The "Dynamics" of Leibnizian Relationism: Reference Frames and Force in Leibniz's Plenum

Edward Slowik

Abstract

This paper explores various metaphysical aspects of Leibniz's concepts of space, motion, and matter, with the intention of demonstrating how the distinctive role of force in Leibnizian physics can be used to develop a theory of relational motion using privileged reference frames. Although numerous problems will remain for a consistent Leibnizian relationist account, the version developed within our investigation will advance the work of previous commentators by more accurately reflecting the specific details of Leibniz's own natural philosophy, especially his handling of the dynamical interactions of plenum bodies.

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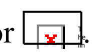
Section 1: Introduction

The details of Leibniz's physics would suggest to many later commentators the structures implicit in his preferred, if not consistently avowed, relationist alternative to Newton's absolute/substantival space: these include, most importantly, the restriction of the spacetime invariants to the relative distances, velocities, and accelerations manifest among the material occupants of the spacetime. The possibility of implementing a consistent Leibnizian physics limited to the relative motions of this spacetime backdrop, a structure that is often labeled "Leibnizian spacetime", will form the basis of our investigation—and the key factor that will emerge in our analysis is the foundational role of the privileged reference frames or perspectives that are tacitly presupposed in Leibniz's handling of the bodily collisions that underwrite his conservation law for . As will be demonstrated, however, a close examination of the many intriguing metaphysical and physical hypotheses that comprise Leibnizian natural philosophy will disclose structures and properties that challenge the suitability of a Leibnizian spacetime designation for Leibniz's own theory.

Admittedly, the non-relational character of Leibniz's world system has been the subject of several recent investigations: see, Gale, 1988; Cook, 1979; Arthur, 1994; Lodge, 2003; and, Roberts, 2003. While these essays have considerably improved our understanding of Leibniz's natural philosophy, it will be argued that the unique, and somewhat peculiar, nature of Leibniz's physics has nonetheless eluded most previous commentators. In particular, there are several interrelated features of Leibniz's overall physical system that, *collectively,* have not been adequately examined in the Leibnizian literature: (a) the local versus the global properties of the spacetime, (b) the foundational role of force (*vis viva*) and privileged spatial perspectives in explicating bodily motions, and (c) the plenum-continuum setting wherein Leibnizian dynamics unfolds. Underlying all of these issues is (d) Leibniz's penchant for blurring the kinematic and dynamic components of his physics, a problematic facet of his approach which will be seen to engulf nearly all of his (quite numerous) physical hypotheses. Throughout our analysis, the closely analogous problems that beset Descartes' physics will be frequently discussed. Not only will the comparison with Descartes prove informative in evaluating the content of Leibniz's hypotheses, but the obvious similarities between their respective programs will likewise betray the allegedly anti-Cartesian intention and outcome of Leibniz's physics.

In section 2, the main elements of Leibniz's relationism will be briefly examined, thus laying the groundwork for the more extensive exploration, in section 3, of the role of force in Leibnizian natural philosophy. Finally, in section 4, the attempt to utilize a privileged system of reference frames to construct a relational Leibnizian physics will be critically assessed.

Section 2: Leibniz and Relationism: An Overview

It is important to specify first the domain of works that comprise our study. The central text in Leibnizian dynamics is the unpublished *Dynamica* (1689-1690), along with the accompanying essay, "Specimen Dynamicum" (1695), but our investigation will include many physical tracts from this general period; i.e., from the mid-1680s to the early-1700s. In the later phase of Leibniz thought, 1705 until 1716, the phenomenal aspects of space and motion come to the foreground, with the integration of his metaphysical doctrines into a viable theory of material body interactions playing a much diminished role. In reconstructing Leibniz's physics, therefore, the later phenomenal essays can be bracketed away from our inquiry (see, Hartz and Cover, 1988).

2.1. Extension and Space. A number of important relationist doctrines are consistently endorsed throughout Leibniz's mature years, the most prominent being (R2), the rejection of space as a form of substance or property that can exist independently of material bodies (loosely following Earman, 1989, p. 12). In 1695, he states: "Extension or space and the surfaces, lines, and points one can conceive in it are only relations of order or orders of coexistence, both for the actually existing thing and the possible thing one can put in its place" (Ariew & Garber, 1989, p. 146). Later, in the correspondence with Clarke, he openly acknowledges that space lacks this form of independent existence: "But if there were no creatures [bodies, persons, etc.], space and time would only be in the ideas of God" (p. 331).

As for matter, Leibniz pointed out that the Cartesian physics of mere extension was incapable of explaining the most basic interactions among bodies; in particular, the fact that bodies "resist" being both penetrated and moved in bodily collisions (which fall under "derivative passive force" in the "Specimen"; pp. 119-122). As for the "primitive passive force", which grounds the resistance of bodies to both penetration and motion, Leibniz likens it to the primary matter of the Scholastics, and tentatively suggests that "matter in this sense [primitive passive force] will not be extended or divisible, though it will be the principle of divisibility or that which corresponds to it in the [material] substance" (letter to Arnauld, 9 October 1687: Loemker, p. 350). Leaving aside the problem of how one can make sense of material interactions *prior* to a concept of extension, a theory that bases spatial notions on the "resistance" properties of bodies is largely in keeping with relationist doctrine, since space and extension are thus reduced to the *physical* properties of resistance to penetration and motion (although Leibniz's entire analysis is, obviously, more metaphysical and phenomenal than physical).

2.2. Motion and Vis Viva. For strict relationists, given a relative change in motion among two or more bodies, all determinations of the individual components of motion (speed, velocity, acceleration) are purely arbitrary. For instance, while the "relative velocity" between bodies A and B may total "5 kilometers per hour", and this exact quantity is measured identically by all observers (i.e., it is an invariant among the allowable reference frames of the spacetime), the measurement of each body's specific velocity is relative to different frames. The thesis that all motion is the reciprocal motion of bodies, or (R1) (Earman, 1989), is a repeated theme in Leibniz's mature natural philosophy, as the following passage from the *Discourse* typifies:

> If we consider only what motion contains precisely and formally, that is, change of place, motion is not something entirely real, and when several bodies change position among themselves, it is not possible to determine, merely from a consideration of these changes, to which body we should attribute motion or rest, as I could show geometrically. . . . (Ariew & Garber, 1989, p. 51)

Alas, there are two general and related problems, which we can label (P1) and (P2), that quickly compromise (R1) relationism given Leibniz's system. As for the first, (P1), Leibniz proceeds to break the relational symmetry of (R1) by positing an internal "force" within one (or possibly both) of the bodies, a force that is causally responsible for the manifest relational change in motion:

> But the force or proximate cause of these changes is something more real, and there is sufficient basis to attribute it to one body more than another. Also, it is only in this way that we can know to which body the motion belongs. (Ariew & Garber, 1989, p. 51; see, also, Loemker, 1969, p. 496)

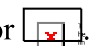
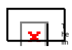
In effect, Leibniz is claiming that bodies do indeed possess individual states of motion, in direct violation of (R1), since "we can know to which body the motion belongs".[i] The rationale for Leibniz's avowal of individual bodily states of motion can probably be traced to his revival of a "substantial forms" metaphysics, along with its implicit assumption that "if motion, or rather the motive force of bodies, is something real, . . . it would need to have a *subject"* (Letter to Huygens, 12/22 June 1694: Ariew & Garber, 1989, p. 308).

The second impediment to (R1) relationism, (P2), stems from Leibniz's *vis viva* conservation principle, [illegible], and its consequences for the spacetime structures required to support this law. From roughly the mid-1680s, Leibniz began to publish a series of influential critiques of the Cartesian conservation law for the "quantity of motion" (or size [illegible] speed), contending instead that the conserved quantity employs the direction and squared speed, [illegible], of the colliding bodies (e.g., "Essay de Dynamique…", Gerhardt, 1962, pp. 215-231). By embracing a robust notion of "velocity" in his conservation law (which he labels, "conatus", see section 3.2), as opposed to "velocity difference", Leibniz would thus seem committed to a spacetime structure much stronger than (R1) relationism can condone. If motion is purely relative, as some of Leibniz's statements seem to endorse, then *all* determinations of motion are equally legitimate, such that the speed and path of any given bodily motion can be measured, with equal meaningfulness or validity, by any and all reference frames. A theory that incorporates velocity into its conservation law, on the other hand, will only allow a limited number of reference frames, namely, the inertially related reference frames, to uphold an invariant measure of the conserved quantity of motion. Consequently, if Leibnizian physics must conserve , then a spacetime that possesses the ability to discern inertial and non-inertial motions must also be adopted, such as Newtonian or Neo-Newtonian spacetimes. Unfortunately, these spacetime structures are incompatible with (R1) relationism, whose spacetime invariants can only include relative measures of motion. The spacetime that is usually associated with a robust (R1) relationism, ironically dubbed "Leibnizian spacetime" in the literature, is thus incapable of discerning the privileged inertial frames required for Leibniz's own conservation law of motion.

2.3. Privileged Reference Frames: A Possible Solution? For the relationist, there may be alternative methods of picking out the needed inertial frames that, ostensibly, can avoid the second of the two problems introduced above. Rather than build the inertial structure into the spacetime, one can retain a weaker structure that only admits relational quantities of motion, yet allows the material contents of the spacetime *themselves* to single out the (inertial) reference frames needed to consistently describe the relevant physical interactions. Put simply, the relationist can claim that, given a system of interacting material bodies, there will always exist certain reference frames from which the interactions among those bodies maintain the conservation law for . As long as these reference frames are construed as co-existent with matter, and the inertial paths of bodies are determined relative to these frames, then the relationist can claim that the need for an absolute or substantivalist conception of spacetime has been averted. Overall, this reconstruction of Leibnizian physics may, or may not, uphold the thesis (R1), which states that all motion is relative, such that there can exist no privileged determinations of motion. (That is, the fact that there exists a privileged set of reference frames, even if defined relative to material bodies, may be construed as a violation of (R1), since inertial structure has been covertly espoused, or so they might claim.) Nevertheless, our Leibnizian can confidently claim that her new hypothesis of "privileged (relational) reference frames", as they may be called, does uphold the equally important thesis (R2), which denies that space (spacetime) is an independently existing entity.

In the recent literature on the prospects for a relational physics, the use of privileged reference frames has found a number of advocates, namely Huggett (1999) and Slowik (1999a). More importantly, there may exist a textual precedent for this maneuver in Leibniz's work: in the document, "On Copernicanism and the Relativity of Motion" (1689), Leibniz proposes that considerations of the "greater intelligibility and simplicity" of an hypothesis, in comparison with a rival hypothesis, can be utilized in deciding between their conflicting judgments of the individual motions of bodies. Although he acknowledges that motion is entirely reciprocal, (R1), one can still regard the Copernican theory as true as long as one understands that truth is being used in a special sense:

> The truth of a hypothesis should be taken to be nothing but its greater intelligibility, . . . , so that henceforth there would be no more distinction between those who prefer the Copernican system as the hypothesis more in agreement with the intellect, and those who defend it as the truth. For the nature of the matter is that the two claims are identical; nor should one look for a greater or a different truth here. And since it is permissible to present the Copernican system as the simpler hypothesis, it would be permissible to teach it as the truth in this particular sense. (Ariew & Garber, 1989, p. 92)

Leibniz is careful to point out that the Ptolemaic system can also be deemed true if viewed in another context; specifically, "the Ptolemaic account is the truest one in spherical astronomy" (p. 92). Consequently, while "motion is not something absolute, but consists in a relation" (p. 92), it is nevertheless the case that other factors (e.g., simplicity) can be brought into play that can select a particular perspective, or reference frame, for breaking the relational symmetry of (R1) motion, and thus ascribing rest and motion to individual bodies. For the Leibnizian intent on establishing a relationally consistent physics, the "purpose" that is best served by adopting a set of inertially related reference frames is, of course, the conservation law for . From the perspective of these privileged reference frames,  is conserved: but, the commitment to a relational theory of space and motion—(R2), and possibly (R1)—is likewise maintained, since there are many other purposes and perspectives that can be selected.

The question remains open, however, if the privileged reference frame hypothesis can fully capture Leibniz's overall approach to, and understanding of, the motion of bodies. As an initial criticism of the privileged frame procedure, it must be immediately acknowledged that it is not ideally suited to resolving our first problem for Leibniz's relationism (P1), namely, that individual bodies possess determinate states of motion. By claiming that the relative motion of bodies is only apparent, such that every body is really at rest or in a determinate state of motion, Leibniz would seem to have inadvertently undermined any attempt to construct a relationist spacetime *directly* on those relative bodily motions—and, unfortunately, relative motions are *the only* invariant quantities open to the relationist utilizing Leibnizian spacetime structure (since the privileged reference frames are grounded directly upon those relative changes in motion). If one admits individual states of motion into the spacetime, then relative motion loses its essential ontological and epistemological status: it is the individual states of motion which now become the important invariant quantity, with relative motions assuming a secondary, epiphenomenal importance (see, Garber, 1995, p. 309). The privileged reference frame technique therefore seems incapable of resolving our first problem of relational motion, (P1), since the force responsible for individual bodily motion is not discernible in a spacetime restricted to relational quantities of motion.

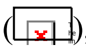
3. The Multifaceted Role of Leibnizian Force.

There are fundamental questions that can be raised with respect to Leibniz's proposal, in "On Copernicanism", that equates truth with the most intelligent hypothesis. These questions inevitably revolve around the status of that all-important Leibnizian physical doctrine, which he entitled, the "equivalence of hypotheses" (hereafter, EH). In particular, the local versus the global scope of EH has not been adequately treated in the literature; and, as will be demonstrated, the lessons drawn from this investigation have the potential to shed some important light on Leibnizian motion and force.

3.1. The Equivalence of Hypotheses. It is difficult to estimate the significance of EH for Leibniz's physics, for at face value it seems to be just a restatement of the reciprocity of motion in the *Discourse,* which we have labeled (R1). Yet, in the "Specimen"*,* Leibniz is quick to draw specific lessons for the laws of motion based on these insights. After chastising Descartes' collision rules for failing to uphold the relativity of motion,[ii] he continues:

> Therefore, we must hold that however many bodies might be in motion, one cannot infer from the phenomena which of them really has absolute and determinate motion or rest. Rather, one can attribute rest to any one of them one may choose, and yet the same phenomena will result. From this follows something that Descartes did not notice, that *the equivalence of hypotheses is not changed even by the collision of bodies with one another,* and thus, the laws of motion must be fixed in such a way

> that the relative nature of motion is preserved, so that one cannot tell, on the basis of the phenomena resulting from a collision, where there had been rest or determinate motion in an absolute sense before the collision. (Ariew & Garber, 1989, p. 131; original emphasis)

The formal presentation of EH occurs, not coincidentally, in the *Dynamica* (particularly, propositions 14 and 19 of part II, section III; GM VI 500-507), which indicates its central role in his physics. Indeed, the EH even turns up as the justification of his “truth = more intelligent hypothesis” maneuver (in “On Copernicanism”, p. 91). Furthermore, Leibniz’s reference to the importance of the EH for his laws of motion pertains not only to the conservation of *vis viva* (), which he calls “absolute” force, but also to his discovery of the conservation of “common progress” (momentum, ), as well as the “respective (relative) speed” that is conserved among colliding bodies (see, Gerhardt, 1962, pp. 215-231, pp. 488-514).

Provided this textual evidence, the emphasis that Leibniz places on the mutual impact of bodies cannot be underestimated in seeking to understand the function of EH. As he declares in a later passage of the “Specimen”: “It also follows from the relative nature of motion that *the mutual action or impact of bodies on one another is the same, provided that they approach one another with the same speed”* (Ariew & Garber, 1989, p. 131, original emphasis). In a revealing example (which continues the preceding quote), Leibniz’s discussion slides subtly from the equivalence of the ascriptions of bodily motion/rest, to the equivalence of the dynamical effects within each of the bodies during the collision:

> That is, if we keep the appearances in the given phenomena constant, then whatever the true hypothesis might finally be, to whichever body we might in the end truly ascribe motion or rest, the same outcome would be found in the resulting phenomena, even as regards the action of bodies on one another. And indeed, this is just what we experience, for we would feel the same pain whether we hit our hand against a stone

> at rest, suspended, if you like, from a string, or whether the stone hit our resting hand with the same speed. (Ariew & Garber, 1989, p. 131)

The rationale for Leibniz's interest in the equivalence of the experienced force effects of impact, as opposed to just the relational motion among the bodies, becomes all the more clear once the metaphysical grounding for Leibnizian motion is taken into account. In the ensuing passages of the "Specimen", Leibniz puts forward his well-known thesis that substances do not causally interact (*"what happens in a substance can be understood to happen of that substance's own accord",* p. 131). The ideality of space and motion is therefore of primary importance in explaining the seemingly fundamental role that force assumes in the description of EH: "space, time, and motion are, to a certain extent, beings of reason, and are true or real not *per se,* but only to the extent that they involve either the divine attributes . . . or the force in created substances" (p. 130). If the forces *within* bodies are ontologically privileged, rather than the mere relations among bodies (such as the geometrical relations of space, and hence motion), then it naturally follows that Leibniz's would stress the equal importance of the equivalence of action/reaction forces among the colliding bodies, as opposed to just the equivalence of the determinations of motion: or, to put it differently, the Leibnizian metaphysics of force apparently sanctions a theory of impact that regards the "equivalence of hypotheses" as pertaining to the purely arbitrary stipulation of which body *strikes,* and which body *reacts,* in a given collision, as well as the equally arbitrary choice of which body moves or remains at rest.

If the foregoing analysis of the EH is an accurate estimate of Leibniz's reasoning, then it casts the relationship between the kinematical and dynamical aspects of his physics in an interesting light (where, as used in this essay, "kinematics" refers to the study of motion *per se,* and "dynamics" to the motion of bodies under forces, whether during a collision or not). Many of Leibniz's contemporaries approached the conserved quantities in bodily collisions from a decidedly kinematical standpoint, such that the forces involved in the actual bodily collision were largely glossed over, since the main interest resided in the conservation, or lack thereof, among the frame-dependent, observable properties of velocity, momentum, etc.[iii] In Leibniz's case, however, what transpires in the actual collision between the bodies assumes a great deal more prominence, almost to the exclusion of any other factor: one might even conjecture that the "equivalence" of the metaphysically-privileged action/reaction forces manifest in the collision (i.e., that determinations of which body strikes/reacts are entirely conventional) serves as the *metaphysical foundation* for the less-basic, phenomenal EH (*since* the motion and spatial properties of bodies that appear in the EH, conceived purely as reciprocal motion, are indeed less basic). On a less contentious reading, it is clearly the case that Leibniz perceives an important parallel between the action/reaction behavior of bodies and the reciprocal nature of motion (EH), and this might explain why the EH figures so prominently in his post-*Dynamica* writings. In essence, the EH is much more than just a restatement, within his study of impact, of his earlier observation of the reciprocity of assignments of individual motion—it is rather an extension of the "equivalence" concept itself to *include* bodily impact in addition to the already well-known (by Leibniz's time) equivalence of bodily rest/motion. It is only on this reading that we can make sense of the passages quoted earlier, in particular, "to whichever body we might in the end truly ascribe motion or rest, the same outcome would be found in the resulting phenomena, *even as regards the action of bodies on one another*" (p. 131; added emphasis).[iv]

While the increased scope of the EH to include the dynamics of bodily impact does not in itself compromise the privileged reference frame technique, it does point to a very different conception of the role of motion in Leibniz's natural philosophy. This different conception assumes a greater prominence once the additional complexities of the Leibnizian system are brought into play; specifically, the plenum setting, and the elasticity of all material particles.

3.2. Rotation and Centrifugal Force. In contrast to most of his Cartesian forebears, Leibniz held that "no body is so small that it is without elasticity" (p. 132). Yet, Leibniz concurred with the Cartesians in insisting on the continuous divisibility of matter, as well as the lack of a vacuum (all nicely summarized in the "Specimen"; pp. 130-132). Given this literally constrained physical domain, the move towards a dynamical conception of bodily phenomena becomes all the more obvious. To better grasp the function and interplay of Leibniz's various physical hypotheses, it will be beneficial to examine a specific instance of a solution to a physical problem: the attempted explanation of circular motion is ideal for this purpose, for not only does it demonstrate how the numerous variables in Leibniz's physics work together, but circular motion had been singled out by the Newtonians as an inexplicable problem for a relationist-based physics.

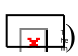
Before launching into the analysis of rotational motion, however, it will be necessary to briefly survey Leibniz's intricate concept of "active derivative force", along with its subsidiary notions. In the "Specimen", he defines "velocity" as a measure of "motio", the latter being correlated with an instantaneous element of motion. "Conatus" ("nisus", "solicitation") arises from conjoining velocity with direction, and together with "bulk" (size) comprise "impetus" (which he also associates with the instantaneous measure of the Cartesian's quantity of motion; p. 120). Finally, "dead force" corresponds to conatus, and is thus the instantaneous measure of force, while "living force" signifies force over time (i.e., "which arises from an infinity of continual impressions of dead force"; p. 122). Overall, Leibnizian rotational motion employs the concept of conatus to explicate the centrifugal tendencies experienced by a rotating body, an approach that draws heavily on the established Cartesian tradition. Like Descartes, Leibniz identifies two instantaneous quantities (either impetus or conatus) in a rotating object, say, a ball in a rotating tube, at a given point in its trajectory: first, the impetus directed along the circular path ("maintaining the same distance from the center"; p. 121), and second, the impetus of the ball directed radially outward along the tube.[v] The concept of dead force therefore figures prominently in the explanation of the origin of centrifugal force-effects, since "motion does not yet exist in it [a body], but only a solicitation to motion [i.e., conatus], as with the ball in the tube, or a stone in a sling while it is still being held in by the rope" (p. 121).

It will be instructive to quote in full Leibniz's ensuing explanation of how these instantaneous dynamical properties of the rotating body account for its centrifugal force.

> Since only force and the nisus arising from it exist at any given moment (for motion never really exists, . . .), and since every nisus tends in a straight line, it follows that *all motion is either rectilinear or composed of rectilinear motions.* From this it not only follows that what moves in a curved path always tries to proceed in a straight line tangent to it, but also—something utterly unexpected—that the *true notion of solidity* derives from this. (Nothing is really solid or fluid, absolutely speaking . . .). For if we assume something we call solid is rotating around its center, its parts will try to fly off on the tangent; indeed, they will actually begin to fly off. But, since this mutual separation disturbs the motion of the surrounding bodies, they are repelled back, that is, thrust back together again, as if the center contained a magnetic force for attracting them, or as if the parts themselves contained a centripetal force. Thus, the rotation arises from the composition of the rectilinear nisus for receding on the tangent and the centripetal conatus among the parts. Thus, all curvilinear motion arises from rectilinear nisuses composed with one another, and at the same time, it is

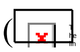
> understood that all solidity is caused by surrounding bodies pushing a body together; if matters were otherwise, then it could not happen that all curvilinear motion is composed of pure rectilinear motions. From this we also get a new argument against atoms that is no less unexpected than the one before. (Ariew & Garber, 1989, pp. 135-136, original emphasis; a similar exposition is given in the *Dynamica,* Gerhardt, 1962, pp. 507-511)

Throughout the remaining portion of this essay, we will attempt to elucidate the significance of this (quite astonishing) hypothesis for the viability of a relationist Leibnizian physics. In short, Leibniz's plan is to utilize the instantaneous inertial tendencies of the (presumably) infinite number of particles that comprise a body to explain *both* the centrifugal force and the solidity of the composite body (and, later, gravity as well: Ariew & Garber, 1989, p. 137). There are a number of crucial elements in this story: the plenum, the continuous divisibility and elasticity of matter, the instantaneous dynamical processes, and the equivalence of action/reaction forces in the impact of matter. As the material particles instantaneously strive to move in uniform rectilinear motion along the tangent of the circle, and "actually begin to fly off", they are "thrust back" by the surrounding plenum particles, an effect that is manifest both in the observed centrifugal force and in the perceived solidity of the rotating body that is comprised from those individual particles.

The reciprocal nature of the action/reaction forces among the component particles of the rotating body and the particles of the surrounding plenum is therefore the key to unraveling the truly unique role that Leibniz assigns the EH in this scenario: since the impact behavior of these particles *explains* the centrifugal force-effects (and solidity) of the composite body, the function of EH cannot be simply equated with the arbitrary assignment of rest and motion to the colliding particles. Besides the reciprocity of motion (R1), it must be the equivalence of the dynamic action/reaction impact behavior of the particles that falls under the EH, and thereby demonstrates why rotational motion does *not* violate the EH.[vi] Put simply, one can regard either the bodily particle as acting on the plenum particle, with the plenum particle reacting in turn, or visa-versa (i.e., plenum particle acting, and bodily particle reacting). In either case, the manifest solidity and perceived force-effects (which we describe as centrifugal) remain the same, as does the conservation law. Once the confined setting of the plenum is taken into account, furthermore, a full understanding of the EH would seem to blur the distinction, if any, between the equivalence of the assignments of bodily motion/rest and the equivalence of action/reaction forces. As there is literally no room in a plenum for the particles of a rotating body to "begin to fly off", there is correspondingly little or no difference between the instantaneous inertial conatus of each rotating particle and its dynamic interaction with a contiguous plenum particle. Leibniz's explanation of centrifugal force thereby nicely displays his penchant for downplaying any foundational role for motion in his physics, while simultaneously favoring the central ontological importance of instantaneous dynamic processes. Indeed, given his repeated declarations of the ideality of motion, as opposed to the reality of force, this is precisely what one should come to expect of Leibnizian physics. This is not to suggest, of course, that motion is irrelevant to his physics; rather, the dynamic forces that *give rise* to the durational dynamic properties (such as living force, ) are the instantaneous properties associated with conatus. Finally, it should be noted that Leibniz's analysis of rotation relies crucially on the elasticity of the particles, since the conservation laws and the equivalence of action/reaction forces are contingently based on this supposition (so as not to violate the principles of continuity, pp. 132-133). In the next section, we will begin to apply these insights into the operation of Leibnizian dynamics to the issue of relational motion and the EH.

4. Relationism in the Leibnizian Plenum.

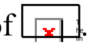
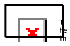
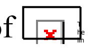
4.1. The Dynamical Domain: Local versus Global. In surveying the results of our examination thus far, there is at least one feature of Leibniz's treatment of centrifugal force that stands out as under-appreciated in the contemporary literature, both in the spacetime and Early Modern circles—namely, the localized, infinitely small setting of the interaction of bodies that falls under the EH. In striving to preserve his relationist commitments, Leibniz attempts to solve the problem of rotational bodies by invoking what he believes to be a relationally acceptable analysis of the behavior of the minute particles that *comprise* those bodies. In short, he seems to be making the inference, which may or may not be an instance of the "fallacy of composition", that if the parts of a body uphold the EH, then the whole body must preserve the EH. Or, put another way, Leibniz is trying to *reduce* the allegedly non-relational behavior of a rotating macroscopic body to the relationally palatable interactions of its microscopic component parts. When viewed in isolation from the composite whole, each collision of a rotating body's particles with a particle of the surrounding plenum can thus be treated as an instance of his general theory of impact as developed in the *Dynamica.* Once again, these localized, infinitely small regions of the plenum constitute the ontological ground floor of Leibnizian physics, for the conservation laws are strictly based on a model of impact utilizing elastic bodies moving at uniform speeds—and these types of collisions can only be presumed to hold in general at the unobservable level of the infinitely small particles that form larger bodies. In the dynamical writings of late 1680s, Leibniz's quick transition from the collisions of inertially-moving elastic bodies to his hypothesis of a universally conserved property of motion for all bodily phenomena would thus seem to gain coherence given our interpretation; i.e., since all manifestations of bodily interactions, whether at the macroscopic or microscopic, can *ultimately* be represented as a physical process involving *just* inertially moving, elastic microscopic particles, it directly follows that all physical processes satisfy the conservation laws. This assumes, once again, that one can also non-problematically integrate the of each individual collision to determine the sum for an entire macroscopic body (more on this below).

Although this general strategy of defending the EH in the case of rotation is quite ingenious, it does raise a number of intriguing questions concerning the ontological levels at which the EH operates: first and foremost, are *all* dynamical interactions of macroscopic bodies intended to be analyzed in this manner; i.e., such that behavior of the macroscopic body can be reduced to the EH-preserving behavior of its constituent parts? Leibniz clearly holds this view with respect to rotating macroscopic bodies, but does it hold, say, for all colliding macroscopic bodies, as well? While there may not be sufficient evidence to decide this issue, our interpretation does gain some confirmation from Leibniz's belief that the force apparently lost in the collisions of inelastic bodies has actually been transferred to the component particles (Gerhardt, 1962, pp. 230-231). The relevant branch of Leibnizian metaphysics that has the potential to answer this question, unfortunately, is also one of the most obscure; namely, aggregates, and the means by which aggregates are formed from simpler substances. In the 1680s and 1690s, Leibniz developed a thesis that sought to explain the unity of substances in terms of souls or entelechies, such that a substance could be divided into an infinity of further, constituent substances, each with its own body and soul (see, especially, the letter to Arnauld, 9 October 1687: Gerhardt, 1965, pp. 118-120). As Garber (1985) has convincingly argued, since Leibniz was also occupied with his dynamic writings at this time, it is therefore not surprising that one can find a ready correlation of his metaphysics and physics in these contemporaneous texts (see, Ariew & Garber, 1989, pp. 119-121); specifically, the matter and form of a substance with the passive and active primitive forces, and the modes of corporeal substance with the derivative active force (living and dead) and derivative passive force (resistance and impenetrability). If this basic analogy is accepted straightforwardly, then it would seem to follow that the living and dead forces ( and conatus) of a macroscopic body are, indeed, a combination of the living and dead forces of its constituent parts. Nevertheless, the resulting aggregate force of the macroscopic body need not be a simple linear summation of its constituent particles, since it is clearly not the case, following the analogy a bit further, that the substance of a person is a simple summation of, or *like,* the infinity of smaller substances that comprise the person (such as kidneys, liver, etc.).

In studying the conservation law on these different ontological/epistemological levels, one can also begin to discern a latent teleological tendency at its global level of operation. As disclosed in section 3.2, Leibniz believes that the EH is confirmed in the collisions of the bodily and plenum particles since one can arbitrarily opt for a view that assigns the action/reaction forces (or rest/motion) to either particle. However, if one does decide to view the body as the passive, or reactive, member of this dynamic interaction, then the plenum begins to accrue some very extraordinary global, or long-range, abilities. In effect, Leibniz's story of the equivalence of action/reaction forces must entail that a resting body can be placed in rotational motion if all of the particles along its surface are simultaneously struck, in the appropriate way, by the contiguous plenum particles. But this explanation, as John Earman correctly points out, "is potentially lame, since without further explanation it is not apparent what would occasion the otherwise miraculous coordination of the microbodies [plenum particles] needed to produce the perceived macroscopic rigid motion when the body is, say, struck on its circumference" (Earman, 1989, p. 72). This miraculous coordination of the plenum must likewise conserve the total of the universe, which demonstrates how the multi-faceted conservation law can even be put to use in resolving the problem of relational motion. A precedent for this handling of a conservation law can be found in Descartes, for quantity of motion was presumed to remain invariant in his universe despite his, often bizarre, hypotheses of bodily interaction. For instance, in trying to literally plug the plenum gaps left in the wake of moving macroscopic bodies, Descartes claimed that innumerably many smaller particles rush in to fill the space (and thus prevent the formation of a void); and although he is not forthcoming with any details on how these particles provide this convenient service, it is assumed that their interactions conserve quantity of motion (see, Descartes, 1991, pp. 55-59). Like Descartes, Leibniz appeals to the global, conservation law preserving qualities of his plenum to meet various difficulties at the local level of bodily interactions, which once again highlights the fundamental importance of force in understanding Leibnizian physics. On the whole, this form of global approach to conserved forces would prove to be immensely important for the development of continuum mechanics (among other areas of science); but whether or not it is an appropriate means of settling the metaphysical problem of relational motion would seem another matter indeed.[vii]

4.2. Reference Frames at the Local Level: A Proposal. Returning to the issue of privileged reference frames, Leibniz's theory of impact also provides crucial information on the preferred application of the frames in the collision of plenum particles. As specifically developed in the *Dynamica* (see, Gerhardt, 1962, p. 226), the reference frames invoked to uphold the EH are the center-of-gravity frames (CG frames), thus an application of the privileged frame technique in the wake of Leibniz's rotation hypothesis seems to entail that each *separate* collision of a minute bodily particle with a plenum particle will require its own, *separate* center-of-gravity privileged frame (*since* each body/plenum particle pair will possess its own unique center-of-gravity). Consequently, on the provision that we accept Leibniz's analysis of rotation as a guide to understanding his dynamical processes in general, the utilization of Leibniz's own CG frames would seem to entail that it is only at the spatial level of the colliding particles that a privileged reference frame can be introduced to preserve relationism, via the EH and the conservation law. The relational merits of the CG frame had been previously demonstrated by Huygens, both as a means of constructing a relationally-successful version of Descartes' collision rules, as well as in Huygens' own relational analysis of rotation (which has also appealed to the linear motions of bodily component parts). It is possible, therefore, that Leibniz may have recalled Huygens' center-of-gravity stratagem while grappling with his own, quite different, attempted reconciliation of rotational motion and relationism.[viii]

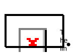
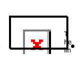
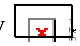
Leaving aside the subject of its conceptual origins, there are ways in which the restriction of the privileged frames to an infinitely small, localized domain of the plenum can actually perform some valuable explanatory work for the relationist. This latent usefulness concerns an unusual trait, often encountered among the Early Modern relationists, that regards the individual states of bodily motion as an arbitrary stipulation (e.g., the EH), yet insists that the actual motions that we arbitrarily assign to the bodies are rectilinear (or inertial). As discussed in section 3.2, Leibniz contends that "since every nisus tends in a straight line, it follows that *all motion is either rectilinear or composed of rectilinear motions"* (Ariew & Garber, 1989, p. 135), which is an hypothesis that runs counter to the relationist denial of privileged trajectories through space. Of course, if Leibniz's hypothesis involves the *real* motions of bodies, i.e., over a succession of instants, then his concurrent sanction of privileged spatial paths and rejection of individual states of motion/rest is, undeniably, a relationist contradiction. However, if we bear in mind that the motion of plenum particles is infinitely small, being generated by an instantaneous conatus, then Leibniz's postulation of a rectilinear basis for all motion makes a great deal more sense. In effect, Leibniz is borrowing an idea, probably stemming from Descartes (Descartes, 1991, pp. 60-61), that the smallest element of an instantaneous conatus is rectilinear, since a curved element would require a combination of two more basic elements: a straight-line conatus and a left- or right-directed conatus whose combination results in the curved conatus (see, Garber, 1992, 285-288). Yet, since the instantaneous conatus is commonly regarded as the *smallest* element, the curved conatus is thereby disqualified as a direct result of its compositional structure. Consequently, Leibniz concludes that all motion generated from these instantaneous rectilinear tendencies must be rectilinear themselves or ultimately composed of (probably infinitely small) rectilinear motions. Whether or not this line of reasoning is compelling, it does demonstrate that the *metaphysical* underpinnings and setting of Leibniz's physics is crucial to a full understanding of his brand of relationism, as well as to the question of his loyalty to the relationist credo.

Conjoining this metaphysical insight with our newly-acquired appreciation of the restricted spatial and temporal scope of the privileged reference frames, it can be seen that the very localized and infinitely small span of these frames makes them ideally suited for determining the infinitely small motions produced by the instantaneous strivings—and, more importantly, these determinations of motion are consistent with a privileged reference frame relationism. In other words, although the privileged frames only cover a small extent of space and time, the foundation of Leibniz physics, i.e., the conservation law, is only required to be invariant for the particle collisions that occur within the domain of these small, localized regions of space and time. Therefore, the privileged frames are capable of securing a relationally satisfactory measure of the rectilinear, uniform motions of the particles involved in each separate impact; and this is accomplished via a unique CG frame for each "relative speed" difference between the minute particles involved in a single collision. Returning to the rotation case, each separate collision of a composite bodily particle with a plenum particles can be treated as an instantiation of Leibniz's conservation law for , wherein rectilinear motion is maintained by the colliding, elastic bodies (both before and after impact) as measured by the CG frame. In this way, Leibniz's analysis of impact, which provides the foundation for his conservation law, can be conveniently harmonized with his claim that rotational motion does not violate relationism—since the particle collisions of the rotating body and plenum are just instances of his relationist model of impact, the local CG frames are sufficient to sustain the desired relational conservation of . Additionally, one must stipulate that the particles do not accelerate, or that the "relative acceleration" difference is 0 (otherwise  may not be conserved).[ix] If the impact only involves infinitely small motions, as the plenum setting and Leibniz's rectilinear motion hypothesis both appear to favor, then ruling-out accelerations at this minute scale might be defensible. Finally, while this strategy can resolve problem (P2), since a relationist measure of  can be constructed, it cannot remedy (P1): that is, the CG frames have been designed to preserve the "relative speed" differences manifest among the colliding particles, but this difference in speed cannot determine individual states of motion.

4.3. Assessing Reference Frame Relationism. If, as Leibniz insists, the instantaneous strivings and motions of all bodies are rectilinear, the critic will likely contend that Leibniz's own views go beyond the relationist's strategy of merely introducing privileged reference frames constructed from "relative speed" differences. One must also assume a spacetime structure that guarantees rectilinear paths, even if those paths are extremely small—and the relationist's Leibnizian structure does not preserve the straightness of paths (and the privileged reference frame method requires this invariant in order to establish the CG frames). There seems little recourse for the relationist but to accept this richer spacetime structure, and forsake the "Leibnizian" structure often attributed to Leibniz's own dynamics. As argued in section 2.3, if the relationist accepts thesis (R2), which rejects the independent existence of space and time, then it is consistent to claim that space can retain any structure, even Newtonian structure—as long as it constantly acknowledged, of course, that these structure cannot exist in the absence of matter. The realization that the underlying spacetime structure may undermine (R1) relationism does have an advantage, however: provided a Full-Newtonian backdrop, the spacetime can now discern both the individual states of all bodily motion and consistently measure in all interactions, thereby resolving both problems of Leibnizian relationism. On the other hand, this strategy apparently concedes that Leibniz's EH is merely phenomenal, such that the underlying ontology of the spacetime system does indeed violate EH—and this concession, as noted above, seems to violate the spirit of Leibniz's approach to motion and space.[x]

Leaving aside the spacetime structure debate, the realization that each plenum interaction may require its own unique reference frame has other potentially embarrassing repercussions for the relationist. Since the privileged frame method would necessitate a potentially infinite set of separate reference frames for the potentially infinite set of particle collisions in Leibniz's plenum, the relationist is faced with the task of finding a coherent means of conserving a *universal* quantity of . This difficulty, to be more precise, is not that numerous CG frames are needed per se, for even the Newtonian must apply a host of CG frames to measure the observable motions of bodies (since substantival space is not empirically accessible, at least for velocity). Rather, on the privileged frame hypothesis, must be determined from a reference frame, but Leibniz does not appear to have endorsed any such "master" perspective from which the individual contributions of each separate collision can be integrated into a conserved universal quantity.

In response, it may be possible simply to stipulate the existence of a sort of “master privileged frame”, especially if one adapts the “truth = most intelligible hypothesis” as a justification (section 3.1). As noted, Leibniz argued that different spatial perspectives are better suited for certain purposes, thereby permitting the use of different astronomical systems depending on the need. Furnished with this precedent, a similar maneuver could be implemented with respect to a master privileged frame, with the “most intelligible hypothesis” amounting to a further privileged frame (or set of frames) from which the of *all* the individual collision frames can be successfully measured to uphold a universally conserved quantity of . Whereas an ordinary privileged frame conserves for the particles in a single collision, the master privileged frame measures the of all bodies in all the other CG frames as well. Because the minute collisions all occur in inertial CG frames, and since is invariant across all inertial frames, the master frame has the capability to measure the total of every individual impact frame (although the actual conserved numerical quantity will depend on the selected master frame). Any individual impact frame could thus serve as a master privileged frame; which further implies that the selection of a master frame is merely a conventional choice among the entire set of individual impact frames (and is thus in keeping with relationist doctrine). Being momentary impact frames, the duration of these designated master frames is likewise equal to the duration of any other separate collision frame. This short temporal span, although potentially troublesome for the process of measurement, need not represent a serious problem: just as long as there exists a never ending succession of such master frames at each succeeding instant (as will be the case in a plenum), the conservation law can be maintained.

Unfortunately, a serious problem looms on the horizon for the privileged reference frame hypothesis; and it surfaces, not coincidentally, within the course of Leibniz's explanation of rotation. As described in section 3.2, Leibniz judges that the conatus of a rotating particle possesses two component parts: an outward radial and a circular conatus, whose combination results in an observed tangential trajectory of the particle upon release. In order to make sense of this story, one must therefore accept that the rotating particle is both striving in a direction radially outward from, and circularly around, the center point of a circle (whereby the circle's circumference could be traced out by the circular-directed conatus if unimpeded). Returning to our search for a master privileged frame, the point (particle) that marks the center of rotation would now seem to be the preferred candidate for role of master frame, for it stands out from the competition due to its unique, *non-local* dynamic scope or influence: i.e., a rotating particle *acts as if* it were rotating around, and fleeing radially outward from, the center point of the rotation. While this may sound like a vindication of the use of privileged frames, it is in actuality a setback for the kind of reference frame dynamics pursued in our investigation. Central to our construction is the foundational role of the collision hypothesis put forth in the *Dynamica,* which relies on the impact of *rectilinear, uniformly* moving, elastic bodies to conserve . Given the independence of each CG collision frame, our goal has been to find a means of connecting these frames in such a manner so as to procure a relationally successful determination of a universally conserved . Nevertheless, in Leibniz's own analysis of rotation, it would seem to be the case that the bodily particle that collides with the contiguous plenum particle *does not* follow a simple rectilinear path both before and after impact; rather, the bodily particle must strike the plenum particle obliquely, since the particle possesses a *circular* conatus in addition to its radial (rectilinear) conatus—and is unclear, given a complex impact of this sort, just how  is conserved, if at all. Furthermore, Leibniz also holds that the conatus of a rotating particle increases incrementally (see endnote 5), thus the stipulation that the colliding particles maintain a constant “relative velocity”, such that the “relative acceleration” is 0, becomes quite difficult to maintain. Bringing into play Leibniz’s tendency to underscore the contribution of motion to his dynamics, as opposed to instantaneous forces, does not help to remedy this situation, either; for it is unclear how to construct a dynamics confined to instantaneous forces alone, while deprived of real motion. In effect, Leibniz’s hypothesis of the compound nature of the instantaneous strivings of a rotating body would appear to rule out its easy incorporation with his conservation law (and, not surprisingly, one can find the same dilemma in Descartes’ physics).[xi]

5. Conclusion

If one were attempting to explain the peculiar failure of Leibnizian dynamics to integrate his various models of impact with his conservation law, a plausible diagnosis of the problem might single out its inherent incapacity to successfully isolate the local behavior of bodies from a host of corrupting global constraints and hypotheses. Like Descartes, the need to explicate a variety of dynamical, large-scale phenomena in a plenum, such as rotation, led Leibniz to posit a host of intricate metaphysical/dynamical properties to individual bodies and their interactions (e.g., conatus, impetus). The inevitable consequence of this rise in dynamic complexity was that the newly postulated bodily properties quickly became so unmanageable that it was no longer clear how to apply the basic conservation laws and rules of impact to a single, localized collision. In effect, as latently expressed in Earman's criticisms (section 4.1), the local behavior of a pair of colliding particles was soon incapable of being isolated and understood apart from a global, synchronized (and quasi-teleological) mass movement of particles, despite the fact that the former was intended to *ground* the latter. In our investigation, we have attempted to construct a Leibnizian dynamics that was both faithful to his professed relationism and the actual details of his plenum physics: if the resulting effort, the privileged reference frame method, is incapable of resolving many of the central inconsistencies and obstacles of his program, at least our endeavors have shed some new light on the array of complicating influences that shaped the course of Leibniz's thought.

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ENDNOTES

[i] It should be noted that Leibniz denies that a body can be entirely at rest, although he does admit that we can assign rest to bodies relative to one another (and/or as an

approximation; Ariew & Garber, 1989, p. 136). The close relationship between force and motion may thus explain the underlying reason for this ban on absolute rest (since bodies are, of course, never without force).

ii For instance, one of Descartes' laws of impact, the fourth, entails that a smaller body could never move a larger one (Descartes, 1991, p. 66).

iii Huygens would provide the model for treating impact in this manner. With respect to Huygens' relationist usage of the center-of-gravity frames, Westfall comments: "From its point of view, nothing whatever happens. Impact is not a dynamic action. It is purely kinematic" (Westfall, 1971, p. 155). Even granting the prominence that bodily forces assume in Leibnizian physics, a kinematical element is discernible in Leibniz's hypotheses as well, which might indicate Huygens' continuing influence. As we have seen, Leibniz accounts for the kinematical features of bodily motion by subsuming them within his dynamics, so there is a sense in which his kinematics is retained in his dynamics. The details of Leibniz's plenum physics, moreover, display a more robust kinematics, if not statics, orientation, for it is relies heavily on the synchronized, mass movements of particles to explain bodily phenomena (much like both Descartes' and Huygens' physics).

iv Translated into Newtonian terms, Leibniz's EH thus seems comparable to grounding a conservation law of motion (e.g., momentum) on Newton's Third Law, which likewise expresses the action/reaction equivalence among bodily interactions.

v Leibniz's handling of the rotating body's instantaneous dynamical "conatus" is thus similar to Descartes' in that he regards the resulting tangential tendency as a composition of both an outward radially-directed and a circularly directed conatus. Nevertheless, Leibniz also attributes an incremental build-up of conatus to the rotating body, which he then identifies with "impetus" (Ariew & Garber, 1989, p. 121).

vi Howard Stein had been one of the few commentators to also suggest this possible connection between the EH and the action/reaction forces in colliding bodies. See, Stein, 1977, p. 32.

vii Another clue to the similar global properties of force in Descartes' and Leibniz's respective systems lies in the dual role of their bodily forces: i.e., the fact that they both refer to forces at the instantaneous conatus level and the durational level of moving bodies. On Descartes, see Slowik 1999b.

viii Huygens, 1950, vol. 16, p. 96. On Huygens' analysis of rotation, see, Earman, 1989, pp. 66-73, and, Bernstein, 1984; and for the importance of CG frames, see, Barbour, 1989, pp. 473-476.

ix Also, if the particles preserved a constant relative distance, but rotated about the CG axis, then the frame would not be inertial. This scenario would present a problem for the

CG reference frame method; but, presumably, a Leibnizian could appeal to the "centrifugal" force that these bodies would manifest, as opposed to a non-rotating pair, to distinguish this frame from the inertial frames.

x Of course, given the many non-relational components in Leibniz analysis of motion, one might simply conclude that Leibniz did not accept (R1), but rather (R2). Problems (P1) and (P2) would seem to support this conclusion as well. Yet, the goal of this investigation is to determine the extent to which a consistent (R1) relationism and Leibnizian spacetime structure can be secured for Leibniz's physics.

xi This problem can also be described as a failure to isolate the local context from the complex material interactions exhibited at the global level, such that the required relative motion invariants do not locally obtain. As suggested by an anonymous referee, the local collisions of the particles can be seen as located within an "open neighborhood" of the larger topological manifold. Yet, since these local settings are just as susceptible to the ostensibly non-relational aspects of the theory (i.e., the failure to manifest purely relational differences in motion, or to clearly follow Leibniz's own analysis of impact as regards rectilinear, uniform motion throughout impact), the open neighborhoods retain the same difficulties that are manifest at the global level.

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