# Structure, NeutroStructure, and AntiStructure in Science

**Florentin Smarandache**

University of New Mexico
Mathematics, Physical and Natural Science Division
705 Gurley Ave., Gallup, NM 87301, USA
E-mail: smarand@unm.edu

**Abstract**

In any science, a classical Theorem, defined on a given space, is a statement that is 100% true (i.e. true for all elements of the space). To prove that a classical theorem is false, it is sufficient to get a single counter-example where the statement is false.

Therefore, the classical sciences do not leave room for *partial truth* of a theorem (or a statement). But, in our world and in our everyday life, we have many more examples of statements that are only partially true, than statements that are totally true.

The NeutroTheorem and AntiTheorem are generalizations and alternatives of the classical Theorem in any science.

More general, by the process of NeutroSophication, we have extended any classical *Structure*, in no matter what field of knowledge, to some *NeutroStructur*e, and by the process of AntiSophication to some *AntiStructure*.

Keywords : Structure, NeutroStructure, and AntiStructure

## 1. The Neutrosophic Triplet *(<A>, <neutA>, <antiA>)*

Let *S* be a given non-empty space (or set) from a universe of discourse *U*.

In neutrosophy, the general neutrosophic triplet *(<A>,<neutA>,<antiA>),* sometimes using the notation *<neutroA>* for the middle term, can be written as:

$(\langle A(1,0,0)\rangle, \langle A(T,I,F)\rangle, \langle A(0,0,1)\rangle)$, where $(T,I,F) \notin \{(1,0,0),(0,0,1)\}$;

i.e. $A(1,0,0)$ means that <A> is 100% true (T = 1), 0% indeterminate (I = 0), and 0% false (F = 0);

$A(T,I,F)$ means that <A> is T% true, I% indeterminate, and F% false, where $(T, I, F) \notin \{(1,0,0),(0,0,1)\}$;

and $A(0,0,1)$ means that <A> is 0% true (T = 0), 0% indeterminate (I = 0), and 100% false (F = 1), respectively.

## 2. Example of Neutrosophic Triplet when <A> = Operation

In the case when <A> is an *Operation* (or Operator, Function, Law), on the given space *S*, then <A(T, I, F)> means:

Operation <A> is T% well-defined (or inner-defined, inside of S),

I% indeterminate-defined (undefined, unknown),

and F% outer-defined (outside of S).

## 3. Neutrosophic Triplet Concepts

We have the following particular neutrosophic triplets of notions defined on *S*:

*(<Theorem>, <NeutroTheorem>, <AntiTheorem>),*
*(<Lemma>, <NeutroLemma>, <AntiLemma>),*
*(<Consequence>, <NeutroConsequence>, <AntiConsequence>),*
*(<Proposition>, <NeutroProposition>, <AntiProposition>),*
*(<Definition>, <NeutroDefinition>, <AntiDefinition>),*
*(<Property>, <NeutroProperty>, <AntiProperty>),*
*(<Function>, <NeutroFunction>, <AntiFunction>),*
*(<Operation>, <NeutroOperation>, <AntiOperation>),*
*(<Axiom>, <NeutroAxiom>, <AntiAxiom>),* etc.

These neutrosophic triplets are referred to *any field of knowledge*, not only to mathematics.

### 4.  Theorem, NeutroTheorem, AntiTheorem

Let's take the first neutrosophic triplet:
*(<Theorem>, <NeutroTheorem>, <AntiTheorem>).*
For the other neutrosophic triplets, it will be similar.
Let $T, I, F \in [0, 1]$ be single-valued numbers representing respectively the degree of truth (T), degree of indeterminacy (I), and degree of falsehood (F).

(i)    A *classical Theorem* is a statement that is true (T) for all elements of the space S. Therefore, (T, I, F) = (1, 0, 0).
(ii)   A *NeutroTheorem* is a statement that is partially (T), partially indeterminate (I), and partially false (F), where (T, I, F) $\notin \{(1, 0, 0), (0, 0, 1)\}$;
(iii)  An *AntiTheorem* is a statement that is false (F) for all elements of the space *S*. Therefore, (T, I, F) = (0, 0, 1).

We can rewrite this neutrosophic triplet as:
$(\langle Theorem(1, 0, 0)\rangle, \langle Theorem(T, I, F)\rangle, \langle Theorem(0, 0, 1)\rangle)$,
where (T, I, F) $\notin \{(1, 0, 0), (0, 0, 1)\}$.

Let T, I, F be intervals (and in general any subsets) from [0, 1].
If a Theorem is, let's say, between 90%-100% true, i.e. Theorem([0.9, 1], 0, 0), it does not satisfy the classical Theorem(1, 0, 0), since there is some uncertainty (unclearness) with respect to its degree of truth [0.9, 1] $\neq$ 1. So, this case goes under NeutroTheorem.

If the Theorem is, let's say again, between 99%-100% false, i.e. Theorem(0, 0, [0.99, 1]), it does not satisfy the AntiTheorem, since similarly there is some uncertainty (unclearness) with respect to its degree of falsehood [0.99, 1] $\neq$ 1. This case goes also under NeutroTheorem.

In conclusion, no matter if T, I, F are single-valued numbers, intervals, and in general any subsets of [0, 1], the neutrosophic triplet of each concept is the same.

*Classical Theorem = <Theorem(1, 0, 0)>*
*NeutroTheorem = <Theorem(T, I, F)>, with (T, I, F)* $\notin \{(1, 0, 0), (0, 0, 1)\}$*;*
*AntiTheorem = <Theorem(0, 0, 1)>.*

### 5.  Remark

Let $T, I, F \in [0, 1]$ and an axiom $\langle A(T, I, F)\rangle$, which means that the axiom has the neutrosophic degree of truth *T*, the neutrosophic degree of indeterminacy *I*, and the neutrosophic degree of falsehood *F*.
If $I > 0$ or $0 < F < 1$, the $\langle A\rangle$ is a NeutroAxiom.

***Proof:***
If $I > 0$, then $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$, because the last two neutrosophic triplets have both $I = 0$.

If $0 < F < 1$, then again $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$ because the last two neutrosophic triplets have $F = 0$ and respectively $F = 1$.

## 6. Elementary Examples of NeutroConcepts and AntiConcepts

### 6.1. NeutroOperation

Let $Z$ be the set of integers, and $Q$ the set of rational numbers that is considered the universal set, with $Z \subset Q$.
Let's define the operation of division:
$$\div : Z \times Z \longrightarrow Z.$$
This is a *NeutroDivision* because:
(i) There exist the integers $15, 5 \in \mathbb{Z}$ such that $15 \div 5 = 3 \in Z$; this is *degree of well-defined (inner-defined)*;
(ii) There exist the integers $7, 0 \in \mathbb{Z}$ such that $7 \div 0 = $ undefined; this is *degree of indeterminacy*;
(iii) There exist the integers $11, 2 \in \mathbb{Z}$ such that $11 \div 2 = 5.5 \notin Z$ or $5.5 \in Q \setminus \mathbb{Z}$; this is *degree of outer-defined*.

### 6.2. AntiOperation

Let $Z^- = \{-1, -2, -3, \dots, -\infty\}$ the set of negative integers,
$\mathbb{C} = \{a + bi; a, b \in \mathbb{R}, i = \sqrt{-1}\}$ the set of complex numbers that acts as the universal set of $Z^-$.
Let's define the operation of square root ($\sqrt{}$):
$\sqrt{}: Z^- \longrightarrow Z^-$.
For any negative integer $-a$, where $a > 0$ is a positive integer,
$\sqrt{-a} = i\sqrt{a} \notin \mathbb{Z}^-$, or $\sqrt{-a} \in C - Z^-$.
Therefore the operation $\sqrt{}$ is *totally outer-defined*.

### 6.3. NeutroFunction

On the set of integers $\mathbb{Z}$ and its universe of discourse $Q$, which is the set of rational numbers, we define the function:
$$f: \mathbb{Z} \longrightarrow \mathbb{Z}, f(x) = \frac{12}{x}.$$

(i) There exists the integer, for example $x = 6$, such that:
$f(6) = \frac{12}{6} = 2 \in \mathbb{Z}$;
this is *degree of well-defined*.

(ii) There exists the integer $0 \in \mathbb{Z}$, such that:
$f(0) = \frac{12}{0} = undefined$;
this is *degree of indeterminacy*.

(iii) There exists the integer, for example $5 \in \mathbb{Z}$, such that:
$f(5) = \frac{12}{5} = 2.4 \notin \mathbb{Z}$; or $2.4 \in Q \setminus \mathbb{Z}$;
this is *degree of outer-defined*.

### 6.4. NeutroTheorem

Let $Z^+ = \{1, 2, 3, \dots, +\infty\}$ be the set of positive integers.
We consider the following:
*Statement*
If $x$ and $y \in \mathbb{Z}^+$, then $x^y$ is a perfect square.

In classical algebraic structures this statement is considered false, because it is not 100% true, and to prove it, it is sufficient to get a single particular counter-example. For example, if $x = 2$ and $y = 3$, then $2^3 = 8$ which is not a perfect square.

The classical algebraic structures do not leave room for *partial truth* of a theorem. But, in our world and in our everyday life, we have many more examples of statements that are only partially true, than statements that are totally true.

The NeutroTheorem and AntiTheorem are generalizations and alternatives of the classical Theorem.

In the above statement, we have:

(i)    *Degree of truth*, when $x = a^2$ or $y = 2b$, where $a, b \in \mathbb{Z}^+$. Since we get $x^y = (a^2)^y = (a^y)^2$ and respectively $x^y = x^{2b} = (x^b)^2$.

Therefore we have two double-infinity many cases when the statement is true.

$x = a^2$ means that $x$ can be written as a perfect square. For example, if $x = 3^8$ we can re-write it as $x = (3^4)^2 = 81^2$.

And $y = 2b$ means $y$ is an even number.

(ii)    *Degree of indeterminacy* is zero, since $x^y$ is always well-defined for non-zero $x, y \in \mathbb{Z}^+$.

(iii)    *Degree of falsehood*, as shown above, for example when $x = 2$ and $y = 3$.

Herein we also have infinitely many cases when the statement is false (for example when $x \neq a^2$ and $y \neq 2b$).

## 7. Structure, NeutroStructure, AntiStructure in any field of knowledge

In general, by NeutroSophication [1, 2, 3, 4], we have extended any classical *Structure*, in no matter what field of knowledge, to *NeutroStructure*, and by AntiSophication to *AntiStructure*.

A classical Structure, in any field of knowledge, is composed of: a non-empty <u>space</u>, populated by some <u>elements</u>, and both (the space and all elements) are characterized by some <u>relations</u> among themselves (such as: laws, operations, operators, axioms, properties, functions, theorems, lemmas, consequences, algorithms, charts, hierarchies, equations, inequalities, etc.), and their <u>attributes</u> (size, weight, color, shape, location, etc.).

## 8. Relation, NeutroRelation, AntiRelation

(i) A classical **Relation** on a given set is a relation that is true for all elements of the set (degree of truth T = 1). Neutrosophically we write *Relation(1,0,0)*.

(ii) A **NeutroRelation** is a relation that is true for some of the elements (degree of truth T), indeterminate for other elements (degree of indeterminacy I), and false for the other elements (degree of falsehood F). Neutrosophically we write *Relation(T,I,F),* where (T,I,F) is different from (1,0,0) and from (0,0,1).

(iii) An **AntiRelation** is a relation that is false for all elements (degree of falsehood F = 1). Neutrosophically we write *Relation(0,0,1)*.

## 9. Attribute, NeutroAttribute, AntiAttribute

(i) A classical **Attribute** of the elements of a given set is an attribute that is true for all elements of the set (degree of truth T = 1). Neutrosophically we write *Attribute(1,0,0)*.

(ii) A **NeutroAttribute** is an attribute that is true for some of the elements (degree of truth T), indeterminate for other elements (degree of indeterminacy I), and false for the other elements (degree of falsehood F). Neutrosophically we write *Attribute(T,I,F),* where (T,I,F) is different from (1,0,0) and (0,0,1).

(iii) An **AntiAttribute** is an attribute that is false for all elements (degree of falsehood F = 1). Neutrosophically we write *Attribute(0,0,1)*.

## 10. Definitions of Structure, NeutroStructure, AntiStructure

(i) A classical **Structure** is a structure whose all elements are characterized by the same given Relationships and Attributes.

(ii) A **NeutroStructure** is a structure that has at least one NeutroRelation or one NeutroAttribute, and no AntiRelation nor AntiAttribute.

(iii) An **AntiStructure** is a structure that has at least one AntiRelation or one AntiAttribute.

### 11. Example of NeutroStructure

In the Christian society the marriage is defined as the union between a male and a female (*degree of truth*).

But, in the last decades, this law has become less than 100% true, since persons of the same sex were allowed to marry as well (*degree of falsehood*).

On the other hand, there are transgender people (whose sex is not well-determined, or whose sex is undetermined), and people who have changed the sex by surgical procedures, and these people (and their marriage) cannot be included in the first two categories (*degree of indeterminacy*).

Therefore, since we have a NeutroLaw (with respect to the Law of Marriage) we have a Christian NeutroStructure.

### Conclusion

A classical Structure, in any field of knowledge, is composed of: a non-empty <u>space</u>, populated by some <u>elements</u>, and both (the space and all elements) are characterized by some <u>relations</u> among themselves, and by some <u>attributes</u>. Classical Structures are mostly in theoretical, abstract, imaginary spaces.

Of course, when analysing a structure, it counts with respect to what relations and attributes we analyse it.

In our everyday life almost all structures are NeutroStructures, since they are neither perfect nor uniform, and not all elements of the structure's space have the same relations and same attributes in the same degree (not all elements behave in the same way).

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