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Spacetime, Ontology, and Structural Realism

Edward Slowik

This essay explores the possibility of constructing a structural realist interpretation of spacetime theories that can resolve the ontological debate between substantivalists and relationists. Drawing on various structuralist approaches in the philosophy of mathematics, as well as on the theoretical complexities of general relativity, our investigation will reveal that a structuralist approach can be beneficial to the spacetime theorist as a means of deflating some of the ontological disputes regarding similarly structured spacetimes.

1. Introduction

Although its origins can be traced back to the early twentieth century, structural realism (SR) has only recently emerged as a serious contender in the debates on the status of scientific entities. SR holds that what is preserved in successive theory change is the abstract mathematical or structural content of a theory, rather than the existence of its theoretical entities. Some of the benefits that can be gained from SR include a plausible account of the progressive empirical success of scientific theorizing (thus avoiding the 'no miracles' argument), while also accommodating the fact that the specific entities incorporated by these distinct, evolving theories often differ quite radically (and thus SR evades the 'pessimistic meta-induction' that results from an ontological commitment to the entities in specific theories). To take a well-known example, the similar 'structure' that underlies the progression in nineteenth-century optics from Fresnel's elastic solid ether theory to Maxwell's electromagnetic field theory can be identified as the mathematical structure exemplified in Maxwell's field equations, since they give Fresnel's equations as a limiting case: 'Fresnel's equations are taken over completely intact into the superseding theory [Maxwell's]—reappearing there newly interpreted but, as mathematical equations, entirely unchanged' (Worrall 1996, 160). In short, the mathematical structure of Fresnel's theory carried over into the new theory, but his

Edward Slowik is a Associate Professor in Philosophy at Winona State University, USA.
Correspondence to: Edward Slowik, Department of Philosophy, Winona State University, Winona, MN 55987-5838, USA. Email: eslowik@winona.edu

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ontological commitment did not (since his mechanical elastic solid differs quite drastically from the non-mechanical interpretation of the electromagnetic field often favored at the end of the nineteenth century).

While quantum mechanics has been a central concern of contemporary SR theorists, it is curious to note that one highly mathematical area of philosophical speculation that has not been accorded the same degree of attention is the longstanding problem of the ontological status of space or spacetime; in particular, whether space exists as a form of entity independent of matter (substantivalism) or is reducible to the relations among material existents (relationism). To rectify this neglect, the goal of our investigation will be to examine the prospects for applying SR to the substantivalist/relationist problem, with special emphasis placed on how spacetime structure differs, if at all, from structuralist approaches in the philosophy of mathematics. Although the structural realist position itself may harbour various difficulties, it will be argued that an SR account can prove beneficial in assessing the plethora of similarly structured, but ontologically divergent, spacetimes that have surfaced in the wake of Earman and Norton's hole argument. By stressing the central importance of the geometric structure of spacetime theories, as opposed to their ontological interpretations, SR is also more successful in accounting for the non-causal role of mathematics in spacetime theories.

2. Structural Realism and Spacetime: Making the Case

2.1. The Hole Argument and Its Structuralist Aftermath

Many previous investigations of spacetime structure can be traced back to Earman's and Norton's reworking of Einstein's 'hole' argument into a negative critique of manifold substantivalism. Put briefly, the hole argument concludes that substantivalism, in its modern general relativistic setting, leads to an unacceptable form of indeterminism. By shifting the metric and matter (stress-energy) fields, g, and, T, respectively, on the spacetime manifold of points, M, with the latter representing the 'substance' of spacetime, one can obtain a new model $\langle M, g^*, T^*\rangle$ from the old model $\langle M, g, T\rangle$ that also satisfies the field equations of the general theory of relativity or GTR (via a 'hole diffeomorphism', h: $\mathrm{h}(g) = g^*$). If the mapping is the identity transformation outside the hole, but a non-identity mapping inside, then the substantivalist will not be able to determine the trajectory of a particle within the hole (despite the observationally identical nature of the two models; see Earman and Norton 1987).

One of the more influential structural-role solutions to the hole argument rejects a straightforward realist interpretation of the individuality of the points that comprise the manifold, M. 'A preferable alternative [to manifold substantivalism] is to strip primitive identity from space-time points: call this view *metric field substantivalism*. The focus of this view is on the metric tensor [g] as the real representor of space-time in GTR' (Hoefer 1996, 24). Since the identity of the points of M is secured by the metric g, any transformation of g, i.e., g^*, does not result in the points of M possessing *different* g-values; rather, g^*, (in conjunction with T^*) simply gives back the very same spacetime points. (Similar structural role solutions are offered in Butterfield 1989, Mundy 1992, and Brighouse 1994 to name just a few.) In Belot and Earman (2001, 228), these structural role constructs are somewhat pejoratively labeled 'sophisticated' substantivalists, and the charge is that the 'substantivalists are helping themselves to a position most naturally associated with relationism' (Belot 2000, 588–589)—the reason being that the identification of a host of observationally indistinguishable models with a single state-of-affairs is the very heart of relationism. What is ironic about these allegations is that, until recently, they have usually gone the other way, with the substantivalists accusing many of the latest relational hypotheses of an illicit use of 'absolute' structure. In the case of motion and its accompanying effects, for instance, it has long been recognized that a physics limited to the mere relational motion of bodies, which (following Earman 1989) we can define as (R1), faces serious difficulties in trying to capture the full content of modern physical theories. In GTR, the formalism of the theory makes it meaningful to determine if a lone body rotates in an otherwise empty universe (or whether the whole universe rotates); and such possibilities are excluded on a strict (R1) construal. A potential relationist rejoinder is to reject (R1), and simply hold that the spatial structures needed to make sense of motion and its effects do not supervene on some underlying, independent entity called 'substantival space'. The rejection of substantival space, which we can define as (R2), allows the relationist to freely adopt any spatial structure required to explicate dynamical behavior, e.g. affine structure, ∇, just as long as they acknowledge that these structures are (somehow) directly instantiated by material bodies (or fields). Yet, by embracing the richer structures, the (R2) relationist is open to the charge of being an 'instrumentalist rip-off' of substantivalism (see section 2.2 below).

GTR likewise displays the inherent difficulties in clearly differentiating the relationist and substantivalist interpretations of ontology. As revealed above, Hoefer views the metric as representing substantival space, but Einstein judged the metric (or gravitational) field as more closely resembling Descartes's relational theory of space:

> If we imagine the gravitational field, i.e. the functions g_{ik}, to be removed, there does not remain a space … but absolutely *nothing*…. There is no such thing as an empty space, i.e. a space without field. Space-time does not claim existence on its own, but only as a structural quality of the field. (Einstein 1961, 155–56)

By holding that all fields, including the metric field g, are physical fields, even the vacuum solutions to GTR no longer correspond to empty spacetimes (thus eliminating a large relationist obstacle; see Rovelli 1997, and Dorato 2000, for similar 'metric field' relationisms). This predicament has led some to question the very validity, or relevance, of the substantival/relational debate in the context of GTR (Rynasiewicz 1996).

2.2. The Structural Realist Spacetime Solution

The foregoing analysis of the GTR substantival/relational debate nicely demonstrates how two divergent ontological interpretations can nonetheless agree on the necessity of a common structure: for both Einstein and Hoefer, the metric field g is the structure identified or associated with spacetime, whether as its 'real representor' (Hoefer) or where spacetime is reckoned to be a 'structural quality of the [*g*] field' (Einstein). In either case, if you remove *g*, then you remove spacetime. On a structural realist construal of the debate, consequently, both Hoefer's substantivalist and Einstein's relationist versions of GTR would appear to constitute different ontological interpretations of the very same underlying physical theory, since the key mathematical structure equated with the nature of spacetime is identical in both cases. In other words, just as the different theories of light proposed by Fresnel and Maxwell embody the same formal structure, and so comprise different ontological perspectives of the same underlying (structural) reality, the substantivalist and relationist readings of GTR likewise capture the same spacetime structure but from the standpoint of diverse ontological assumptions. Moreover, a host of diverse philosophical evaluations of GTR (or Newtonian physics) would fall under the same SR category, since these allegedly distinct hypotheses all accept the theory's spacetime structure in a fairly straightforward manner. For example, the (R2) relationists that posit spatiotemporal structure as a sort of 'property' (Sklar 1974; Teller 1987), or the modal relationist renditions of (R2) (such as Manders 1982; Teller 1991; Hinckfuss 1975), presumably do not differ on the mathematical structure and predictive scope of the relevant spacetime theory. Since they all reject (R1), they all can agree on the meaningful possibility of, say, a lone rotating body in an empty universe.[1] Hence, if we group all of the sophisticated (R2) relationist hypotheses according to their structure, an SR analysis has the decided advantage of reducing a vast array of disparate ontological speculation into a single SR theory that, rather conveniently, also includes most substantivalist readings of the spacetime. In brief, the rationale for the SR approach is based on the apparently irresolvable ontological dispute between the substantivalists and relationists: the SR theorist maintains that all we can ever know about spacetime is its structure, and not the competing claims of its substantival or material foundation.[2]

More carefully, any ontological interpretation of a spacetime theory that puts forward the same mathematical structure constitutes the same SR spacetime theory. So, given the fact that most (R2) relationists and sophisticated substantivalists (1) accept the standard formalism of the relevant spacetime structure, such as *M* and *g* from the set $\langle M, g, T\rangle$, for the relativistic spacetime of GTR, as well as (2) accept the implications of these structures (e.g. our lone rotating body), it follows that SR must regard these apparently different theories as identical.[3] The essential criterion for an SR approach to spacetime is the structure actually *utilized* in the theory, and neither the ontological ranking of those structures nor the attempt to prove that some structures are more privileged is relevant to the theory's SR classification.[4] Even those ontological interpretations of GTR that strive to identify spacetime with *M* or *g* (and not *M* and *g*), will consequently fall under the same SR category that endorses the joint *M* and *g* structure. Since these interpretations do not eliminate, but rather still employ, the other structures, they fit the same SR spacetime category as those models less concerned with embracing both *M* and *g*: e.g. Hoefer does not claim *M* can be dropped altogether, since 'it represents the continuity of space-time and the global topology' (Hoefer 1998, 24).[5]

SR can also be helpful in explicating our evolving understanding of the importance of structure in spacetime theories considered historically. In the same manner that the success of both Fresnel's and Maxwell's theories can be explained as due to a common structure, despite their different ontologies, so the evolution of spacetime theories demonstrates how conflicting spacetime commitments may, or may not, incorporate the same necessary structures; and, by this means, explain the strengths (and weaknesses) of the respective theories. For instance, whereas Newton's conception of space and time can be faulted for postulating an unnecessarily rigid structure (absolute position and velocity), Descartes's competing (R1) conception lacks the necessary structure required to make sense of his own laws of motion. It was only much later that the spacetime structure mandated by Newtonian mechanics came to be fully recognized—and, not coincidentally, the newly discovered structure can be seen as combining facets of relationism (in the symmetry group that eliminate absolute position and velocity) alongside the more 'absolutist' insights of the substantivalists (as in the affine structure). Likewise, the striking resemblance of many post-hole argument substantivalist and current (R2) relationist hypotheses can be seen as a further manifestation of this structuralist evolutionary tendency. As the analysis of spacetime theories progresses (the insufficiency of (R1) relationism, the hole argument, etc.), the structures put forward by the competing ontologies draw ever more closer, and may have reached a point where there is no longer any significant difference. It is this capacity of SR to reduce the seemingly irresolvable ontological conflicts, and focus on the crucial role of structure, that marks its true advantage in the spacetime debates.

3. Assessing the SR Spacetime Hypothesis

In examining the potential stratagems for an SR interpretation of spacetime theories, it will prove invaluable to review the parallel developments towards a structuralist account of mathematics. As will be demonstrated, the structuralist standpoint in the philosophy of mathematics bears a close resemblance to any SR spacetime philosophy, and even suggests that there exists no real difference between the mathematical and spacetime applications of a structuralist outlook.

3.1. Structuralism in Spacetime Theories and Mathematics

An obvious initial dilemma that faces the prospective SR theorist is the ontological status of the spacetime structures themselves. Do they exist as a sort of Platonic universal, independent of all physical objects or events in spacetime, or are they dependent on matter/events for their very existence or instantiation? This problem arises for the mathematician in an analogous fashion, since they also need to explicate the origins of mathematical structures (e.g. set theory, arithmetic). Consequently, a critic of the SR spacetime project might seem justified in regarding this dispute in the philosophy of mathematics as a re-emergence of the traditional substantivalist versus relationist problem, for the foundation of all mathematical structures, including the geometric spacetime structures, is once again either independent of, or dependent on, the physical.

Nevertheless, a survey of the various positions in the mathematical ontology dispute may work to the advantage of the SR spacetime theory, especially when the relevant mathematical and spacetime options are paired together according to their analogous role within the wider ontology debate. First, mathematical structuralism can be classified according to whether the structures are regarded as independent or dependent on their instantiation in systems (*ante rem* and *in re* structuralism, respectively), where a 'system' is loosely defined as a collection of 'objects' and their interrelationships. *Ante rem* structuralism, as favored by Resnik (1997) and Shapiro (1997, 2000), is thus closely akin to the traditional 'absolute' conception of spacetime, for a structure is held to 'exist independent of any systems that exemplify it' (Shapiro 2000, 263). Yet, since 'system' (and 'object') must be given a broad reading, without any *ontological assumptions* associated with the *basis* of the proposed structure, it would seem that substantivalism would not fit *ante rem* structuralism, as well. The structure of substantivalism is a structure *in* a substance, namely, a substance called 'spacetime', such that this unique substance 'exemplifies' the structure (whereas *ante rem* structure exists in the Platonic sense as apart from any and all systems that exemplify it). The substantivalist might try to avoid this implication by declaring that their spacetime structures are actually closer in spirit to a pure absolutism, without the need of any underlying entity (substance) to house the structures (hence, 'substantivalism' is simply an unfortunate label). While this tactic may be more plausible for interpreting Newtonian spacetimes, it is not very convincing in the context of GTR, especially for the sophisticated substantivalist theories. Given the reciprocal relationship between the metric and matter fields, it becomes quite mysterious how a non-substantival, 'absolute' structure, g, can be effected by, and effect in turn, the matter field, T. For the *ante rem* structuralist, mathematical structures do not enter into these sorts of quasi-causal interrelationships with physical things; rather, things 'exemplify' structures (see also note 8). Accordingly, one of the initial advantages of examining spacetime structures from within a mathematical ontology context is that it drives a much needed wedge between an absolutism about quantitative structure and the metaphysics of substantivalism, although the two are typically, and mistakenly, treated as identical.

In fact, as judged against the backdrop of the ontology debate in the philosophy of mathematics, the mathematical structures contained in *all* spacetime theories would seem to fall within a nominalist classification. If, as the nominalists insist, mathematical structures are grounded on the prior existence of some sort of 'entity', then both the substantivalists and relationists would appear to sanction mathematical nominalism (with *in re* structuralism included among nominalist theories, as argued below): whether that entity is conceived as a unique non-material substance (substantivalism), physical field (metric-field relationism), or actual physical objects/events (relationism, of either the modal (R2) or strict (R1) type), a nominalist reading of mathematical structure is upheld. This outcome may seem surprising, but given the fact that traditional substantivalist and relationist theories have always based spatiotemporal structure on a pre-existing or co-existing ontology—either on a substance (substantivalism) or physical bodies (relationism)—a nominalist reading of spacetime structure has been implicitly sanctioned by both theories. Consequently, if both substantivalism and relationism fall under the same nominalist category in the philosophy of mathematics, then the deeper mathematical Platonist/nominalist issue does not give rise to a corresponding lower-level substantival/relational dichotomy as regards the basis of those spacetime structures (e.g. with substantivalism favoring a Platonic realism about mathematical structures, and relationism siding with a nominalist anti-realism). This verdict could change, of course, if a non-substantival 'absolute' conception of spacetime becomes popular in GTR; but this seems unlikely, as argued above.

As there are a number of nominalist reconstructions of mathematics, a closer examination of their content reveals that the different versions can be paired to different substantival and relational theories. For instance, a reductive (R1) spacetime relationism can be linked to some strict nominalist reconstructions of mathematics, as in Field's (1980) attempt to treat mathematical objects and structures as entirely dispensable, or 'fictional'. Field posits a continuum of spacetime points, conceived physically in the manner of a manifold substantivalist, in his effort to rewrite Newtonian gravitation theory along mathematically anti-realist lines. Modal (R2) relationists, like Teller (1991), would not constitute the spacetime analogue of Field's program, accordingly, since (as discussed above) this form of relationism sanctions modal spacetime structures that can transcend the structures exhibited by the actually existing physical objects, e.g. the affine structure ∇ instantiated by a lone rotating body. Whereas Field requires an infinity of physical spacetime points (isomorphic to $\Re^4$) in order to capture the full content of the mathematician's real numbers, the (R2) relationist can allow modal structures to serve this function, thus releasing the ontology of such extravagant demands. More importantly, if Field's nominalist program is committed to manifold substantivalism, *M*, then it is susceptible to the hole argument. All spacetime theories that utilize the metric in the identity of spacetime points, such as sophisticated substantivalism and metric-field relationism, would thereby incorporate a divergent set of structural scenarios (since they are not susceptible to the hole argument). Therefore, Field's nominalist mathematics entails a spacetime structure that comprises a different SR theory than sophisticated substantivalism and metric-field relationism.

The mathematical equivalent of both sophisticated substantivalism and (R2) relationism is, rather, any of the less stringent nominalist theories that reject Field's strict nominalism, as in, e.g. Chihara (2004) or Azzouni (2004), or Hellman's (1989) *in re* structuralism. Much like the modal (R2) relationist theories surveyed above, the 'minimal nominalists' (as we will dub them) do not allow structures to exist independently of the systems they exemplify, yet they do not believe that these structures can be dismissed as mere fictions, either. Contra Field, the minimal mathematical nominalists deny a purely instrumentalist construal of mathematical structures (while simultaneously rejecting a Platonic absolutism): they all insist, for instance, that mathematical structures cannot be excised from scientific theories without loss of valuable physical content (e.g. Chihara 2004, 317–49). Hellman's *in re* structuralism, moreover, employs 'possible structures' as a means of avoiding a commitment to an infinite background ontology, a feature that helps to explain its common nominalist classification (see Hellman 1989, 47–52, and Chihara 2004, 107–115). The minimal nominalist theories often differ on how the mathematical structures are constructed from their basic ontology, as well as how to construe the truth-values of mathematical statements about mathematical structures (see Azzouni 2004, 30–48). But these differences, such as in the kind of modality sanctioned, etc., can be quite subtle, and do not lend themselves to any drastic distinctions in type or basic intent. Hence, any attempt to rekindle the substantival/relational distinction among the competing minimal nominalist theories would seem implausible. A firm reliance on some form of modality and a non-instrumental construal of mathematical structures is the common, and crucial, similarity among these theories; and it is these aspects that are most important for the spacetime structuralist, whether of the sophisticated substantivalist or relationist variety.

These last observations are not meant to downplay the importance of the ongoing research in the philosophy of mathematics on the origin of structures, for it is always possible that substantial problems will arise for some of the minimal nominalist theories, thus eliminating them from contention. From the SR standpoint, in fact, the philosophy of mathematics would likely be considered a more proper arena for assessing the structures employed by spacetime theories, at least as opposed to the apparently unverifiable metaphysics of 'substance versus body'. Not only has the traditional spacetime dichotomy failed to explain how these mathematical structures arise from their basic ontology, but, as we have seen, the underlying structures advocated by the sophisticated versions of both substantivalism and relationism are identical when judged within the wider philosophy of mathematics framework. Whether that foundational entity is called a substance or a physical object is irrelevant, and probably a conventional stipulation, since the real work, as judged from the mathematical perspective, concerns *how* the structures are constructed from the underlying entity—and the competing claims of substance or physical existent do not effect this mathematical construction. In essence, the only apparent difference between the sophisticated substantivalists and (R2) relationists is *where* those mathematical structures are located: either *internal* to the substance or field (for the substantivalists and metric-field relationists, respectively) or external to bodies/events (for non-field formulations of (R2) relationism, such as Teller's). Needless to say, this internal/external distinction does not provide any information on how the mathematical structures are built up; rather, it reveals the pervasive influence of the age-old substance/property dichotomy within the philosophy of science community, an unfortunate legacy that the SR theorist regards as hindering the advancement of the debate on spacetime theories.[6]

Finally, since the competing minimal nominalist constructions are being judged not solely from a mathematical perspective, but from a scientific and empirical standpoint as well, a few words are in order on the relevance of empirical evidence in assessing spacetime structure. This issue will be addressed further in the remaining sections, but in brief, it is unlikely that any physical evidence could provide a strong confirmation of any one of the competing nominalist theories. In effect, these nominalist constructions are only being utilized to explain the origins of the spacetime structures, such as M or g, that *do* appear in our best physical theories, with the important qualification that these nominalist constructions do not commit the physical theory to any problematic or meaningless physical outcomes (e.g. Field's nominalism and the hole argument, as noted above). As for the sophisticated brands of both substantivalism and relationism, all of the minimal nominalist constructions are apparently identical as regards their implications for possible spacetime scenarios and meaningful physical states. Unless other reasons are brought forward, the choice among the competing minimal nominalist constructions could thus be viewed as conventional, since it is difficult to conceive how empirical evidence could reach the deep mathematical levels where the differences in nominalist constructions of spacetime structures come into play.

3.2. Geometric Structure, Causation, and 'Instrumental Rip-Offs'

The benefit of the various structuralist approaches in the philosophy of mathematics is that it allows both the mathematical realist and anti-realist to use mathematical structures without obligating a Platonism about mathematical objects, such as numbers—one can simply accept that, say, numbers exist as places in a larger structure, like the natural number system, rather than as some sort of independently existing, transcendent entities (see Shapiro 2000, 258). Accordingly, a variation on a well-known mathematical structure, such as exchanging the natural numbers '3' and '7', does not create a new structure, but merely gives the same structure 'relabelled' (with '7' now playing the role of '3', and vice versa). This structuralist tactic is familiar to spacetime theorists, as mentioned previously, for not only has it been adopted by substantivalists to undermine an ontological commitment to the independent existence of the manifold points of M, but it is tacitly contained in all relational theories, since they would count the initial embeddings of all material objects and their relations in a spacetime as isomorphic.

A critical question remains, however: since spacetime structure is *geometric* structure, how does the SR approach to spacetime differ in general from mathematical structuralism? Is the theory just mathematical structuralism as it pertains to geometry (or, more accurately, differential geometry), rather than arithmetic or the natural number series? While it may sound counter-intuitive, the SR theorist should answer this question in the affirmative—the reason being, quite simply, that the puzzle of how mathematical spacetime structures apply to reality, or are exemplified in the real world, is identical to the problem of how all mathematical structures are exemplified in the real world. Philosophical theories of mathematics, especially nominalist theories, commonly take as their starting point the fact that certain mathematical structures are exemplified in our common experience, while others are excluded. To take a simple example, a large collection of coins can exemplify the standard algebraic structure that includes commutative multiplication (e.g. $2 \times 3 = 3 \times 2$) but not the more limited structure associated with, say, Hamilton's quaternion algebra (where multiplication is non-commutative; $2 \times 3 \neq 3 \times 2$). In short, not all mathematical structures find real-world exemplars (although, for the minimal nominalists, these structures can be given a modal construction). The same holds for spacetime theories: empirical evidence currently favors the mathematical structures utilized in GTR, such that the physical world exemplifies, say, g, but a host of other geometric structures, such as the flat Newtonian metric, h, are not exemplified.

The critic will likely respond that there is a substantial difference between the mathematical structures that appear in physical theories and the mathematics relevant to everyday experience. For the former, and not the latter, the mathematical structures will vary along with the postulated physical forces and laws; and this explains why there are a number of competing spacetime theories, and thus different mathematical structures, compatible with the same evidence: in Poincaré fashion, Newtonian rivals to GTR can still employ *h* as long as special distorting forces are introduced (we will return to this topic). Yet, underdetermination can plague even simple arithmetical experience, a fact well known in the philosophy of mathematics and in measurement theory. For example, in Chihara, an assessment of the empiricist interpretation of mathematics prompts the following conclusion: 'the fact that adding 5 gallons of alcohol to 2 gallons of water does not yield 7 gallons of liquid does not refute any law of logic or arithmetic ["5 + 2 = 7"] but only a mistaken physical assumption about the conservation of liquids when mixed' (Chihara 2004, 237). While obviously true, Chihara could have also mentioned that, in order to capture our common-sense intuitions about mathematics, the application of the mathematical structure in such cases requires coordination with a physical measuring convention that preserves the identity of each individual entity, or unit, both before and after the mixing. In the mixing experiment, perhaps atoms should serve as the objects coordinated to the natural number series, since the stability of individual atoms would prevent the sort of blurring together of the individuals ('gallon of liquid') that led to the arithmetically deviant results. By choosing a different coordination, the mixing experiment can thus be judged to uphold, or exemplify, the statement '5 + 2 = 7'. What all of this helps to show is that mathematics, for both complex geometrical spacetime structures and simple non-geometrical structures, cannot be empirically applied without stipulating *physical* hypotheses and/or conventions about the objects that model the mathematics. Consequently, as regards real-world applications, there is no difference in kind between the mathematical structures that are exemplified in spacetime physics and in everyday observation; rather, they differ only in their degree of abstractness and the sophistication of the physical hypotheses or conventions required for their application. Both in the simple mathematical case and in the spacetime case, moreover, the decision to adopt a particular convention or hypothesis is normally based on a judgment of its overall viability and consistency with our total scientific view (aka the scientific method): we do not countenance a world where macroscopic objects can, against the known laws of physics, lose their identity by blending into one another (as in the addition example), nor do we sanction otherwise undetectable universal forces simply for the sake of saving a cherished metric.

Another significant shared feature of spacetime and mathematical structure is the apparent absence of causal powers or effects, even though the relevant structures seem to play some sort of 'explanatory role' in the physical phenomena. To be more precise, consider the example of an 'arithmetically challenged' consumer who lacks an adequate grasp of addition: if they were to ask for an explanation of the event of adding five coins to another seven, and why it resulted in twelve, one could simply respond by stating, '5 + 7 = 12', which is an 'explanation' of sorts, although not in the scientific sense. On the whole, philosophers since Plato have found it difficult to offer a satisfactory account of the relationship between general mathematical structures (arithmetic/'5 + 7 = 12') and the physical manifestations of those structures (the outcome of the coin adding). As succinctly put by M. Liston: 'Why should appeals to mathematical objects [numbers, etc.] whose very nature is non-physical make any contribution to sound inferences whose conclusions apply to physical objects?' (Liston 2000, 191). One response to the question can be comfortably dismissed, nevertheless: mathematical structures did not *cause* the outcome of the coin adding, for this would seem to imply that numbers (or '5 + 7 = 12') somehow had a mysterious, platonic influence over the course of material affairs.

In the context of the spacetime ontology debate, there has been a corresponding reluctance on the part of both sophisticated substantivalists and (R2) relationists to explain how space and time differentiate the inertial and non-inertial motions of bodies; and, in particular, what role spacetime plays in the origins of non-inertial force effects. Returning once more to our universe with a single rotating body, and assuming no other forces or causes, it would be somewhat peculiar to claim that the causal agent responsible for the observed force effects of the motion is either substantival spacetime or the relative motions of bodies (or, more accurately, the motion of bodies relative to a privileged reference frame, or possible trajectories, etc.).[7] Yet, since it is the motion of the body *relative* to either substantival space, other bodies/fields, privileged frames, possible trajectories, etc., that *explains* (or identifies, defines) the presence of the non-inertial force effects of the acceleration of the lone rotating body, both theories are therefore in serious need of an explanation of the relationship between space and these force effects.[8] The strict (R1) relationists face a different, if not less daunting, task; for they must reinterpret the standard formulations of, say, Newtonian theory in such a way that the rotation of our lone body in empty space, or the rotation of the entire universe, is not possible. To accomplish this goal, the (R1) relationist must draw upon different mathematical resources and adopt various physical assumptions that may, or may not, ultimately conflict with empirical evidence: for example, they must stipulate that the angular momentum of the entire universe is 0.[9]

To sum up, all participants in the spacetime ontology debate are confronted with the nagging puzzle of understanding the relationship between, on the one hand, the empirical behavior of bodies, especially the non-inertial forces, and, on the other hand, the apparently non-empirical, *mathematical* properties of the spacetime structure that are somehow inextricably involved in any adequate explanation of those non-inertial forces—namely, for the substantivalists and (R2) relationists, the affine structure, that lays down the geodesic paths of inertially moving bodies. The task of explaining this connection between the empirical and abstract mathematical or quantitative aspects of spacetime theories is thus identical to elucidating the mathematical problem of how numbers relate to experience (e.g. how '5 + 7 = 12' figures in our experience of adding coins). Likewise, there exists a parallel in the fact that most substantivalists and (R2) relationists seem to shy away from positing a direct causal connection between material bodies and space (or privileged frames, possible trajectories, etc.) in order to account for non-inertial force effects, just as a mathematical realist would recoil from ascribing causal powers to numbers so as to explain our common experience of adding and subtracting.

An insight into the non-causal, mathematical role of spacetime structures can also be of use to the (R2) relationist in defending against the charge of instrumentalism, as, for instance, in deflecting Earman's criticisms of Sklar's 'absolute acceleration' concept. Conceived as a monadic property of bodies, Sklar's absolute acceleration does not accept the common understanding of acceleration as a species of relative motion, whether that motion is relative to substantival space, other bodies, or privileged reference frames (Sklar 1974, 225–34). Earman's objection to this strategy centers upon the utilization of spacetime structures in describing the primitive acceleration property: 'it remains magic that the representative [of Sklar's absolute acceleration] is neo-Newtonian acceleration

$(d^2x^i / dt^2) + \Gamma^i_{jk}(dx_j / dt)(dx_k / dt)$ [i.e. the covariant derivative, or ∇ in coordinate form]' (Earman 1989, 127–28).[10] Ultimately, Earman's critique of Sklar's (R2) relationism would seem to cut against all sophisticated (R2) hypotheses, for he seems to regard the exercise of these richer spacetime structures, like ∇, as tacitly endorsing the absolute/substantivalist side of the dispute: 'the Newtonian apparatus can be used to make the predictions and afterwards discarded as a convenient fiction, but this ploy is hardly distinguishable from instrumentalism, which, taken to its logical conclusion, trivializes the absolute-relationist debate' (Earman 1989, 128).

In the wake of our lengthy examination of the parallel function of mathematical and spacetime structures, the weakness of Earman's argument should be readily apparent, since, to put it bluntly, does the equivalent use of mathematical statements, such as '5 + 7 = 12', likewise obligate the mathematician to accept a realist conception of numbers (such that they exist independently of all exemplifying systems)? Yet, if the straightforward employment of mathematics does not entail either a realist or nominalist theory of mathematics (as most mathematicians would likely agree), then why must the equivalent use of the *geometric* structures of spacetime physics, e.g. ∇, require a substantivalist conception of ∇ as opposed to an (R2) relationist conception of ∇? Put differently, does a substantivalist commitment to $(d^2x^i / dt^2) + \Gamma^i_{jk}(dx_j / dt)(dx_k / dt)$, whose overall function is to determine the straight-line trajectories of, here, Neo-Newtonian spacetime, also necessitate a substantivalist commitment to its components, such as the vector, *d*/*dt*, along with its limiting process and mapping into $\Re$? In short, how does a physicist read off the physical ontology from the mathematical apparatus? A non-instrumental interpretation of some component of a theory's quantitative structure is often justified if that component can be given a plausible causal role (as in subatomic physics)—but, as noted above, ∇ does not appear to cause anything in spacetime theories.[11] All told, Earman's argument may prove too much, for if we accept his reasoning at face value, then the introduction of any mathematical or quantitative device that is useful in describing or measuring physical events would saddle the ontology with a bizarre type of entity (e.g. gross national product, average household family, etc.). A nice example of a geometric structure that provides a similarly useful explanatory function, but whose substantive existence we would be inclined to reject as well, is provided by Dieks's example of a three-dimensional colour solid:

> Different colours and their shades can be represented in various ways; one way is as points on a 3-dimensional colour solid. But the proposal to regard this 'colour space' as something substantive, needed to ground the concept of colour, would be absurd. (Dieks 2001, 230)

3.3. Underdetermination Problems

In summary, it has been argued that the geometric structures of spacetime theories should be regarded as a species of mathematical structure, and hence the ontological and epistemological puzzle associated with the function of these geometric structures is, in essence, a question about how mathematics relates to the physical world. If spacetime structures are simply mathematical structures, furthermore, then the proper arena for disputes concerning the ontological foundations of these spacetime structures is not the substantival/relational dichotomy in the philosophy of physics, but the realist/nominalist debate in the philosophy of mathematics. As discussed in section 2.1, the advantage of translating the spacetime ontology debate into a mathematical context is that it treats as identical the currently favored forms of both substantivalism and relationism that have emerged in the wake of the hole argument and in the context of GTR—namely, sophisticated substantivalism and (R2) relationism—since both theories require the same minimal nominalist mathematical construction (i.e. as also noted in section 2.1, GTR appears to be hostile to a non-substantival, absolutist spacetime conception, which is the equivalent of a platonic realism concerning mathematics; and a strict nominalism along the lines of Field requires a realist commitment to spacetime points that runs afoul of the hole argument). Accordingly, with both the sophisticated substantivalists and (R2) relationists embracing the same foundational conception of the origin of geometric spacetime structure, namely, a minimal nominalism, the old substantival versus relational dichotomy has been dissolved at the higher level of mathematical ontology. The evidence for regarding spacetime structures as purely mathematical structures, put forward in section 2.2, is the non-causal role that geometric spacetime structures play in our understanding of dynamical phenomena, which is identical to the non-causal role that mathematical structures in general assume in our understanding of everyday occurrences.

A further point raised in section 2.2, and which now needs to be discussed at greater length, is the underdetermination that accompanies all real-world applications of mathematical structures, whether simple arithmetic ('adding 5 gallons of alcohol to 2 gallons of water does not yield 7 gallons of liquid') or geometric spacetime structure (à la Poincaré (1952): flat metric with distorting universal forces versus curved metric with no distorting universal forces). If many different geometric–physical combinations are consistent with the same empirical evidence, as Poincaré's examples suggest, then a troubling implication is that the geometric structure of spacetime is open to conflicting interpretations. But, how can the geometry of spacetime be open to alternative construction without undermining the 'realism' in 'structural realism'? The structural realist program was intended to overcome the pessimistic meta-induction by obtaining some form of invariant mathematical quantity among competing scientific theories. The realization that the evidence underdetermines the geometry of physical theories, such that the geometry of a theory cannot be isolated against experience, might suggest that there are no such invariants as regards geometric structure.[12]

The SR theorists could fall back upon one of two general strategies in responding to the underdetermination problem. First, they might simply reject the possibility of a radical underdetermination of this sort, insisting that there remains a single, fundamental geometric structure underlying our best spacetime theories, presumably from Aristotle all the way to GTR (and beyond), that underwrites their success. An invariance of geometric structure across the development of Newtonian theories has been documented in Friedman (1983) and may lend credence to the SR theorist's belief that the continuing evolution of spacetime theories will confirm a core geometric structure. In moving from the classical type of Newtonian theory to the more complex neo-Newtonian formulation (which eliminates both absolute place and absolute velocity, besides incorporating a 'geometrized' gravitational potential), three geometric structures are featured in all versions of the theory: an affine connection, ∇, which provides the inertial trajectories (and may be either absolute or dynamic); a co-vector field, dt, which represents absolute time; and the metric tensor, h, which secures a Euclidean measure of distance (Friedman 1983, 71–124; a fourth structure, the point manifold, M, should also be included). In all versions of the Newtonian theory, these structures play an integral role in the spacetime structure, so they cannot be removed without undermining the overall Newtonian theory and its various physical laws (such as the laws of motion or, in the latter case, the gravitational law). In moving to the spacetime of GTR, the geometric structures now shrink to just $\langle M, g \rangle$, since the affine and temporal structures of the Newtonian theories, ∇ and dt, are now provided by the semi-Riemannian metric tensor g (which replaces the Euclidean metric, h; Friedman 1983, 177–215).

The real test of this strategy, of course, will come if a successor theory to GTR is formulated, and empirically confirmed. Even if the postulated successor theory coordinates an entirely different set of geometric structures to its physical processes, the combined 'geometry + physics' of this new theory, $T = G + P$, must at least approximate the 'geometry + physics' of the older theory, $T' = G' + P'$, under the appropriate circumstances. Of course, one never knows which geometric components of a spacetime theory will be retained, transformed, or dropped, altogether. For example, if the proposal for an alternative (R1) construction of GTR mentioned in Barbour (1999, 349–50) were experimentally confirmed, then supposedly the conformal structure of space (the geometry of shapes and angles) would form the core geometric component of the overall theory (in conjunction with an action principle), since the metrical notion of 'distance' would be purged from GTR. Barbour's theory will allegedly show how to eliminate this excess structure, while simultaneously explaining how distance was (mistakenly) viewed as crucial to the geometric structure of all past theories. Barbour's project may ultimately fail; but, for our purposes, it does conform to this structural realist strategy of countering the underdetermination problem.

The second strategy for dealing with a Poincaré-style underdetermination is to invoke Laudan's general distinction between its non-unique and egalitarian forms (Laudan 1996). If, on the egalitarian option, all geometric structures are compatible with same physical evidence, then a structural realist interpretation of spacetime is indeed in trouble; but, if the underdetermination is limited to just 'more than one' coordination of geometry and physical theory, as the nonunique variety insists, then underdetermination does not represent a serious problem. The SR theorist can contend that, given the complexity of physical phenomena and the latitude in the construction of $T = G + P$, there can be no guarantee of a unique final spacetime theory—rather, it is possible that several (but not all) different combinations of 'geometry plus physics' ($T = G + P$) equally satisfy the protocols of the scientific method. This interpretation upholds a realism about structure by admitting only a select number of different geometric structures, implying that the vast majority of structures are not exemplified in physical phenomena—a point made earlier, in section 2.2, about simple non-geometric structures as well. In order to defeat the pessimistic meta-induction, however, each of these different structures must, *individually,* be interpreted as an invariant feature across all the other past and present spacetime theories: that is, as in our first response to the underdetermination problem, all other successful $T' = G' + P'$ combinations must be approximately contained in *each* of the other $T = G + P$ combinations that makeup the nonunique class of successful spacetime theories. Returning to Poincaré's examples, the 'flat space plus forces' theory must approximate empirically the 'curved space without forces' theory, and visa versa—as is indeed the case. What this second response to the underdetermination problem contends is that, in the limit of scientific theorizing, there may remain a select handful of geometry plus physics combinations (and not just one) that save the phenomena. To take another example, it is conceivable (albeit highly unlikely) that both the standard curved space (tensors on manifold) version of GTR and Barbour's Machian instantaneous 3-space formulation (as above) might ultimately account for the empirical data equally well (such that both theories are identically consistent with other well-confirmed physical laws, etc.). Given this outcome, a nonunique response to the underdetermination problem seems warranted. Yet, for those sympathetic to the first response to the underdetermination problem (i.e. that there is a unique solution for each alleged underdeterminism case), the possibility that two or more theories could account equally for the empirical data, along with all of the other virtues of the scientific process (consistency, fruitfulness, etc.), will appear far-fetched.

4. Conclusion

By way of conclusion, it would be helpful to examine the SR approach to spacetime theories against the backdrop of the current debate on ontological and epistemological implications of the structures themselves. Epistemic structural realism, as its name implies, regards the mathematical structures of our best scientific theories as only providing epistemological information, and not information on the ontology that underlies the observed structural relationships. The ontic structural realists, in contrast, claim that structures do reveal facts or truths about the underlying ontology and may, in fact, *be* the underlying ontology (see, e.g. Chakravartty 2003).

Overall, the view of spacetime structure advocated in our investigation would seem to favor the epistemic strand of SR, since the mathematical form of structuralism that has served as the basis of our understanding of spacetime structure is a minimal nominalism (and includes the *in re* modal structuralists). As described in section 2.2, both the sophisticated substantivalists and (R2) relationists predicate spacetime structure on some form of 'entity'—whether that entity is a unique substance (spacetime), body, or field, the structure is grounded upon that entity. Accordingly, in agreement with Psillos (forthcoming), an SR spacetime cannot uphold the more radical ontic structuralist conception that 'all there is' may be structure. On the other hand, the more mild forms of ontic structuralism, which regards structure as just revealing facts or truths about the underlying ontology, could also be judged as consistent with the general SR approach to spacetime theories (although it remains unclear what type of ontological facts or truths can be divulged). More work on the specific details of the ontic and epistemic varieties of SR would be required to make a more informed decision on this matter. However, there is one conclusion that can be confidently reached; namely, that the ontic/epistemic dichotomy does not constitute a surrogate substantival/relational distinction. The ontic/epistemic debate does not divide along the lines of attributing to physical reality either the existence or non-existence of some alleged ontological member (spacetime structure). Rather, the ontic/epistemic dispute turns on the type of information, either epistemic or ontological, that scientific theories convey (e.g. one could accept the epistemic SR theory, yet accept the arguments for sophisticated substantivalism).

Finally, to recap the advantages of the SR approach to spacetime theories over the traditional (substantival versus relationist) ontological dichotomy, it has been argued above that: (1) the SR approach more successfully captures the importance of geometric structure to spacetime theories; (2) by treating spacetime structure as mathematical structure, one gains an insight into the non-causal relevance of spacetime structure for understanding dynamical phenomena (i.e. all mathematical structures are non-causal); and (3) the SR spacetime approach treats the top two contenders in the ontological dispute (i.e. sophisticated substantivalism and (R2) relationism) as the very same type of SR spacetime theory, thus eliminating a great deal of unproductive quarreling. All told, spacetime physics may actually prove to be one of the more congenial environments for the application of SR.[13]

Notes

[1] An objection to grouping the similarly structured substantival and relationist theories might draw on Maudlin (1993), whose classificational scheme attempts to formulate both relationist and substantivalist versions of specific spacetime structures, e.g. Newtonian substantivalism, Newtonian relationism, etc. For many (R2) relationist theories, however, both the structures advocated, *and* the physical implications of those structures (such as a lone rotating body), seem identical to those espoused by the substantivalists. A case in point is Teller's (1991) 'liberalized' relationism, which not only employs the same inertial structure as the substantivalist (381), but also may only require the existence of one object or event (396) to ground these structures. The vacuum solutions to GTR need not be seen as violating this 'one object/event' rule, moreover. Following Earman and Norton (1987, 519), one can identify the gravity waves in these models as material objects/events (since they can in principle be converted to other forms of energy); and, for the solutions without gravity waves, one could either accept Harré's suggestion that they have 'no reasonable physical interpretations' (1986, 131), or simply side with Einstein's hypothesis that the *g* field is a physical field.

[2] In the succeeding sections (Sections 3.2 and 4), the reason for not advocating the stronger claim, 'all that exists is structure', will become apparent.

[3] This is a fairly informal presentation of the identity of structuralist theories. Formulating an identity criterion for SR is a work in progress, as noted in Da Costa and French (2003, 122), but one could utilize their method of (partial) isomorphisms among the sub-structures of models (48–52). Shapiro (1997, 91–93) also describes several means of capturing 'sameness of (mathematical) structure' across different systems through the use of a full, or partial, isomorphism of the objects and relations of the compared structures.

[4] Some of the different mathematical formulations of spacetime theories can be regarded in this manner, i.e. they may employ the same structures, but simply disagree on which structure is more fundamental. For instance, in the twistor formalism of Penrose, conformal structure is basic, with other structures as derivative, whereas the standard tensor formalism would consider conformal structure as derivative of the manifold and metric structures. The necessity of the mathematical structure to the function of the theory is the key point, here, regardless of its primary or derived status. This point is reminiscent of Quine's famous critique of Carnap. Quine argued that the conventional element in Carnap's treatment of geometric truth arose, not in determining the truths of geometry (since 'the truths were there'), but in selecting from that interrelated set of pre-existing truths which ones would serve as the fundamental Euclidean axioms, and which ones would serve as the derived results (Quine 1966, 108–109). In section 2.3, we will return to this topic, with the competing mathematical formulations of a spacetime theory (such as GTR) subsumed under the more general problem of underdeterminism, such that there are many different theoretical combinations of 'geometry coordinated to physical processes' ($T = G + P$) that save the phenomena.

[5] One of Hoefer's reasons for singling out the *g*-field is that a metric field without a global topology is possible 'for at least small patches of space-time', although a manifold M without a metric cannot supply even a portion of spacetime (Hoefer 1998, 24). But, once again, the overall structure of GTR (i.e. for more than just small patches) mandates *both* M and g (in the standard formalism).

[6] The view of structure advocated in Brading and Landry (forthcoming), called 'minimal structuralism', would seem to be in accord with the 'minimal nominalist' structuralist position advocated in this essay. 'What we call minimal structuralism is committed only to the claim that the kinds of objects that a theory talks about are presented through the shared structure of its theoretical models and that the theory applies to the phenomena just in case the theoretical models and the data models share the same kind of structure. No ontological commitment—nothing about the nature, individuality or modality of particular objects—is entailed' (Brading and Landry, forthcoming, 20). If one interprets 'objects' as including substantival space, bodies, fields, etc., then this characterization of structure could apply to the minimal nominalist approach to spacetime structure described above.

[7] It should be noted that the problem of incongruent counterparts (Kant's 'handed' objects) could stand as a counter-argument to the claims of the causally inert status of spacetime. On the other hand, one could always invoke a maneuver similar to that advocated in Sklar (1985, 234–48), which accepts an 'intrinsic' property of handedness (i.e. a primitive property defined via continuous rigid motions) to avoid the commitment to substantival space.

[8] See Teller (1991), Sklar (1990), Bricker (1990), and Azzouni (2004, 196–212), for similar arguments about the causal irrelevance of spacetime structures for explaining accelerated motions and effects. An important early critique is Einstein (1923, 112–13), which labels absolute space as 'a fictitious cause' since rotation with respect to absolute space is not 'an observable fact of experience'. Furthermore, with respect to the interrelationship between the metric and matter fields in GTR, (i) this relationship is not normally presented as 'causal', (ii) nor does it explain the non-inertial forces associated with accelerated motion (rather, the interrelationship is relevant to explaining the metric curvature).

[9] See Barbour (1982). The attempt to explain non-inertial force effects as due to the relative motion of matter through the Higgs field (in contemporary quantum field theories) would thus constitute an exception to this line of reasoning, for a direct causal explanation is intended. However, we are dealing with the classic GTR models in this essay.

[10] Earman's text quoted above erroneously contains an extra $1/dt^2$ term in the left-hand side of the equation for the covariant derivative in coordinate form, as verified through personal correspondence with Earman.

[11] Since the minimal nominalists reject Platonism, they do not claim, of course, that mathematical entities and their relationship enter into spacetime theories. But, spacetime structures are not just the *physical relationships* between the physical objects, either, since (as argued above) causation is not a spacetime relationship (whereas causation is most definitely a physical relationship among physical objects). Spacetime structures, like all other mathematical structures, are unique in this regard: systems can exemplify the structures, but the structures are not identical to the systems, nor (for the nominalists) can they exist in the absence of systems.

[12] A viewpoint remarkably similar to the SR thesis developed above can be found in the work of Stein (1977) and DiSalle (1995). Not only do they reject the standard substantival versus relational classification, but they place special emphasis on denying a causal role to spacetime: 'A spacetime theory does not *causally explain* phenomena of motion, but uses them to construct physical *definitions* of basic geometrical structures by coordinating them with dynamical laws' (DiSalle 1995, 317). The underdetermination problem poses a threat to this conception, nevertheless, since DiSalle leaves open the potential for a viable coordination of the physical and geometrical aspects of theories alternative to general relativity: e.g. the Brans–Dicke theory, but only if a physical process can be located for the needed coordination with the long-range scalar field (geometrical component), which provides a (relational) Machian conception of mass (DiSalle 1995, 334–35).

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References

Azzouni, J. 2004. *Deflating existential consequence.* Oxford: Oxford University Press.

Barbour, J. 1982. Relational concepts of space and time. *British Journal for the Philosophy of Science* 33: 251–274.

Barbour, J. 1999. *The end of time.* Oxford: Oxford University Press.

Belot, G. 2000. Geometry and motion. *British Journal for the Philosophy of Science* 51: 561–595.

Belot, G., and J. Earman. 2001. Pre-Socratic quantum gravity. In *Physics meets philosophy at the Planck scale,* edited by C. Callender and N. Huggett. Cambridge: Cambridge University Press.

Brading, K., and E. Landry. Forthcoming. A minimal construal of scientific structuralism. *Philosophy of Science.*

Bricker, P. 1990. Absolute time versus absolute motion: Comments on Lawrence Sklar. In *Philosophical perspectives on Newtonian science,* edited by P. Bricker and R. I. G. Hughes. Cambridge, MA: MIT Press.

Brighouse, C. 1994. Spacetime and holes. In *PSA 1994,* vol. 1, edited by D. Hull, M. Forbes, and R. Burian. East Lansing, MI: Philosophy of Science Association.

Butterfield, J. 1989. The hole truth. *British Journal for the Philosophy of Science* 40: 1–28.

Chakravartty, A. 2003. The structuralist conception of objects. *Philosophy of Science* 70: 867–878.

Chihara, C. 2004. *A structural account of mathematics.* Oxford: Oxford University Press.

Da Costa, N. C. A., and S. French. 2003. *Science and partial truth.* Oxford: Oxford University Press.

Dieks, D. 2001. Space and time in particle and field physics. *Studies in History and Philosophy of Modern Physics* 32: 217–241.

DiSalle, R. 1995. Spacetime theory as physical geometry. *Erkenntnis* 42: 317–337.

Dorato, M. 2000. Substantivalism, relationism, and structural spacetime realism. *Foundations of Physics* 30: 1605–1628.

Earman, J. 1989. *World enough and space-time: Absolute versus relational theories of space and time.* Cambridge, MA: MIT Press.

Earman, J., and J. Norton. 1987. What price space-time substantivalism? The hole story. *British Journal for the Philosophy of Science* 38: 515–525.

Einstein, A. 1923. The foundations of the general theory of relativity. In *The principle of relativity.* Translated by W. Perrett and G. B. Jeffery. New York: Dover.

Einstein, A. 1961. Relativity and the problem of space. In *Relativity: The special and the general theory.* New York: Crown.

Field, H. 1980. *Science without numbers.* Princeton, NJ: Princeton University Press.

Friedman, M. 1983. *Foundations of space-time theories.* Princeton, NJ: Princeton University Press.

Harré, R. 1986. *Varieties of realism.* Oxford: Blackwell.

Hellman, G. 1989. *Mathematics without numbers.* Oxford: Oxford University Press.

Hinckfuss, I. 1975. *The existence of space.* Oxford: Oxford University Press.

Hoefer, C. 1996. The metaphysics of space-time substantivalism. *Journal of Philosophy* 93: 5–27.

Hoefer, C. 1998. Absolute versus relational spacetime: For better or worse, the debate goes on. *British Journal for the Philosophy of Science* 49: 451–467.

Laudan, L. 1996. *Beyond Positivism and Relativism.* Boulder, CO: Westview Press.

Liston, M. 2000. Review of M. Steiner, *The applicability of mathematics as a philosophical problem. Philosophia Mathematica* 8: 190–207.

Manders, K. 1982. On the spacetime ontology of physical theories. *Philosophy of Science* 49: 575–590.

Maudlin, T. 1993. Buckets of water and waves of space: Why spacetime is probably a substance. *Philosophy of Science* 60: 183–203.

Mundy, B. 1992. Space-time and isomorphism. In *PSA 1992,* vol. 1, edited by D. Hull, M. Forbes, and K. Okruhlik. East Lansing, MI: Philosophy of Science Association.

Poincaré, H. 1952. *Science and hypothesis.* Translated by W. J. Greenstreet. New York: Dover, originally published 1905.

Psillos, S. Forthcoming. *The* structure, the *whole* structure and nothing *but* the structure? *Philosophy of Science.*

Quine, W. V. O. 1966. Carnap and logical truth. In *The ways of paradox,* edited by W. V. O. Quine. New York: Random House.

Resnik, M. 1997. *Mathematics as a science of patterns.* Oxford: Oxford University Press.

Rovelli, C. 1997. Halfway through the woods: Contemporary research on space and time. In *The cosmos of science,* edited by J. Earman and J. Norton. Pittsburgh, PA: University of Pittsburgh Press.

Rynasiewicz, R. 1996. Absolute versus relational space-time: An outmoded debate? *Journal of Philosophy,* 93: 279–306.

Shapiro, S. 1997. *Philosophy of mathematics: Structure and ontology.* Oxford: Oxford University Press.

Shapiro, S. 2000. *Thinking about mathematics: The philosophy of mathematics.* Oxford: Oxford University Press.

Sklar, L. 1974. *Space, time, and spacetime.* Berkeley, CA: University of California Press.

Sklar, L. 1985. *Philosophy and spacetime physics.* Berkeley, CA: University of California Press.

Sklar, L. 1990. Real quantities and their sensible measures. In *Philosophical perspectives on Newtonian science,* edited by P. Bricker and R. I. G. Hughes. Cambridge, MA: MIT Press.

Stein, H. 1977. On space-time and ontology: Extract from a letter to Adolf Grunbaum. In *Foundations of space-time theories,* edited by J. Earman, C. Glymour, and J. Stachel. Minneapolis, MN: University of Minnesota Press.

Teller, P. 1987. Space-time as physical quantity. In *Kelvin's Baltimore lectures and modern theoretical physics,* edited by P. Achinstein and R. Kargon. Cambridge, MA: MIT Press.

Teller, P. 1991. Substance, relations, and arguments about the nature of space-time. *Philosophical Review,* 100: 362–397.

Worrall, J. 1996. Structural realism: The best of both worlds? In *The philosophy of science,* edited by D. Papineau. Oxford: Oxford University Press.