a principle of recombination to get from a global principle to a local, intra-world principle. My (6) plays a role similar to that played by their principle of recombination. See also [Moyer 2008].

3. Justifying (6)

I gave a brief justification of (6) in the previous section. Here I want to discuss its justification in more detail.

(6) is a necessary truth about isolated laboratories in deterministic worlds because it follows from the definition of "isolated laboratory" and the fact that the laws are deterministic. But giving an adequate definition of "isolated laboratory" is a little tricky. The difficulties can be sidestepped for one special case of (6), and so I will look at that case first. If there is no action at a distance, then one way for a laboratory to be isolated is for it to be far away from everything else, out in empty space. That is, satisfying the following two conditions is sufficient for a laboratory *L* to be isolated:

(7a) There is an open region of spacetime *R* such that *L* occupies a subregion of *R* and *R* is otherwise completely empty;

(7b) The laws are local. For a field theory, this means that the laws pick out the dynamically possible fields on spacetime by giving conditions on the state of the field and (finitely many of) its derivatives at each spacetime point. Then the laws have the *global to local* property: we can restrict the laws to any open region of spacetime *U*; then if *w* is a world permitted by the original laws, the state on *U* in *w* is permitted by the restricted laws.[16]

If we restrict our attention to laboratories that are isolated by satisfying conditions (7a) and (7b), then (6) follows immediately. For suppose that *L1* and *L2* are two laboratories satisfying the hypotheses in (6) in the same region of spacetime in different physically possible worlds *w1* and *w2*. By (7a) we can find an open region *U* small enough that it contains *L1* in *w1* and contains *L2* in *w2* and is otherwise completely empty in both worlds. Then by (7b) the state on *U* in both *w1* and *w2* satisfies the laws when restricted to *U*. But the state on *U* in *w1* and the

16. This definition resembles Earman's [Earman 1987: 455] locality principle (L6). (Thanks here to Gordon Belot.)

state on U in $w2$ agree at the initial time (the same type of experiment is carried out in $L1$ and $L2$); since the laws are deterministic, it follows that the state on all of U is the same in both worlds. But then the outcome of $L1$ in $w1$ is the same as the outcome of $L2$ in $w2$.

But (7a) and (7b) are not necessary for a laboratory to be isolated. Suppose that (7a) fails: the laboratory is surrounded by all sorts of other physical goings-on. The laboratory may be still isolated if those other physical goings-on do not affect what happens in the laboratory. (Perhaps the laboratory is surrounded by a kind of matter that does not interact with any of the equipment in the laboratory.) Or suppose that (7b) fails: the laws are non-local and so allow for action at a distance. A laboratory far away from everything else, out in empty space, may still be isolated, if no distant causes ever act on it.

I am afraid that defining "isolated laboratory" so that it covers these cases will require me to use the word "cause" in the definition. It is a messy business, finding a causal definition of "isolated laboratory," but it will be important for my discussion of proposed counterexamples to the claim that (5) entails (3). So let us proceed. Things work best if I explicitly take into account the fact that a laboratory may be isolated for some parts of its career and not for others. So I will give necessary and sufficient conditions for a laboratory to be isolated while it occupies some spacetime region R. Intuitively, a laboratory is isolated while it is in R just in the case that, once the laboratory has entered R, nothing outside the laboratory has any influence on what goes on inside the laboratory. Now, since R is a region the lab occupies during one stage of its career, R has an "initial time": there is some (flat) spacelike hypersurface S that overlaps the boundary of R, with the rest of R on the future side of S. We are interested in the outcomes of experiments conducted in isolated laboratories; this initial time is the time at which the experiment begins. I propose that the laboratory is isolated while it occupies R if and only if it meets this condition:

(8) If for any events C and E, (*i*) E is an event that occurs inside the laboratory while the laboratory occupies R; (*ii*) C is a cause of E; and (*iii*) C occurs outside of R; then C is a cause of E only because it is also a cause of an event that occurs inside the lab at the initial time of R.

That is my precisification of "isolated laboratory" that uses causal language. It covers the examples I gave of isolated laboratories that do not satisfy (7a) or (7b). The laboratory surrounded by matter it does not interact with is still isolated, because no event involving the surrounding matter is a cause of any event in the laboratory. And the laboratory in the world with non-local laws is still isolated because no far away event is a direct cause of any event in the laboratory (after the "initial event," that is).

I proceed now to the argument that (6) is true of any pair of laboratories that satisfy (8). The argument requires some way to connect laws and causation. The intuitive idea is simple: in a deterministic world, the set of all causes of an event determines what that event is like. A more useful statement makes use of the concept of a *complete set of causes* of an event.[17] A complete set of causes of an event is just a set of causes of that event such that *every* cause of that event is a cause of a member of that set. The idea is that the events that occur inside an isolated laboratory from the beginning of an experiment onward form a complete set of causes of the outcome of that experiment: by (8), any other cause of the outcome is a cause of the event that began the experiment.

My principle connecting laws and causation, then, is:

(9) If (*i*) $E1$ and $E2$ occur in the same region of spacetime in different possible worlds with the same deterministic laws, and (*ii*) there is a complete set of causes for $E1$, and a complete set of causes for $E2$, and a correspondence between the sets of causes such that corresponding causes are the same type of event and occur in the same region of spacetime, then $E1$ and $E2$ are the same type of event.

17. I take this phrase from [Lange 2002: 13–17].

(To avoid obvious counterexamples, "type of event" here must mean "maximally specific type, intrinsically characterized." I require the events to occur in the same region of spacetime to avoid begging any questions. (9) is a premise in a proof of a relativity principle and so had better not presuppose it.)

I take it that (9) is a necessary truth. Then (8) and (9) together say that once the role of the events that occur inside an isolated laboratory has been taken into account, no other events are relevant to determining the outcome of the experiment. Even if there were no other events anywhere else — even if the rest of the universe consisted only of empty spacetime — the outcome of the experiment would be the same. It is straightforward to see that (8) and (9) entail (6).

That completes my proof that (5) entails (3). But my use of causal language in the justification of (6), though necessary, comes with a price. The causal facts are not fundamental facts, and the fundamental facts at some world may leave it indeterminate just what the causal facts are at that world. And if it is indeterminate what the causal facts are in some world, then it may be indeterminate whether a given laboratory in that world is an isolated laboratory. So it can be indeterminate whether (3) is true at some world. (5), though, is not infected with indeterminacy (or, at least, not in the same way). Perhaps, then, there are cases where (5) is true in some world but it is indeterminate whether (3) is. (We will see a case like this in the next section.) When I say that I have proved that (5) entails (3), then, I mean to claim that in every world in which (5) is true and (3) is either determinately true or determinately false, (3) is determinately true.

4. Alleged Counterexamples

Although I have given a proof that (5) entails (3), some doubts might remain that (5) really does entail (3). In this section I discuss some apparent counterexamples to the entailment.

We might try to construct a counterexample to the claim that (5) entails (3) by brute force.[18] Start with just one possible world in which (3) is false. In that world, $E1$ and $E2$ are the same type of experiment, both experiments are in inertial motion, but they have different outcomes. (A spacetime diagram of this world will closely resemble the diagram of world 1 in figure 1.) Then consider the physical theory according to which just that world and all Lorentz boosts of that world are physically possible. (3) is false but (5) is true according to this theory, since Lorentz boosts are dynamical symmetries by construction.

I think there's something fishy going on here. We have picked out a set of possible worlds; but is there really a physical theory according to which all and only those worlds are physically possible? There is if just any set of possible worlds (or any set of models) constitutes a physical theory — but is that claim true? What about the set containing just the actual world? If *that* set constitutes a physical theory, shouldn't it be the best, most interesting one? (Surely it is not.) It sometimes seems to me that if T is a physical theory it must be possible to express T by giving the laws of T, including the dynamical laws. After all, what is a physical theory but a statement of some (possible) laws of nature? And it also seems to me that there are no dynamical laws that are true just in the actual world. Perhaps, for similar reasons, there is no theory according to which just the worlds mentioned in the previous paragraph are physically possible. If so, then we have not actually found a theory according to which (5) is true and (3) is false.

But I do not want to rest much weight on this objection. Perhaps there is a weak sense of "dynamical laws" according to which for any set of worlds, there are laws that are true in just the members of that set. I do not want to try to argue for an account of laws of nature according to which this is not so. (Still, the fact that it is difficult to make sense of these worlds' laws will continue to make trouble.)

I have a better objection. Suppose there really is a theory according to which the possible worlds mentioned are the physically possible worlds. Even so, I don't think that (3) is false in that theory. Instead, I think that (3) is vacuously true in that theory. It is vacuously true because there are no isolated experiments in those worlds. For consider: in those worlds, it is *physically necessary* that if experiment $E1$ is conducted, then $E2$ is also conducted. It hardly seems that $E1$ is being

18. I learned this technique from [Arntzenius 1994].

conducted in isolation if it is impossible to conduct it without E2's also being conducted.

That is what I believe. But, unfortunately, I do not think it can be conclusively established that *E1* is or is not isolated. One way to show that *E1* is isolated is to show that it meets the sufficient conditions (7a) and (7b). But it does not meet those conditions: the laws do not satisfy (7b). They are not locally deterministic.

(One might try to argue that *E1* is isolated because the laws factor into locally deterministic dynamical laws, and an additional, strange law that limits the admissible initial conditions. But this cannot work. If there were such locally deterministic dynamical laws, they would be Lorentz invariant. So my argument would apply to show that (3) is true relative to those dynamical laws, considered apart from the law restricting the initial conditions. But then (3) is also true in any theory that adds a restriction on initial conditions to those dynamical laws.)

Another way to show that *E1* is or is not isolated is to inspect the causal facts in that possible world and use (8). But I cannot figure out what the causal facts are in enough detail to arrive at a verdict. I suspect that this is because the causal facts are just not determinate enough to yield a verdict. Is any event in experiment *E2* a direct cause of any event in *E1* (that occurs after the initial time)? One way to figure out whether there is a causal connection between them is to survey the events in *E1* and *E2*, trying to find events *A* and *B* in them that make true the counterfactual: if *B* had not occurred but experiment *E1* had still been started in the same initial state, then *A* would not have occurred. But I do not know whether any of these counterfactuals are true or false. I do not know how to evaluate them. The problem, again, is that it is not clear what the dynamical laws of this possible world are. So it is very difficult to say how things would have evolved forward in time in *E1*, had things been different in *E2*.

I conclude, therefore, that this first example is not a clear counterexample to the claim that (5) entails (3). Now, this first example is simple, artificial, and contrived. Other, more complicated and less contrived counterexamples have been proposed. But they suffer from essentially the same defect. In fact, it is even clearer that these other examples fail. Since they are more detailed there is less room for the kind of indeterminacy that makes the first example difficult to evaluate.

Tim Budden [1997] proposes a theory according to which (he alleges) (5) is true but (3) is false. His theory is set in a novel spacetime structure. Start with Minkowski spacetime, then remove one piece of structure and add another: remove the built-in unit for (spatiotemporal) length by allowing dilations to be symmetries of the spacetime; then add a preferred congruence of parallel null lines *N*. (Adding this congruence removes some symmetries from the structure.) This spacetime cannot be represented using a metric on a manifold, since a metric provides a built-in unit for length and so is not invariant under dilations. Instead this spacetime can be represented using the methods of synthetic geometry. In standard synthetic presentations of flat relativistic spacetime, the spacetime structure is characterized using just one intrinsic relation on spacetime points: *x and y are lightlike related*.[19] Using this relation we can define the standard spatial congruence and temporal congruence relations. (Intuitively, in metrical terms, *x* and *y* are spatially congruent to *z* and *w* iff *x* and *y* are spacelike related, *z* and *w* are spacelike related, and the spatiotemporal interval between *x* and *y* is the same as between *z* and *w*. There is a similar intuitive explanation of temporal congruence. But these are not the official definitions of these relations in terms of lightlike connectability.)

Budden uses the extra structure in his spacetime — the congruence of null lines *N* — to define another temporal congruence relation, which he calls "$\sim_{-1}$." First he defines $\sim_{-1}$ for pairs of pairs of points *(a,b)*, *(a,c)* that have a point in common: $ab \sim_{-1} ac$ iff *a* is timelike related to *b* and to *c* and "*b* and *c* lie on the same lobe of a light cone which has as apex a point on the element of *N* which passes through *a*" (498). Figure 3 shows the set of points that are as far into the future of *a*, according to $\sim_{-1}$, as *b* is, in a two-dimensional spacetime. The set of points that are as far into the future of *a* as *b* is, according to the *standard*

19. The classic presentation is in [Robb 1914]. A more recent presentation is [Winnie 1997].

definition, forms a hyperbola; this set forms a ray. (The element of *N* that passes through *a* is the purple lightlike geodesic leaving *a* towards the right. Budden extends his definition of $\sim_{-1}$ to cover pairs of pairs of points that do not have a point in common, using parallelograms; but I will not need the extended definition.)

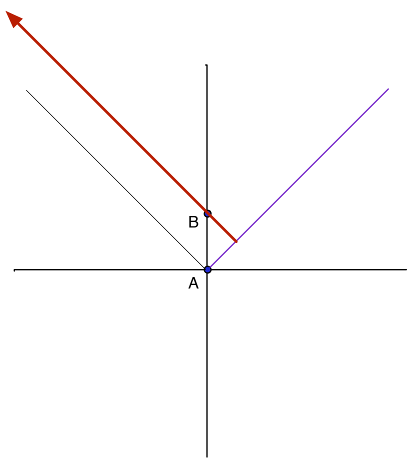

Figure 3: The set of points x such that $ax \sim_{-1} ab$, shown in red

That is the spacetime in which Budden's theory is set. Now for the theory's laws. In that theory, measuring rods behave just as they do in ordinary Minkowski spacetime. But (ideal) clocks do not. The law governing the behavior of clocks is:

> If two (ideal) clocks move inertially, one from point *a* to point *b*, the other from point *c* to point *d*, then the same amount of time elapses according to both clocks if and only if $ab \sim_{-1} cd$.

(Clocks also observe obvious laws like: if a clock moves inertially from *a* to *c* and *b* is between *a* and *c* then more time elapses according to that clock between *a* and *c* than between *a* and *b*.)

That is the theory Budden proposes as a counterexample. Here is the argument that (3) is false according to this theory. We are going to perform an experiment to measure the round-trip speed of light, in two different inertial frames. We do this by attaching a mirror to the end of a measuring rod, and timing how long it takes light to get from one end of the rod to the other and back. Figure 4 is a spacetime diagram of these two experiments: the first experiment occurs at rest in this diagram, and the other is moving at half the speed of light, relative to the first experiment.

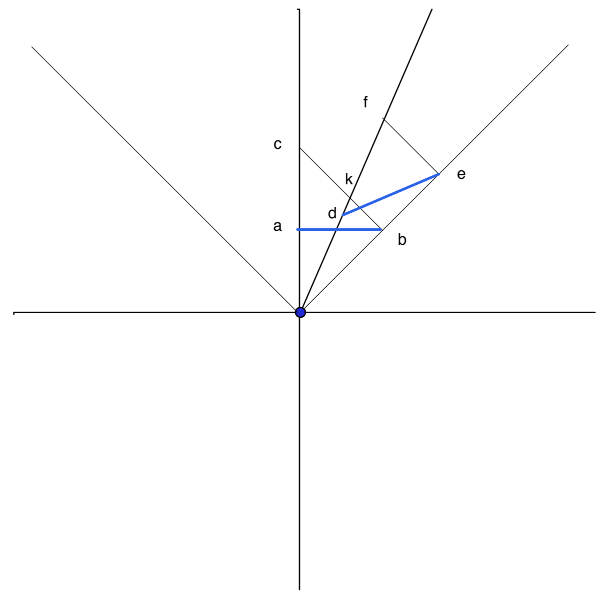

Figure 4: Two Experiments in Budden's Theory

(The diagram shows the experiments happening in overlapping regions of spacetime. I only do this for convenience. The second experiment actually occurs shifted over to the right, in a nonoverlapping region of spacetime.)

In the diagram *ab* is a timeslice of the measuring rod for the experiment conducted in the rest frame. The light ray travels from the origin to *b* to *c* while the clock travels from the origin to *c*. At the end of the experiment, at point *c*, the clock reads some number *m* — the amount of time that has elapsed since the beginning of the experiment, according to that clock. (In this spacetime, as in Minkowski spacetime, light rays travel on null geodesics.) And *de* is a timeslice of the measuring rod for the experiment in the moving frame. The light ray in this experiment travels from the origin to *e* to *f* while the clock travels from the origin to *f*. At the end of the experiment, at point *f*, the clock reads some number *n*. (I leave it open, for now, whether $m = n$.)

Now, for this pair of experiments to show that (3) is false, they must be experiments of the same type but have different outcomes. It is clear that they are experiments of the same type: both experiments measure the round-trip speed of light using measuring rods that are the same proper length. (In fact, the points *d,e,* and *f* are the images under a Lorentz boost of *a,b,* and *c*, respectively.)[20] And the experiments do have different outcomes. The line *cb* in the diagram is a subset of the set of points *x* such that $ox \sim_{-1} oc$ (*o* is the origin). So when the moving clock reaches point *k*, it has recorded the same amount of time as the stationary clock has when it reaches point *c*. But the moving experiment doesn't end until the moving clock reaches point *f*, which is past *k*.[21] So in the moving experiment, the clock ends up recording more time than the clock in the stationary experiment. (That is, $n > m$.) And that means that the experiments have different outcomes.

(There is a way in which my presentation here is misleading, which I should clear up before going on. It will be important later. Given the initial state, it does not look like Budden's laws determine just which numbers the two clocks display; the laws only determine the ratio between those numbers. Does this mean that Budden's laws are not

20. If *a,b,* and *c* have *(x,t)* coordinates (0,1),(1,1), and (0,2), then *d, e,* and *f* have coordinates $(1/\sqrt{3}, 2/\sqrt{3})$, $(\sqrt{3}, \sqrt{3})$, and $(2/\sqrt{3}, 4/\sqrt{3})$.

21. The equation for the line through *b* and *c* is $t = 2\text{-}x$. Since $(2/\sqrt{3}, (2\sqrt{3}-2)/\sqrt{3})$ lies on that line, $f = (2/\sqrt{3}, 4/\sqrt{3})$ lies above it.

deterministic? In fact it does not. To the extent that the laws look indeterministic, it is because we are misled by the appearance of clocks that read particular numbers in the models of Budden's theory. (I have followed Budden in presenting them this way; see page 501 of his paper.) It is easier to see, at first, how the theory works when its models are presented in that way. Now that we have seen how it works, I can sketch a more perspicuous presentation of the example. This presentation will make it clear that the theory is deterministic. The models look like this: clocks do not display numbers at all. Instead the two "clocks" work like this: each has a "start/stop/reset" button, and each can display one of five words: "ready"; "running"; "same"; "faster"; "slower." It is physically necessary that one clock is in its ready state if and only if the other is in its ready state. At the beginning of the experiments, the start/stop buttons are pushed on the clocks, and the displays switch from "ready" to "running." At the end of the experiments the buttons are pushed again. Then the clocks display "same" if the initial and final spacetime points are $\sim_{-1}$ congruent; if not, the clock that traveled the $\sim_{-1}$-longer path reads "slower," and the other reads "faster." So in the experiment I discussed, the moving clock reads "slower" and the stationary clock reads "faster." In this presentation, the experiments still have different outcomes, but just what each clock says at the end of the experiment is completely determined.)

That is Budden's argument that (3) is false in this theory. He also argues that (5) is true. I accept his argument and will not reproduce it here. (It is clear that (5) is true in two-dimensional models of his theory, but the proof is trickier in more dimensions.) If he is right about all of these claims, then (5) does not entail (3).

There are a couple of things about this example that should raise suspicion. Could the clock law really be a law? You might think that to see how clocks behave in this spacetime, we must first write down some dynamical laws governing how the parts of clocks move, and then see whether that dynamics validates the clock law.[22] It is not clear

22. Brown [2005] is an extended articulation and defense of this point of view.

that this can be done. Certainly some kinds of devices we might build in this spacetime that we might have thought were pretty good clocks turn out not to be. For example, the light clocks that appear in relativity texts are not ideal clocks in this theory. Budden's experiment shows this directly. (To build a light clock, you take a measuring rod, attach mirrors to both end, and bounce a light ray back and forth between them; the clock "ticks" each time the light ray hits, say, the mirror on the left. So Budden's experiments amount to two comparisons of a light clock with a co-moving ideal clock. Since the outcomes of the experiments are different, light clocks do not always tick in sync with ideal clocks.)

Someone might reply, on behalf of Budden, that it doesn't matter whether it is physically possible to build good clocks. It is enough that $\sim_{-1}$ be the temporal congruence relation, even if it is physically impossible for a clock to keep correct time. We can understand the claim about the behavior of ideal clocks as an indirect way to say that $\sim_{-1}$, and not the standard congruence relation on timelike pairs of points definable from lightlike connectability alone, is the temporal congruence relation.

I am sympathetic with a lot of this reply. I agree that it need not be physically possible to build an accurate clock. The laws of physics could conspire against us. But I have a related worry. Is it really possible that $\sim_{-1}$ be the temporal congruence relation? If it is not possible, then Budden's clock law could not be a law. However, I think it is very difficult to know how to go about settling whether $\sim_{-1}$ could be the temporal congruence relation. Budden does not do much to argue that it could be, but I do not know how to argue that it could not.

Those are some reasons to be suspicious about Budden's alleged counterexample. But I do not rest my case against it on those suspicions. I accept, for the sake of argument, that the clock law could be a law and that there could be clocks that obey that law. I reject Budden's claim that (3) is false in his theory. I think, instead, that (3) is true in Budden's theory.

Three things must be the case for the experiments described above

to falsify (3). First, the experiments must be of the same type. Second, the experiments must have different outcomes. And third, the experiments must be conducted in isolated laboratories. The first two conditions are met but not the third. These experiments are not conducted in isolated laboratories. Call the world in which the described pair of experiments take place "world 3"; call the stationary experiment in that world "*SE3*," and the moving experiment "*ME3*." Consider a world (world 4) that is boosted relative to world 3.[23] That world contains an experiment, *SE4*, that is the boosted version of *SE3*. World 4 is boosted in such a way that *SE4* is moving at half the speed of light (relative to the frame picked out by the diagram of world 3 — I leave this implicit from now on.) Since (5) is true in Budden's theory, world 4 is physically possible. Also, *SE4* is the same type of experiment as *ME3*: the clock, rod, and light rays in *SE4* occupy just the same regions of spacetime as the clock, rod, and light rays in *ME3*. But these experiments have different outcomes. We are given that the clock in *ME3* reads *n* at the end of the experiment. Now at the end of the experiment the clock in *SE4* is the image under a Lorentz transformation of the clock in *SE3*; since the clock in *SE3* reads *m*, so does the clock in *SE4*. (To go through this more slowly: since the two clocks are related by a Lorentz transformation, and Lorentz transformations are dynamical symmetries, the two clocks end up in the same physical state at the end of the experiment, and so both read the same number.) But *m* does not equal *n*. Given (6), this means that the experiments are not isolated.

Let me dwell a little longer on the question whether there are isolated laboratories in Budden's theory. As in the first alleged counterexample, Budden cannot use the sufficient conditions (7a) and (7b) to argue that the laboratories are isolated. That is because Budden's clock law is non-local: it enforces correlations between the readings of different clocks, no matter how far apart those clocks are, or what

23. Actually, since the experiments do not occur in overlapping regions of spacetime, world 4 differs by a boost and a spatial shift. This does not matter since spatial shifts are also dynamical symmetries.

insulating walls are built between them.[24] The laws cannot be used to predict the outcome of an experiment with one clock without using information about what other clocks are being used in other experiments, no matter how far away those experiments are.

We can use the non-locality of the clock law to argue directly that the laboratories in Budden's theory are not isolated. The kind of lawfully enforced correlation that appears in that law is sufficient for causation. It is sufficient because it entails counterfactual dependence, and counterfactual dependence is sufficient for causation.[25] Given Budden's law, if one of the clocks had registered a different elapsed time, the other one would have as well. So there is some kind of continuous direct interaction between the clocks, maintaining their coordination.

Someone might hesitate to accept that there is direct interaction between the clocks; he might want first to see the details. He might describe two clocks, perhaps clocks built from completely different materials and operating according to completely different principles, and demand an account of how the interaction works, how the one clock signals the other. Only when a satisfying account has been presented would he accept that there is such an interaction.

Of course, I cannot give such an account. That is not my fault; it is Budden's. The clock law is the only law we are given. Either it is supposed to be a fundamental law, or it is not. If it is a fundamental law (as I assumed it was in my argument in the previous paragraph), then the demand that the direct interaction between the clocks be explained is misplaced, since it amounts to the demand that a fundamental law be derived from some other laws. If it is not a fundamental law, then we do not know what the fundamental laws of Budden's theory are. But I do not think that there is a way to specify more fundamental laws governing the movements of the clocks' parts that entail Budden's clock law without having those clocks interact.[26]

I conclude, therefore, that Budden has not produced a possible physical theory according to which (5) is true but (3) is false.

5. Concluding Remark

I have argued that (5) entails (3). My argument is meant to undermine doubts that (5) deserves its current status as the standard formulation of the special principle of relativity. But in giving the argument we have seen new reasons to think that it, rather than (3), does deserve that status. First, in a theory that does not allow for isolated experiments (other than the "experiment" that is the evolution of the entire universe in time), (3) is vacuously true. Yet it seems that the special principle of relativity is not automatically true in that theory. That theory might still treat one frame of reference as special. (5) is the principle that lets us separate that theory out from theories according to which states of inertial motion really are all indistinguishable. And second, the principle of relativity, whatever it is, expresses some important and fundamental truth about the way the world is. But (3) contains the word "isolated," which is defined in causal terms. As I said earlier, causation is a messy thing, and the causal facts about the world are not fundamental facts. So we should keep causation out of the principle of relativity.[27]

24. Budden knows that his theory is non-local — see page 503.

25. No claim about the connection between causation and counterfactuals is uncontroversial, but this is one of the less controversial claims. I should note that dependence is only sufficient for causation when the two events are logically distinct. Socrates' death does not cause Xanthippe to become a widow. But in this case this condition seems to be met (though see footnote 26).

26. For those who are still skeptical that Budden's clocks interact, there may be another interpretation of his example that also has it fail to be a genuine counterexample. It was suggested by an anonymous referee. On this interpretation, the experiments fail to be isolated because the state of the pair of clocks is *nonseparable*. That is, for any two spacelike separated points x and y on the worldlines of the stationary and moving clocks, respectively, the state of the pair of clocks on the instant containing x and y is not completely specified by specifying the intrinsic physical state of the clock at x and the intrinsic physical state of the clock at y. (In my discussion, and in my definition of "isolated laboratory," I presupposed that things could not be in this kind of nonseparable state.)

27. Thanks to Frank Arntzenius, Phillip Bricker, Chris Wuthrich, two anonymous referees, and the MIT Work in Progress Seminar for helpful comments.

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