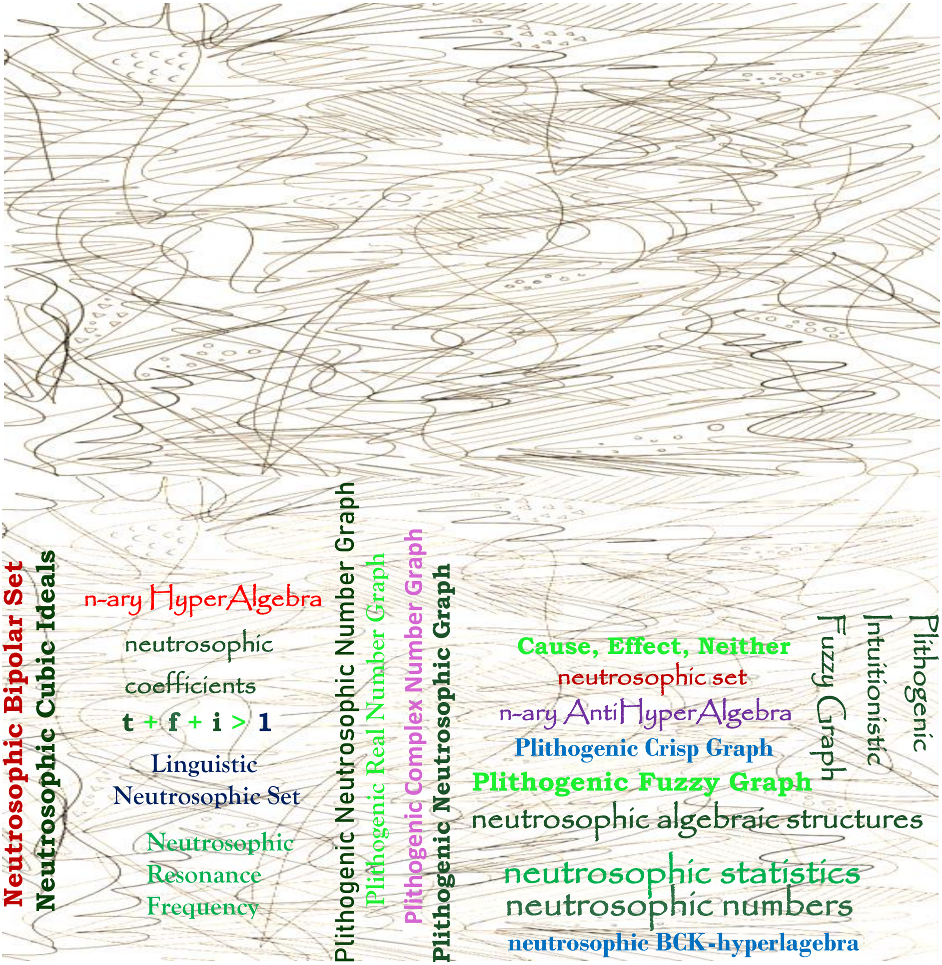
scilogs, IX

florentin smarandache

nidus idearum

neutrosophia perennis



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Florentin Smarandache

Nidus idearum.
Scilogs, IX: neutrosophia perennis

Grandview Heights, Ohio, USA, 2022

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FOREWORD

Welcome into my scientific lab!

My **lab**[oratory] is a virtual facility with non-controlled conditions in which I mostly perform scientific meditation and chats: *a nest of ideas* (**nidus idearum**, in Latin).

I called the jottings herein *scilogs* (truncations of the words *scientific*, and gr. Λόγος – appealing rather to its original meanings "ground", "opinion", "expectation"), combining the welly of both science and informal (via internet) talks (in English, French, and Romanian).

*

In this *ninth book of scilogs* collected from my nest of ideas, one may find new and old questions and solutions, – in email messages to research colleagues, or replies, and personal notes, some handwritten on the planes to, and from international conferences, about topics on Neutrosophy and its applications, such as: Neutrosophic Bipolar Set, Linguistic Neutrosophic Set, Neutrosophic Resonance Frequency, n-ary HyperAlgebra, n-ary NeutroHyperAlgebra, n-ary AntiHyperAlgebra, Plithogenic Crisp Graph, Plithogenic Fuzzy Graph, Plithogenic Intuitionistic Fuzzy Graph, Plithogenic Neutrosophic Graph, Plithogenic Real Number Graph, Plithogenic Complex Number Graph, Plithogenic Neutrosophic Number Graph, and many more.

Neutrosophy [1998], as a new branch of philosophy and a generalization of dialectics, is based on the dynamics of opposites and the neutralities between them, and it has been extended to Refined Neutrosophy, and consequently the Neutrosophication was extended to Refined Neutrosophication.

Whence, Regret Theory, Grey System Theory, and Three-Ways Decision are particular cases of Neutrosophication and of Neutrosophic Probability. We have further extended the Three-Ways Decision to n-Ways Decision, the last one is a particular case of Refined Neutrosophy.

Neutrosophy is also an extension of the international movement called Paradoxism (based only on contradictions in science and literature) [1980].

Neutrosophic Set, defined on three components {membership (T), indeterminacy (I), and nonmembership (F)}, is a generalization of Crisp Set, Fuzzy Set, Intuitionistic Fuzzy Set, Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set, q-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, and n-HyperSpherical Fuzzy Set. Neutrosophic Set has been further extended to Refined Neutrosophic Set. Further on, as extension and alternative there was defined the Plithogenic Set [2017] based on MultiVariate Analysis.

*

I coined the words "neutrosophy" and "neutrosophic" in my 1998 book: Florentin Smarandache, *Neutrosophy. Neutrosophic Probability, Set, and Logic*, ProQuest Information & Learning, Ann Arbor, Michigan, USA, 105 p., 1998; http://fs.unm.edu/eBook-neutrosophics6.pdf (last edition online).

Etymologically, "neutro-sophy" (noun) [French neutre < Latin neuter, neutral, and Greek sophia, skill/wisdom] means knowledge of neutral thought, while "neutrosophic" (adjective), means having the nature of, or having the characteristic of Neutrosophy.

THE MOST IMPORTANT PUBLICATIONS IN THE DEVELOPMENT OF NEUTROSOPHICS

1995-1998

- Smarandache generalizes the Yin Yang and Dialectics to Neutrosophy as a new branch of philosophy: http://fs.unm.edu/Neutrosophy-A-New-Branch-of-Philosophy.pdf ;
- introduces the Neutrosophic set / logic / probability / statistics;
- introduces the single-valued Neutrosophic Set (pp. 7-8); http://fs.unm.edu/eBook-Neutrosophics6.pdf

1998 & 2019

- Nonstandard Neutrosophic Logic, Set, Probability https://arxiv.org/ftp/arxiv/papers/1903/1903.04558.pdf

2002

- introduces special types of sets / probabilities / statistics / logics, such as:
    - intuitionistic set, paraconsistent set, faillibilist set, paradoxist set, pseudo-paradoxist set, tautological set, nihilist set, dialetheist set, trivialist set;
    - intuitionistic probability and statistics, paraconsistent probability and statistics, faillibilist probability and statistics, paradoxist probability and statistics, pseudo-paradoxist probability and statistics, tautological probability and statistics, nihilist probability and statistics, dialetheist probability and statistics, trivialist probability and statistics;
    - paradoxist logic (or paradoxism), pseudo-paradoxist logic (or pseudo-paradoxism), tautological logic (or tautologism);

http://fs.unm.edu/DefinitionsDerivedFromNeutrosophics.pdf

2003

- introduction of Neutrosophic numbers (a+b*I*, where I = indeterminacy)
- introduction of *I*-Neutrosophic algebraic structures
- introduction to Neutrosophic Cognitive Maps
  http://fs.unm.edu/NCMs.pdf

2005

- introduction of Interval Neutrosophic Set/Logic
  http://fs.unm.edu/INSL.pdf

2006

- introduction of the Degree of Dependence and Degree of Independence between T, I, and F
  http://fs.unm.edu/eBook-Neutrosophics6.pdf [p. 92]
  http://fs.unm.edu/NSS/DegreeOfDependenceAndIndependence.pdf

2007

- The Neutrosophic Set was extended [Smarandache, 2007] to Neutrosophic Overset (when some Neutrosophic component is > 1), and to Neutrosophic Underset (when some Neutrosophic component is < 0), and to and to Neutrosophic Offset (when some Neutrosophic components are off the interval [0, 1], i.e. some Neutrosophic component > 1 and some Neutrosophic component < 0).

Then, similarly, the Neutrosophic Logic / Measure / Probability / Statistics etc. were extended to respectively Neutrosophic Over/Under/Off Logic, Measure, Probability, Statistics etc.

http://fs.unm.edu/NSS/DegreesOf-Over-Under-Off-Membership.pdf

http://fs.unm.edu/SVNeutrosophicOverset-JMI.pdf

http://fs.unm.edu/IV-Neutrosophic-Overset-Underset-Offset.pdf

http://fs.unm.edu/NeutrosophicOversetUndersetOffset.pdf

- introduction of the Neutrosophic Tripolar Set and Neutrosophic Multipolar Set [Smarandache], and consequently
- the Neutrosophic Tripolar Graph and Neutrosophic Multipolar Graph

http://fs.unm.edu/eBook-Neutrosophics6.pdf [p. 93]
http://fs.unm.edu/IFS-generalized.pdf

2009

- introduction of N-norm and N-conorm

http://fs.unm.edu/N-normN-conorm.pdf

2013

- development of Neutrosophic Probability:
(chance that an event occurs, indeterminate chance of occurrence, chance that the event does not occur).
http://fs.unm.edu/NeutrosophicMeasureIntegralProbability.pdf
- Smarandache refined the components (T, I, F) as (T_1, T_2, ...; I_1, I_2, ...; F_1, F_2, ...)
http://fs.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf

2014

- introduction of the Law of Included Multiple-Middle (<A>; <neutA$_1$>, <neutA$_2$>, ..., <neutA$_k$>; <antiA>)
http://fs.unm.edu/LawIncludedMultiple-Middle.pdf

- development of Neutrosophic Statistics (indeterminacy is introduced into classical statistics with respect to the sample/population, or with respect to the individuals that only partially belong to a sample/population, with respect to the probability distributions, with respect to statistical inference methods, etc.)
http://fs.unm.edu/NS/NeutrosophicStatistics.htm
http://fs.unm.edu/NeutrosophicStatistics.pdf

2015

- introduction of Neutrosophic Precalculus and Neutrosophic Calculus
http://fs.unm.edu/NeutrosophicPrecalculusCalculus.pdf
- Refined Neutrosophic Numbers ($a + b_1 I_1 + b_2 I_2 + \ldots + b_n I_n$)
- Neutrosophic Graphs
- Thesis-Antithesis-Neutrothesis, and Neutrosynthesis, Neutrosophic Axiomatic System, Neutrosophic dynamic systems, symbolic Neutrosophic logic, (t, i, f)-Neutrosophic Structures, *I*-Neutrosophic Structures, Refined Literal Indeterminacy, Quadruple Neutrosophic Algebraic Structures, Multiplication Law of Subindeterminacies:
http://fs.unm.edu/SymbolicNeutrosophicTheory.pdf
- Introduction of the Subindeterminacies of the form $(I_0)_n = k/0$, for $k \in \{0, 1, 2, \ldots, n-1\}$, into the ring of modulo integers Z_n - called natural neutrosophic indeterminacies (Vasantha-Smarandache)
http://fs.unm.edu/MODNeutrosophicNumbers.pdf
- Introduction of Neutrosophic Crisp Set <A, B, C> and Topology (Salama & Smarandache)
http://fs.unm.edu/NeutrosophicCrispSetTheory.pdf

2016

- Neutrosophic Multisets (as generalization of classical multisets)

  http://fs.unm.edu/NeutrosophicMultisets.htm

- Neutrosophic Triplet structures (F. Smarandache & M. Ali) and Neutrosophic Extended Triplet structures (Smarandache)

  http://fs.unm.edu/NeutrosophicTriplets.htm

- Neutrosophic Duplet structures

  http://fs.unm.edu/NeutrosophicDuplets.htm

2017 - 2020

- Neutrosophic Score, Accuracy, and Certainty Functions form a total order relationship on the set of (single-valued, interval-valued, and in general subset-valued) neutrosophic triplets (T, I, F); and these functions are used in MCDM (Multi-Criteria Decision Making): http://fs.unm.edu/NSS/TheScoreAccuracyAndCertainty1.pdf

2017

- In biology Smarandache introduced the Theory of Neutrosophic Evolution: Degrees of Evolution, Indeterminacy or Neutrality, and Involution, http://fs.unm.edu/neutrosophic-evolution-PP-49-13.pdf

- Introduction by F. Smarandache of Plithogeny (generalization of Dialectics and Neutrosophy), and Plithogenic Set/Logic/Probability/Statistics (generalization of fuzzy, intuitionistic fuzzy, neutrosophic set/logic/probability/statistics) http://fs.unm.edu/PPS/

- Introduction of Plithogeny (as generalization of Yin-Yang, Dialectics, and Neutrosophy), and Plithogenic Set / Plithogenic Logic as generalization of MultiVariate Logic / Plithogenic Probability and Plithogenic Statistics as generalizations of MultiVariate Probability and Statistics (as generalization of fuzzy, intuitionistic fuzzy, neutrosophic set/logic/probability/statistics):
https://arxiv.org/ftp/arxiv/papers/1808/1808.03948.pdf
http://fs.unm.edu/Plithogeny.pdf

2018

- Introduction to Neutrosophic Psychology (Neutropsyche, Neutropsychic Personality, Eros / Aoristos / Thanatos, Refined Neutrosophic Memory: conscious, aconscious, unconscious, Neutropsychic Crisp Personality, Neutrosophic Body-Soul-Mind Functioning)
http://fs.unm.edu/NeutropsychicPersonality-ed3.pdf
- Generalization of the Soft Set to HyperSoft Set
http://fs.unm.edu/NSS/ExtensionOfSoftSetToHypersoftSet.pdf

2019

- Theory of Spiral Neutrosophic Human Evolution (Smarandache - Vatuiu):
http://fs.unm.edu/SpiralNeutrosophicEvolution.pdf
- Introduction to Neutrosophic Sociology (Neutrosociology) [neutrosophic concept, or (T, I, F)-concept, is a concept that is T% true, I% indeterminate, and F% false]
http://fs.unm.edu/Neutrosociology.pdf
- Refined Neutrosophic Crisp Set
http://fs.unm.edu/RefinedNeutrosophicCrispSet.pdf

- Generalization of the classical Algebraic Structures to NeutroAlgebraic Structures (or NeutroAlgebra)[whose operations and axioms are partially true, partially indeterminate, and partially false] and AntiAlgebraic Structures (or AntiAlgebra) [with operations and axioms totally false]:
http://fs.unm.edu/NA/NeutroAlgebra.htm

2020

- Introduction to Neutrosophic Genetics
http://fs.unm.edu/IJNS/NeutrosophicGenetics.pdf

2021

- As alternatives and generalizations of the Non-Euclidean Geometries, Smarandache introduced in 2021 the NeutroGeometry & AntiGeometry. While the Non-Euclidean Geometries resulted from the total negation of only one specific axiom (Euclid's Fifth Postulate), the AntiGeometry results from the total negation of any axiom and even of more axioms from any geometric axiomatic system (Euclid's, Hilbert's, etc.), and the NeutroAxiom results from the partial negation of one or more axioms [and no total negation of no axiom] from any geometric axiomatic system.
http://fs.unm.edu/NSS/NeutroGeometryAntiGeometry31.pdf

- Introduction of Plithogenic Logic as a generalization of MultiVariate Logic
http://fs.unm.edu/NSS/IntroductionPlithogenicLogic1.pdf

- Introduction of Plithogenic Probability and Statistics as generalizations of MultiVariate Probability and Statistics respectively http://fs.unm.edu/NSS/PlithogenicProbabilityStatistics20.pdf

1998 – 2021

Neutrosophic Applications in:

Artificial Intelligence, Information Systems, Computer Science, Cybernetics, Theory Methods, Mathematical Algebraic Structures, Applied Mathematics, Automation, Control Systems, Big Data, Engineering, Electrical, Electronic, Philosophy, Social Science, Psychology, Biology, Genetics, Biomedical, Engineering, Medical Informatics, Operational Research, Management Science, Imaging Science, Photographic Technology, Instruments, Instrumentation, Physics, Optics, Economics, Mechanics, Neurosciences, Radiology Nuclear, Medicine, Medical Imaging, Interdisciplinary Applications, Multidisciplinary Sciences etc.

[Xindong Peng and Jingguo Dai, A bibliometric analysis of neutrosophic set: two decades review from 1998 to 2017, Artificial Intelligence Review, Springer, 18 August 2018; http://fs.unm.edu/BibliometricNeutrosophy.pdf].

CONTENTS

TOPICS

Universal NeutroAlgebra and Universal AntiAlgebra

Florentin Smarandache

The *Universal NeutroAlgebra* studies common properties of the *NeutroAlgebra* structures. The *Universal AntiAlgebra* studies common properties of the *AntiAlgebraic* structures.

— *A universe of discourse, a set, some operations, and some axioms*

Let's consider a non-empty set S included in a universe of discourse U, or $S \subset U$.

The set S is endowed with n operations, $1 \leq n \leq \infty$, $*_1, *_2, \ldots, *_n$.

Each operation $*_i$, for $i \in \{1, 2, \ldots, \infty\}$, is an m_i-ary operation, where $0 \leq m_i \leq \infty$. {A o-ary operation, where "0" stands for zero (or null-ary operation), simply denotes a constant.}

Then a number of α axioms, $0 \leq \alpha \leq \infty$, is defined on S.

The axioms may take the form of identities (or equational laws), quantifications {universal quantification ($\forall$) except before an identity, existential quantification ($\exists$)}, inequalities, inequations, and other relations.

With the condition that there exist at least one m-ary operation, with $m \geq 1$, or at least one axiom.

We have taken into consideration the possibility of infinitary operations, as well as infinite number of axioms.

- *Operation, NeutroOperation, AntiOperation*
- A classical Operation ($*_m$) is an operation that is well-defined for all elements of the set *S*, i.e. $*_m(x_1, x_2, \ldots, x_m) \in S$ for all $x_1, x_2, \ldots, x_m \in S$.
- An AntiOperation ($*_m$) is an operation that is not well-defined (i.e. it is outer-defined) for all elements for the set S; or $*_m(x_1, x_2, \ldots, x_m) \in U \setminus S$ for all $x_1, x_2, \ldots, x_m \in S$.
- A NeutroOperation ($*_m$) is an operation that is partially well-defined (the degree of well-defined is *T*), partially indeterminate (the degree of indeterminacy is *I*), and partially outer-defined (the degree of outer-defined is *F*); where $(T, I, F) \neq (1, 0, 0)$ that represents the classical Operation, and $(T, I, F) \neq (0, 0, 1)$ that represents the AntiOperation. An operation ($*_m$) is indeterminate if there exist some elements $a_1, a_2, \ldots, a_m \in S$ such that $*_m(a_1, a_2, \ldots, a_m)$ = undefined, unknown, unclear, etc.

- *Axiom, NeutroAxiom, AntiAxiom*
- A classical Axiom is an axiom that is true for all elements of the set *S*.
- An AntiAxiom is an axiom that is false for all elements of the set *S*.
- A NeutroAxiom is an axiom that is partially true (the degree of truth is T), partially indeterminate (the degree of indeterminacy is I), and partially

false (the degree of falsehood is F), where $(T, I, F) \neq (1,0,0)$ representing the classical Axiom, and $(T, I, F) \neq (0,0,1)$ that represents the AntiAxiom.

— *Algebra, NeutroAlgebra, AntiAalgebra*
— A classical Algebra (or Algebraic Structure) is a set *S* endowed only with classical Operations and classical Axioms.
— An AntiAlgebra (or AntiAlgebraic Structure) is a set *S* endowed with at least one AntiOperation or one AntiAxiom.
— A NeutroAlgebra (or NeutroAlgebraic Structure) is a set *S* endowed with at least one NeutroOperation or one NeutroAxiom, and no AntiOperation and no AntiAxiom.

NeutroAlgebra

Florentin Smarandache

Let *U* be a universe of discourse and a non-empty set *S* included in *U*. Let $n, m \geq 1$ be given integers.

Let $H = (S, *_1, *_2, \ldots, *_n)$ be a classical *n*-ary hyper-structure, that has *n* hyper-operations and *m* hyper-axioms.

In order to construct a *NeutroAlgebra* which is a derivative of *H*, we neutro-sophicate at least one operation, or we change the set *S* (by removing elements from it, or adding elements into it in order to get some neutro-operations or neutro-axioms).

Florentin Smarandache

The idea is alike the neutrosophy (opposites <*A*> and <anti*A*>, and the neutral <neut*A*>). So, we split the space (or set) that the algebraic structure is defined on, into three parts, with respect to a given axiom:

— elements for which the axiom is *true*;

— elements for which the axiom is *false*;

— elements for which the axiom is *indeterminate* (if any, since there may be space where there is no indeterminacy with respect to the axiom).

And we get a *NeutroAxiom*.

For a given operation $*$ defined on the space (or set) S we do the same — split the space into three parts:

— elements for which the operation is well-defined (or inner-defined) (i.e. $x * y$ in S); equivalent to truth;

— elements for which the operation is outer-defined (i.e. there exist x, y in S such that $x * y$ not in S);

— elements for which the operation is indeterminate (i.e. $x * y$ = undefined, unknown).

And we get a *NeutroOperation*.

Florentin Smarandache

A Law (Operation) defined on a set, in classical algebraic structures, is well-defined *(T)* for all set's elements.

But this is idealistic, perfect. While our world is imperfect, non-idealistic.

A *NeutroLaw* (or *NeutroOperation*) is a law (or operation) that is:

—well-defined *(T)* for some set's elements,

—indeterminate *(I)* for other set's elements,

—and undefined *(F)* for the other set's elements.

Similarly, an Axiom defined on a set is in classical algebraic structure true *(T)* for all set's elements.

But this is idealistic, perfect too. Yet our world is imperfect, with partialities.

Thus, a *NeutroAxiom* defined on a set is true *(T)* for some set's elements, indeterminate *(I)* for other set's elements, and false *(F)* for the other set's elements.

It is like a proposition in neutrosophy, that has a degree of truth, a degree of indeterminacy, and a degree of falsehood.

Whence a *NeutroAlgebra* is an algebra that has at least one *NeutroLaw* (*NeutroOperation*) or at least one *NeutroAxiom*.

Florentin Smarandache

to Akbar Rezaei

In order to have a *NeutroAlgebra*,

— at least one axiom should be partially true (and partially indeterminate, and/or partially false);

— or at least one operation should be partially defined (and partially indeterminate, and/or partially undefined).

Let's say we take the Axiom you employed in your paper:

$$(B)(\forall x \in X)(x * x = 0)$$

then we re-define it as:

— $x * x = 0$ for some elements x in X,
— and $x * x$ is not equal to 0 for other elements in X.

The other axioms may remain the same as done into the paper.

Or, you may have them only partially true for some elements, while for other elements they should be either false or indeterminate.

When introducing new terms, just use as a suffix *NeutroAlgebra*, for example: *Sub-BI-NeutroAlgebra*, etc.

Therefore, in conclusion, try to have at least one axiom which is partially true, or at least one operation which is partially defined.

Lagrange's NeutroTheorem

Florentin Smarandache

With the new development of *NeutroAlgebra*, we can have theorems, in particular Lagrange's Theorem that are partially true. Lagrange's Theorem may be:

— partially true (true for some elements into the space),
— partially false (false for some elements into the space),
— and maybe partially indeterminate.

So, we may have a Lagrange's NeutroTheorem.

For each classical Algebra, there are many NeutroAlgebras and many AntiAlgebras

Florentin Smarandache

Let us state that for each classical Algebra, there are many *NeutroAlgebras* and many *AntiAlgebras* (one Algebra, many *NeutroAlgebras*, many *AntiAlgebras*).

Distinctions between Neutrosophic Numbers

Florentin Smarandache

Distinctions between the neutrosophic numbers:

— (t,i,f), $a+bI$ (I = literal indeterminacy),

— and $a+bI$ (I = numerical (subset) indeterminacy).

(t,i,f) represent the neutrosophic components of an element with respect to a set, i.e. they represent the degrees of membership / indeterminacy / nonmembership of an element with respect to a set.

$N = a+bI$ represents a number such that its determinate part is "a" and its indeterminate part is "aI".

"I" may be literary, i.e. $I^2 = I$.

For example:

$$N_1 = 2+3I$$

Or "I" = a numerical subset.

For example:

$$N_2 = 2+3[0.1,0.2] = [2.3, 2.6].$$

NeutroOperation vs. PartialOperation

Florentin Smarandache

A PartialAlgebra has at least one PartialOperation, meaning an operation which is partially well-defined and partially undefined (indeterminate). But it does not say anything about the operation to be outer-defined. For example:

Let $N = \{0, 1, 2, \ldots\}$, then the set $(N, \div)$, where "÷" means division, is endowed with a *NeutroOperation*, since:

— there exist elements, let's say $6, 2 \in N$ such that $6 \div 2 = 3 \in N$ (*degree of well-defined*);

— there exist elements, let's say $7, 0 \in N$ such that $7 \div 0 =$ undefined (*degree of indeterminacy*);

— and there exist elements, let's say $8, 5 \in N$ such that $8 \div 5 = 1.6 \notin N$ (*degree of outer-defined*).

When a *NeutroAlgebra* has no *NeutroAxiom*, and no operation is outer-defined, then it coincides with the *PartialAlgebra*.

Definition of Fuzzy Point [Pu's idea]

Florentin Smarandache

A fuzzy set A included into a set X is called a *fuzzy point* if it takes the value 0 for all $y \in X$ except one, say, $x \in X$. If its value at x is λ, $(0 < \lambda \leq 1)$, we denote the fuzzy point by p_x^λ, where the point x is called its support, by $supp(p_x^\lambda)$, that is $supp(p_x^\lambda) = x$.

— *Example of Fuzzy Point*

We can construct an example, as below:

Let *X* be a set, and a fuzzy set *A* included in *X*.

Say *X* = { *a, b, c, d* }, and *A* = { *a, b, c* } included in *X*. Then, if *A* = {*a*(0), *b*(0), *c*(0.7), *d*(0)},

then *A* is a fuzzy point, because it has only an element (= point) that belongs to *A*, i.e. *c*(0.7), whose membership is 0.7 > 0. The other elements do not belong to *A* since their membership = 0.

— *Example of Neutrosophic Point*

A neutrosophic point should then be defined similarly: Say *X* = { *a, b, c, d* } is a set, and *A* = { *a, b, c* } included into *X*.

If *A* = {*a*(0, 1, 1), *b*(0, 1, 1), *c*(0.7, 0.1, 0.4), *d*(0, 1, 1)}, then *A* is a neutrosophic point, since *a, b, c* do not belong to *A* {because their neutrosophic membership degree (0, 1, 1) is the opposite of the truth (1, 0, 0) }, while *c*(0.7, 0.1, 0.4) does neutrosophically belong to *A*.

A neutrosophic point is a neutrosophic set that contains only one element element whose appurtenance (truth) degree is > 0, and all other elements from its superset have the neutrosophic components (0, 1, 1).

References:

[1] Pu. Pao-Ming and Liu Ying-Ming, *Fuzzy topology. I. Neighbourhood structure of a* fuzzy *point and Moore-Smith convergence,* J. Math. Anal. Appl. 76 (1980), 571{599.

[2] Pu. Pao-Ming and Liu Ying-Ming, *Fuzzy topology. II. Product and quotient spaces,* J. Math. Anal. Appl. 77 (1980), 20-37.

Neutrosophic Algebraic Structures

Florentin Smarandache

I proposed several types of neutrosophic algebraic structures in this book (from page 107 and on):

http://fs.unm.edu/SymbolicNeutrosophicTheory.pdf

One can do this:

$n_1(t_1, i_1, f_1) * n_2(t_2, i_2, f_2) = (n_1*n_2)((t_1, i_1, f_1) * n_2(t_2, i_2, f_2))$,

where * is an operation (+, -, x, /).

Similarly for scalar multiplication and power.

Neutro-sophication and Anti-sophication

Florentin Smarandache

Neutro-sophication of an Axiom on a given set *X* means to split the set *X* into three regions such that:

— on one region the Axiom is true (we say degree of truth *T* of the Axiom),

— on another region the Axiom is indeterminate (we say degree of indeterminacy *I* of the Axiom),

— and on the third region the Axiom is false (we say degree of falsehood *F* of the Axiom),

such that the union of the regions covers the whole set, while the regions may or may not be disjoint, where *(T,I,F)* is different from (1,0,0) and from (0,0,1).

*

Anti-sophication of an Axiom on a given set *X* means to have the Axiom false on the whole set *X* (we say total degree of falsehood *F* of the Axiom), or (0,0,1).

*

Similarly for the *Neutro-sophication* of an Operation defined on a given set *X*, meaning to split the set *X* into three regions such that:

— on one region the Operation is well-defined (or inner-defined) (we say degree of truth *T* of the Operation),
— on another region the Operation is indeterminate (we say degree of indeterminacy *I* of the Operation),
— and on the third region the Operation is outer-defined (we say degree of falsehood *F* of the Operation),

such that the union of the regions covers the whole set, while the regions may or may not be disjoint, where *(T,I,F)* is different from (1,0,0) and from (0,0,1).

*

Anti-sophication of an Operation on a given set *X* means to have the Operation outer-defined on the whole set *X* (we say total degree of falsehood *F* of the Axiom), or (0,0,1).

NeutroAlgebraic Structures lay between Algebraic Structures and AntiAlgebraic Structures

Florentin Smarandache

Algebra can be all kinds of *algebras*: groupoid, semigroup, group, ring, field, vector space, BCK-algebra, BCI-algebra, etc.

There are several researchers (Agboola and his students, Hamidi, Rezaei and his friend Saeed, so on) working on *NeutroAlgebras*.

The generalization of all classical Algebraic Structures are their corresponding NeutroAlgebraic Structures and AntiAlgebraic Structures.

What is a (classical) *Algebraic Structure*:

a set *S*,
endowed with several operations,
that satisfy some axioms.

On the set *S*, all operations are 100% well-defined, i.e. (1, 0, 0), and all axioms are 100% true, i.e. (1, 0, 0).

An *AntiAlgebraic Structure* means that at least one operation is 100% (outer-defined), i.e. (0, 0, 1) or at least one axiom is 100% false, i.e. (0, 0, 1).

*

A *NeutroAlgebra* is in between the previous two.

A *NeutroAlgebra* means that:

— at least one operation is partially well-defined *(T),* partially indeterminate *(I),* and partially outer-defined *(F),* with *(T,I,F)* different from the previous (1,0,0) and (0,0,1);
— or at least one axiom is partially true *(T),* partially indeterminate *(I),* and partially false *(F),* with *(T,I,F)* different from the previous (1,0,0) and (0,0,1).

Therefore, at least one of all together (operations and axioms) has some partial neutrosophic degree.

NeutroGroups

Florentin Smarandache

Normally the *NeutroGroup* (*G*, *) should be defined as follows:

— *NO1 (NeutroOperation):* * is a neutro-operation {partially well-defined *(T),* partially indeterminate *(I)* and partially outer-defined *(F),* where *(T, I, F)* is different from (1,0,0) and from (0,0,1)}.

— *NA1 (NeutroAxiom1):* NeutroAssociativity

— *NA2 (NeutroAxiom2):* NeutroIdentity

— *NA3 (NeutroAxiom3):* NeutroInverse

A *NeutroGroup* has at least one of *NO1, NA1, NA2, NA3* and *no* AntiOperation *nor* AntiAxiom.

NeutroAxioms, NeutroOperations

Florentin Smarandache
to Akbar Rezaei

In order to avoid confusion, the below definition in easy term should be employed into the paper we have for *International Journal of Neutrosophic Science* (IJNS):

— *NeutroAxiom*

An axiom that is

— true for some elements (degree of true = T),

— indeterminate for other elements (degree of indeterminacy = I),

— and false for the other elements (degree of falsehood = F),

where *T, I, F* are in [0, 1], and *(T, I, F)* different from (1, 0, 0) {i.e. different from totally true axiom, or classical Axiom}, and *(T, I, F)* different from (0, 0, 1) {i.e. different from totally false axiom, or AntiAxiom}.

Similarly for *NeutroOperation*.

Neutro-K-algebra

Florentin Smarandache
to Mohammad Akram

What about we do something related to you paper, considering the *Neutro-K-algebra* where an axiom of the *k*-algebra is neutrosophicated, i.e. this axiom be true for some elements, indeterminate for others, and false for the others?

Let's say, we take the axiom

"$s \odot s = e$",

and we say: there exist s in G such that $s \odot s = e$, and p in G such that "$p \odot p \neq e$" or "$p \odot p$ = *indeterminate*".

Neutro BCK-algebras

Florentin Smarandache

Just get one or more axioms A_1, A_2, ... of the *BCK* algebras, and consider on a given set (space), the set's elements form which for example A_1 is true, the set's elements for which A_1 is indeterminate (unknown, unclear, undefined), and the set's elements for which A_1 is false. Then get some examples (as I did with Akbar Rezaei).

The Neutrosophic Triplet

Florentin Smarandache

Let us introduce the *n-ary HyperOperation*, *n-ary HyperAxiom*, *n-ary HyperAlgebra*, also the *n-ary NeutroHyperOperation*, *n-ary NeutroHyperAxiom*, *n-ary NeutroHyperAlgebra*, and the *n-ary AntiHyper-Operation*, *n-ary AntiHyperAxiom*, *n-ary AntiHyper-Algebra* respectively.

We form the following neutrosophic triplets:

— *(n-ary HyperOperation, n-ary NeutroHyperOperation, n-ary AntiHyperOperation),*

— *(n-ary HyperAxiom, n-ary NeutroHyperAxiom, n-ary AntiHyperAxiom),*

— and *(n-ary HyperAlgebra, n-ary NeutroHyperAlgebra, n-ary AntiHyperAlgebra)*.

Let U be a universe of discourse, a nonempty set $S \subset U$. Let $\mathcal{P}(S)$ be the power set of S (i.e. all subsets of S, including the empty set ϕ and the whole set S), and an integer $n \geq 1$.

— *n-ary HyperOperation (n-ary HyperLaw)*

A *n-ary HyperOperation (n-ary HyperLaw)* $*_n$ is defined as:

$*_n : S^n \to \mathcal{P}(S)$, and

$\forall a_1, a_2, ..., a_n \in S$ one has $*_n(a_1, a_2, ..., a_n) \in \mathcal{P}(S)$.

The *n-ary HyperOperation* (*n-ary HyperLaw*) is well-defined.

*

— *n-ary HyperAxiom*

A *n-ary HyperAxiom* is an axiom defined of *S*, with respect the above *n-ary* operation $*_n$, that is true for all *n-plets* of S^n.

— *n-ary HyperAlgebra*

$(S,*_n)$ is the *S* endowed with the above n-ary well-defined *HyperOperation* $*_n$.

— *Types of n-ary HyperAlgebras*

Adding one or more *n-ary HyperAxioms* to *S* we get different types of *n-ary HyperAlgebras*.

— *n-ary NeutroHyperOperation (n-ary NeutroHyperLaw)*

A *n-ary NeutroHyperOperation* is a *n-ary Hyper-Operation* $*_n$ that is well-defined for some *n-plets* of S^n [i.e. $\exists(a_1,a_2,...,a_n)\in S^n, *_n(a_1,a_2,...,a_n)\in P(S)$], and indeterminate [i.e. $\exists(b_1,b_2,...,b_n)\in S^n, *_n(b_1,b_2,...,b_n)=$ *indeterminate*] or outer-defined [i.e. $\exists(c_1,c_2,...,c_n)\in S^n, *_n(c_1,c_2,...,c_n)\notin P(S)$] (or both), on other *n-plets* of S^n.

— *n-ary NeutroHyperAxiom*

A *n-ary NeutroHyperAxiom* is an *n-ary HyperAxiom* defined of *S*, with respect the above *n-ary* operation $*_n$, that is true for some *n-plets* of S^n, and indeterminate or false (or both) for other *n-plets* of S^n.

— *n-ary NeutroHyperAlgebra*

The *n-ary NeutroHyperAlgebra* is an *n-ary HyperAlgebra* that has some *n-ary NeutroHyper-Operations* or some *n-ary NeutroHyperAxioms*.

— *n-ary AntiHyperOperation (n-ary AntiHyperLaw)*

A *n-ary AntiHyperOperation* is a *n-ary HyperOperation* $*_n$ that is outer-defined for all *n-plets* of S^n [i.e. $\forall(s_1, s_2, ..., s_n) \in S^n, *_n(s_1, s_2, ..., s_n) \notin P(S)$].

— *n-ary AntiHyperAxiom*

A *n-ary AntiHyperAxiom* is an *n-ary HyperAxiom* defined of *S*, with respect the above *n-ary* operation $*_n$, that is false for all *n-plets* of S^n.

— *n-ary AntiHyperAlgebra*

A *n-ary AntiHyperAlgebra* is an n-ary HyperAlgebra that has some *n-ary AntiHyperOperations* or some *n-ary AntiHyperAxioms*.

Neutrosophic Triplet of Algebras

Florentin Smarandache

We get the neutrosophic triplet of algebras:

(Algebra, NeutroAlgebra, AntiAlgebra).

Algebra = classical algebra, i.e. all operations and all axioms are true 100%.

AntiAlgebra = an algebra that has at least one operation or at least one axiom that is false 100% [such operation is called *AntiOperation*, and respectively such axiom is called *AntiAxiom*].

NeutroAlgebra = algebra in between the above ones, i.e. algebra that has at least one operation or at least one axiom that is partially true *(T),* partially indeterminate *(I),* and partially false *(F),* with *(T,I,F)* different from (1,0,0) and from (0,0,1).

Such operation is called *NeutroOperation* and respectively such axiom is called *NeutroAxiom.*

NeutroAlgebra should have no *AntiOperation* and no *AntiAxiom.*

Broadest possible sense of Indeterminacy

Said Broumi

Gramatically what is the difference between *indeterminacy* and *uncertainty*?

Florentin Smarandache

As you said, they are grammatically almost synonymous.

But, in the neutrosophic environment, we interpret Indeterminacy in the broadest possible sense, i.e. :

— everything which is in between the opposites *T* (Truth) and *F* (Falsehood) {denoted by *'I'* in Neutrosophic Logic, Set, Probability} ;

— or respectively everything which is in between the opposites <*A*> and <anti*A*> {denoted by <neut*A*> in Neutrosophy}, where <*A*> is an item, idea, concept etc. such that the neutrosophic triplet (<*A*>, <neut*A*>, <anti*A*>) makes sense.

Even more, Indeterminacy (*"I"* or <neut*A*>) can be split/refined (within the Refined Neutrosophic environment) into: Uncertainty ($T \vee F$, or respectively <*A*> ∨ <anti*A*>), Contradiction ($T \wedge F$, or respectively <*A*>∧<anti*A*>), Unknown, Incompleteness, etc.

The Difference between Indeterminacy and Uncertainty

Florentin Smarandache

Indeterminacy = <neut*A*>, as in neutrosophy (branch of philosophy) that is based on the neutrosophic triplet (<*A*>, <neut*A*>, <anti*A*>).

Hence, Indeterminacy is everything that is in between opposites.

One should not take Indeterminacy *ad litteram*, i.e. as a definition from an English Dictionary, but everything that fulfil the hole between opposites.

As such, *Uncertainty* is part of *Indeterminacy*; vagueness, confused, conflicting, unclear information, etc. are all parts of *Indeterminacy*.

*

Let us cite some simple examples where indeterminacy is omnipresent.

Nothing is perfect, not even the perfect!

See this paradox:

http://mathworld.wolfram.com/SmarandacheParadox.html

Therefore, everything has some indeterminacy…

Combination of either Crisp, Fuzzy, Intuitionistic, or Neutrosophic Degrees

Florentin Smarandache

to Nivetha Martin

The idea is good, never done before: the combination of either crisp, fuzzy, intuitionistic, or neutrosophic degrees into the plithogenic appurtenance function.

Actually, we can have more types of degrees of appurtenance, in addition to the previous ones: Pythagorean Fuzzy, Picture Fuzzy, Spherical Fuzzy, n-Hyperspherical Fuzzy, and the most general Refined Neutrosophic (T_1, T_2, ...; I_1, I_2, ...; F_1, F_2, ...).

A problem may occur when using the aggregate operators $\wedge_P$ (plithogenic intersection), $\vee_P$ (plithogenic union), etc.

One solution would be to firstly transform all types of degrees into the largest degree, and then combine them.

For example, all crisp, fuzzy, intuitionistic to be converted to neutrosophic, and then use the plithogenic neutrosophic operators.

Neutrosophic set, extended to Plithogenic Set

Florentin Smarandache

While the crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets and other fuzzy extensions are sets whose elements x are characterized by a single attribute, called "appurtenance", whose attribute values are: "membership" (for crisp sets and fuzzy sets), or "membership" and "nonmembership" (for intuitionistic fuzzy set), or "membership" and "nonmembership" and "indeterminacy" (for neutrosophic set), a *plithogenic set* is a set whose elements x are characterized by many attributes, and each attribute may have many values.

*

I extended the *Neutrosophic set* to *plithogenic set* in 2017. A simple *example*:

Let's consider a set $M = \{x_1, x_2, x_3\}$, such that each element is characterized by two attributes:

— C = color,

— and S = size.

Suppose the attribute values of C = {white (w), blue (b), green (g)} and of size are S = {small (s), medium (m)}.

Thus, each x element of M is characterized by the all five attribute values: white, blue, green, small, tall, i.e.:

$$M = \{x_1(w, b, g; s, m), x_2(w, b, g; s, m), x_3(w, b, g; s, m)\}.$$

Therefore, each element x belongs to the set M with a degree of white d(w), a degree of blue d(b), a degree of green d(g), a degree of small d(s), and a degree of medium d(m).

Thus,

$$M = \{ x_1 (d_1(w), d_1(b), d_1(g); d_1(s), d_1(m)),$$
$$x_2(d_2(w), d_2(b), d_2(g); d_2(s), d_2(m)),$$
$$x_3(d_3(w), d_3(b), d_3(g); d_3(s), d_3(m)) \}$$

where $d_1(.)$, $d_2(.)$, and $d_3(.)$ are the degrees of appurtenance of x_1, x_2, and x_3 respectively to the set M with respect to each of the five values.

But the degree of appurtenance may be: classical degree { whose values are 0 or 1 }, fuzzy degree { whose values are in [0,1] }, intuitionistic fuzzy degree { whose values are in $[0,1]^2$ }, neutrosophic degree { whose values are in $[0,1]^3$ }, or any fuzzy extension degree.

Therefore, we may get:

A Plithogenic Classical Set:

$M = \{ x_1(0, 1, 0; 0, 1),$
$x_2(1, 0, 0; 0, 0),$
$x_3(1, 0, 0; 1, 0) \},$

which means that: x_1 is not white, x_1 is blue, x_1 is not green, x_1 is not small, x_1 is medium; similarly for x_2 and x_3.

A Plithogenic Fuzzy Set:

$M = \{ x_1(0.2, 0.7, 0.5; 0.8, 0.3),$
$x_2(0.5, 0.1, 0.0; 0.9, 0.2),$
$x_3(0.5, 1, 0.6; 0.4, 0.3) \},$

which means that: x_1 has the fuzzy degree of white equals to 0.2, x_1 has the fuzzy degree of blue equals to 0.7, x_1 has the fuzzy degree of green equals to 0.1, x_1 has the fuzzy degree of small size equals to 0.8, and x_1 has the fuzzy degree of medium size equals to 0.3; similarly for x_2 and x_3.

A Plithogenic Intuitionistic Fuzzy Set:

$M = \{ x_1 ((0.4,0.1), (0.2,0.7), (0.0,0.3); (0.8,0.5), (0.2,0.3)),$
$x_2((0.7,0.2), (0.2,0.6), (1.0,0.0); (0.6,0.4), (0.1,0.5)),$
$x_3((0.4,0.4), (0.5,0.6, (0.5,0.1); (0.5,0.6), (0.3,0.3));$

which means that: x_1 has the truth-degree of white equals to 0.4 and the false-degree of white equals to 0.1; x_1 has the truth-degree of blue equals to 0.2 and the false-degree of blue equals to 0.7; x_1 has the truth-degree of green equals to 0.0 and the false-degree of green equals to 0.3; x_1 has the truth-degree of small size equals to 0.8 and the false-degree of small size equals to 0.5; x_1 has the truth-degree of medium size equals to 0.2 and the false-degree of white equals to 0.3; similarly for x_2 and x_3.

A Plithogenic Neutrosophic Set:

M = { x_1((0.2,0.4,0.3), (0.5,0.2,0.7), (0.6,0.4,0.3);
(0.9,0.6,0.5), (0.1,0.2,0.3)),
x_2 ((0.1,0.7,0.2), (0.3,0.2,0.7), (0.0,0.2,1.0);
(0.6,0.6,0.1), (0.0,0.1,0.6)),
x_3((0.7,0.4,0.4), (0.5,0.6, (0.3,0.5,0.1); (0.0,0.5,0.6),
(0.8,0.3,0.2));

which means that: x_1 has the truth-degree of white equals to 0.2, the indeterminacy-degree of white equals to 0.4, and the false-degree of white equals to 0.3; x_1 has the truth-degree of blue equals to 0.5, the indeterminacy-degree of blue equals to 0.2, and the false-degree of blue equals to 0.7; x_1 has the truth-degree of green equals to 0.6, the indeterminacy-degree of green equals to 0.4, and the false-degree of green equals to 0.3; x_1 has the truth-degree of small size equals to 0.9, the indeterminacy-degree of small size equals to 0.6, and the false-degree of small size equals to 0.5; x_1 has the truth-degree of medium size equals to 0.1, the indeterminacy-degree of minimum size equals to 0.2, and the false-degree of minimum size equals to 0.3; similarly for x_2 and x_3.

Of course, we have considered the Single-Valued Plithogenic Set, i.e. when all degrees are single-valued (crip) numbers from [0,1].

But similarly we may define: Interval-Valued Plithogenic Set (when the degrees are intervals included into [0,1]), or Hesitant Plithogenic Set (when the degrees are discrete finite subsets included into [0,1]), or in the most general case Subset Plithogenic Set (when the degrees are any subsets included into [0,1]).

*

Using generic notations one has:

Plithogenic Fuzzy Set

$x(v_1(t_1), v_2(t_2), \dots, v_n(t_n))$

Plithogenic Intuitionistic Fuzzy Set

$x(v_1(t_1, f_1), v_2(t_2, f_2), \dots, v_n(t_n, f_n))$,

with $0 \le t_j + f_j \le 1$, for all $j \in \{1, 2, \dots, n\}$.

Plithogenic Neutrosophic Set

$x(v_1(t_1, i_1, f_1), v_2(t_2, i_2, f_2), \dots, v_n(t_n, i_n, f_n))$,

with $0 \le t_j + i_j + f_j \le 3$, for all $j \in \{1, 2, \dots, n\}$.

where $t_j, i_j, f_j \in [0, 1]$ are degrees of membership, indeterminacy, and nonmembership respectively.

Plithogenic Set is much used in Multi-Criteria Decision Making.

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Concrete Example of Plithogenic Fuzzy Set

Florentin Smarandache

Let $U = \{a, b, c, d, e\}$ be a universe of discourse, and let $P = \{b, c, e\}$ be a plithogenic set included in U.

Let the attributes be:

— color *(c)*,

— weight *(w)*,

— size *(s)*,

whose attribute values are respectively:

— c = {white, yellow, blue};

with the degree of contradiction

- c_c (white, yellow) = 0.20,
- c_c (white, blue) = 0.40,
- c_c (yellow, blue) = 0.20;

— w = {thin, fat};

with the degree of contradiction

- c_w (thin, fat) = 1.

— s = {small, medium, big, verybig},

with the degree of contradiction

- c_s (small, medium) = 0.25,
- c_s (medium, big) = 0.25,
- c_s (big, verybig) = 0.25.

Then:

P = { b (white, yellow, blue; thin, fat; small, medium, big, verybig),

c (white, yellow, blue; thin, fat; small, medium, big, verybig),

e (white, yellow, blue; thin, fat; small, medium, big, verybig) },

and because we use the plithogenic "fuzzy" set, each element b, c, e will have a "fuzzy" degree of membership to the set P with respect to each of its attribute values.

*

As an example, we may have:

P = { b (white(0.2), yellow(0.1), blue(0.4); thin(0.7), fat(0.0); small(0.9), medium(0.1), big(0.0), verybig(0.1)),

c (white(0.3), yellow(0.5), blue(0.0); thin(0.6), fat(0.1); small(0.3), medium(0.3), big(0.6), verybig(0.2)),

e (white(0.2), yellow(0.1), blue(0.4); thin(0.7), fat(0.0); small(0.9), medium(0.1), big(0.0), verybig(0.4)) }.

A Plithogenic Set P is simply a set whose each element is characterized by many attribute values.

Florentin Smarandache

For example:

P = {x_1(white, yellow, blue, green, red),
x_2(white, yellow, blue, green, red),
x_3(white, yellow, blue, green, red)}.

Each element has a degree of white, degree of yellow, degree of blue, degree of green, and degree of red.

For example:

x_1(0.2, 0.4, 0.5, 0.1, 0.3),

which means that x_1's fuzzy degree of white is 0.2, x_1's fuzzy degree of yellow is 0.4, x_1's fuzzy degree of blue is 0.5, x_1's fuzzy degree of green is 0.1, and x_1's fuzzy degree of red is 0.3.

In this example we assigned a FUZZY degree to each attribute.

So this is a *Plithogenic Fuzzy Set.*

*

But we may assign, for example, a neutrosophic degree to each attribute:

For example:

x_1((0.2, 0.1, 0.3), (0.4, 0.0., 0.2), (0.5, 0.3, 0.3), (0.1, 0.9. 0.0), (0.3, 0.4, 0.4)),

which means that x_1's neutrosophic degree of white is (0.2, 0.1, 0.3) , x_1's neutrosophic degree of yellow is (0.4, 0.0., 0.2), x_1's neutrosophic degree of blue is (0.5, 0.3, 0.3), x_1's neutrosophic degree of green is (0.1, 0.9. 0.0), x_1's neutrosophic degree of red is (0.3, 0.4, 0.4).

This is a *Plithogenic Neutrosophic Set.*

*

Then, the operators (union, intersection) between plithogenic sets also use the t-norm and t-conorm from fuzzy set, but they depend of the degree of contradictions between the attribute values.

Thus a such operator (union or intersection) becomes a linear combination of t-norm and t-conorm depending on the degrees of contradictions *c*(.) between attribute values.

Plithogenic Number.

Florentin Smarandache

Let U be a universe of discourse, and a non-empty set M included in U.

Let x be a generic element from M.

Let us consider the attributes A_1, A_2, ..,. A_n, for $n \geq 1$.

The attribute A_1 has the attribute values A_{1_1}, A_{1_2}, ..., $A_{1_{m1}}$, where $m_1 \geq 1$.

The attribute A_2 has the attribute values A_{2_1}, A_{2_2}, ..., $A_{2_{m2}}$, where $m_2 \geq 1$.

…

The attribute A_n has the attribute values A_{n_1}, A_{n_2}, …, $A_{n_{m_n}}$, where $m_n \geq 1$.

The fuzzy plithogenic set:

$$M = \left\{ x \begin{pmatrix} A_{1_1}(t_{1_1}), A_{1_2}(t_{1_2}), \dots, A_{1_{m1}}(t_{1_{m1}}); \\ A_{2_1}(t_{2_1}), A_{2_2}(t_{2_2}), \dots, A_{2_{m2}}(t_{2_{m2}}); \\ \dots \\ A_{n_1}(t_{n_1}), A_{n_2}(t_{n_2}), \dots, A_{n_{m_n}}\left(t_{n_{m_n}}\right) \end{pmatrix}; \text{ with } x \text{ in } U \right\}$$

where

— t_{1_1} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{1_1},

— t_{1_2} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{1_2},

etc.

The intuitionistic fuzzy plithogenic set:

$$M = \left\{ x \begin{pmatrix} A_{1_1}(t_{1_1}, f_{1_1}), A_{1_2}(t_{1_2}, f_{1_2}), \dots, A_{1_{m1}}(t_{1_{m1}}, f_{1_{m1}}); \\ A_{2_1}(t_{2_1}, f_{2_1}), A_{2_2}(t_{2_2}, f_{2_2}), \dots, A_{2_{m2}}(t_{2_{m2}}, f_{2_{m2}}); \\ \dots \\ A_{n_1}(t_{n_1}, f_{n_1}), A_{n_2}(t_{n_2}, f_{n_2}), \dots, A_{n_{m_n}}\left(t_{n_{m_n}}, f_{n_{m_n}}\right) \end{pmatrix}; \text{ with } x \text{ in } U \right\}$$

where

— t_{1_1} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{1_1},

— and f_{1_1} is the degree of non-appurtenance of element x to the set M with respect to the attribute value A_{1_1};

— t_{1_2} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{1_2},

— and f_{1_2} is the degree of non-appurtenance of element x to the set M with respect to the attribute value A_{1_2};

etc.

The neutrosophic plithogenic set:

$$M = \left\{ x \begin{pmatrix} A_{1_1}(t_{1_1}, i_{1_1}, f_{1_1}), A_{1_2}(t_{1_2}, i_{1_2}, f_{1_2}), \dots, A_{1_{m1}}(t_{1_{m1}}, i_{1_{m1}}, f_{1_{m1}}); \\ A_{2_1}(t_{2_1}, i_{2_1}, f_{2_1}), A_{2_2}(t_{2_2}, i_{2_2}, f_{2_2}), \dots, A_{2_{m2}}(t_{2_{m2}}, i_{2_{m2}}, f_{2_{m2}}); \\ \dots \\ A_{n_1}(t_{n_1}, i_{n_1}, f_{n_1}), A_{n_2}(t_{n_2}, i_{n_2}, f_{n_2}), \dots, A_{n_{m_n}}\left(t_{n_{m_n}}, i_{n_{m_n}}, f_{n_{m_n}}\right) \end{pmatrix}; \text{ with } x \text{ in } U \right\}$$

where

— t_{1_1} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{1_1},

— i_{1_1} is the degree of indeterminacy of element x to the set M with respect to the attribute value A_{1_1},

— and f_{1_1} is the degree of non-appurtenance of element x to the set M with respect to the attribute value A_{1_1};

— t_{1_2} is the degree of appurtenance of element x to the set M with respect to the attribute value A_{1_2},

— i_{1_2} is the degree of indeterminacy of element x to the set M with respect to the attribute value A_{1_2},

— and f_{1_2} is the degree of non-appurtenance of element x to the set M with respect to the attribute value A_{1_2};

etc.

In general:

The D-Plithogenic Set, where “D” may be any of Fuzzy / Intuitionistic Fuzzy / Neutrosophic / Picture Fuzzy / Pythagorean Fuzzy / any fuzzy extension, etc.:

$$M = \left\{ x \begin{pmatrix} A_{1_1}(d_{1_1}), A_{1_2}(d_{1_2}), \dots, A_{1_{m1}}(d_{1_{m1}}); \\ A_{2_1}(d_{2_1}), A_{2_2}(d_{2_2}), \dots, A_{2_{m2}}(d_{2_{m2}}); \\ \dots \\ A_{n_1}(d_{n_1}), A_{n_2}(d_{n_2}), \dots, A_{n_{m_n}}\left(d_{n_{m_n}}\right) \end{pmatrix}; \text{ with } x \text{ in } U \right\}$$

where

— d_{1_1} is the *D*-degree of appurtenance of element x to the set *M* with respect to the attribute value A_{1_1};

— d_{1_2} is the *D*-degree of appurtenance of element x to the set *M* with respect to the attribute value A_{1_2}; etc.

Again, where *"D"* may be any of Fuzzy / Intuitionistic Fuzzy / Neutrosophic / Picture Fuzzy / Pythagorean Fuzzy / any fuzzy extension, etc.

Plithogenic Graph

Florentin Smarandache

to W. B. Vasantha Kandasamy

Plithogenic Crisp Graph, when we have the row matrix as 0's and 1's, such as: 0, 1, 1, 0, 0, 1;

Plithogenic Fuzzy Graph, when we have the row matrix of numbers between 0 and 1 (as in fuzzy set), such as 0.2, 0.6, 0.9;

Plithogenic Intuitionistic Fuzzy Graph, when we have the row matrix of duplets of numbers between 0 and 1, such as: (0.3, 0.5), (01, 0.7) etc.

Plithogenic Neutrosophic Graph, when we have the row matrix of triplets of numbers between 0 and 1, such as: (0.3, 0.4, 0.5), (01, 0.2, 0.7) etc.

{The above are all Single-Values; but we may consider as well Interval-Values, Hesitant-Values, or any subset etc.}

Plithogenic Real Number Graph, when we have the row matrix by real numbers, such as: 3, 37, -6.

Plithogenic Complex Number Graph, when we have the row matrix formed by complex numbers, such as: 3+2i, 37-7i, -6-i, where i = squareroot(-1).

Plithogenic Neutrosophic Number Graph, when we have the row matrix formed by neutrosophic numbers, such as: 3+2*I*, 37-7*I*, -6-*I*, where *I* = literal Indeterminacy.

When Indeterminacy is zero

Florentin Smarandache

Let $T, I, F \in [0, 1]$ be neutrosophic components.

If Indeterminacy $I = 0$, the neutrosophic components $(T, 0, F)$ are still more flexible and more general than fuzzy components and intuitionistic fuzzy components.

Because we get:

1-2) for *fuzzy set* and for *intuitionistic fuzzy set* (they coincide):

$$T + F = 1.$$

3) for *neutrosophic set*:

$$0 \leq T + F \leq 2.$$

Therefore, the neutrosophic set is more flexible and more general than the other sets, no matter the value of indeterminacy.

T under some conditions, F under other conditions

Florentin Smarandache

> *Is it logical to expect the value of T under some conditions, and expect the value of F under other conditions?*

It depends on the application.

Let's see a simple example:

Two teams A and B compete against each other.

Andrew is a supporter of team A, while Brenda is a supporter of team B. The two sources of information, Andrew and Brenda, are independent (they do not communicate with each other). Andrew bits that his team A will win ($T = 70\%$). But Brenda bits that her team B will win ($F = 60\%$).

This is a very frequently met example in our everyday life.

In consequence, it is logical to have T evaluated by a source (Andrew), and F evaluated by another source (Brenda) in various applications.

Included Multiple-Middles Law is not in general reducible to Included One-Middle Law

Florentin Smarandache

Multiple-middles may reduce to a single one in some cases, but in others not.

If you consider a voting process, T = voting for, F = voting against, and I = neither one, then here the middle "I" can be split into three parts:

I_1 = not going to vote;

I_2 = casting a black vote (cutting all candidates on the list);

I_3 = casting a white vote (not choosing any candidate on the list).

But if people are not interested in such refinement of Indeterminacy I, and only in this case, then you can collapse all three-middles into one (as in Included Single-Middle).

In other cases it is not possible. For example, if the opposites are T = white, and F = black, then inside of them you have multiple-middles, such as for example: I_1 = yellow, I_2 = green, I_3 = blue, I_4 = red, I_5 = violet, etc.

You cannot collapse (mix) all of them, because you get something totally different.

Neutrosophic Measures

Florentin Smarandache

to Akbar Rezaei

Since there are many types of indeterminacies, it is possible to define many types of neutrosophic measures.

And, in general, because of dealing with many types of indeterminacies, we can extend any classical scientific or cultural concept from various indeterminate/neutrosophic viewpoints.

SuperHyperGraphs

Florentin Smarandache

HyperGraph means an edge connects many vertices (not only two). SuperGraph has SuperVertex (i.e. many vertices put together, or a group of vertices all together), which is as in our everyday life when we have many organizations, teams, clubs etc. (each having many individuals; an individual = a vertex).

Then I combined them and I got the SuperHyperGraph, many vertices put in groups as SuperVertices, and HyperEdges connecting many vertices or even connecting many HyperVertices (because in our everyday life we have relationships between teams, between clubs, between countries etc.).

Mohammad Hamidi

SuperHyperGraphs have applications in computer engineering, such as networking, complex networks, complex hyper networks, machine learning.

Florentin Smarandache

Examples:

a citizen = a Vertex

a city = a SuperVertex (all city's citizen put together)

a region = a Super(SuperVertex), since a region is formed by many cities (SuperVertices),

or we say a 2-SuperVertex;

a country = a 3-SuperVertex...

a continent = a 4-SuperVertex...

a planet = a 5-SuperVertex, and so on.

Taking into consideration the relationships vertices (edges and hyper-edges), then between n-super-vertices, we construct n-HyperEdges.

So, the whole graph is an n-SuperHyperGraph.

Dependent and Independent Components

Florentin Smarandache
to Mary Jansi

Dependent components means when the components influence each other, i.e. if one changes its value the others change too.

Independent components means when the components do not influence each other, i.e. if one component changes its value it is not necessarily for the other components to change.

Real life examples

1) Suppose you watch a football match.

You have: T = chance that your team wins, I = chance of tie game, and F = chance that your team loses.

You are a mathematician and you know that the sum of all space probabilities is 1. So: $T + I + F = 1$.

Herein, all T, I, F are dependent on each other.

For example, if $T = 0.6$, $I = 0.1$, $F = 0.3$, you have $T + I + F = 0.6+0.1+0.3 = 1$.

But let's say, you change your mind and you predict that the chance of winning is $T = 0.7$; then mandatory I and/or F should change their values since the sum has to still be 1.

2) Suppose three people, which do not communicate with each other, and they stay in places far from each other.

The first one John predicts the chance that your team wins is $T = 0.8$.

The second one George predicts the chance that the match is tie $I = 0.3$.

The third one predicts the chance that your team loses is $F = 0.5$.

Clearly *T, I, F* are independent, since the three people do not communicate with each other and do not stay together, so they think independently of each other.

Now, if you sum $T+I+F$ you cannot get 1, only in extremely rare coincidences.

Suppose now that John changes his prediction and says that $T = 0.7$, but he does not say anything to the other two.

The other two may change or may not.

That's why in a neutrosophic set/logic/probability we have $T+I+F \leq 3$, and each one *T, I, F* can be any number in [0, 1]. *T I, F* are independent.

In intuitionistic fuzzy set, $T+I+F = 1$, so *T, I* (called "hesitancy" in this case), *F* are dependent.

3) Suppose *T* and *F* are dependent. Then *T* and *F* depend on each other, or $T+F = 1$ (as in fuzzy logic).

Another real life example

Suppose you strongly believe that in the above football game one of the teams will win, so you do not believe in a tie game.

Then if you believe that $T = 0.60$, automatically $F = 1- 0.60 = 0.40$.

Example: Somebody else, not in touch with you and not knowing your belief (therefore a source that is independent from you) strongly believes that there will be a tie game, suppose he says that $I = 0.90$.

"I" does not depend on T and F. If you change $T = 0.70$, F should be changed too because it is dependent of T, so $F = 1-0.70 = 0.30$.

But *"I"* is independent of both T and F, so *"I"* may remain the same.

Surely, the second source may change *"I"* too, but it does not depend on none of T and F.

HyperGroupoid Homomorphism

Florentin Smarandache

HyperGroupoid Homomorphism is similar to the Groupoid Homomorphism.

a) either inclusion (weak homomorphism) - in this paper is called just homomorphism;

b) or equality (strong homomorphism) - in this paper it is called good homomorphism.

We may define for each of them the Neutro and Anti using the same definitions as for Operations and Axioms, going by degree of inclusion, degree of indeterminacy (or not knowing), and degree of non-inclusion.

So we get:

Weak-Homomorphism, NeutroWeak-Homomorphism, AntiWeak-Homomorphism.

Similarly we get a neutrosophic triplet of the strong homomorphisms:

(Strong-Homomorphism, NeutroStrong-Homomorphism, AntiStrong-Homomorphism.

They make perfect sense.

No matter what definition of hyper-homomorphism we use, we just apply the same Neutro-sophication and Anti-sophistication of the definition.

Left Weak Distributivity

Florentin Smarandache

The *Left (Strong) Associativity* and *Left Weak Associativity* apply to HyperAlgebras, where instead of equality *(=)*, one uses non-empty intersection *(∩)*.

Let *U* be a universe of discourse, a non-empty set $X \subseteq U$, and *P(X)* the power-set of *X* (all subsets of *X*, except the empty set ϕ).

A HyperAlgebra *(X, *, #)* with two well-defined binary operations:

$$*: X^2 \to P(X) \text{ and } \#: X^2 \to P(X)$$

$$\forall x, y, z \in X, \; x*(y\#z) = (x*y)\#(x*z)$$

(left strong distributivity); (1)

while

$$\forall x, y, z \in X, \; x*(y\#z) \cap (x*y)\#(x*z) \neq \phi,$$

(left weak distributivity). (2)

But these are totally different from NeutroAxiom, because both the Left Strong Distributivity and the Left Weak Distributivity are satisfied (are true) for all elements *(100%)* of the set *X*. So, they are classical Axioms.

We can define a Left NeutroWeakDistributivity and a Left Neutro(Strong)Distributivity {and correspondingly Left AntiWeakDistributivity and a Left Anti(Strong)Distributivity} for a HyperAlgebra...

The Left WeakDistributivity (1) is a Left *Neutro*StrongDistributivity if there is at least one equal sign *(=)* for some x, y, z {i.e. $\exists x, y, z \in X$, $x*(y\#z) = (x*y)\#(x*z)$}.

If there is no equal *(=)* sign for no triplet *x, y, z,* {i.e. $\forall x, y, z \in X$, $x*(y\#z) \neq (x*y)\#(x*z)$} then the Left WeakDistributivity (1) is a Left *Anti*StrongDistributivity.

Neutrosophic 3D-Image Processing and Identification

Florentin Smarandache
to Yanhui Guo

I wanted to ask you, or to propose you the following idea: now there is a 3D-scanning, so 3D-image, so 3D-image processing and identification. Could you approach the neutrosophic 3D-image processing / identification? Did you also do colored image processing / identification?

For example you have 5 colors into an image, C_1, C_2, C_3, C_4, C_5.

Each pixel (or element) is characterized by degrees of C_1, C_2, C_3, C_4, C_5.

We may write:

$$x(d_1, d_2, d_3, d_4, d_5),$$

where d_1 = degree of color C_1 that characterized x, ..., d_5 = degree of color C_5 that characterizes x.

The degree may be fuzzy degree, or intuitionistic fuzzy degree, or neutrosophic degree.

Later on, you may need to combine (union, intersection, etc.),

$$x(d_1, d_2, d_3, d_4, d_5) \text{ with } y(e_1, e_2, e_3, e_4, e_5),$$

where e_1 = degree of color C_1 that characterized y, ..., e_5 = degree of color C_5 that characterizes y.

In such case you can employ the plithogenic set, that allows an element x be characterized by many attribute values.

How did you deal with colored image processing?

Neutrosophic Distribution as Triple Probability Distribution

Florentin Smarandache
to Terman Frometa-Castillo

Neutrosophic probability of an event E is:

- chance that event E occurs,
- chance that event E does not occur,
- indeterminate-chance of event E to occur or not.

In the future maybe, besides computing the probability that the event occurs, try to see the other two probabilities (if the event does not occur), or indeterminate (neutral, unknown) chance of the event occurring or not. You then bring more information to the study of tumor neutrosophic probability.

There also may be the cases that according to some human variable, there is bigger tumor probability, while with respect to other human variable a smaller tumor probability.

Or, the probability of the tumor to advance, regress, or stagnate (i.e. neutrosophic probability).

See also the neutrosophic statistics, where indeterminacy is taken into consideration: when the probability distributions are not well known, or we have a triple probability distribution: the first curve presenting the chance that the tumor advances, second curve the chance that the tumor regresses, while the third curve the chance that the tumor stagnate...

I see your paper is more focused on medical treatments.

If one takes a 3D-scan of the tumor area, this can be interpreted as a neutrosophic set, each pixel is an element of the set.

Then we compare this neutrosophic set S towards a database of neutrosophic sets representing different tumor areas, calculating the similarity measures between *S* and each of the other sets, to see which one is the closes one. (We did this procedure in neutrosophic image recognition.)

Neutrosophic Boolean Lattice

Florentin Smarandache

What about considering the propositional calculus, but instead of Boolean Lattice, one would propose a Neutrosophic Boolean Lattice, i.e. instead of only 0

(False) and 1 (True) from Boolean Lattice, to consider 0 (False), 1 (True), and I { Indeterminacy, meaning we are not sure if it is 0 or 1, or we may have 0 and 1 simultaneously (superposition) }?

One can make a table of neutrosophic truth values, using 0, 1, and I.

We can consider in a pessimistic way: $F > I > T$ for neutrosophic intersection, the bigger absorbs the smaller, then for neutrosophic union we have $F < I < T$.

Neutrosophic Hypergraph

Florentin Smarandache
to Said Broumi

Try to extend to "neutrosophic hypergraph" further when you can.

Hypergraph means when the vertexes are groups of elements (not only one), for example:

vertex $V_1 = \{e_1, e_2, \ldots, e_5\}$, vertex $V_2 = \{e_6, e_7\}$ etc.

Totally Ordered / Partially Ordered / Totally Unordered Set

Florentin Smarandache

Let U be a universe of discourse, and a nonempty set S included in U. The given set S endowed with a binary order relation < may be:

1) totally ordered (meaning that for any two distinct elements $x, y \in S$, either $x < y$ or $y < x$);

2) partially ordered (meaning that:

i) there exist at least two distinct elements $a, b \in S$ such that either $a < b$ or $b < a$;

ii) and there exist at least two distinct elements $c, d \in S$ such that neither $c < d$ nor $d < c$);

3) totally unordered (meaning that for any two distinct elements $x, y \in S$, neither $x < y$ nor $y < x$).

The spherical fuzzy set membership, nonmembership, indeterminacy are NOT independent from each other

Florentin Smarandache

I read the paper "Spherical Fuzzy Linear Assignment Method for Multiple Criteria Group Decision-Making Problems" [1], where the authors say that:

> "Spherical fuzzy sets theory is useful and advantageous for handling uncertainty and imprecision in multiple attribute decision-making problems by considering membership, nonmembership, and indeterminacy degrees *independently* for each element".

But unfortunately their assertion that in spherical fuzzy set the membership, nonmembership, and indeterminacy are independent is *UNTRUE*.

See [2] where I proved that they are not independent.

A counterexample, if the membership = 0.8, then nonmembership cannot be 0.7, because $0.8^2 + 0.7^2 > 1$. So, nonmembership depends on membership, in this counterexample: nonmembership ≤ 0.6.

Another counterexample, if membership = 0.5, nonmembership = 0.6, then indeterminacy cannot be 0.9, since $0.5^2 + 0.6^2 + 0.9^2 > 1$.

Thus indeterminacy is dependent on membership and on nonmembership, in this case, indeterminacy ≤ $(1-0.5^2-0.6^2)$.

Neither t-spherical fuzzy set has the components independent, since as a counterexample, if membership = 1, then mandatory nonmembership = indeterminacy = 0 (because $1^t + 0^t + 0^t = 1$).

Neither picture fuzzy set, nor Pythagorean fuzzy set have independent components (see the second paper).

The only set whose components are 100% independent from each other is the neutrosophic set, where it does not matter what values have *T* (truth-membership), *F* (falsehood-membership), and *I* (indeterminacy) take in [0, 1] their sum is allowed to be up to 3.

References

[1] Yaser Donyatalab, Seyed Amin Seyfi-Shishavan, Elmira Farrokhizadeh, Fatma Kutlu Gündoğdu, Cengiz Kahraman, Spherical Fuzzy Linear Assignment Method for Multiple Criteria Group Decision-Making Problems, *Informatica*, 2020, Vilnius University.

[2] F. Smarandache, Neutrosophic Set is a Generalization of Intuitionistic Fuzzy Set, Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set (Atanassov's Intuitionistic Fuzzy Set of second type), q-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, and n-HyperSpherical Fuzzy Set, while Neutrosophication is a Generalization of Regret Theory, Grey System Theory, and Three-Ways Decision (revisited), arXiv, Cornell University, New York City, NY, USA, (Submitted on 17 Nov 2019 (v1), last revised 29 Nov 2019 (this version, v2)), https://arxiv.org/ftp/arxiv/papers/1911/1911.07333.pdf https://arxiv.org/abs/1911.07333 (Abstract)

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What is the physical meaning of Score Function?

Florentin Smarandache
to Harish Garg

I agree with you for the following reason:

T = positive quality;

F = negative quality,

whence $1 - F$ = positive quality;

I = negative quality,

whence $1 - I$ = positive quality;

Hence, the score function:

$$s = \frac{\{T + (1 - I) + (1 - F)\}}{3}$$

= the average of positive quality.

Degrees of membership, indeterminacy or nonmembership; with a real life example

Florentin Smarandache
to Tareq Al-Shami

Indeterminacy *(I)* is not a complement to truth *(T)* and falsehood *(F)*, since all three components are independent. *"I"* can be any number in [0, 1], no matter what *T* and *F* are equal too.

> For example, it is not logic that someone expects that *A* will win with a chance of 40% and will fail with a chance of 70%.

Yes, it is possible, if you evaluate the chances from different criteria.

Let A be a soccer team, playing against the team *B*.

For example, *A* will have a chance of winning 40% considering the value of its players, but a chance of losing 70% considering the fact that *A* will play away from home on the field of *B* team.

So,

— first criterion is: value of *A* team's players;

— second criterion: team *A* playing at home (which is an advantage), or playing away from home at the adversary field (which is a disadvantage).

> Is it logical to expect the value of T under some conditions and expect the value of F under other conditions?

It depends on the application.

If you evaluate a student John in general.

John can be 40% good in mathematics, but 70% bad in linguistics.

Therefore, you cannot have only one condition (one discipline) to judge John, since John studies many disciplines.

Any event/application has to be analyzed with respect to all conditions [or as many as possible], not with respect to only one. Some conditions may be positive for the event/application, others negative, and others neutral (or indeterminate) for the event / application - as in neutrosophy.

For your information, there are cases when the degrees of membership, indeterminacy or nonmembership may be each of them > 1 or < 0, and these are in our real life applications.

See this book and papers below:

F. Smarandache, *Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset. Similarly for Neutrosophic Over-/Under-/Off-Logic, Probability, and Statistics*, 168 p., Pons Editions, Brussels, Belgium, 2016,
http://fs.unm.edu/NeutrosophicOversetUndersetOffset.pdf ,
on Cornell University's website:
https://arxiv.org/ftp/arxiv/papers/1607/1607.00234.pdf
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And articles:

http://fs.unm.edu/SVNeutrosophicOverset-JMI.pdf
http://fs.unm.edu/IV-Neutrosophic-Overset-Underset-Offset.pdf
http://fs.unm.edu/NeutrosophicOversetUndersetOffset.pdf

An elementary real example, that often occur in our everyday life.

Suppose a worker has the duty to work 40 hours/week.

If he works 40 hours this week, his membership degree is 1.

But many workers work overtime. Assume this worker worked 42 hours this week.

So, it is normal that his degree of membership be greater than somebody's who worked only 40 hours.

Thus, his membership degree is 42/40 = 1.05 > 1.

Therefore, neutrosophic component > 1, or sum of neutrosophic components > 1, are REAL and needed in applications.

Addition of numbers with Neutro-Components

Florentin Smarandache

3(0.7,0.4,0.2) + 4(0.4,0.7,0.1)

It depends on how one interprets 3(0.7, 0.4, 0.2).

If one considers that 0.7 is the degree of confidence (truth) that the number is 3, 0.4 indeterminate-degree of confidence of the number to be 3, and 0.2 degree of nonconfidence (falsehood) that the number is 3, then one intersects the neutrosophic degrees:

$$3(0.7,0.4,0.2) + 4(0.4,0.7,0.1) =$$
$$= (3+4)(\ (0.7,0.4,0.2) \wedge (0.4,0.7,0.1)\ ) =$$
$$= 7(0.4,\ 0.7,\ 0.2),$$

we took min/max/max of components.

Type-2 Neutrosophics

Florentin Smarandache

I expected that *Type-2 Fuzzy* is: fuzzy of the fuzzy.

Example of Fuzzy Set of Type-1:

$$A_{FuzzyType\text{-}1} = \{\ (x(\ 0.5\ ),\ y(\ 0.6\ )\ \}.$$

Example of Fuzzy Set of Type-2:

$$A_{FuzzyType\text{-}2} = \{\ (x(\ 0.5(0.4)\ ),\ y(\ 0.6(0.3)\ )\ \}.$$

I would say: $A_{FuzzyType\text{-}2}$ = the fuzzy membership of the element x is 0.5 (but the possibility of this fuzzy membership to be correct is 0.4), and the fuzzy membership of the element y is 0.6 (but the possibility of this fuzzy membership to be correct is 0.3).

I observed that interval-valued fuzzy set was also called type-2 fuzzy set, which I did not find to be appropriate.

The *Type-2 Neutrosophic* is: fuzzy / intuitionistic fuzzy / neutrosophic of the neutrosophic.

Example of Neutrosophic Set of Type-1:

$A_{NeutrosophicType\text{-}1} = \{ (x(0.5, 0.4, 0.2), y(0.6, 0.1, 0.3) \}.$

Example of Fuzzy Neutrosophic Set of Type-2:

$$A_{FuzzyNeutrosophicType\text{-}1} = \{ (x(0.5(0.6), 0.4(0.7), 0.2(0.9)), y(0.6(0.6), 0.1(0.7), 0.3(0.5)) \}.$$

Which means: for the element x its membership is 0.5 (but the possibility of this membership to be correct is 0.6), its indeterminate-membership is 0.4 (but the possibility of this indeterminate-membership to be correct is 0.7), and its nonmembership is 0.2 (but the possibility of this nonmembership to be correct is 0.9);

— and similarly for the element y.

Example of Intuitionistic-Fuzzy Neutrosophic Set of Type-2:

$$A_{Intuitionistic\text{-}FuzzyNeutrosophicType\text{-}1} = \{ (x(0.5(0.6, 0.1), 0.4(0.7, 0.3), 0.2(0.9, 0.8)), y (0.6(0.6, 0.9), 0.1(0.7, 0.7), 0.3(0.5, 0.2)) \}.$$

Which means: for the element x its membership is 0.5 (but the possibility of this membership to be correct is 0.6, while the possibility of this membership to be incorrect is 0.1),

- its indeterminate-membership is 0.4 (but the possibility of this indeterminate-membership to be correct is 0.7, while the possibility of this indeterminate-membership to be incorrect is 0.3),

- and its nonmembership is 0.2 (but the possibility of this nonmembership to be correct is 0.9, while the possibility of this nonmembership to be incorrect is 0.8);

— and similarly for the element y.

Example of Neutrosophic Neutrosophic Set of Type-2:

$$A_{NeutrosophicNeutrosophicType\text{-}1} =$$
$$\{ (x\ (\ 0.5(0.6,\ 0.0\ 0.1),\ 0.4(0.7,\ 0.5,\ 0.3),\ 0.2(0.9,\ 0.7,\ 0.8)\ ),$$
$$y\ (\ 0.6(0.6,\ 0.8,\ 0.9),\ 0.1(0.7,\ 0.6,\ 0.7),\ 0.3(0.5,\ 0.1,\ 0.2)\ )\ \}.$$

Which means: for the element x its membership is 0.5 (but the possibility of this membership to be correct is 0.6, the possibility of this membership to be indeterminate is 0.0, while the possibility of this membership to be incorrect is 0.1),

- its indeterminate-membership is 0.4 (but the possibility of this indeterminate-membership to be correct is 0.7, the possibility of this indeterminate-membership to be indeterminate is 0.5, while the possibility of this indeterminate-membership to be incorrect is 0.3),

- and its nonmembership is 0.2 (but the possibility of this nonmembership to be correct is 0.9, the possibility of this membership to be indeterminate is 0.7, while the possibility of this nonmembership to be incorrect is 0.8);

— and similarly for the element y.

Florentin Smarandache

One follows the neutrosophic operators for *T, I, F* (non-refined), therefore whatever you do on *"I"* for non-refined, you do on the refined I_1, I_2, ...

$$(TA, IA_1, IA_2, FA) \vee ((TB, IB_1, IB_2, FB) =$$
$$= (TA \vee TB, IA_1 \wedge IB_1, IA_2 \wedge IB_2, FA_1 \wedge FB_2)$$

The first $\vee$ is the neutrosophic union; on the right-hand side of the = sign the symbols are fuzzy union and fuzzy intersection.

Bipolar Neutrosophic Set Union

Christy Vincent

The union of two bipolar neutrosophic sets into the universe of discourse S:

$$BN_1(S) \cup BN_2(S) =$$
$$\left\langle \begin{matrix} \max(T^+_{BN}(1), T^+_{BN}(2)), \min(I^+_{BN}(1), I^+_{BN}(2)), \min(F^+_{BN}(1), F^+_{BN}(2)) \\ \max(T^-_{BN}(1), T^-_{BN}(2)), \min(I^-_{BN}(1), I^-_{BN}(2)), \min(F^-_{BN}(1), F^-_{BN}(2)) \end{matrix} \right\rangle$$

From the above bipolar neutrosophic union of two sets, whether it is possible to form same maximum and minimum for the positive membership degrees of the truth, indeterminacy, and falsehood:

$$\text{Max}(T^+_{BN}(1), T^+_{BN}(2)), \text{Min}(I^+_{BN}(1), I^+_{BN}(2)), \text{Min}(F^+_{BN}(1), F^+_{BN}(2))$$

and negative degree of the truth, indeterminacy, and falsehood:

$$\text{Max}(T^-_{BN}(1), T^-_{BN}(2)), \text{Min}(I^-_{BN}(1), I^-_{BN}(2)), \text{Min}(F^-_{BN}(1), F^-_{BN}(2))$$

Whether the above Union is right?

Or:

Whether we must form like this?

Positive membership degrees of the truth, indeterminacy, and falsehood:

$Max(T^{+BN}(1), T^{+BN}(2)), Min(I^{+BN}(1), I^{+BN}(2)), Min(F^{+BN}(1), F^{+BN}(2))$

Negative degree of the truth, indeterminacy, and falsehood:

$Min(T^{-BN}(1), T^{-BN}(2)), Max(I^{-BN}(1), I^{-BN}(2)), Max(F^{-BN}(1), F^{-BN}(2))$

Florentin Smarandache

As you know the (fuzzy, intuitionistic fuzzy, neutrosophic) operators are approximations, and there are classes of such operators for union [not only a single operator], classes of operators for intersection, etc.

The chosen ones depend on the application and on expert.

In your example the best is:

max/min/min for positive memberships

and

max/min/min for negative memberships.

Example:

If $T_1+ = 0.5$ and $T_2+ = 0.6$, we take clearly for union the most optimistic, i.e. $\max(T_1+, T_2+) = 0.6$.

If $T_1- = -0.2$ and $T_2- = -0.3$ we also take the most optimistic, i.e. $\max(T_1-, T_2-) = -0.2$.

The second version or yours also works, but is less accurate.

Example of a Linguistic Neutrosophic Set

Florentin Smarandache

Let $U = \{a, b, c, d\}$ be a universe of discourse.

Let $M = \{a, b\}$ be a subset of U.

And let the set of labels be: $L = \{L_1, L_2, L_3, L_4\}$ = {small, medium, big, very big}.

Then, the following set is a linguistic neutrosophic set:

$$M = \{ a(L_3, L_1, L_2), b(L_2, L_2, L_4) \}.$$

Therefore:

— the element *a*'s appurtenance degree to the set *M* is *L3 (big)*,
— its indeterminate-appurtenance degree to the set *M* is *L1 (small)*,
— and its non-appurtenance degree to the set *M* is *L2 (medium).*

Similarly for the element *b*:

— the element *b*'s appurtenance degree to the set *M* is *L2 (medium).*
— its indeterminate-appurtenance degree to the set *M* is *L2 (medium)*,
— and its non-appurtenance degree to the set *M* is *L4 (very big).*

A real-life example for neutrosophic bipolar vague set

Florentin Smarandache

Suppose a doctor prescribes to a patient the medication *M* in order to cure his disease. But the medication has two effects:

— the positive effect (that cures his disease), let's say $T+ = 0.6$ degree of health improvement, $I+ = 0.2$ indeterminate (unknown) degree of health improvement, and $F+ = 0.3$ degree of no health improvement;

— and a negative effect (or side effect, medication *M* causing another disease), let's say *T*- = -0.7 degree of new disease development, *I*- = 0.1 indeterminate (unknown) degree of development of the new disease, and *F*- = 0.4 degree of no new disease development.

This is a *neutrosophic bipolar set* example.

Now, because *T*+ = 1 – *F*- and *F*+ = 1 – *T*- we also have a vague set.

Example of different parameters applied in everyday life

Florentin Smarandache

In the court of law there are three distinct and opposed parameters:

- *defense lawyer* (that says positive things about inculpate),
- *persecutor* (who say negative things about inculpate),
- and *jury* (who is neutral).

Another example

Some medications have, besides the *positive effect* against a malady, also a *secondary (negative) effect.*

Simple and common definitions of Fuzzy-Type Sets

Florentin Smarandache

See below the simple and common-sense definitions:

1) Fuzzy Set is a set whose each element *x* is characterized by only a truth-membership degree *(T)* with respect to the set. We write *x(T).*

But *T* can be: a single number, or an interval, or a hesitant subset, or in general any subset (all included in [0, 1]). That's why we may have: single-valued fuzzy set (many people call it only: fuzzy set), interval-valued fuzzy set, hesitant fuzzy set, etc.

Examples:

x(0.6), or *x*([0.5, 0.8]), or *x*({0.1, 0.2, 0.4, 0.9}), etc.

2) Intuitionistic Fuzzy Set is a set whose each element *x* is characterized by a truth-membership degree *(T)* and a false-membership degree *(F)* with respect to the set. We write *x(T, F).*

But *T* and *F* can be single numbers, or intervals, or hesitant subsets, or in general any subsets (all included in [0, 1]).

That's why we may have: single-valued intuitionistic fuzzy set (many paople call it only: intuitionistic fuzzy set), interval-valued intuitionistic fuzzy set, hesitant intuitionistic fuzzy set, etc.

Examples:

x(0.6, 0.7), or *x*([0.5, 0.8], [0.0, 0.1]),

or *x*({0.1, 0.2, 0.4, 0.9}, {0.0, 0.4}), etc.

3) Neutrosophic Set is a set whose each element x is characterized by a truth-membership degree *(T),* an indeterminacy (neutrality) membership *(I),* and a false-membership degree *(F)* with respect to the set.

We write *x(T, I, F).* But *T, I* and *F* can be single numbers, or intervals, or hesitant subsets, or in general any subsets (all included in [0, 1]).

That's why we may have: single-valued neutrosophic set, interval-valued neutrosophic set, hesitant neutrosophic set, subset neutrosophic set (the last one not much used).

Examples:

$x(0.6, 0.7, 0.1)$, or $x([0.5, 0.8], [0.0, 0.1], [0.4, 0.5])$, or $x(\{0.1, 0.2, 0.4, 0.9\}, \{0.0, 0.4\}, \{0.6, 0.7\})$, etc.

T, I, F can play an independent, or dependent, or partially independent and partially dependent role in algebraic structures. NOT only independent.

*

In the (singled-valued) neutrosophic set the inequality is:

$$0 \leq T+I+F \leq 3.$$

This relationship (inequality) says that the sum $T+I+F$ can be equal to any number between 0 and 3.

a) If $T+I+F = 3$, then the components *T, I, F* are 100% independent from each other.

b) If $T+I+F = 1$, then the components *T, I, F* are 100% dependent on each other (as in classical fuzzy, and intuitionistic fuzzy sets).

c) If $T+I+F \in (1, 3)$, then *T, I, F* are partially independent and partially dependent all together.

You may have for example: $T+I+F = 2.8$ (these degrees of dependence/independence result from the given problem/application to solve and from the experts).

And here there are many cases: if $T+F = 1$ (hence *T, F* are 100% dependent), and "*I*" is totally independent, we may have $T+I+F = 2$.

And so on.

In NeutroAlgebras, we may have operations and axioms that are only partially well-defined/true, partially indeterminate, partially outer-defined/false.

Degrees of Independence and Dependence of the Neutrosophic Components

Florentin Smarandache

When $T + I + F = 1$ the three components are *100%* dependent altogether, and if $T + I + F = 3$ the three components are *100%* independent altogether.

In the middle between 1 and 3, when $T + I + F = 2$, then the three components are *50%* dependent and *50%* independent.

The *general formulas* are: Let's consider the single-valued neutrosophic components $T, I, F \in [0, 1]$, where $T + I + F \in [1, 3]$.

The Degree of Independence of T, I, F altogether is:

$$d_{ind}(T, I, F) = \frac{T + I + F - 1}{2} \in [0,1]$$

The Degree of Dependence of T, I, F altogether is:

$$d_{dep}(T, I, F) = \frac{3 - T - I - F}{2} \in [0,1]$$

Theorem.

$$d_{ind}(T, I, F) + d_{dep}(T, I, F) = 1.$$

Known Part and Unknown Part of a Neutrosophic Quadruple Number

Florentin Smarandache

Let $N = a + bT + cI + dF$ be a neutrosophic quadruple number.

The "a" is the known part of the neutrosophic quadruple number N.

While "$bT + cI + dF$" is the unknown part of the neutrosophic quadruple number N.

Then the unknown part is split into three subparts:

- degree of confidence (T),
- degree of indeterminacy of confidence-nonconfidence (I),
- and degree of nonconfidence (F).

N is a four-dimensional vector that can also be written as: $N = (a, b, c, d)$.

T, I, F are herein literal parameters.

We may consider them as sets as well, using the absorbance (prevalence) law that one parameter absorbs (includes) another.

There are transcendental, irrational etc. numbers that are not well known, they are only partially known and partially unknown, they may have infinitely many decimals with no repetition.

Not even the most modern supercomputers can compute more than a few thousands decimals, but the infinitely many left decimals still remain unknown.

Therefore, such numbers are very little known (because only a finite number of decimals are known), and infinitely many decimlas unknown (because an infinite number of decimals are unknown).

Operations with Neutrosophic Quadruple Numbers

Let a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2 be real or complex numbers.

— *Addition of the Neutrosophic Quadruple Numbers:*

$(a_1 + b_1T + c_1I + d_1F) + (a_2 + b_2T + c_2I + d_2F) =$
$= (a_1 + a_2) + (b_1 + b_2)T + (c_1 + c_2)I + (d_1 + d_2)F.$

— *Subtraction of the Neutrosophic Quadruple Numbers:*

$(a_1 + b_1T + c_1I + d_1F) - (a_2 + b_2T + c_2I + d_2F) =$
$= (a_1 - a_2) + (b_1 - b_2)T + (c_1 - c_2)I + (d_1 - d_2)F.$

— *Real/Complex Scalar Multiplication of a Neutrosophic Quadruple Number:*

Let α be a real or complex number.

Then:

$\alpha(a_1 + b_1T + c_1I + d_1F) = \alpha a_1 + \alpha b_1T + \alpha c_1I + \alpha d_1F.$

The notions of subset of a set and of power set have the same definitions as in classical set theory.

When are we using different parameters to evaluate the same event?

Florentin Smarandache

Many times in our everyday life.

1) Let's say there is the following event E = { A street protest in Minneapolis }.

From the point of view of the Human Rights the protest is positive, because people have the right to express their view, and consequently the *CNN* television station (reflecting the *left politics*) joys it. Let's say *T*(positiveness) = 0.8

But, from the point view of the Police, the protest is negative, and the Fox News television station (reflecting the *right politics*) presents the negative side of the protests: violence, destruction, arson, chaos. Let's say *F*(negativeness) = 0.8.

As you see, it's not fair to analyze something from only one point of view (from only one parameter).

Therefore, the same event *E* is interpreted, in our world, from two different parameters: the left-wing, and the right-wing.

And you always see such different opinions emerging from different sources of information, such as: TV stations, radio stations, newspapers, books, propagandas, etc.

2) During our long human history, always the *Double-Standard* and *Hypocrisy* have been used:

- interpreting an enemy from a negative point of view (and completely ignoring his positive components);
- and a friend from a positive point of view (and completely ignoring his negative components).

3) Nothing is perfect, it has positive and negative sides, what does matter is: from what side you analyze it. That's why the neutrosophy allows us to analyze anything from all points of view: positive, negative, and even neutral ones.

Refined Sentiment Analysis

Florentin Smarandache

We can extend the sentiment analysis from

<positive, neutral, negative> analysis

to

<high positive, medium positive,
low positive; neutral; low negative,
medium negative, high negative>

(or other types of splitting of the positive and/or of the negative).

Trapezoidal Neutrosophic Set

Florentin Smarandache

For trapezoidal neutrosophic set we can adapt the Score, Accuracy, and Certainty functions of single-valued neutrosophic number:

$S(a) = (a_1+a_2+a_3+a_4)[T+(1-I) + (1-F)]/12$

$A(a) = (a_1+a_2+a_3+a_4)[T - F]/8$

$C(a) = (a_1+a_2+a_3+a_4)T/4$

Resonance: when two entities vibrate at the same frequency.

Florentin Smarandache

Each physical entity has resonance vibration frequency, where it can yield large amplitude (magnitude of displacement due to vibration)

There is a story of pedestrian bridge in England went collapse few years ago. It collapsed because there are many people stands on that bridge during the opening ceremony of the bridge.

It happens that the vibration of people's feet on the bridge deck, resonates with the resonance vibration of the bridge structures.

Because the bridge reaches resonance vibration, its structure gets large amplitude of vibration. Then it collapses.

Therefore, each tower structure should be designed such that if there is earthquake, the vibration is different from its resonance frequency. Otherwise it will collapse.

That is basic principle of earthquake engineering in structural design.

Every particle in this universe has its own (neutrosophic) resonance frequency which can make them explode.

Neutrosophic Physical Constants

Florentin Smarandache
to Robert Neil Boyd

The physical constants in the mathematical abstract spaces, are actually variables in the physical real spaces. A physical constant C (where C is a number) is mostly a Neutrosophic Physical Constant, i.e. an approximation, or an interval (C_1, C_2) that may include C.

The degree of constancy of C is inversely proportional to the interval length, i.e. the larger is the interval the less constancy degree has C.

My question, are there some physical constants where the interval may tend to infinity, i.e. (C_1, ∞) ?

Robert Neil Boyd

I think a Neutrosophic infinite interval between *C*+ and *C*- has application in quantum physics in considerations of the quantum event potential, for example.

There are also meta-stable and bi-stable conditions, saddle points, "strange attractors", and avalanche situations in physical behaviors of physical systems, which can be modeled as existing in the interval between *C*+ and *C*- . I think the folks over at the Santa Fe Institute might be interested in such studies.

These can be put in terms of probabilities regarding the observable physical behaviors of the physical system, with NeutroSophic (NS) behaviors and studies thus becoming a predictive tool for physical behaviors. I see you have already considered many such things, but now we are finding real world physical applications for the NeutroSophic approach.

There are also situations where the NeutroSophic *C*+ infinity *C*- behavior of system *A*, interacts bi-directionally with system *B*. This is descriptive of many information-based physical behaviors resulting in physical applications of the quantum information field.

This has many physical applications, ranging from the macroscopic to nano-scopic behaviors of physical systems.

From astrophysics to electromagnetism, and in biophysics and artificial intelligence, just to name a few. I'm aiming for a Sentient Automobile, which is self-aware and Alive. (Many species of ETs have Sentient Spacecraft, so this kind of car is not out of the question.)

Florentin Smarandache

We have Neutrosophic Probability, where the neutrosophic probability of an event *E* has three components (as in neutrosophy):

NP(*E*) = (chance that the event *E* occurs, indeterminate-chance regarding the event *E* to occur or not, and chance that the event *E* does not occur).

The sum of these three neutrosophic probability components is between 0 and 3.

I think this NP may very well apply in quantum physics, and in other physical phenomena.

Classical Statistics vs. Neutrosophic Statistics

Florentin Smarandache

Classical Statistics deals with *determinate data* only, while Neutrosophic Statistics deals mostly with *indeterminate data*, i.e. data that has any kind of indeterminacy (unclear, vague, partially unknown, contradictory, incomplete, etc.) and indeterminate (approximate) statistical inference procedure.

If all data is determinate, then Neutrosophic Statistics coincides with Classical Statistics.

But, in our world, we have more indeterminate data than determinate data.

Neutrosophic Statistics vs. other Statistics

Florentin Smarandache

$X_N = 2+3I_N$; $I_N = [0, 0.01]$

How can we show that *Neutrosophic Logic* (NL) is more efficient than the fuzzy logic and interval based analysis?

First, Neutrosophic Logic is more general than fuzzy logic and interval based analysis, since NL may not necessarily be an interval, but any subset.

For example $I_N = \{0.000, 0.004, 0.008, 0.010\}$, therefore only four discrete values, not infinitely many as in the interval [0, 0.01].

X_N is also better structured, since we know that 2 is the determinate part, and $3I_N$ is the fluctuating part around 2.

Neutrosophic Triplet of (Structure, NeutroStructure, AntiStructure)

Florentin Smarandache

Working with Akbar Rezaei, we developed a neutrosophic triplet of algebras, i.e. (BI-algebra, Neutro-BI-algebra, Anti-BI-algebra). NeutroAlgebras are generalizations of Partial Algebras, because in a Partial Algebra there is only Partial Operations, while in NeutroAlgebra in addition there also are

NeutroAxioms (partially true, partially indeterminate, and partially false on the algebraic set).

The Partial Operation was also extended to NeutroOperation, where indeterminacy (vague, unkwnown, conflicting, etc.) was taken into consideration, and also there are many cases in many structures when the operations are not outer-defined. For each classical Structure in any field, not only in algebraic structures, we can construct a neutrosophic triplet: (Structure, NeutroStructure, AntiStructure) by Neutro-sofication of the Structure's some operations or axioms we get the NeutroStructure, and respectively by Anti-fication of the Structure's some operations or axioms we get the AntiStructure.

Refined Neutrosophic Logic

Florentin Smarandache
Letter to Drs. B. De Baets, D. Dubois, L. Godo, and E. Hüllermeier, chief-editors of *Fuzzy Sets and Systems journal*

Thank you for the message and discussion about *Neutrosophic Logic* and Belnap and Dunn's logic.

I also was in touch with Umberto Rivieccio and I appreciated his paper and since then the neutrosophic community has improved the neutrosophic operators and neutrosophic order relationship in the maintime as required by him.

I am aware of Belnap and Dunn's logic on 4 values: Truth *(T)*, Falsehood *(F)*, and Uncertainty *(U)* and Contradiction *(C)*.

But Belnap and Dunn's logic is a particular case of *REFINED Neutrosophic Logic* (and similarly we have refined neutrosophic set, and refined neutrosophic probability), and this is explained in the first cited paper (or the link below).

In a general *Refined Neutrosophic Logic* (Set, Probability), T (truth) can be split into subcomponents $T_1, T_2, \ldots, T_p$, and I (indeterminacy) into $I_1, I_2, \ldots, I_r$, and F (falsehood) into $F_1, F_2, \ldots, F_s$, where $p+r+s = n \geq 4$; $p, r, s \geq 0$ are integers and at least one of p, r, s is ≥ 2.

Even more: T, I, and/or F (or any of their subcomponents T_j, I_k, and/or F_l) can be countable or uncountable infinite sets.

Please see the the below link from *arXiv*:

Florentin Smarandache, n-Valued Refined Neutrosophic Logic and Its Applications in Physics. In *Progress in Physics*, 143-146, Vol. 4, 2013; https://arxiv.org/ftp/arxiv/papers/1407/1407.1041.pdf .

Belnap and Dunn's logic is a particular case, when *"I"* (Indeterminacy) is split into only two subcomponents (sub-indeterminacies):

— I_1 = Uncertainty,
— and I_2 = Contradiction,
— while T and F are not split.

I gave other examples where *"I"* is split into three subcomponents, and so on.

*

Refined Neutrosophic Set/Logic is used in U.S. electoral voting, since for each state we need to know the winner, therefore T (truth, or votes for a candidate) is split into $T_1, T_2, \ldots, T_{50}$.

See section 4.8 into the below link or second attached paper.

Florentin Smarandache: Refined Neutrosophy and Lattices vs. Pair Structures and YinYang Bipolar Fuzzy Set. *Mathematics* 2019, 7, 353; DOI:10.3390/math7040353; http://fs.unm.edu/neut/RefinedNeutrosophyAndLattices.pdf .

Therefore, as of today, Refined Neutrosophic Logic is the largest logic.

*

The editors-in-chief have monopolized the *Fuzzy Sets and Systems* journal, rejecting all papers related to extensions of the fuzzy theory, such as: intuitionistic fuzzy, picture fuzzy, Pythagorean fuzzy, spherical fuzzy, neutrosophic, refined neutrosophic, plithogenic theories, etc.

Thank you again.

09/28/2020

NeutroTopology

Florentin Smarandache

For example, we can develop a NeutroTopology, considering one of the topological axioms (let's say that about union) as: partially true, partially indeterminate, and partially false.

Florentin Smarandache
to Mani Parimala

NeutroTopology, i.e. a classical topology defined on a set S where one (or more) classical axiom is transformed into a NeutroAxiom, i.e. NeutroAxiom = axiom that is true for some elements into the given set (degree of truth *T*), it is indeterminate for other elements (degree of Indeterminacy *I*), and false for other elements (degree of falsehood *F*).

Florentin Smarandache

For example, you take a classical algebraic structure. Let's say topology = a family Gamma of neutrosophic sets. Several axioms have to satisfy the topology. Let's take one of them: intersection of any two neutrosophic sets from Gamma is a neutrosophic set that also belongs to Gamma. This is in classical sense. In NeutroTopology, we may have some neutrosophic sets whose intersection belong to Gamma, other neutrosophic sets whose intersection does no belong to Gamma, and others whose intersection is indeterminate. Therefore, an axiom is not 100% true as in classical topology, but only partially true, partially indeterminate, and partially false (as in neutrosophy). Then you give examples of NeutroTopology, get some properties, etc.

Neutro-AG-groupoid

Florentin Smarandache

A groupoid $(G, *)$ is called an *AG-groupoid* if it holds the left invertive law, that is, for all $a, b, c \in G$, $(a * b) * c = (c * b) * a$.

We may transform the *AG-groupoid* to the *Neutro-AG-groupoid*, withing the newly introduced NeutroAlgebra structures.

A *Neutro-AG-groupoid* is a groupoid (G, ∗), such that the left invertive law is true for some elements in G [degree of truth T], indeterminate (undefined, unknown, unclear etc.) for other elements [degree of indeterminacy I], and false for other elements in G [degree of falsehood F];
where (T,I,F) is different from (1,0,0) which represents the classical AG-groupoid, and from (0, 0, 1) which represents the Anti-AG-groupoid (i.e. when the left invertive law fails for all elements of G).

References

Madad Khan, Florentin Smarandache, Saima Anis, *Theory of Abel Grassmann's Groupoids*, Education Publisher, Columbus, 208 p., 2015.

Madad Khan, Florentin Smarandache, Tariq Aziz, *Fuzzy Abel Grassmann Groupoids [second updated and enlarged version]*, 110 p., Educational Publisher, Columbus, 2015.

NeutroIsomorphism & AntiIsomorphism

Florentin Smarandache

Let's have two algebraic structures S_1 and S_2.

Define a function $f_1 : S_1 \to S_2$ that is an isomorphism;

- then a function f_2: $S_1 \to S_2$ that is partially an isomorphism (its condition is verified for some elements, and not verified or indeterminate for other elements); this is a Neutro-Isomorphism;

- then a function f_3: $S_1 \to S_2$ that is an Anti-Isomorphism; it is not verified for no elements.

Extended Neutrosophic Duplets

Florentin Smarandache

A Neutrosophic Triplet *(x, y, z)* can be split into two Neutrosophic Duplets *(x, y)* and *(z, y).*

In a Neutrosophic Duplet *(x, y),* where *y* is the neutral, there is no inverse for *x*.

Therefore, we should call them *Extended* Neutrosophic Duplets (since there is an inverse for *x*, that inverse is *z*).

Total order on Neutrosophic Integers and their Factorials

Florentin Smarandache
to Yilmaz Ceven

Congratulations again for the paper *Some Properties of Neutrosophic Integers*.

I like the way you defined the primes, division etc.

I have two questions that I am unable to answer:

1) Can you define a total order on $Z(I) = \{a+bI$, where a, b are integers, and $I^2 = I\}$.

2) How to define the factorial of a neutrosophic number?

$(a+bI)!$ = the product of all numbers of the form $k+jI$, where $1 \leq k <= a$ and $0 \leq j \leq b$.

But what about if $b \leq 0$?

Fermatean Fuzzy Set is a particular case of the Neutrosophic Set

Florentin Smarandache

Let's use Latin letters for the positive membership *(T)* and negative membership *(F)* and indeterminacy membership *(I)*.

The Single-Valued Fermatean Fuzzy Set is defined as follows: Let *U* be a universe of discourse and a set *A* included in *U*, such that:

$$A = \{x, (T_A(x), F_A(x)), x \in U\},$$

where $T_A(x), F_A(x) \in [0, 1]$,

and $0 \le T_A(x)^3 + F_A(x)^3 \le 1$,

while indeterminacy (hesitancy):

$$I_A(x) = \sqrt[3]{1 - T_A(x)^3 + F_A(x)^3} \in [0,1].$$

Since $T_A(x), F_A(x) \in [0, 1]$, then also $T_A(x)^3, F_A(x)^3 \in [0, 1]$. If we denote, $T_A^{NS}(x) = T_A(x)^3$ and $F_A^{NS}(x) = F(x)^3$, and $I_A^{NS}(x) = I_A(x)$,

where NS means Neutrosophic Set, then we get:

$0 \le T_A(x) + F_A(x) + I_A(x) \le 1.$

*

But this is a particular case of the neutrosophic set, where the sum of components T, I, F can be any number between 0 and 3, and for the Fermatean Fuzzy Set it is it taken to be up to 1. Therefore, any Single-Valued Fermatean Fuzzy Set is also a Neutrosophic Set, but the reciprocal is not true.

See the next *counterexample*.

$T_A(x) = 0.8$, $F_A(x) = 0.9$, and $I_A(x) = 0.3$,

then the triplet $(T_A(x), I_A(x), F_A(x)) = (0.8, 0.3, 0.9)$ is a neutrosophic triplet since $0.8, 0.3, 0.9 \in [0, 1]$ and of course $0 \leq 0.8 + 0.3 + 0.9 \leq 3$,
but it is not a Fermatean fuzzy triplet since:

$$0.8^3 + 0.9^3 = 1.241 > 1.$$

Also, by replacing $q = 3$, the Fermatean Set is a particular case of the q-Rung Orthopair Fuzzy Set, which in its turn is a particular case of the Neutrosophic Set (see [2], section 43.p. 29).

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UNM Digital Repository:
https://digitalrepository.unm.edu/math_fsp/21

Finitely or Infinitely Refined Neutrosophic Set / Logic / Probability

Florentin Smarandache

In the Finitely or Infinitely Refined Neutrosophic Set (Logic, Probability) introduced in 2013 [1],

T can be split into sub-truths $T_1, T_2, \ldots, T_p$,

and I into sub-indeterminacies $I_1, I_2, \ldots, I_r$,

and F into sub-falsehoods $F_1, F_2, \ldots, F_s$,

where $p, r, s \in \{0, 1, 2, \ldots, \infty\}$
and $p + r + s = n \in \{0, 1, 2, \ldots, \infty\}$.

By index = *0*, of a neutrosophic component *T, I,* or *F*, or of any of their corresponding sub-components, we denote the empty set, i.e. $T_0 = \phi$, $I_0 = \phi$, $F_0 = \phi$. The case (T_0, I_0, F_0) is the most degenerated one.

Example of Infinitely Refined Neutrosophic Set:

Let *T* = *White*, and its opposite *F* = *Black*. Then in between *T* and *F* we have infinitely many nuances of colors: $I_1, I_2, \ldots, I_\infty$. We get: $(T; I_1, I_2, \ldots, I_\infty; F)$.

References

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[2] F. Smarandache, S. Broumi, True-False Set is a particular case of the Refined Neutrosophic Set, *International Journal of Neutrosophic Science* (IJNS), Vol. 12, No. 1, pp. 9-12, 2020; http://fs.unm.edu/IJNS/TrueFalseSet.pdf

Neutrosophic Topological Space

Florentin Smarandache

Let *U* be a universe of discourse, Φ be the empty set, and *N(U)* be the neutrosophic sets over *U*. Let $\tau \subseteq N(U)$ be a collection of neutrosophic sets over *U*. Then τ is called a neutrosophic topology on *U* if:

i. *U* and Φ belong to τ ;

ii. If $A_1, A_2, ..., A_n \in \tau$, for any finite or infinite integer $n = 1, 2, ..., \infty$, then their union $\bigcup_{i=1}^{n} A_i \in \tau$;

iii. If $A, B \in \tau$, then their intersection $A \cap B \in \tau$.

The above pair (*U, τ*) is called a neutrosophic topological space over *U*.

All members of τ are called *Neutrosophic Open Sets*.

If $M \in N(U)$ and its complement $M^C \in \tau$, then M is called a *Neutrosophic Closed Set.*

Neutrosophic Manifold

Florentin Smarandache

A neutrosophic manifold is a neutrosophic topological space that, on the small scales, near each point, resembles a Neutrosophic Euclidean Space (i.e., an Euclidean Space that has some indeterminacy with respect to its points, or lines, curves, planes, surfaces, etc.).

Or each point has a neighborhood (with or without indeterminacy) that is homeomorphic to an open Neutrosophic Euclidean Set), where a Neutrosophic Homeomorphism means a neutrosophic function (function with some indeterminacy) that is neutrosophically continuous and bijective and whose inverse is also continuous. It is an extension of the classical Manifold onto an indeterminate space. The neutrosophic manifold can be equipped with more structures, giving birth to more types of neutrosophic manifolds.

Neutrosophic Submanifold

It is a subset of a neutrosophic manifold, which is itself a neutrosophic manifold.

Neutrosophic Differential Manifold

It allows the neutrosophic calculus to be used.

Type-3 Neutrosophic Number

Florentin Smarandache

To Jun Ye

The Type-1 Neutrosophic Number, $N_1 = u+vI$, where all u, v, I are fixed. The Neutrosophic Number $N_2 = u+vI$ is more general, in the sense that u and v have many values in a given set ℝ, while $I = [I^-, I^+]$.

You may call it Type-2 Neutrosophic Number (since there are two variables u and v). And further on, define the most general one: Type-3 Neutrosophic Number, $N_3 = u+vI$, where all three u, v, I are variables (they change).

Approximate a unclear crisp number by a Neutrosophic Number

Florentin Smarandache

The experts, when solving a practical problem, do not convert from a crisp number to a neutrosophic number, but they approximate an unclear crisp number by a neutrosophic number.

The Ambiguous Set

Florentin Smarandache

The Ambiguous Set [1], defined by four components denoted as *(T, TA, FA, F),* is a particular case of the Refined Neutrosophic Set [2], when into the neutrosophic components *(T, I, F)* the middle neutrosophic component *"I"* (Indeterminacy) is split into two neutrosophic sub-indeterminacies, I_1 and I_2, that in the ambiguous set are named as *TA* (Ambiguous Truth) and respectively *FA* (Ambiguous Falsehood). In the Refined Neutrosophic Set, the neutrosophic components *(T, I, F),* *T* = Truth, *I* = Indeterminacy, and *F* = Falsehood, all or some of them are refined/split into many neutrosophic sub-components: $T_1, T_2, \ldots; I_1, I_2, \ldots; F_1, F_2, \ldots$ respectively.

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Refined Neutrosophic Bi-Topology

to Riad Khidr Alhamido

You may try to extend your research to Refined Neutrosophic Topology, and then Refined Neutrosophic Bi-Topology. See the Refined Neutrosophic Set:

http://fs.unm.edu/RefinedNeutrosophicSet.pdf.

Non-Standard Numbers and Non-Standard Sets in Neutrosophic Logic

Florentin Smarandache

Not to worry about non-standard numbers and non-standard subsets, since they are used only in philosophy to make distinction between the Relative Truth (which is truth in at least one world, according to Leibniz), and whose neutrosophic logic value is:

NL(Relative Truth) = 1,

and Absolute Truth (which is truth in all possible worlds, according to Leibniz), and whose neutrosophic logic value is:

NL(Absolute Truth) = 1+ (which means 1+ε, where ε is a positive infinitesimal).

The researchers that are in a technical or scientific program, not in philosophy, do need only standard numbers and standard sets.

NeutroTheorem for NeutroGeometry

Florentin Smarandache

to W.B. Vasantha Kandasamy

It is enough to consider a classical geometric theorem [not necessarily an axiom, since the theorem is based on some axioms anyway], and then construct a geometric space where this theorem is partially true, partially indeterminate, and partially false (therefore, we transform the Theorem into a NeutroTheorem) in order to get a NeutroGeometry.

Similarly we can do it in any field, for example in Algebraic Structures, science *X*, science *Y*, etc.

Neutrosophicate a classical concept

Florentin Smarandache

One could take a concept, for example Corsini Hypergroup (or other Corsini concept) and you neutrosophicate it. Or one could get another algebraic (or geometric, or any field) concept, and neutrosophicate it (split it into 3 parts: part of *true*, part of *indeterminate*, and part of *false* concepts).

Adjust to Neutrosophy

Florentin Smarandache

One could adjust the following to neutrosophy:

1) *cathode*, *anode* in the battery; anything neutral in between these opposites?

2) chemistry: salt = $Na^{+}Cl^{-}$; something in between these opposites?

*

Also, Nutrosophic Set could be extendend to *Doubt Neutrosophic Set*, also *Doubt Refined Neutrosophic Set*.

How to compute the T, I, F Neutrosophic Components

Florentin Smarandache

How to compute the *T, I, F* neutrosophic components for each element *x* that belongs to a neutrosophic set?

It depends on each application and on the experts.

For some applications, you may use a subjective neutrosophic probability for getting the neutrosophic components of the element *x(T,I,F)* that belongs to a given set, i.e.:

T = the chance that *x* occurs (*x* belongs to a given set);

I = the indeterminate-chance that *x* occurs or not (*x* belongs or not to a given set);

F = the chance that *x* does not occur (*x* does not belong to a set).

See this book on *Neutrosophic Probability*:
http://fs.unm.edu/NeutrosophicMeasureIntegralProbability.pdf

Distinctions between A-IFS and Neutrosophic Set

Florentin Smarandache

"Notably, the most obvious differentiation between intuitionistic fuzzy logic and neutrosophic logic lies in whether the three membership degrees are independent or dependent" [1].

This is not the only one. Another more important one is that the neutrosophic aggregation operators are different from the intuitionistic fuzzy operators, since in the neutrosophic aggregation operators the Indeterminacy *(I)* is involved in the formulas, while in the intuitionistic fuzzy operators the hesitancy (indeterminacy) *(H)* is NOT used in the formulas. And the results of aggregation for the same inputs (for the case where the sum of the components $T+I+F = 1$) give different outputs.

Independence and Dependence of Neutrosophic Components

Florentin Smarandache

"in neutrosophic logic, the assumption that the grades of positive membership and negative membership meet independence is not appropriate for decision-making problems" [1].

I feel you do not perceive the exact meaning of the neutrosophic set.

The neutrosophic components may be: 1) either totally independent, or 2) partially independent and partially dependent, or 3) totally dependent.

Therefore, NS does not include ONLY the case when all neutrosophic components are independent, but also the cases when there is some (or total) dependence between components.

Also, ironically the greatest number of applications of the neutrosophic set are in decision making;

See such sites:

http://fs.unm.edu/neutrosophy.htm,
http://fs.unm.edu/NSS/Articles.htm.

Reference

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Literal and Numerical Indeterminacy

Florentin Smarandache

1) In $a+bI$, *"I"* is a letter (not a number) that we call "literal indeterminacy" and indeed $I^n = I$, where n is a positive integer, $n \geq 1$.

This is used in neutrosophic algebraic structures.

2) $N = a+bI$, where I = subset, is called "numerical indeterminacy". Where *"a"* is the determinate part of N, and *"bI"* is the indeterminate part of N.

Mostly I = interval, or hesitant set {0.1, 0.3, 0.8}, etc.

It is used in neutrosophic Statistics (http://fs.unm.edu/NS/NeutrosophicStatistics.htm), in neutrosophic MCDM, and in other applications.

Herein I^2 is not equal to I.

For example, there are irrational numbers, transcendental numbers etc., that have infinitely many decimals with no repetition.

Neutrosophic Multi-Variate Logic

Florentin Smarandache

to Bouzina Salah

1) See first the Refined Neutrosophic Logic (and Set): http://fs.unm.edu/RefinedNeutrosophicSet.pdf
where the truth T is refined/split into T_1, T_2, ...; then similarly the indeterminacy I was refined/split into I_1, I_2, ...; and finally the falsehood F was refined/split into F_1, F_2, ... : http://fs.unm.edu/RefinedNeutrosophicSet.pdf.

2) Another way, the most general form of logic of today is the Plithogenic Logic (see the link below),

which is a MultiVariate Truth-Value Logic, i.e. a proposition P is evaluated by many sources (or parameters, or random variables), therefore it is a MULTI-LOGIC (double, triple, ..., *n*-folded logic): Plithogenic Logic (= MultiVariate Logic, i.e. logic value upon many random variables, the most general logic of today):
http://fs.unm.edu/NSS/IntroductionPlithogenicLogic1.pdf

3) Since I see you are a philosopher from Algeria (I also speak French very well), you may be interested in the study of Neutrosophy, as a new branch of philosophy:
http://fs.unm.edu/Neutrosophy-A-New-Branch-of-Philosophy.pdf.

Neutrosophic Logic is Tripolar

Florentin Smarandache

Neutrosophic Logic is Tripolar, while the Bipolar Neutrosophic Logic Sextan-Polar. Unlike the fuzzy and intuitionistic fuzzy logics, Neutrosophic Logic is tripolar, since the degree of truth *(T)*, the degree of falsehood *(F)*, and the degree of indeterminacy *(I)* are all totally (100%) independent two by two, i.e. $0 \leq T + I + F \leq 3$, where T, I, F are in [0,1]. For example, we can have $T = 1$, $I = 1$, $F = 1$, therefore three-poles.

In addition, the Bipolar Neutrosophic Logic has 6 poles, the three positive poles denoted by: $T+$, $I+$, $F+$ in [0, 1] and three negative poles, denoted by $T-$, $I-$, $F-$ in [-1, 0], where all six poles are totally independent two by two, i.e. $0 \leq (T+) + (I+) + (F+) \leq 3$ and $-3 \leq (T-) + (I-) + (F-) \leq 0$.

I think it is possible to use them for the feedforward and feedback circuits for the neural networks.

Projective NeutroGeometry

Florentin Smarandache
to Erik Gonzalez

Very good idea to consider various types of geometries, such as: Projective Geometry, Affine Geometry, Finite Geometry, Differential Geometry, etc. and their neutrosophications to Projective NeutroGeometry (or Projective AntiGeometry), etc.

Another strategy: you can consider just in the Euclidean Geometry an ordinary theorem, for example:

On a plane from a point exterior to a line one can draw only one perpendicular on that line.

And you may design a model where this theorem can be partially true, partially indeterminate, and partially false.

The T-Spherical Fuzzy Set is a particular case of the Neutrosophic Set

Florentin Smarandache

In order to avoid their heavy notations we use descriptive notations such as T = truth (membership), I = indeterminacy {called uncertainty or abstinence by the authors of [1]}, and F = falsehood (nonmembership) {called dissatisfaction by the authors of [1]} to adjust the *Definitions* 6 and 7, page 36, from paper [1].

Definition 6 from [1]

The **T-Spherical Fuzzy Set** (T-SFS) *A* in the universe of discourse *U* is scripted as:

$A = \{(x, <T_t(x), I_t(x), F_t(x)>; x \in U\}$,

where for any $x \in U$ one has: $T_t(x), I_t(x), F_t(x) \in [0, 1]$, and

$$0 \le T_t(x)^n + I_t(x)^n + F_t(x)^n \le 1$$

while $R_t(x) = \sqrt{1 - T_t(x)^n - I_t(x)^n - F_t(x)^n}$ that is called Refusal Degree, for $n = 1, 2, 3, \dots$.

The T-Spherical Fuzzy Set is a particular case of the Neutrosophic Set {see the proof in [2]}.

*

In the above definition, it is not clear if the exponent *n* is a finite number or it is even infinite,

but do not matter how big is *n*, a triplet of the form (1, 1, 0.3) for example that is a neutrosophic triplet, cannot be a T-Spherical Fuzzy triplet, since

$$1^n + 1^n + 0.3^n > 1.$$

I had already defined the **n-HyperSpherical Fuzzy Set** (**n-HSFS**) in 2019 as a generalization of Spherical Fuzzy Set (see [2]).

The Neutrosophic set has no restriction with respect to the sum of the components $T + I + F$

Definition 7 from [1]

The **Spherical Linear Diophantine Fuzzy Set** (SLDFS) B in the universe of discourse U is defined as:

$B = \{(x, <T(x), I(x), F(x)>, <\alpha(x), \beta(x), \eta(x)>); x \in U\}$, where for any $x \in U$ one has:

$$T(x),\ I(x),\ F(x),\ \alpha(x), \beta(x), \eta(x) \in [0,1],$$

and

$$0 \le T(x) + I(x) + F(x) \le 1$$

$$0 \le \alpha(x) + \beta(x) + \eta(x) \le 1,$$

where T, I, F are called by these authors as membership, uncertainty (or abstinence), and non-membership,

while

$$\alpha(x), \beta(x), \eta(x)$$

are called reference parameters corresponding to these grades respectively.

Also, the Refusal Part is defined as:

$$\pi = 1 - \alpha(x)T(x) - \beta(x)I(x) - \eta(x)F(x).$$

*

The authors [1] say that:

> "we found that neutrosophic sets, T-SFSs, PiFSs, and SFSs have various restrictions on satisfaction, abstinence, and degrees".

While it is true with respect to the T-SFSs, PiFSs, and SFSs of having some restrictions with respect to the sum of their components, it is false with respect to the neutrosophic set that has no restriction, where the sum of the components T+I+F can be any number between 0 and 3, for example one can have neutrosophic triplets of the form (1, 1, 1), (0, 0, 0), and anything in between, for example (0.4, 0.9, 0.8) etc.

No connection between the Linear Diophantine Equation and the Spherical Linear Diophantine Fuzzy Set

Further on they continue:

> "In the field of number theory, we have the concept of linear Diophantine equation for three variables given as $ax + by + cz = d$. The intended structure has a correspondence with this equation, so we described it as SLDFS."

The denomination of "Diophantine" is inappropriate for this type of fuzzy set, because *linear Diophantine equation* is an equation (of any number of variables, not only three) which is solved only in the set of integers (i.e. their accepted solutions are only integer numbers).

Or, it is not the case for the SLDFS, since the components T, I, F and even its reference parameters α, β, η may be any numbers in the interval [0, 1], not necessarily integers. Only in the case when these components and parameters were 0 and 1 only, the set could have been named "Diophantine".

*

Let's take a simple example of linear Diophantine equation with three variables:

$$2x - 3y + 8z = 9 .$$

Let's solve it, and we get its double-infinite general integer solution:

$$\begin{Bmatrix} x = 3u - 4v + 3 \\ y = 2u - 1 \\ z = v \\ u, v \in Z \end{Bmatrix}$$

where Z is the set of integers, Z = {…, -3, -2, -1, 0, 1, 2, 3, …}.

Some particular cases we obtain by parameterizing *u* and *v*.

Let's assign *u* = 0 and *v* = 1, then *x* = -1, *y* = -1, z = 1.

Or *u* = 4, *v* = -3, then *x* = 27, *y* = 7, z = -3.

We see no connection whatsoever between this linear Diophantine equation of three variables or its solution and the Spherical Linear Diophantine Fuzzy Set!

Spherical Fuzzy Set does not have totally independent components

In [3]:

> "Gundogdu et al. recently developed the spherical fuzzy sets theory as a fusion of Pythagorean fuzzy sets […] and Neutrosophic sets […]. The theory was developed based on the notion that a decision maker's hesitancy can be defined independently of related and non-related degrees"

The above sentence is false, since the spherical fuzzy set and the Pythagorean fuzzy set are both particular cases of the neutrosophic set.

Please read this paper [2]:
http://fs.unm.edu/Raspundatan.pdf.

Also, the components of the spherical fuzzy set are NOT completely/totally independent.

Only in neutrosophic set the components T, I, F are completely independent.

Because, in the spherical fuzzy set, if let's say T = 0.8, then the components I and F have to be less than or equal to $\sqrt{1-0.8^2} = \sqrt{1-0.64} = \sqrt{0.36} = 0.6$, so I and F are dependent of T.

Even so, for example, if T = 0.9, then I and F cannot be for example 0.4 and 0.5 respectively, because $0.9^2 + 0.4^2 + 0.5^2 = 1.22 > 1$.

While in the neutrosophic set, if T = 0.9 { or any other number in [0, 1] }, "I" and F can also be any numbers in the interval [0, 1]. As such (0.9, 0.4, 0.4) is a neutrosophic triplet, but not a spherical fuzzy triplet. The spherical fuzzy set has a degree of independence greater than the intuitionistic fuzzy set's, but strictly less than the neutrosophic set's.

Although I published a paper [2] and I told many authors by emails that the spherical fuzzy set is a particular case of the neutrosophic set, there are high rank journals from prestigious publishing houses that wrongly publish papers asserting that the spherical fuzzy set has independent components and that it is a generalization of the neutrosophic set – both of them are false.

These mathematical errors are propagating in journals with high impact factor…

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Florentin Smarandache

Istanbul, Turkey

June 1989

Neutrosophic Life

That's how the life is... Life is neutrosophic (good and bad simultaneously, actually each time with a degree of good and a degree of bad, tending towards indeterminacy).

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