THE FATE OF MATHEMATICAL PLACE: OBJECTIVITY AND THE THEORY OF LIVED-SPACE FROM HUSSERL TO CASEY

EDWARD SLOWIK

This essay explores continental/postmodern theories of place, or lived-space, as regards the role of mathematics, objectivity, and the relativist dilemma that afflicts the lived-space movement. By employing a geometric approach, such as Minkowski pioneered, it is argued that the lived-space theorists can gain a better insight into objectivity of spatial relationships.

1. Introduction

This essay explores space in contemporary continental philosophy and the philosophy of the social sciences, a popular movement often dubbed the study of "place", or "lived-space", due to its emphasis on the human experience of space, both personal and social. Among analytic philosophers of science, it is not widely recognized that there have been many contributions to the debate on the ontology and epistemology of space from this diverse field, which includes: contemporary philosophers of place (e.g., Edward Casey), prominent continental philosophers from the second half of the twentieth century (e.g., Deleuze, Derrida), and many renowned phenomenological investigations in the first half of the twentieth century (e.g., Husserl, Heidegger, and Merleau-Ponty). Many of these studies have sanctioned, often inadvertently, a form of relativism or social constructivism (Casey), or even metric conventionalism (Merleau-Ponty) as regards the ontology/epistemology of space. Accordingly, this essay will explores these highly popular works in order to determine both the general content of their claims and the overall philosophy of space either implicitly or explicitly advanced in their philosophies. As will be demonstrated, the theories of lived-space put forward by these philosophers, from the later Husserl to Casey, bare a number of uncanny similarities with work in the analytic study of space and spacetime, such as an emphasis on objectivity and an interest in structuralist forms of explanation.

Much of the examination will focus, however, on the role of mathematics within the lived-space approach to space, since a misunderstanding or mistrust of mathematics, which can be traced in part to the influence of the early phenomenologists, has been a major factor in the relativist dilemma that afflicts the lived-space movement (sections 2 and 3). By incorporating various geometrical concepts within the analysis of place, it will be argued that the lived-space theorists can better grasp the nature of objective spatial relationships—and, more importantly, that this appeal to mathematical content need not be construed as undermining the basic tenants of the lived-space approach (section 4). In the final section, Deleuze's unconventional foray into differential geometry will serve as a means of demonstrating the inherent limitations of the lived-space conception of mathematics. Overall, the geometric approach to spacetime, as exemplified in Minkowski's interpretation of Special Relativity, is ideally suited to capture the objectivity of the spatial component of physical systems, unlike the contemporary lived-space school. Indeed, it will be argued that Minkowski's utilization of the group concept set the stage for the numerous philosophical investigations that later explored the subjectivity-objectivity issue (and which are based on these geometric techniques). Finally, it should be noted that one of the additional goals of this examination is to open up a largely unexplored field for researchers interested in the ontology and epistemology of space and spacetime, especially given the fact that this field, i.e., lived-space, has exhibited such a broad and popular appeal among present-day philosophers.

2. The Place Theory and the Subjective/Objective Dichotomy.

In the more practice-oriented disciplines and philosophical schools of the late-twentieth century, considerable attention has been devoted to the concept of "place", or lived-space; which, put roughly, denotes the study of space (spatiality) as manifest within a human, usually social, order or practice (as in, dwelling, abode, local). The place theory of space relies on the insights gathered from a host of twentieth century philosophers and philosophical movements traditionally categorized as continental: for the philosophers, e.g., Husserl, Heidegger, Deleuze, and for the philosophical movements, e.g., phenomenology, environmental studies, literary theory, social geography. In particular, many researchers of place attempt to shed light on the relationship between our subjective, i.e., human, social, and practical experience of space, and the epistemological/ontological notion of objectivity.

2.1. Radical Spatial Subjectivism. Nevertheless, these studies have largely failed to address two important, and somewhat obvious, interrelated problems associated with the objectivity of space:

Problem (1): Can a theory of place successfully counter any radically subjectivist interpretation of the epistemology and ontology of space? As employed in this context, a "radical spatial subjectivist" rejects any objective or invariant spatial structure, and thus the geometric structure of space is *entirely relative* to different persons, cultures, or practices—example: space is Euclidean relative to geometer A, and space is non-Euclidean relative to geometer B, although both inhabit the same world.

Problem (2): How does the subjective or social aspect of the experience of space connect or interface with the underlying ontology of the physical world?

The relevance of space to the vexed subjective/objective problem assumes an obvious importance in the lived-space field, moreover: a subjective space is "a space that is tied to some feature of the creature's own awareness or experience . . . —the space of awareness within which it acts and with respect to which its actions are oriented and located" (Malpas 1999, 50). Objective space, conversely, "is a grasp of space that, while it requires a grasp of one's own perspective and location, is a grasp of space that is not centered on any particular such perspective or on any particular location . . . " (66). Yet, apart from a few instances, such as Malpas' forthright analysis, most of the investigations of place do not address adequately the exact structure or relationship between place and its objective and subjective components. If, indeed, any trend can de detected in this field, it would seem that many authors favor an interpretation of place that posits subjective space as primary, with objective space being derived, or "stitched together", from subjective experience. In a popular text, Edward Casey seems to endorse this reduction, viewing the objective, infinite, mathematical "space" of the Modern era as derived from the earlier, human-centered concept of bodily and social "place": "In a dramatic reversal of previous priorities, space is being reassimilated into place, . . . as a result of this reversal, spacing not only eventuates in placing but is seen to *require it to begin with"* (Casey 1997, 340; original emphasis); and, commenting on the social theorist Nancy, he proclaims, "spaces comes from places, not the other way around" (341). Among other examples, one can cite various difficult passages in Tuan's environmental study, where a person's experience "constructs a reality" (1977, 8), and Entrikin's appeal to "narrative" to connect the subjective and objective aspects of place (1991, 132-134), since "narrative" has strong subjectivist overtones.

However, without some set of *constraints* on the acceptable methods of explicating or constructing the global, objective structure of space (place) via the local, subjective spaces, the inevitable and unfortunate outcome is a radically divergent set of competing objective spatial structures. An obvious example is the spatial beliefs common to ancient Middle-Eastern civilizations, who interpreted the world as both flat and centered upon their home civilization—two "hypotheses" apparently confirmed through simple *bodily experience* and *common social practice.* Consequently, the subjectivist interpretation of space would seem to lack the conceptual resources needed to defuse problem (1): Was the earth "really" flat for ancient Middle-Eastern civilizations (e.g., the Genesis creation story in the Bible), but "really" spherical for modern Western societies? It is tempting to claim that modern science provides the "true" explanation of space; but, of course, modern science is just another social practice or narrative.[i]

2.2. Internal and External Constraints. Some place theorists strive to avoid the impending conflict of spatial schemes by means of constraints imposed either internally or externally to all potential subjectivist theories: either by openly endorsing the "irreducibility" of the objective aspect of spatial experience, or by acknowledging the intervention of an underlying physical space in the subjective act of spatial construction. One of the more notable efforts to address issues related to our problem (1) appears in Malpas (1999), which also draws upon both of the above methods for undermining a radical subjectivism. First, Malpas persistently rejects the view that objective space can be derived "from a mere concatenation of subjective spaces" (61; as does Campbell 1994, 5-37). By claiming that the two are "correlative concepts" (Malpas 1999, 36), or have a "complex interconnection" (70), the suggestion would seem to be that the irreducibly objective facet of spatial experience sets up barriers, an internal constraint, concerning different subjective formulations of space (i.e., the proposed objective-subjective irreducibility prevents, say, the flat and spherical models of earth's geometry from being equally successful constructions). Yet, this strategy seems consonant with a full-blown objectivism regarding space—and, of course, the intention was to develop a theory that shuns a strong objectivism through the utilization of an irreducibly subjective aspect of spatial experience. Since it is the objective aspect of space that, returning to our example, rules out the flat-earth case, the subjective element would appear to be idle. Moreover, this form of response does not explain *how* the interrelated objectivity-subjectivity of place prevents conflicting constructions of place.

Possibly in response to such worries, Malpas invokes an external constraint on the subjective constructions of space by means of a supervenience relationship between place and the underlying physical space: "In some sense place must 'supervene' upon physical space, and upon the physical world in general, such that the structure of a particular place will reflect, in part, the structure of the physical region in relation to which that place emerges" (34). Yet, since no further details are offered on this quite mysterious form of supervenience, which must be an ontological relationship of some sort, this attempt to resolve problem (1) comes at the considerable expense of inflaming problem (2): i.e., what does it mean to say that the subjective, socially-oriented conception of place supervenes on physical space?

2.3. Physical Space and Merleau-Ponty's Metric Conventionalism. Since few in the lived-space tradition address problems (1) and (2), it is possible that, given the continental/postmodern leanings of this movement (see section 3 below), a radical spatial subjectivism is, in fact, acceptable to many place theorists. As a means of undermining the primacy of objective space, a radical subjectivist might, for instance, appeal to Quine's "indeterminacy of translation" in order to claim that each separate subjective space is incommensurable, i.e., not communicable, with other subjective spaces. Most analytic philosophers of physics will no doubt find this type of argument inadequate, since it turns on an uncritical acceptance of a (controversial) theory of reference that may not hold in the more quantitative, mathematical domain of physics and, hence, physical geometry (as opposed to common language). While the role of mathematics will be taken up in more detail below, an interesting question does arise in the context of this hypothetical response to a "spatial incommensurability": Have the place theorists employed any philosophical arguments utilizing mathematical/physical evidence or premises in support of radical spatial subjectivism?

In short, the answer is apparently negative, at least among the contemporary advocates of the lived-space school. Merleau-Ponty, who foreshadows the place theorists, did invoke a Poincaré-style metric conventionalist argument to undermine the "reality" of physical geometry, if not its objectivity. Metric conventionalist arguments attempt to reveal the underdetermination that plagues the ascription of spatial geometry, in conjunction with the physical hypotheses, for any would-be geometer: e.g., Poincaré's disc-world (1905), where the measurements conducted by the hypothetical inhabitants disclose a non-Euclidean metric structure. On Poincaré's disc, two theories are consistent with the evidence: (a) that the geometry is Euclidean but "universal forces" distort the measuring apparatus, or (b), the geometry is non-Euclidean and there are no such universal forces. Does this outcome support a radical spatial subjectivism, as some place theorists might contend?—No, because not all aspects of the choice between (a) and (b) are conventional. If one chooses to preserve a flat space, then one *must* postulate forces that distort the measuring instruments. Alternatively, if one accepts the non-Euclidean measurements, then one *must* conclude that the space is curved. Given a strong form of spatial subjectivism, however, any geometry, and any stipulation on spatial measuring instruments, should apply equally: for instance, option (c), where the geometry is Euclidean but no universal forces alter the measuring device.[ii] Consequently, option (c), which is available to the radical spatial subjectivist, but not the metric conventionalist, thereby demonstrates that the latter cannot serve as an argument to support radical spatial subjectivism.

Turning to Merleau-Ponty's analysis, he comments: "'Real', i.e., perceived, triangles, do not necessarily have, for all eternity, angles the sum of which equals two right angles, if it is true that the space in which we live is no less amenable to non-Euclidean than to Euclidean geometry" (1962, 391); and, in more detail:

> It is impossible to relate this or that proposition concerning space to the structure of space, and some other [proposition] to a physical influence. . . . The same physico-geometrical ensemble is capable of covering both flat space and curved space. . . . If we take relativistic science seriously, we must say that Riemannian space is not real, but objective to the extent that it allows for Einstein: it allows for better integrating the results of modern physics than does Euclidean space. We can thus speak of a closed space, such that in pursuing it we return to the same place. The experimental verification is relative to it. If space is closed, it is clear that there can be a double image of the same star, the whole difficulty being only to identify them. . . . In this sense, the idea of closed space must not be considered . . . as an overcoming of Kantian relativism, but on the contrary, as its accomplishment . . . . (2003, 103)

Briefly, Merleau-Ponty's estimate is misleading in that he seems to imply that any geometry is straightforwardly consistent ("no less amenable", 1962, 391) with the totality of physical evidence. Rather, as described above, specific physical assumptions are required in order to render a particular geometry consistent with empirical data, and these assumptions can be challenged in numerous ways: e.g., the peculiar universal forces needed to retain Euclidean geometry may be inconsistent with our best, well-confirmed physical theories. So, when all of the evidence is taken into consideration, many alternative theories of "geometry plus physics" may be excluded, such that only a handful, or just one, may be supported by the evidence.

Merleau-Ponty's claim that, "the same physico-geometrical ensemble is capable of covering both flat space and curved space", thus can only be maintained if one takes a rather impoverished view of the criteria for constructing and evaluating the “physico-geometrical ensemble”. His own example of a Riemannian (spherical) space, where the experimental verification could be “a double image of the same star”, practically makes this point: Is Merleau-Ponty suggesting that a flat space interpretation of the same evidence would be as equally plausible and consistent as the spherical depiction (since “the experimental verification is relative to it”)? One can easily imagine evidence that would begin to unravel this assumption, such as the simultaneous super-nova explosion of both stars (i.e., the same star), or simply measuring the angles of a (very large) triangle, à la Gauss, in order to determine if the interior angles match the Euclidean prediction. So, unless wildly ad hoc and implausible physical hypotheses are invoked, the evidence hardly seems “relative” to the particular geometry used in the theory.

Ironically, Merleau-Ponty relies on a similar tactic—namely, the constraints imposed by the world/evidence—to dispel his own version of our problem (1), which he describes as a potential solipsism that may ensue from a subjective-based conception of space: “Since there are as many spaces as there are distinct spatial experiences, . . . are we not imprisoning each type of subjectivity, and ultimately each consciousness, in its own private life” (1962, 291-292)? Answer: invoke physical space as an external constraint, which, presumably, prevents spatial solipsism by connecting all of our separate spatial experiences to the same spatial world. He states that, “I never wholly live in varieties of human space, but am always ultimately rooted in a natural and non-human space” (293); and that “Human spaces present themselves as built on the basis of natural space, . . .” (294). Merleau-Ponty is quick to add, however, that “natural and primordial space is not geometrical space” (294), which accords with his other claims, cited earlier, that the objectivity of Riemannian space only "allows for better integrating the results of modern physics than does Euclidean space", and thereby does not overcome a "Kantian relativism". In essence, Merleau-Ponty's own phenomenological theory appeals to physical space in order to counter problem (1), radical spatial subjectivism—but this maneuver is no different than the scientist who appeals to the physical evidence, in conjunction with the consistency of our best physical theories, to counter the metric underdetermination brought about by a host of divergent "geometry plus physics" combinations. Put differently, how can Merleau-Ponty be so sure that subjective spatial experience is somehow constrained by the physical world, but that the determination of metric properties (in conjunction with the best physics) is not? Indeed, if metric conventionalism does hold true, such that the physical component is powerless to help (in the manner advocated by Merleau-Ponty), then falling back on the physical world cannot free a subjective spatial theory of the same underdetermination.

Finally, like Merleau-Ponty, some of the other phenomenological investigations that inspired the contemporary lived-space movement may have employed physical space as a form of external constraint. In Husserl's theory, since the phenomenal realm of the subject presupposes a physical body, a pre-existing "continuum of places" is postulated for the body's occupation (see, Husserl 1981, 225). As for Heidegger, the complexities of the relationship between Dasein (roughly, human existence) and spatiality are enormous (see, e.g., Vallega 2003, Malpas 2006), but a similar dependence on a pre-given world may be in evidence: "space is . . . 'in' the world in so far as space has been disclosed by that being-in-the-world which is constitutive of Dasein . . ." (1962, 146). Overall, these early phenomenological theories of the human and social construction of space—which are not modern lived-space theories, by the way—run afoul of problem (2); namely, the manner by which the underlying physical ontology interacts with, and thus *constrains,* subjective space constructions.

3. The Place Theory and The Mathematization of Space.

Unlike the early phenomenologists, contemporary exponents of the place theory seldom appeal to either the underlying ontology or the objectivity of space to resolve problems (1) and (2), likely due to the perception that it situates the human/social element of space in a decidedly inferior and subordinate status with respect to the more quantitative and mathematical, and thus less qualitative and subjective, aspects of space and science. Ironically, the modern bias against the use of mathematics in attempting to meet the relativist challenge can be traced, at least in part, to these same early twentieth century phenomenologists, most notably, Heidegger and the later Husserl. Unlike recent treatments of place, which either ignore or quickly dismiss mathematics as relevant to the place theory, these early phenomenological tracts openly discussed the relationship between mathematics, especially geometry, and their new conception of a subjective, lived-space (much like Merleau-Ponty above). Husserl, in particular, will comprise a major part of the remainder of our investigation, for the difficulties associated with Husserl's theory of subjective space in his later work are identical to the problems just described for the contemporary practitioners of the lived-space theory, and hence Husserl's more forthright analysis of the interrelationship of objectivity and mathematics will serve as an ideal basis for diagnosing the viability of contemporary place theory. As will be disclosed, one of the more intriguing puzzles that emerges in the early phenomenological works concerns the status of mathematics, especially geometry, in its seemingly unavoidable mediating role between, on the one hand, physical space, and on the other, subjective lived-space.

3.1. Husserl and the Early Phenomenological influence. Despite the presence of an a priori factor in spatial experience, which allows an immediate grasp of general geometric truths ("essential seeing"), objective space and geometry in Husserl's middle period (e.g., *Ideas* I (1982)) are ultimately constructions based on subjective experience, much like the earlier theories put forward by, among others, Helmholtz, Mach, Wundt, and Lotze.[iii] The geometry of our subjective experience is Euclidean, furthermore, whether in a single intuited act of spatial perception/imagination (as just described), or as one goes beyond these single acts to construct the larger space that results from the accumulation of spatial experience through bodily motion (and spatial variations in imagination).[iv] In his late period, a more subjectivist tone is supposedly struck in several of Husserl's works that cover space and geometry, foremost being, *The Crisis in European Sciences* (1970), along with its associated appendices ("The Origin of Geometry", in particular). These writings would prove a source of inspiration for the later place school, for they bring to the forefront several concepts central to the contemporary approach to place: principally, the "life-world", and the "mathematization of nature". The life-world, as defined in the *Crisis,* is "the spatiotemporal world of things as we experience them in our pre- and extra-scientific life" (1970, 138). The emerging mathematization of the world, which takes the form of Euclidean geometry, cannot capture the life-world in its entirety, however, for mathematical idealizations and abstractions can only indirectly apply to the purely qualitative aspects of the life-world (32-37). Nevertheless, Husserl does not question the objectivity of physical geometry, for he repeatedly rejects any historicist, relativist conception that would regard space and geometry as merely contingent constructs of a particular society: "geometry, with all its truths, is valid with unconditioned generality for all men, all times, all peoples, and not merely for all historically factual ones but all conceivable ones" (377). In effect, Husserl grounds the unconditioned validity of all geometric practices on an invariant human feature common to all individuals and societies. This invariant feature, which we will explore further in section 4, would thereby preclude our problem (1), since it acts as a form of internal constraint on the construction of geometric schemes.

Returning to the topic of Husserl's impact on the later place theory, in particular, for the prospects of a mathematical conception of lived-space, it was probably his methodology of "bracketing off" the objective sciences that would prove to be most influential. The process of bracketing, also termed the epochē in the *Crisis,* is designed to isolate the objective sciences in order to ascertain the unique or principle characteristics of the life-world, which "must have their own 'objectivity', even if it is in a manner different from our [objective] sciences . . ." (133). This theme, that the proposed objective principles of the life-world may be "different" than the developed sciences of the day, persists throughout the *Crisis:*

> A certain idealizing accomplishment is what brings about the higher-level meaning-formation and ontic validity of the mathematical and every other objective a priori on the basis of the life-world a priori. . . . What is needed, then, . . . [is] a division among the universal inquiries according to the way in which the "objective" a priori is grounded in the "subjective-relative" a priori of the life-world . . . . (140)

By separating the different "a prioris" of the objective sciences and the life-world, the implication is that mathematics and geometry must be, or should be, confined to the objective a priori (via the epochē) in order to ascertain the true nature of the life-world. Not surprisingly, ensuing generations of place theorists would almost certainly interpret Husserl's late research as advocating a complete and total ban on the use of mathematical techniques in their study of the "subjective-relative" sphere of human spatial practices. In fact, with respect to space, Husserl is quite clear that geometric content is not "internal" to the life-world: "Prescientifically, the world is already a spatiotemporal world: to be sure, in regard to this spatiotemporality there is no question of ideal mathematical points, of 'pure' straight lines or planes, no question at all of mathematically infinitesimal continuity or of the 'exactness' belonging to the sense of the geometrical a priori" (139-140). In other words, the life-world has its own kind of space, a space which is radically different from the space utilized in mathematical physics, i.e., physical geometry.

Finally, Heidegger's *Being and Time* also expounds a subjectivist-based hypothesis of space, yet his skeptical critique of the concept of objectivity arguably influenced the place school in a more profound and radical fashion. Despite a general similarity of content between Heidegger's and Husserl's theories—Heidegger's "Dasein" and the "Mathematical Projection of Nature" functioning somewhat analogously to Husserl's life-world and mathematization of nature—the type of a priori science of the life-world championed in Husserl's later work would seem quite incompatible with Heidegger's finite, historical understanding of human experience. In *Being and Time,* Heidegger refers to "the manifold questionableness of the phenomenon of 'validity', which since Lotze has been fondly passed off as a not further reducible 'basic phenomenon'", and he proceeds to outline various meanings of "validity": "as manner of being of the ideal, as objectivity, and as bindingness [for all people]" (1927, 155-156). Therefore, any attempt to locate an invariant structure underlying all human spatial practices would likely draw the Heideggerian charge of invoking timeless "essences"; or, Husserl's project errs by trying to explicate our social engagements in the world, the "ready-to-hand", by means of the "present-to-hand", which are the theoretical idealizations derived from those practices—but this turns Heidegger's philosophy exactly on its head, for the defining trait of Dasein is its "being-in-the-world" (existence).[v]

3.2. Contemporary Social Trends. There are a number of themes in these major phenomenological tracts that, directly or indirectly, shaped the course of the place theory's approach to space and mathematics: first and foremost is the primacy of subjective lived-space, which thus serves as the basis for deriving objective geometric space; second, that subjective space is essentially qualitative, and *not* quantitative, geometrical or mathematical; and third, as a direct result of the rise of mathematical physics in the Early Modern period, that objective geometrical space is Euclidean, infinite, and homogeneous.

To demonstrate the mathematical aversion that is prevalent among many place theorists, one need only consult Casey's influential history, *The Fate of Place* (1997), which is representative of much contemporary work on the topic of lived-space.

> "The ultimate reason for the apotheosis of space as sheerly extensional is that by the end of the seventeenth century place has been disempowered, deprived of its own dynamism. . . . The triumph of space over place is the triumph of space in its endless extensiveness, its coordinated and dimensional spread-outness, over the intensive magnitude and qualitative multiplicity of concrete places. . . . Space on the modernist conception ends by failing to locate things or events in any sense other than that of pinpointing positions on a planiform geometric or cartographic grid. Place, on the other hand, situates, and it does so richly and diversely. It locates things in regions whose most complete expression is neither geometric nor cartographic" (200-201).

Presumably, the motivation for this line of thought is derived from many sources, but Husserl's later work may have played a major role: prior to quoting from the *Crisis* (where Husserl declares that in the life-world "we find nothing of geometrical idealities, no geometrical space or mathematical time with all their shapes"; 1970, 50), Casey explains that "the organic body singled out by Husserl opens onto the 'primary world' that is not amenable to direct mathematization" (223). Furthermore, in Casey's chronological survey, Husserl is one of the first philosophers examined who supposedly favors a view, like Casey's, concerning the (alleged) non-mathematical essence of subjective place.

While these extracts help to corroborate the importance of Husserl, other passages make an explicit link with a Heideggerian brand of subjectivism, such that mathematics, logic and language are relative, at least in part, to culture or practice (i.e., place): "Treatments of logic and language", he cautions, "are . . . place-blind, as if speaking and thinking were wholly unaffected by the locality in which they occur" (xii). He also hints at a theme common among many in the lived-space movement, specifically, an attempt to link an objective, mathematical conception of space with various forms of social and political totalitarianism or exploitation: "Is it accidental that the obsession with space as something infinite and ubiquitous coincided with the spread of Christianity, a religion with universalist aspirations" (xii)? In Casey's defense, some exponents of subjective space go much further, as in the case of Henri Lefebvre, who categorizes "abstract" space, which is geometric, with a "phallic" attribute that "symbolizes force, male fertility, masculine violence" (1991, 287).

Leaving aside the gross implausibility of these last few allegations, what is equally troubling in these texts is the woeful treatment of the historical development of the concept of subjective space. Casey's treatise, which claims to be a history of place, discusses neither the rise of the empirical approach to geometry and space that began with Helmholtz and Mach (among many others), nor the *Lebensphilosophie* movement that drew encouragement from these nineteenth century mathematical developments (with the *Lebensphilosophie* school serving as the starting point for Heidegger). For many of the place theorists, there is a (postmodern/continental) tendency to interpret modern, or post-Kantian, philosophy as having began with the later Husserl and Heidegger, hence contributing to an impoverished conception of the significance of mathematics in the evolution of the subjective space idea. That a long "dry spell" came between the German Idealists and the phenomenologists is evident in Casey's book: "Starting with Kant and continuing in Husserl and Whitehead and Merleau-Ponty, place is considered with regard to living organisms and, in particular, the lived human body" (332)—which suggests that nothing of importance for the development of subjective spatial theories occurred between Kant and Husserl!

4. Towards a Mathematical Conception of Subjective Space.

As outlined in sections 1 and 2, an interpretation of spatial objectivity that, in some fashion, includes a subjective component might possibly provide a means of combating the radical subjectivist dilemma, problem (1), while simultaneously upholding the subjective experience of the individual, culture, or practice. Despite being largely ignored by contemporary place theorists, late nineteenth century mathematicians actually developed many techniques that can be seen as offering just this kind of strategy, and it is discussed in the philosophical writings of Eddington, Weyl, and a host of others. These later philosophical explorations, moreover, were likely spurred by Minkowski's singular achievement in 1908.

4.1. Geometry and the Subjective. The lived-space theorists are fond of characterizing geometrical space as the flat, lifeless plain of Euclidean geometry (as the above quotes by Casey indicate), but the history of geometric theories and constructions undermines this simplistic assumption. For our purposes, two of the most important innovations concern the analytic method of geometric construction and the investigation of the intrinsic structure of manifolds (differential geometry), which originated in the pioneering work of Gauss and Riemann, in particular. A Euclidean understanding of geometrical objects, such as "point" or "line", was no longer necessary given the new analytic methods, since algebraic equations are essentially neutral and uninterpreted as regards their geometric meaning. The analytic approach allowed, in turn, the creation of differential geometry, which could furnish a characterization of surfaces in terms of their intrinsic, as opposed to extrinsic, curvature (where "intrinsic" curvature is determined from a perspective confined entirely to that surface, and "extrinsic" from outside). In short, curvature could now be characterized intrinsically for each point on a surface without requiring a larger, Euclidean space in which to embed the surface. The intrinsic geometry of a surface can be regarded, roughly, as its geometry as determined by geometers confined to that surface using (idealized mathematical) measuring procedures (e.g., comparing vectors between neighboring points, etc.). Consequently, by conceiving geometric structures from a local, surface-bound basis, there is a tacit affinity between the intrinsic methodology of differential geometry and the theory of lived-space.

A second point of comparison between the place theory and the geometric techniques invented in the nineteenth century can be found in the latter's utilization of coordinate frames, the transformations rules that link these frames, and the invariants preserved among these translations; a branch of differential geometry known as tensor analysis (and which is intimately connected with the intrinsic geometric method just described). The kinship with the place theory's idea of subjective space is immediately apparent in tensor analysis, since this branch of mathematics can be roughly characterized as the study of "what remains the same" (invariant) under different spatial perspectives (frames). On the Euclidean plane, distance is an invariant feature, such that the distance between any two given points is measured to be the same regardless from which position, or coordinate point, one measures it: if $u = (x_1, y_1)$ and $v = (x_2, y_2)$, then the distance between these points is $d(u,v) = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$, which will be an identical numerical value from all perspectives. The transformations on the plane (the space $\hat{A}^2$ of ordered pairs of real numbers) that leave distance invariant includes all rotations, $\mathbf{U}$, and translations, $\mathbf{a}$, such that: $t(\mathbf{x}) = \mathbf{U}(\mathbf{x}) + \mathbf{a}$, for the vector $\mathbf{x} = (x, y)$. This distance (metric) function can be generalized to incorporate different coordinate systems and different geometries (Euclidean, Spherical non-Euclidean, etc.) as given in the well-known formula for the line element, $ds^2 = g_{ij} dx^i dx^j$ (for Riemannian and semi-Riemannian spaces). Overall, the group of allowable transformations on a space specify the type of geometry—consequently, the *same* space (say, $\hat{A}^2$) can allow different groups of transformations, and thus different invariants, and thus different geometries. That is, some perspectives in

$\hat{A}^2$ will reveal an invariant quantity that other frames will not uphold. Only a limited number of transformations among frames will preserve the invariants of Euclidean geometry (length and angle), for example, whereas a wider class of transformations will preserve the ratios along parallel lines (affine geometry).[vi] Unlike the monotonous, uniform geometry caricatured by the place theorists, the picture that differential geometry presents is quite complex and varied, with a host of different geometrical structures and invariants all residing in the very same space. More importantly, differential geometry constructs these invariants from the subjective perspective of diverse coordinate positions or frames (and the transformations among frames), thereby revealing an indispensable, or *non-reducible,* contributing role for a subjective (i.e., perspectival and non-global) component of space and geometry in securing the objective invariants.

Finally, this methodology resolves both problems (1) and (2) in a more consistent and plausible manner than the (non-mathematical) lived-space approach can supply. The relativism of subjective space, problem (1), is resolved since many subjective perspectives (frames) in a space are *not* incorporated within any particular group of transformations: that is, the long sought after “constraint” on possible spatial constructions is a direct consequence of the type of transformation group and its corresponding invariant, which thus accounts for the absence of incommensurable geometries (i.e., it explains why there is only a determinate number and order of interrelated, non-incommensurable geometries). Likewise, the fixed interrelationship between the invariants of the geometry and the group of transformations also resolves our problem (2). The geometrical invariants are often regarded as representing the *objective* features of the underlying spatial ontology, although these features can only be accessed through the subjective-bound group of transformations. In short, the groups of transformations among frames secure the needed constraints, and constraints indicate, or correspond to, the world's "real" structure.

In the realm of spacetime theories, Minkowski (1964 [1908])) offered one of the first applications of these differential geometric techniques, providing a formulation of Special Relativity that emphasized the invariance of the spacetime interval, $c^2dt^2 - dx^2 - dy^2 - dz^2$, under a group of transformations (often dubbed, Lorentz transformations) required to preserve the laws of physics (i.e., the constancy of the speed of light, c, and the independence of the laws of classical physics over the choice of inertial system): "the existence of the invariance of natural laws for the relevant group *Gc"* (301). Put roughly, Minkowski was able encapsulate or encode a specific domain of our experience of the material world, both actual and potential, via a geometric method that relates these content of this experience (that is, it "saves the phenomena" from multiple perspectives). His comments on the "pre-established harmony between mathematics and physics" (312) can therefore be seen, in retrospect, as a milestone in the application of these geometrical techniques to model actual phenomena. More importantly, Minkowski makes an explicit link between the spacetime group invariant and "independent reality", i.e., objectivity, in the famous opening of his 1908 paper: "only a kind of union of the two [space and time] will preserve an independent reality" (297). As a consequence, Minkowski's achievement, which is seldom examined from this perspective, can be viewed as a foreshadowing the goals of the lived-space theory, since a group of transformations naturally incorporates the objective and the subjective (as outlined above).

It is important to note that Minkowski himself never actually discussed the ramifications of his spacetime group of transformations as regards the metaphysics of the objectivity/subjectivity divide, but a host of others philosophers and physicists, inspired by his work, soon would, most notably, the brand of neo-Kantian inspired structuralism exhibited in the work of Weyl, Eddington, and Cassirer (albeit Weyl was more directly inspired by Husserlian phenomenology; see Ryckman 2005, and section 5). With Minkowski's achievement as a guide, and given the tensor calculus framework of General Relativity as well, it naturally led to a new appraisal of the subjective/objective relationship, as the following comments by Weyl indicate: "[The] *objective* world is of necessity *relative* [subjective]; it can be represented by definite things (numbers or other symbols) only after a system of coordinates has been arbitrarily carried over into the world" (1949, 116). This objective-subjective interrelationship, moreover, "contains one of the most fundamental epistemological insights which can be gleaned from science", since "whoever desires the absolute must take the subjectivity and egocentricity into the bargain" (116). In a similar vein, Eddington emphasizes "the subjectivity of the universe described in physical science" (1958, 85). It is worth quoting Eddington's argument at length:

> Relativity theory allows us to remove (if we wish) the subjective effects of . . . *personal* characteristics of the observer; but it does not remove the subjective effects of *generic* characteristics common to all "good" observers [the allowable transformations] . . . . [The mathematician] has invented a transformation process which enables us to pass very quickly from one [possible] observer's account to another's. The knowledge is expressed in terms of tensors which have a fixed system of interlocking assigned to them; so that when one tensor is altered all the other tensors are altered, each in a determinate way. . . . A tensor may be said to symbolize absolute knowledge; but that is because it stands for the subjective knowledge of all possible subjects at once. (85-87)

Eddington, like the lived-space theorists, also cautions against envisaging the universe from a subject-less, "view from nowhere": "There does not seem to be much difficulty in conceiving the universe as a three-dimensional structure viewed from no particular position", but he notes that "it is perhaps rather unfortunate that it is, or seems to be, so easy to conceive; because the conception is liable to be mischievous from the observational point of view" (86)—that is, it is mischievous as judged from the new subjective-based physics that utilizes tensor calculus.

4.2. The Problem of Quantifying the Qualitative. The lived-space theorists may nevertheless reject any application of mathematics, such as the one outlined above, on the grounds that the essentially qualitative nature of subjective space—the "intensive magnitude and qualitative multiplicity of concrete places", quoting Casey—is just not amenable to mathematical analysis. Eddington's conclusion draws upon a distinction between the "personal versus the generic" understanding of subjectivity, so it might be claimed that subjective space is really the personal (which Eddington denies as relevant for the new physics). Unfortunately, interpreting a practice-based (or praxis) theory of space as akin to personal experience is quite problematic, for the manner by which space, as a social practice, acquires this individual, "personal" trait is left unexplained—and, it may contradict the very idea of a *social practice,* which must rise above the experience of individual practitioners in some fashion. Put another way, the difficulty with associating these non-quantitative, non-mathematical "intensive magnitudes" with subjective spatial *experience,* either at the personal or social level, is that it fails to provide a rationale for criticizing the mathematization of *physical* space—unless, of course, they hold that physical space actually possesses irreducibly qualitative properties, like color or pain, which is patently absurd.

Moreover, the foundational role that the purely qualitative magnitudes play for many lived-space theorists inevitably yields anxieties over an impending radical subjectivism. Husserl's *Crisis* provides a clear example of this dilemma, for he denies that the life-world a priori is geometrical (see section 3.1) while simultaneously rejecting an historicist, relativist interpretation of life-world schemes. He contends that the relativism worry disappears "as soon as we consider that the life-world does have, in all its relative features, a *general structure",* such that "this general structure, to which everything that exists relatively is bound, is not itself relative" (139). Finally, "as life-world the world has, even prior to science, the 'same' structure that the objective sciences presuppose" and "are the same structures that they presuppose as a priori structures and systematically unfold in a priori sciences"(139). In other words, there is an invariant structure that underlies both the life-world a priori and the a priori of the objective sciences.

Deprived of mathematics, however, it is not exactly clear what Husserl has in mind in declaring that the life-world and the objective sciences share the "same" structure. Given the epochē, mathematics has been bracketed away from the life-world a priori, so the similarity of structure cannot be mathematical/geometrical structure—yet, what "structure" remains? It is possible that Husserl has in mind a basic similarity between the scientific a priori's mathematical structures, on the one hand, and the relational structure of the mental content associated with kinesthetic awareness (of the life-world a priori), on the other; where "kinesthetic" refers to the experience of one's body in moving or resting, "each being an 'I move', 'I do' [etc.]" which "are bound together in a comprehensive unity" (106). Nevertheless, a relational similarity of this sort would seem to warrant an analysis employing some form of deeper mathematical structure (such as set theory, topology, or category theory?), since *one of the relata* is, in fact, the mathematics of the natural sciences. But, any non-life-world idealization, like set theory, is apparently ruled out by the epochē; that is, these more abstract structures are also idealizations ultimately derived from the life-world (43-48), so there can be no more basic structure that *underlies both* social practices and mathematics.[vii]

Another tactic might be to simply assert that this similar structure is a metaphysical primitive or unanalyzable notion, a maneuver that Heidegger may have exploited in his later works.[viii] Alas, recourse to metaphysical expedients of this type would seem incompatible with Husserl's claims for the scientific status of the life-world a priori, nor does a primitive metaphysical concept really explain how the impending relativism has been averted. Moreover, a purely metaphysical means of overcoming the radical subjectivism quandary leaves the relationship between the subjective aspect of space and the underlying ontology a mystery, our problem (2). Husserl's *Crisis,* a notable precursor to the modern place theories, thus demonstrates the inherent vulnerability of any position that seeks objective scientific status for a subjective/practice oriented conception of space while simultaneously bracketing away mathematical methods.

Part of what may be driving Husserl's "bracketing away" of mathematics from the life-world a priori can be labeled the "circularity argument": since mathematics is a by-product of human practices, thus mathematics cannot be used to explain its own origin in human practices. Yet, this argument is fallacious, since the manner by which mathematics came about does not provide any information on the domain of mathematical application. Indeed, the fact that human practices can be given a fairly sophisticated mathematical description serves as direct counter-example to the circularity argument. Specifically, there is a long tradition of attempts to integrate modern mathematical techniques with the psychological and social aspects of space,[ix] such as the child psychologist Jean Piaget, who used many of the geometrical structures explained above: e.g., topological, projective, and Euclidean (Piaget 1967). Later researchers have extended these geometrically-informed hypotheses to the larger social realm as well (see, Sack 1980, Hillier and Hanson 1996, to name a few).[x] For example, the spatial constructions of different cultures could be tied to different geometrical invariants. Since these invariants, and their associated geometries, are nested within one another in a natural and determinate way, the diversity of geometrical practices does not entail, therefore, problem (1). Moreover, since the geometric invariants are normally construed as providing a link to, or a representation of, the underlying physical ontology— via the invariant relationships among frames and their associated constraints on possible frames—problem (2) is also resolved.[xi] In conclusion, the utilization of mathematical techniques and concepts to capture the spatial (and temporal) aspects of experience might lead to new breakthroughs, as Minkowski successfully demonstrated long before the later Husserl, Heidegger, or the modern lived-space movement.

5. A Concluding Case Study: Deleuze on Differential Geometry.

The case presented thus far can be briefly summarized: by unfairly purging mathematical/geometrical concepts, the practice-oriented philosophers of place have unwittingly deprived their theory of a useful means of answering both the relativism problem (1), as well as the problematic relationship between the subjective experience of space and the underlying physical ontology, our problem (2).

Not all place theorists have ignored mathematics, however. Therefore, by way of conclusion, we will examine what is probably the most famous (or infamous) instance of the application of mathematics within the theory of lived-space. In *A Thousand Plateaus,* Giles Deleuze and Felix Guattari invoke a plethora of modern geometrical concepts in their exposition of "smooth" space and "striated" space, which, more or less, corresponds to the qualitative and quantitative aspects of space, respectively.[xii] On the whole, Deleuze and Guattari make some interesting claims that are relevant to the potential utilization of mathematics within the place theory. Smooth space, in various passages, is described as topological, non-metrical, local, and "is therefore a vector, a direction and not a dimension or metric determination" (1987, 478); whereas striated space is characterized as metrical. Deleuze and Guattari cite Riemann's theory of manifolds as a model for viewing smooth space; and, although their analysis is rather incongruous mathematically, one of the goals seems to be a sketch of the relationship between the subjective, qualitative space of the individual (smooth space) and a formal quantitative method of objectifying that space (striated space, which they link with, not surprisingly, Euclidean space; 371). Their main contention is that smooth space, as the environs of the individual, need not be necessarily conceived as merely a part of a larger metrical, striated space. They introduce Riemann's theory to demonstrate that a point of the manifold (smooth space) can be connected to an adjacent point in a number of ways, such that a metrical connection need not be assumed, a process they call "accumulation" (485): put in the modern mathematical parlance, while the infinitesimal neighborhood (or tangent space) of each manifold point is Euclidean, a vector in this tangent space can be compared with another vector in a separate tangent space in such a way that their (locally defined) Euclidean properties are lost in the transfer. Consequently, Deleuze and Guattari have relied upon the intrinsic approach to geometry and its concept of a manifold (as outlined in section 4) to correlate separate subjective spaces; albeit without really laying to rest the relativism issue addressed in this essay, since the many possible connections among infinitesimal neighborhoods raises problem (1) in a new guise.[xiii]

Finally, Deleuze and Guattari attribute to Riemann the (quite implausible) legacy of "the beginning of a typology and topology of multiplicities", i.e., an alternative method of conceiving quantities, broadly construed, which would ultimately come to fruition in Henri Bergson's qualitative concept of "duration" ("as a type of multiplicity opposed to metric multiplicity or the multiplicity of magnitude", 483). On these grounds, they conclude that "we consider Bergson to be of major importance (much more so than Husserl, or even Meinong or Russell) in the development of the theory of multiplicities" (483). Yet, in an ironic twist, the results in differential geometry that Deleuze and Guattari refer to, on the multiple connections among points on a manifold, are largely the product of Hermann Weyl—and Weyl's motivation in these mathematical investigations was, in part, to adapt Husserl's phenomenological work on subjective space perception to the new conception of physical space that followed in the wake of the General Theory of Relativity. Each point of the manifold, for Weyl, is linked to the infinitesimal Euclidean space of a hypothetical observer, such that it guarantees the kinesthetic experience of the free mobility (in three-dimensions) of objects in that infinitesimal neighborhood (via the Helmholtz-Lie theorem). Each point of the manifold is, accordingly, a separate

Husserlian (subjective) space, as well as the remnant of Kant's synthetic a priori "form of intuition" of space. Yet, the mutual orientation of the Euclidean metrics located at separate points may differ, and thus the overall space (manifold) may not be Euclidean, a fact that can only be determined by experience (see, Ryckman 2005, chapters 5 and 6, and endnote 4). Deleuze and Guattari may have been unaware of this bit of mathematical history, or of the direct relevance of Husserl's philosophy for the story. Yet, given their very peculiar efforts to proclaim Bergson's importance for the evolution of their quasi-mathematical concept of "multiplicity" (see, 482-483), a more sinister take on this entire discussion is that Deleuze and Guattari favor Bergson because he is a more faithful exponent of the *Lebensphilosophie* movement, as opposed to the more objectivist, scientifically-oriented project of Husserl (with its sharp repudiation of any relativism or historicism). If this interpretation is correct, it would reveal a deeper tension within the lived-space approach as a whole, namely, the continuing battle between its objectivist, scientifically inclined and subjectivist, non-scientifically inclined contingents—and the outcome of this contest will largely determine the role of mathematics for the practitioners of the theory of place. Interestingly, in a recent collection of articles entitled, *Deleuze and Space* (Buchanan and Lambert 2005), none of the mathematical themes in *A Thousand Plateaus* are taken up, which would indicate that the prospects for an integration of mathematical methods within the place theory are, at least for the foreseeable future, not very promising.

i The place theorists might appeal to a deeper, non-metrical invariant to counter radical spatial subjectivism, such as a topological invariant, or a common geometric axiom (defined set-theoretically for all geometries). Yet, as will be explained, the lived-space school's antipathy to mathematics would almost certainly preclude this option. Throughout this essay, moreover, we will refer to objectivity as a joint epistemological and ontological notion, since lived-space theorists tend to blur the distinction between the two; see, Rescher (1997), on the many forms of objectivity, including ontological. Furthermore, "subjective", as used in this essay will refer to either a personal ("egocentric" in some texts) or social conception of space; i.e., as a non-objective conception. Finally, references to "place", or "lived-space", theory and theorists signify the contemporary, largely continental or continental-influenced, approach to space (e.g., Casey, Malpas, Lefebvre, etc.), and not the early phenomenologists.

ii That is, since radical spatial subjectivism is based on a strong form of relativism (either to personal experience, society, practice, etc.), it follows that the physical assumptions implicit in the construction and application of the measuring apparatus are also relative—if the spatial subjectivist were to deny this, and claim instead that the measurement devices are somehow non-relative, it would open up the spatial subjectivist to the charge of inconsistency. Moreover, as argued by, among others, Einstein (1949), since our understanding of the physical constitution of these measuring devices also relies on geometrical assumptions (via the force laws that govern their behavior), the spatial subjectivist can always claim that these lower-level applications of geometry are likewise subjective (relative), thus opening the way for option (c).

iii Husserl studied Lotze's theories of space and geometry in his early years; see, Mohanty (1995, 51). On empiricist theories of space and geometry, see, Torretti (1983).

iv Husserl (1997) is his first extended treatment of these issues. It is also worth noting that Husserl's student, Oskar Becker, strived to remove the apparent contingency associated with the geometry of Husserl's theory. Influenced by Weyl's work, Becker relies on a group-theoretic argument to prove that our subjective experience of moving freely through space (via the Helmholtz-Lie theorem) singles out Euclidean geometry as the only candidate for objective space. Weyl's own theory also employs a group-theoretic approach, but only preserves a Euclidean structure infinitesimally for each point of the spacetime manifold, while repudiating a Euclidean global structure for physical space (as mandated by the variably curved spacetime of General Relativity; see, Mancosu and Ryckman 2005, and section 4). Finally, Becker's own student, Elizabeth Ströker, would develop a theory like Becker's in her (1987) text, a work that is often cited approvingly by contemporary place theorists.

v In addition, see, Friedman (2000, 13-23), for the development of Heidegger's quite hostile attitude towards modern mathematical logic and physics.

vi More carefully, The Euclidean transformations are a subgroup of the affine transformations, hence Euclidean geometry is a subgeometry of affine geometry (and both, in turn, are subgroups and subgeometries of the larger projective transformations and projective geometry). Also, $\hat{A}^{2}$ is the vector space of ordered pairs of real numbers allowing addition and scalar multiplication. Parts of this discussion are based on Brannan, Esplen, and Gray (1999). See, e.g., Nozick (2001) and Debs and Redhead (2007), for similar approaches to objectivity and invariance.

[vii] It is not being claimed, here, that psychological/social factors, such as a language or conceptual scheme, cannot provide a structural foundation from which mathematics emerges (see, e.g., Lakoff and Nuñez 2001). Yet, these attempts to derive mathematics from human practices then run into another version of problem (1), since it would appear that different human practices might then generate conflicting mathematical schemes. Husserl's long antipathy to "psychologism" was based on this very concern.

[viii] In Joseph Kockelmans' excellent survey, the relativism problem for Heidegger is discussed with respect to the "aboriginal Event", the *Ereignes,* which is "ontologically prior to Being as well as to time, because it is that which grants to both what they properly are" (1992, 162-163; from *Zur Sache Des Denkens,* 1988). Given the *Ereignes,* "one understands, or perhaps more accurately stated, *experiences* that the various epochs [different manifestations of Being's history] are no longer mysteries, but are the necessary consequence of the inherent finitude of an aboriginal Event which presents the Open [the bestowing of past, present, future] and grants Being" (167). Yet, it difficult to understand how Heidegger's appeal to this sort of quasi-mystical insight can constitute a serious resolution of the relativism problem. In fact, seeking divine revelation from *Ereignis* in this manner would seem to be just another way of introducing the "God of the philosophers", which is a strategy that he thoroughly repudiates.

[ix] A notable declaration of the need for mathematical investigations in the social studies of space is the following: "It is clear that environmental 'objects' and human 'subjects' are deeply entangled with each other . . . . Nor is it the case that the object side of the urban system can be dealt with mathematically and the subject side only qualitatively. The fact that the city is shaped by the human cognitive subject does not lessen its mathematical content . . . . [T]he cognitive processes by which the subject intervenes reflect mathematical laws. . . . The project for space syntax research must now be to engage with the problematics of both the mathematical and humanistic paradigms in the hope and expectation that by finding how each is present in the other we will progress towards synthesis" (Hillier 2003, 19).

[x] In one of Cassirer's last works, he also advocates the expansion of these geometrical concepts to other disciplines. Cassirer notes that psychologists are "not especially interested in mathematical speculation", and that mathematicians do "not care about psychological problems"; yet, he insists that this "separation is of questionable value" (1979, 285). He continues: "Of course we cannot mix up the two fields of investigation; we must make a sharp distinction between the mathematical and psychological problem of space. But that ought not to prevent us from looking for a connecting link between the

two problems; and I think that the concept of a group [of transformations] may be regarded as such a connecting link" (285).

xi A recent interpretation of Husserl's philosophy of space/geometry, which also utilizes transformation groups, is Tieszen (2005, chapter 3), although the problematic issues pertaining to the life-world (as argued above) would seem to pose an obstacle to his reconstruction of Husserl's later philosophy. See, also, Carr (1977), on the complexities of Husserl's life-world.

xii On Deleuze and Guattari on chaos theory, see Sokal and Bricmont (1998). While controversial in their own right, these types of critiques of postmodern thought do shed light, at least tangentially, on an apparent trend among some contemporary theorists of lived-space; namely, the appropriation of mathematical and scientific *terms* or *ideas,* such that they are no longer used in their strictly technical sense, but rather are exploited to present an array of different meanings or notions (many possibly literary in origin). Furthermore, this essay cannot explore all of the discussions of geometry in Deleuze's work, but merely examines a notable instance.

xiii That is, there is an underdetermination of the exact method of connecting the separate smooth (tangent) spaces. Another problem is that a non-metrical connection, such as an affine connection that preserves linearity but not length, does not guarantee a metrical connection, although the latter does contain the former. Deleuze and Guattari might think, erroneously, that the two concepts are necessarily and sufficiently conjoined, since they claim that "the two [non-metrical "accumulation" and "Euclidean conjunction"] are linked and give each other impetus" (486). But, an affine connection does not require a metrical connection at all. In trying to capture the alleged interdependence of smooth and striated spaces sought by Deleuze and Guattari, a better case from differential geometry might be found in the distinction between tangent vectors (or contravariant vectors) and 1-forms (or covariant vectors), which are inter-defined and equally necessary for the mathematical presentation (see, e.g., Burke 1980, chapter 2): the 1-forms, which provide the gradient for smooth functions, are often given the pictorial representation of a contour map, and thus its role in "numbering" the tangent vectors would nicely fit the category of striated space, while the tangent vectors obviously play the role of smooth space.

BIBLIOGRAPHY

Buchanan, I. and Lambert, G., eds. (2005). *Deleuze and Space.* Edinburgh: Edinburgh University Press.

Burke, W. L. (1980). *Spacetime, Geometry, Cosmology.* Mill Valley, Cal.: University Science Books.

Brannan, D., Esplen, M., Gray, J. (1999). *Geometry.* Cambridge: Cambridge University Press.

Campbell, J. (1994). *Past, Space and Self.* Cambridge, Mass.: MIT Press.

Carr, D. (1977). "Husserl's Problematic Concept of the Life-World", in *Husserl: Expositions and Appraisals,* edited by F. Elliston and P. McCormick, Notre Dame, IN: University of Notre Dame Press, 202-212.

Casey, E. (1997). *The Fate of Place.* Berkeley: University of California Press.

Cassirer, E. (1979, [1945]). "Reflections on the Concept of Group and the Theory of Perception", in *Symbol, Myth, and Culture,* ed. by D. P. Verene. New Haven: Yale University Press.

Debs, T. A. and Redhead, M. (2007). *Objectivity, Invariance, and Convention: Symmetry in Physical Science.* Cambridge, MA.: Harvard University Press.

Deleuze, G. and Guattari, F. (1987). *A Thousand Plateaus,* trans. by B. Massumi. Minneapolis: University of Minnesota Press.

Eddington, A. (1958, [1939]). *The Philosophy of Physical Science.* Ann Arbor: University of Michigan Press.

Einstein, A. (1949). "Remarks on the Essays". In P. Schilpp (Ed.), *Albert Einstein, Philosopher-Scientist* (pp. 663-688). La Salle, Ill.: Open Court.

Entrinkin, J. N. (1991). *The Betweeness of Place.* Baltimore: Johns Hopkins University Press.

Friedman, M. (2000). *A Parting of the Ways: Carnap, Cassirer, and Heidegger.* Chicago: Open Court.

Heidegger, M. (1962, [1927]). *Being and Time,* trans. by J. Macquarrie and E. Robinson. New York: Harper and Row.

Heidegger, M. (1988). *Zur Sache des Denkens.* Tübingen: Verlag.

Hillier, B., and Hanson, J. (1984). *The Social Logic of Space.* Cambridge: Cambridge University Press.

Hillier, B. (2003). "The Knowledge that Shapes the City: The Human City beneath the Social City", Proceedings of the 4th International Space Syntax Symposium, London, 1-20.

Husserl, E. (1970, [1937]). *The Crisis of European Sciences and Transcendental Phenomenology,* trans. by D. Carr. Evanston: Northwestern University Press.

Husserl, E. (1981, [1934]). "Foundational Investigations of the Phenomenological Origin of the Spatiality of Nature", trans. by F. Kersten, in *Husserl: Shorter Writings,* ed. by P. McCormick and F. Elliston (South Bend: University of Notre Dame Press).

Husserl, E. (1982, [1913]). *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy, First Book,* trans. by F. Kersten. Dordrecht: Kluwer.

Husserl, E. (1997, [1907]). *Thing and Space,* trans. by R. Rojcewitz. Dordrecht: Kluwer.

Kockelmans, J. (1992). "Heidegger on Time and Being", in *Martin Heidegger: Critical Assessments,* ed. by C. Macann. London: Routledge.

Lakoff, G. and Nuñez, R. (2001). *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being.* New York: Basics Books.

Lefebvre, H. (1991). *The Production of Space,* trans. by D. Nicholson-Smith. Oxford: Blackwell.

Mancosu, P. and Ryckman, T. A. (2005). "Geometry, Physics, and Phenomenology: Four Letters of O. Becker to H. Weyl", in *Oskar Becker und die Philosophie der Mathematik,* edited by V. Peckhaus, Munich: Verlag, 153-227.

Malpas, J. E. (1999). *Place and Experience.* Cambridge: Cambridge University Press.

Malpas, J. E. (2006), *Heidegger's Topology: Being, Place, World.* Cambridge, MA: MIT Press

Merleau-Ponty, M. (1962, [1945]). *Phenomenology of Perception,* trans. by C. Smith. London: Routledge.

Merleau-Ponty, M. (2003). *Nature,* edited by D. Séglard, translated by R. Vallier, Evanston, IL: Northwestern University Press.

Minkowski, H. (1964, [1908]). "Space and Time", trans. by W. Perrett and G. B Jeffery, in *Problems of Space and Time,* ed. by J. J. C. Smart. New York: Macmillan, 297-312.

Mohanty, J. N. (1995). "The Development of Husserl's Thought", in *The Cambridge Companion to Husserl,* ed. by B. Smith and D. W. Smith. Cambridge: Cambridge University Press.

Nozick, R. (2001). *Invariances: The Structure of the Objective World.* New York: Belknap

Piaget, J. (1967). *The Child's Conception of Space.* New York: W. W. Norton.

Rescher, N. (1997). *Objectivity: The Obligations of Impersonal Reason.* Notre Dame, In.: University of Notre Dame Press.

Ryckman, T. (2005). *The Reign of Relativity.* Oxford: Oxford University Press.

Sack, D. (1980). *Conceptions of Space in Social Thought.* Minneapolis: University of Minnesota Press.

Sokal, A., and Bricmont, J. (1998). *Fashionable Nonsense: Postmodern Intellectuals' Abuse of Science.* New York: Picador.

Ströker, E. (1987). *Investigations in Philosophy of Space,* trans. by A. Mickunas. Athens, OH: Ohio University Press.

Tieszen, R. (2005). *Phenomenology, Logic, and the Philosophy of Mathematics.* Cambridge: Cambridge University Press.

Torretti, R. (1978). *Philosophy of Geometry from Riemann to Poincaré.* Dordrecht: D. Reidel.

Tuan, Yi-Fu (1977). *Space and Place.* Minneapolis: University of Minnesota Press.

Vallega, A. (2003). *Heidegger and the Issue of Space.* University Park: Pennsylvania State University Press.

Weyl, H. (1949, [1927]). *Philosophy of Mathematical and Natural Science.* Princeton: Princeton University Press.