



Practical Applications of IndetermSoft Set and IndetermHyperSoft Set and Introduction to TreeSoft Set as an extension of the MultiSoft Set

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Abstract: The IndetermSoft Set is as an extension of the Soft Set, because the data, or the function, or the sets involved in the definition of the soft set have indeterminacy - as in our everyday life, and we still need to deal with such situations.

And similarly, IndetermHyperSoft Set as extension of the HyperSoft Set, when there is indeterminate data, or indeterminate functions, or indeterminate sets.

Herein, 'Indeterm' stands for 'Indeterminate' (uncertain, conflicting, incomplete, not unique outcome).

We now introduce for the first time the TreeSoft Set as extension of the MultiSoft Set.

Several applications are presented for each type of soft set.

Keywords: Soft Set, IndetermSoft Set, HyperSoft Set, IndetermHyperSoft Set, MultiSoft Set, TreeSoft Set

1. Introduction

We have extended the Soft Set to HyperSoft Set [2, 3] in 2018, then both of them to IndetermSoft Set and IndetermHyperSoft Set [4, 8] respectively in 2022, and we have introduced Indeterminate Soft and HyperSoft operators.

The operations (complement, intersection, union) for IndetermSoft Set and IndetermHyperSoft Set respectively are to be done in the future research.

And in this paper a new type of soft set, called TreeSoft Set, is introduced for the first time as an extension of the MultiSoft Set.

Several applications are presented for each type of soft set.

2. Definition of Soft Set

Let U be a universe of discourse, H a non-empty subset of U , with $P(H)$ the powerset of H , and a an attribute (parameter, factor, etc.), with its set of attribute-values denoted by A . Then, the pair (F, A) , with $F: A \rightarrow P(H)$, is called a (Classical) Soft Set over H .

Molodtsov [1] has defined in 1999 the Soft Set.

3. Real Example of (Classical) Soft Set

Let $H = \{h_1, h_2, h_3, h_4\}$ be a set of houses, and a an attribute, $a = color$, and its set of attribute-values $A = \{white, green, red\}$. The function $F : A \rightarrow P(H)$, as:

$$F(white) = \{h_1, h_2, h_4\}, F(green) = h_3, F(red) = \emptyset \text{ (no house)}.$$

4. Definition of IndetermSoft Set

Smarandache [4, 8] introduced it in 2022.

Let \mathcal{U} be a universe of discourse, H a non-empty subset of \mathcal{U} , and $P(H)$ the powerset of H . Let a be an attribute, and A be a set of this attribute-values.

Then $F: A \rightarrow P(H)$ is called an IndetermSoft Set if:

- i) the set A has some indeterminacy;
- ii) or the set $P(H)$ has some indeterminacy;
- iii) or there exist at least an attribute-value $v \in A$, such that $F(v) = \text{indeterminate}$ (unclear, incomplete, conflicting, or not unique);
- iv) or any two or all three of the above situations.

The IndetermSoft Set has some degree of indeterminacy, and as such it is a particular case of the NeutroFunction [5, 6], defined in 2014 – 2015, which is a function that is only partially well-defined (inner-defined), partially indeterminate, and partially outer-defined. The NeutroFunction is a generalization of the classical function, that is totally well-defined.

IndetermSoft Set, as extension of the classical (determinate) Soft Set, deals with indeterminate data, because there are sources [4, 8] unable to provide exact or complete information on the sets A , H or $P(H)$, and on the function F .

We did not add any indeterminacy, we found the indeterminacy in our real world. Because many sources give approximate/uncertain/incomplete/conflicting information, not exact information as in the Soft Set, as such we still need to deal with such situations.

For more information on IndetermSoft Set consult [4, 8].

5. Real Example of IndetermSoft Set:

Assume a town has many houses.

1) *Indeterminacy with respect to the function.*

1a) You ask a source:

- What houses have the red color in the town?

The source:

- I am not sure, I think the houses h_1 or h_2 .

Therefore, $F(red) = h_1 \text{ or } h_2$

(indeterminate / uncertain answer).

1b) You ask again:

- But, what houses are yellow?

The source:

- I do not know, the only thing I know is that the house h_5 is not yellow because I have visited it.

Therefore, $F(yellow) = \text{not } h_5$

(again indeterminate / uncertain answer).

1c) Another question you ask:

- Then what houses are blue?

The source:

- For sure, either h_8 or h_9

Therefore, $F(blue) = \text{either } h_8 \text{ or } h_9$

(again indeterminate / uncertain answer).

2) Indeterminacy with respect to the set H of houses.

You ask the source:

- How many houses are in the town?

The source:

- I never counted them, but I estimate their number to be *between 100-120 houses*.

3) Indeterminacy with respect to the set A of attributes.

You ask the source:

What are all colors of the houses?

The source:

I know for sure that there are houses of colors *red, yellow, and blue*, but I do not know if there are houses of other colors (?)

This is the IndetermSoft Set.

6. Definition of HyperSoft Set

Smarandache has extended in 2018 the Soft Set to the HyperSoft Set [3, 4, 8] by transforming the function F from a uni-attribute function into a multi-attribute function.

Let \mathcal{U} be a universe of discourse, H a non-empty set included in \mathcal{U} , and $P(H)$ the powerset of H . Let a_1, a_2, \dots, a_n , where $n \geq 1$, be n distinct attributes, whose corresponding attribute-values are respectively the sets A_1, A_2, \dots, A_n , with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \dots, n\}$.

Then the pair $(F, A_1 \times A_2 \times \dots \times A_n)$, where $A_1 \times A_2 \times \dots \times A_n$ represents the Cartesian product, with $F: A_1 \times A_2 \times \dots \times A_n \rightarrow P(H)$ is called a HyperSoft Set.

In other words, for any $(e_1, e_2, \dots, e_n) \in A_1 \times A_2 \times \dots \times A_n$, $F(e_1, e_2, \dots, e_n) \in P(H)$

7. Real Example of HyperSoft Set

Let $H = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7\}$ a set of houses, and two attributes a_1 and a_2 , where $a_1 = \text{color}$, and its set of attribute-values $A_1 = \{\text{white, green, red}\}$, and $a_2 = \text{size}$, and its attribute-values

$A_2 = \{\text{small, big}\}$. The function $F: A_1 \times A_2 \rightarrow P(H)$, as :

$F(\text{white, small}) = \{h_1, h_2\}$, $F(\text{green, big}) = \{h_4, h_6, h_7\}$, $F(\text{red, big}) = \{h_3, h_5\}$.

8. Definition of IndetermHyperSoft Set

Smarandache [4, 8] introduced it in 2022.

Let \mathcal{U} be a universe of discourse, H a non-empty subset of \mathcal{U} , and $P(H)$ the powerset of H . Let a_1, a_2, \dots, a_n , where $n \geq 1$, be n distinct attributes, whose corresponding attribute-values are respectively the sets A_1, A_2, \dots, A_n , with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \dots, n\}$.

Then the pair $(F, A_1 \times A_2 \times \dots \times A_n)$, where $A_1 \times A_2 \times \dots \times A_n$ represents the Cartesian product, with $F: A_1 \times A_2 \times \dots \times A_n \rightarrow P(H)$ is called an IndetermHyperSoft Set if:

- i) at least one of the sets A_1, A_2, \dots, A_n has some indeterminacy;
- ii) or the set $P(H)$ has some indeterminacy;
- iii) or there exist at least one n -plet $(e_1, e_2, \dots, e_n) \in A_1 \times A_2 \times \dots \times A_n$ such that $F(e_1, e_2, \dots, e_n) = \text{indeterminate (unclear, uncertain, conflicting, or not unique)}$;
- iv) or any two or all three of the above situations.

The IndetermHyperSoft Set has some degree of indeterminacy, and it is as extension of the (determinate) HyperSoft Set.

Similarly, we did not add any indeterminacy, we found the indeterminacy in our real world. Because many sources give approximate/uncertain/incomplete/conflicting information, not exact information as in the Soft Set and in the HyperSoft Set, as such we still need to deal with such situations.

9. Real Example of IndetermHyperSoft Set

Assume a town has many houses.

1) *Indeterminacy with respect to the function.*

1a) You ask a source:

2) What houses are of red color and big size in the town?

The source:

3) I am not sure, I think the houses h_1 or h_2 .

Therefore, $F(\text{red}, \text{big}) = h_1 \text{ or } h_2$

(indeterminate / uncertain answer).

1b) You ask again:

4) But, what houses are *yellow and small*?

The source:

5) I do not know, the only thing I know is that the house h_5 is neither yellow nor small because I have visited it.

Therefore, $F(\text{yellow}, \text{small}) = \text{not } h_5$

(again indeterminate / uncertain answer).

1c) Another question you ask:

- Then what houses are *blue and big*?

The source:

- For sure, *either h_8 or h_9*

Therefore, $F(\text{blue}, \text{big}) = \text{either } h_8 \text{ or } h_9$

(again indeterminate / uncertain answer).

2) *Indeterminacy with respect to the set H of houses.*

You ask the source:

- How many houses are in the town?

The source:

- I never counted them, but I estimate their number to be *between 100-120 houses*.

3) *Indeterminacy with respect to the set A of attributes.*

You ask the source:

What are all colors and sizes of the houses?

The source:

I know for sure that there are houses of colors of *red, yellow, and blue*, but I do not know if there are houses of other colors (?)

About the size, I saw many houses that are *small*, but I do not remember to have seen *big* houses.

This is the IndetermHyperSoft Set.

10. Definition of MultiSoft Set [7]

Let U be a universe of discourse, and H a non-empty subset of U .

And $P(H)$ is the power set of H . Let A_1, A_2, \dots, A_n be $n \geq 2$ sets of attributes (parameters) whose intersection $A_1 \cap A_2 \cap \dots \cap A_n = \phi$.

Let $A = A_1 \cup A_2 \cup \dots \cup A_n$ and $P(A)$ be the power set of A .

Then $F : P(A) \rightarrow P(H)$ is a MultiSoft Set over H .

For $\varepsilon \in P(A)$ one considers that $F(\varepsilon)$ is the set of ε -approximate sets of the multisoft set $(F, P(A))$.

11. Extension of the MultiSoft Set to a HyperSoft Set

One introduces the empty-element ϕ to each set of attribute-values, and let denote

$$A'_1 = A_1 \cup \{\phi\}, A'_2 = A_2 \cup \{\phi\}, \dots, A'_n = A_n \cup \{\phi\}.$$

$$\text{Let } \varepsilon = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n) \in A'_1 \times A'_2 \times \dots \times A'_n,$$

then $\varepsilon_1 \in A'_1 = A_1 \cup \{\phi\}$ means that either $\varepsilon_1 \in A_1$ or $\varepsilon_1 = \phi$ (discarded);

similarly for all $\varepsilon_i \in A'_i = A_i \cup \{\phi\}$, $1 \leq i \leq n$.

Thus, $F : A'_1 \times A'_2 \times \dots \times A'_n \rightarrow P(H)$ is a hypersoft set.

12. Real Example of MultiSoft Set

We retake the previous example and adjust it to a MultiSoft Set.

Let $H = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7\}$ a set of houses, and two attributes a_1 and a_2 , where $a_1 = color$, and its set of attribute-values $A_1 = \{white, green, red\}$, and $a_2 = size$, and its attribute-values

$A_2 = \{small, big\}$. Let $A = A_1 \cup A_2 = \{white, green, red; small, big\}$, and $P(A)$ be the power set of A .

Then $F : P(A) \rightarrow P(H)$ is defined as follows:

$$F(white) = \{h_1\}, F(green, big) = \{h_4, h_6\}, F(big) = \{h_3, h_5\}.$$

13. Real MultiSoft Set extended to a HyperSoft Set

Let's enlarge A_1 and A_2 :

$$A'_1 = \{white, green, red, \phi\}, \text{ and } A'_2 = \{small, big, \phi\}$$

$$\text{Then } F' : A'_1 \times A'_2 \rightarrow P(H)$$

$$F'(white, \phi) \equiv F(white) = \{h_1\} \quad (\text{since the attribute-value } \phi \text{ was discarded}).$$

$$F'(green, big) \equiv F(green, big) = \{h_4, h_6\}.$$

$$F'(\phi, big) \equiv F(big) = \{h_3, h_5\} \quad (\text{since the attribute-value } \phi \text{ was discarded}).$$

14. Generalization of MultiSoft Set to the TreeSoft Set

Let U be a universe of discourse, and H a non-empty subset of U , with $P(H)$ the powerset of H .

Let A be a set of attributes (parameters, factors, etc.),

$$A = \{A_1, A_2, \dots, A_n\}, \text{ for integer } n \geq 1,$$

where A_1, A_2, \dots, A_n are attributes of first level (since they have one-digit indexes).

Each attribute $A_i, 1 \leq i \leq n$, is formed by sub-attributes:

$$A_1 = \{A_{1,1}, A_{1,2}, \dots\}$$

$$A_2 = \{A_{2,1}, A_{2,2}, \dots\}$$

.

.

.

$$A_n = \{A_{n,1}, A_{n,2}, \dots\}$$

where $A_{i,j}$ are sub-attributes (or attributes of second level) (since they have two-digit indexes).

Again, each sub-attribute $A_{i,j}$ is formed by sub-sub-attributes (or attributes of third level):

$$A_{i,j,k}$$

And so on, as much refinement as needed into each application, up to sub-sub-...-sub-attributes (or attributes of m-level (or having m digits into the indexes):

$$A_{i_1, i_2, \dots, i_m}$$

Therefore, a graph-tree is formed, that we denote as $Tree(A)$, whose root is A (considered of *level zero*), then nodes of *level 1*, *level 2*, up to *level m*.

We call *leaves* of the graph-tree, all terminal nodes (nodes that have no descendants).

Then the TreeSoft Set is:

$$F : P(Tree(A)) \rightarrow P(H).$$

$Tree(A)$ is the set of all nodes and leaves (from *level 1* to *level m*) of the graph-tree, and $P(Tree(A))$ is the powerset of the $Tree(A)$.

All node sets of the *TreeSoft Set of level m* are:

$$Tree(A) = \{A_{i_1} \mid i_1 = 1, 2, \dots\} \cup \{A_{i_1, i_2} \mid i_1, i_2 = 1, 2, \dots\} \cup \{A_{i_1, i_2, i_3} \mid i_1, i_2, i_3 = 1, 2, \dots\} \cup \dots \cup \{A_{i_1, i_2, \dots, i_m} \mid i_1, i_2, \dots, i_m = 1, 2, \dots\}$$

The first set is formed by the *nodes of level 1*, second set by the *nodes of level 2*, third set by the *nodes of level 3*, and so on, the last set is formed by the *nodes of level m*.

If the graph-tree has only two levels ($m = 2$), then the TreeSoft Set is reduced to a MultiSoft Set.

15. Example of TreeSoft Set of Level 3

Node of *level 0* (the graph-tree root): A .

Nodes of *level 1*: A_1, A_2 .

Nodes of *level 2*: $A_{11}, A_{12}; A_{21}, A_{22}$.

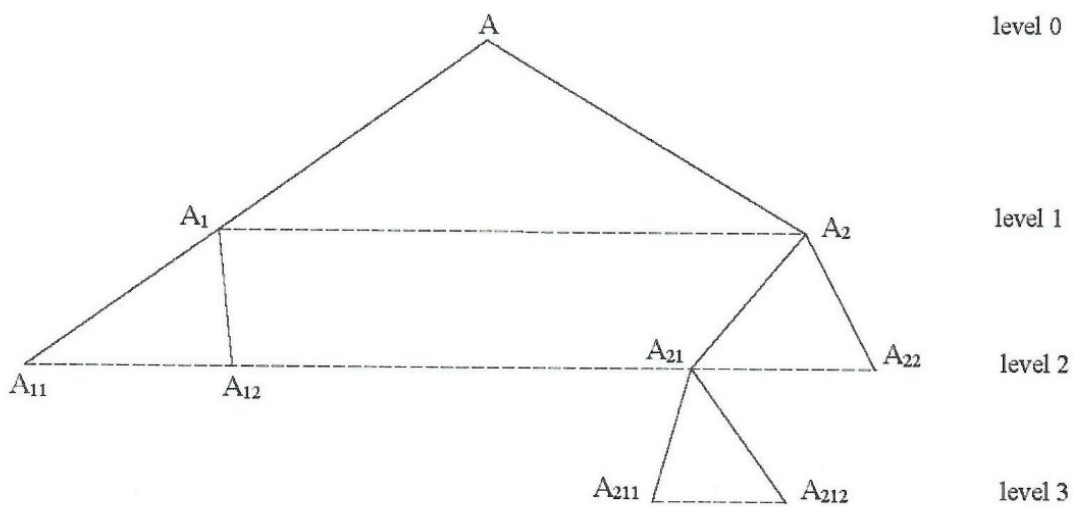
Nodes of *level 3*: A_{211}, A_{212} .

Whence $Tree(A) = \{A_1, A_2; A_{11}, A_{12}; A_{21}, A_{22}; A_{211}, A_{212}\}$.

The *leaves* are: $A_{11}, A_{12}; A_{211}, A_{212}; A_{22}$. As we see, the leaves may have various levels, in this case: 2, or 3.

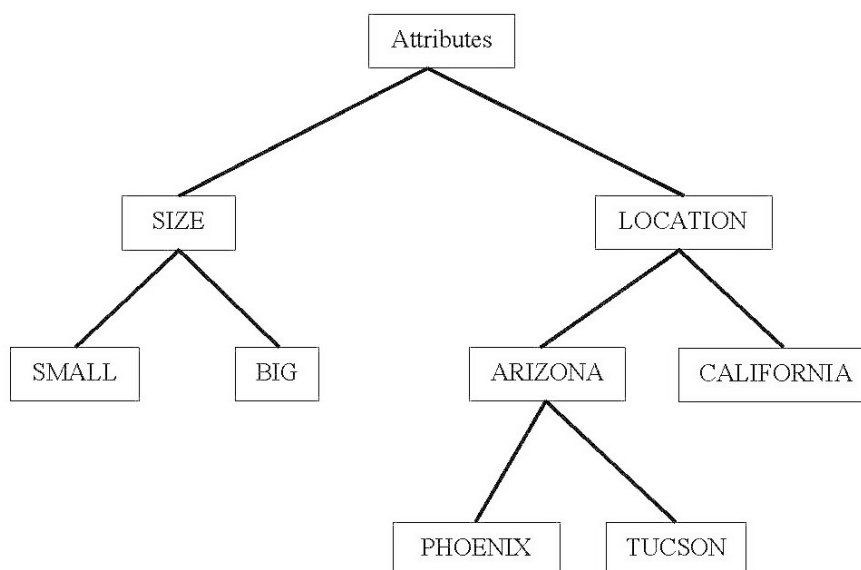
$P(Tree(A))$ is the powerset of $Tree(A)$.

$F : P(Tree(A)) \rightarrow P(H)$ is a TreeSoft Set of Level 3.



Graph 1: TreeSoft Set of Level 3

16. Practical Example of TreeSoft Set of Level 3



Graph 2: Practical TreeSoft Set of Level 3

et’s consider $H = \{h_1, h_2, \dots, h_{10}\}$ be a set of houses, and $P(H)$ the powerset of H .

And the set of attributes: $A = \{A_1, A_2\}$,

where $A_1 = size$, $A_2 = location$.

Then $A_1 = \{A_{11}, A_{12}\} = \{small, big\}$.

$A_2 = \{A_{21}, A_{22}\} = \{Arizona, California\}$, American states.

Further on, $A_{21} = \{A_{211}, A_{212}\} = \{Phoenix, Tucson\}$, Arizonian cities.

Let’s assume that the function F gets the following values:

$$F(big, Arizona, Phoenix) = \{h_9, h_{10}\}$$

$$F(big, Arizona, Tucson) = \{h_1, h_2, h_3, h_4\}$$

$$F(big, Arizona) = \text{all big houses from both cities, Phoenix and Tucson,}$$

$$= F(big, Arizona, Phoenix) \cup F(big, Arizona, Tucson) = \{h_1, h_2, h_3, h_4, h_9, h_{10}\}.$$

17. Properties of the TreeSoft Set

17.1. Theorem 1

$F(node)$ includes all node’s descendants, and sub-descendants, then sub-sub-descendants, and so on up to the corresponding leaves.

From previous Example 15, one has:

$$F(A_{21}) = F(A_{211}) \cup F(A_{212}),$$

and consequently

$$F(A_{12}, A_{21}) = F(A_{12}, A_{211}) \cup F(A_{12}, A_{212}).$$

17.2. Theorem 2

Let $N \in Tree(A)$ be a node.

N generates a $SubTree(N)$ whose root is N itself.

$$\text{Then } F(N) = \bigcup_{\varphi(i)} F(N_{\varphi(i)})$$

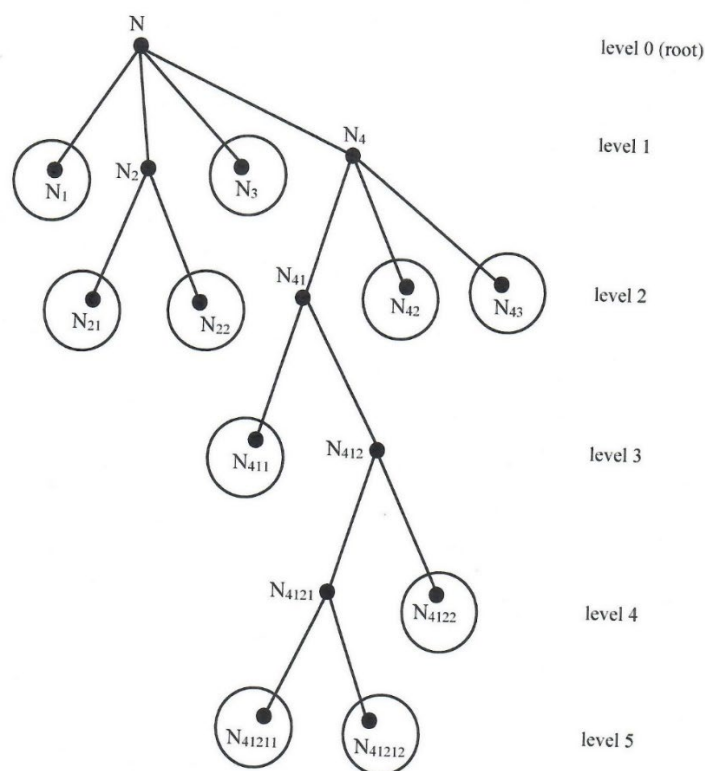
where $N_{\varphi(i)}$ are all leaves of the $SubTree(N)$.

From previous Example 15:

$$F(A_2) = F(A_{21}) \cup F(A_{22}) = (F(A_{211}) \cup F(A_{212})) \cup F(A_{22}) = F(A_{211}) \cup F(A_{212}) \cup F(A_{22})$$

where A_{211}, A_{212}, A_{22} are all leaves of the SubTree whose root is A_2 {i.e. $SubTree(A_2)$ }.

The proof of Theorem 2 is obvious, no matter what graph-tree one has, and it is similar to the below Example:



Graph 3: Tree(N)

The circled nodes are the leaves.

$$\begin{aligned} F(N) &= F(N_1) \cup F(N_2) \cup F(N_3) \cup F(N_4) \\ &= F(N_1) \cup [F(N_{21}) \cup F(N_{22}) \cup F(N_3) \cup F(N_{41}) \cup F(N_{42}) \cup F(N_{43})] \\ &= F(N_1) \cup F(N_{21}) \cup F(N_{22}) \cup F(N_3) \cup [F(N_{411}) \cup F(N_{412})] \cup F(N_{42}) \cup F(N_{43}) \\ &= F(N_1) \cup F(N_{21}) \cup F(N_{22}) \cup F(N_3) \cup F(N_{411}) \cup [F(N_{4121}) \cup F(N_{4122})] \cup F(N_{42}) \cup F(N_{43}) \\ &= F(N_1) \cup F(N_{21}) \cup F(N_{22}) \cup F(N_3) \cup F(N_{411}) \cup F(N_{41211}) \cup F(N_{41212}) \cup F(N_{42}) \cup F(N_{43}) \end{aligned}$$

which is the union of the soft-values $F(.)$ of all leaves of the $SubTree(N)$.

Actually Theorems 1 and 2 are equivalent.

17.3. Theorem 3

$$F(N_{i_1}, N_{i_2}, \dots, N_{i_p}) = F(N_{i_1}) \cap F(N_{i_2}) \cap \dots \cap F(N_{i_p}),$$

where $N_{i_1}, N_{i_2}, \dots, N_{i_p}$ are nodes of various levels into the TreeSoft Set of N .

The proof results from the fact that $F(N_{i_1})$ represents the subset H_1 of elements in H that have the attribute-value N_{i_1} , and $F(N_{i_2})$ represents the subset H_2 of elements in H that have the attribute-value N_{i_2} , and so on $F(N_{i_p})$ represents the subset H_p of elements in H that have the attribute-value N_{i_p} , therefore to get the elements that have all these attribute-values one needs to intersect these subsets $H_1 \cap H_2 \cap \dots \cap H_p$.

18. Future Research

To define the operations (complement, intersection, union) for IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set respectively and to use them in real applications.

19. Conclusion

We introduced the TreeSoft Set as an extension of the MultiSoft Set. We presented simple practical applications of IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set respectively for better understanding.

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