Alethic Pluralism and Logical Consequence

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Abstract: It has been argued that alethic pluralists—who hold that there are several distinct truth properties—face a problem when it comes to defining validity. Via consideration of the classical concept of logical consequence, and of strategies for defining validity in many-valued logics, this paper proposes two new kinds of solution to the problem.

Keywords: alethic pluralism, truth pluralism, mixed inferences, validity, logical consequence, truth preservation, logical form, many-valued logic

1   Introduction

What does truth consist in: correspondence, coherence, warranted assertibility...? Alethic pluralists think that there is not just one correct answer to this question, but that truth consists in different things for different kinds of claims: e.g. provability for mathematical claims, coherence for ethical claims, and correspondence for empirical claims.[2]

An influential objection to alethic pluralism is that there is no adequate definition of validity (logical consequence) that is compatible with the pluralist position. In this paper I respond to this objection. My interest is not so much in defending alethic pluralism as in elucidating the notion of consequence: seeing what is wrong with the objection reminds us of some points about this notion that are of general importance.

The objection to pluralism was presented by Tappolet (1997, 209–10):

> Consider the following inference: (1) Wet cats are funny. (2) This cat is wet. *Ergo*, this cat is funny. The validity of an inference

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[2]There is a variety of sometimes subtly but significantly different alethic pluralist views. For maps of the literature see e.g. Pedersen (2012) and Pedersen and Wright (2013).

requires that the truth of the premises necessitates the truth of the conclusion. But how can this inference be valid if we are to suppose... that two different kinds of truth predicates are involved in these premises? For the conclusion to hold, some unique truth predicate must apply to all three sentences. But what truth predicate is that? And if there is such a truth predicate, why isn't it the only one we need? [¶] Mixed inferences remind us of a central platitude about truth, namely that truth is what is preserved in valid inferences. Moreover, they show that all sentences which can appear in such inferences are assessable in terms of the same truth predicate. The upshot is that only a truth predicate shared by all sentences which can appear in inferences will satisfy the platitude relating truth to inferences.

The problem arises with *mixed inferences*, where at least two of the component propositions are (as the pluralist sees it) in the domains of different truth properties. Some such inferences appear to be valid and some do not. The challenge for the pluralist is to define validity in such a way as to maintain these appearances, without departing (too radically) from the classical understanding of validity as involving necessary truth preservation.

A natural thought for the pluralist is to turn to many-valued logics (MVL) for leads on how to define validity when we have multiple kinds of truth—for the presence of multiple truth values is certainly no object to giving reasonable definitions of validity in MVL. There are at least three standard ways of defining validity in MVL (this list is not exhaustive—these are the most common options): (1) Pick a single one of the truth values and define validity in terms of preservation of that value. (2) Pick a subset of the truth values as *designated* values and define validity in terms of preservation of designatedness. (3) Specify an ordering on the truth values and define validity in terms of that ordering. I shall discuss these options in the order (2)–(3)–(1) in sections 2, 3 and 4 respectively. My discussion of option (2) will be brief because this option has already been proposed in the literature as a model for a pluralist definition of validity—whereas the idea of modelling such a definition on options (1) or (3) is new.

2 Designated values

Beall (2000, 382) writes:

> In the jargon of many-valued logic, validity is to be understood in terms of *designated values*, these being the different ways of being true, as it were. Specifically, an argument is valid iff (necessarily) if all the premises are designated, then the conclusion is designated.

This proposal faces a dilemma however. Designatedness is a *generic* property: it can be possessed by claims from (what the pluralist sees as) different alethic domains (e.g. the component propositions in a mixed inference). If designatedness is a truth property (a kind of truth) then we have no solution to the problem of defining validity for *strong* alethic pluralists, who hold that there are no generic truth properties or that all truth properties are domain-specific. If designatedness is not a kind of truth then we have departed too far from the classical understanding of validity as involving necessary truth preservation.

Rather than discuss these issues further, I shall move on to options (1) and (3)—with the aim of defining validity in a way acceptable even to strong alethic pluralists.

3 Ordering the values

On option (3) we define on ordering on the truth values and say that an argument $\alpha_1, \ldots, \alpha_n / \therefore \beta$ (with premises α_1 through α_n and conclusion β) is valid iff

> (R) there is no model on which the truth value of the conclusion is less than the truth values of all the premisses

where 'less than' refers to the ordering. E.g. consider fuzzy logics in which propositions are assigned as truth values real numbers between 0 and 1 inclusive. One standard way of defining fuzzy consequence is to order the truth values in the usual way and apply recipe (R); this kind of consequence relation is sometimes referred to as 'no drop' or 'salvo gradu' consequence.[3]

In the context of alethic pluralism, *how* we might order the truth values (kinds of truth—and falsity) will depend on how many kinds of truth we suppose there to be and on why we suppose there to be these multiple kinds of truth (i.e. what the various truth values represent and what work they are supposed to do). Note that nothing in recipe (R) requires the truth values

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[3]For further details see Font (2003) and Smith, N. J. J. (2015, 1265–6).

to be ordered in a particular way (e.g. linearly ordered rather than partially ordered). If we have (e.g.) three kinds of truth and one kind of falsity, we might want to (but do not have to) order them in any of the following sorts of way (where an arrow from x to y indicates that x is less than y and the less-than relation is irreflexive and transitive):
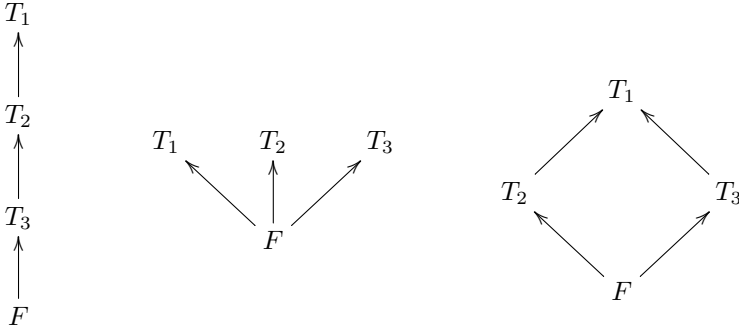
In general, facts about the ordering of the truth values—and facts about how the logical operators behave when given as inputs propositions with various different kinds of truth value—will affect the particular kind of logic that we get when we define consequence in terms of recipe (R). Our concern at present, however, is simply the conceptual task of defining validity. My point is that once we have an ordering on the kinds of truth, we can define validity in terms of preservation of this ordering.[4]

This recipe for defining validity provides a natural implementation of the classical idea of consequence as involving *preservation of truth*. In the classical context, a valid argument has the property that the premises cannot be true without the conclusion being true. There are two interpretations of the idea that this involves 'preserving truth'. One is that there is a single value—truth—such that if all the premises have it then the conclusion has it. The

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[4]Cotnoir (2013) (which I discovered after formulating the arguments of this paper) also presents a definition of validity for alethic pluralists that makes use of an ordering on truth values. Cotnoir commits himself to an algebraic semantics in which the truth values are n-tuples, each *component* of which corresponds to one of the kinds of truth countenanced by the pluralist. In my proposal, by contrast, the various truth values simply are (or represent directly) the various kinds of truth and falsity (or in §3.1 below, the various possible truth statuses) and we define an ordering on them directly. Cotnoir's proposal is therefore unnecessarily complex and involves unnecessary additional commitments—e.g. to a certain form of algebraic semantics—and so is dialectically less effective than my proposal: it gives opponents of alethic pluralism additional targets at which to aim. A similar point applies to Pedersen (2006), who presents a definition of validity for alethic pluralists that involves a commitment to plural quantification.

other is that the conclusion is never less true than the premises (where falsity is thought of as less true than truth). Both interpretations are acceptable. They coincide in the classical case—so consideration of that case cannot be used to favour one of them. They can come apart, however, in the context of MVL or alethic pluralism—and my proposal in this section is that pluralists can define a valid argument as one that is truth preserving in the second sense: the conclusion can never be less true than all the premises.[5] This is not to say that the first interpretation of 'preserving truth'—involving a single value that must be preserved—has to be abandoned in the context of alethic pluralism: we shall return to it in §4. Tappolet writes (recall §1) that "only a truth predicate shared by all sentences which can appear in inferences will satisfy the platitude relating truth to inferences". The platitude is that valid inferences preserve truth. Contra Tappolet, it is only on one interpretation of this platitude that there must be a single value or kind of truth that is preserved from premises to conclusion. On a second and equally acceptable interpretation, the core point is that in a valid argument, the conclusion can never be less true than the premises.

3.1 Possessing multiple truth properties

Implicit in the approach of §3 is the assumption that each proposition has exactly one truth value (on each model). What if we have a pluralist view according to which a proposition may possess multiple truth properties at the same time? There are two approaches we can take in this case. One is to pursue a *relational* semantics, in which propositions may be associated with more than one truth value.[6] I shall focus here on a different approach, which is to model a situation in which propositions may possess multiple truth properties using a formal setup in which each proposition possesses exactly one truth value. This kind of approach is generally technically simpler and is widespread in MVL.[7] For example, it is very common to start with the idea that there are two truth values (Truth and Falsity) and some sentences may possess neither of them, and then model this formally using three truth values—one for each of the three possible truth statuses (as opposed to the two truth values) envisaged in the original motivating story: 'having the

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[5]By analogy, we might say that height is preserved across the generations in a given family tree if no child is shorter than both parents, rather than requiring that there be a single particular height that all the generations possess.

[6]Cf. e.g. the relational semantics for FDE in Priest (2008, Ch.8).

[7]See Smith, N. J. J. (2012b) on the distinction between many-valued semantics in the *strict* and *loose* senses.

value True' (T), 'having the value False' (F) and 'having no value' (N). In this case, a natural ordering of the values is as follows:

$$T$$
$$\uparrow$$
$$N$$
$$\uparrow$$
$$F$$

Similarly, if one starts with a motivating story that has two truth values and allows not only truth gaps but also gluts, then it is quite standard to model the situation formally using four truth values, one for each of the four envisaged truth statuses: 'having the value True' (T), 'having the value False' (F), 'having no value' (N) and 'having both values' (B). In this case, there are two natural ways of ordering these values (Belnap, 1977)—the truth ordering (on the left) and the knowledge ordering:
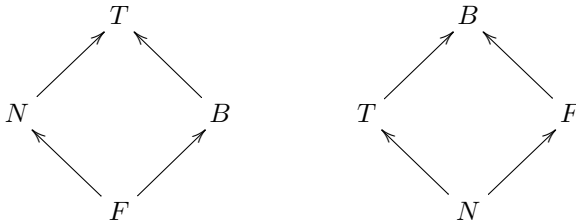

In the case of alethic pluralism, the story might go as follows. We posit (e.g.) three domain-specific truth properties and a generic truth property[8] and allow four possible statuses for propositions: having the first/second/third domain-specific truth property and the generic truth property (T_1^g/T_2^g/T_3^g) or having none of the truth properties (N). This is then modelled using a system that has four truth values—one for each of the four possible truth statuses (not one for each of the four truth properties). Because the four values correspond to the statuses (not the properties) in the original motivating story, it makes perfect sense for each proposition to possess exactly one of

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[8]Pluralists who posit a generic truth property have the option of defining validity in terms of preservation of this property—but the present example is merely an example: the general strategy illustrated here is applicable to any pluralist view according to which a proposition may possess multiple truth properties at the same time.

them: the statuses are mutually exclusive and jointly exhaustive (while the properties are such that a proposition might possess two of them—or none of them). The story then proceeds as in §3: we order the values and define consequence using (R). A natural ordering of the values might be:

$$T_1^g \qquad T_2^g \qquad T_3^g$$

$$N$$
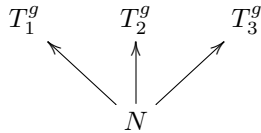
but of course this is just an example—we might want to have a different number of truth statuses/values and we might want to order them in different ways. None of this affects my point about the conceptual task of defining validity in terms of recipe (R), which requires only some truth values and an ordering on them.

4 Defining validity in terms of one value

One might think that a natural option for an alethic pluralist would be to define multiple notions of validity: one for each kind of truth, defined in terms of preservation of that kind of truth. This brings us to option (1): picking one of the truth values and defining validity in terms of preservation of that value—for if we can do this for one of the truth values then we can do it once for each of them (yielding multiple notions of validity, one for each kind of truth). Of course, the pluralist *may* prefer just to single out one notion of truth and define validity in terms of it (rather than having a plurality of kinds of validity to match the plurality of kinds of truth) or to define further notions such as 'valid in all (or some) of the senses corresponding to the kinds of truth *in a specified class*'—where a special case of this would be a notion of 'super validity': truth-preserving for every kind of truth. In any case, the issue now is whether a pluralist can pick a *single* non-generic notion of truth and define validity in terms of preservation of that kind of truth. Tappolet's thought seems to be that the pluralist cannot (recall §1):

> some unique truth predicate must apply to all three sentences.... Mixed inferences... show that all sentences which can appear in such inferences are assessable in terms of the same truth predicate.... only a truth predicate shared by all sentences which can appear in inferences will satisfy the platitude relating truth to inferences.

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Pedersen (2012, 591–2) spells out Tappolet's line of thought as follows:

> The inference [(1) If drunk driving is illegal, there are trees.
> (2) Drunk driving is illegal. ∴ (3) There are trees.]—an instance
> of modus ponens—is clearly valid [as standardly understood in
> terms of necessary truth preservation], but the pluralist seems
> to be unable to account for this. For what property is preserved
> in the inference? The truth of (2) is given by coherence, but for
> (3) truth is given by correspondence. So, neither coherence nor
> correspondence will do. But what property, then, is it? This is
> the problem of mixed inferences.

What precisely is the argument here? Here's one thought:

> (T1) You cannot define validity in terms of preservation of the
> value X if there are to be valid arguments in which the compo-
> nent propositions cannot all have the value X.

(T1) is false: witness the classical validity of the argument $A, \neg A / \therefore B$.
We define classical validity in terms of preservation of the value True—and
here is a valid argument in which the premises cannot all have this value.
However, each premise individually can have the value True—whereas in
a mixed inference, there is no notion of truth countenanced by the strong
pluralist such that each component proposition, taken individually, may be
true in that sense. So perhaps the thought is this:

> (T2) You cannot define validity in terms of preservation of the
> value X if there is to be a valid argument in which one of the
> component propositions cannot have the value X.

(T2) is also false: witness the classical validity of $A \wedge \neg A / \therefore B$. We define
classical validity in terms of preservation of the value True—and here is a
valid argument in which the premise cannot have this value.[9]

The idea that validity involves necessary truth preservation is often spelt
out with a conditional: necessarily, if the premises are true, then the con-
clusion is true. The key point to note here is that—at least as far as the

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[9]Someone might still feel that $A \wedge \neg A$ is "in the running" to be true (or truth-apt), it just
doesn't get there, so to speak—whereas 'Drunk driving is illegal' is not even in the running
(or apt) to be correspondence true. I leave it as a challenge for anyone who finds this thought
appealing to try to spell it out clearly and precisely and turn it into a cogent objection to the
strategy of the present section for defining validity in the context of alethic pluralism.

standard classical definition of validity is concerned—this is a material conditional. The idea is that necessarily, it is not the case that the premisses are true and the conclusion is not—or it is impossible for the premisses but not the conclusion to be true. It is not part of this conception of validity that it must be possible for the premisses (all) to be true. On the contrary, if the premisses cannot (all) be true, then the material conditional is (necessarily) true. So turning to strong alethic pluralism, the fact that in a mixed inference there is no notion of truth such that the component propositions can all be true in this sense does *not* immediately prevent us defining validity in terms of preservation of truth in this sense.

Of course, a problem looms. If we define validity as necessary preservation of truth of kind X then it seems that *every* inference involving a premise that can only possess truth of some *other* kind will automatically be valid. For example, if we define validity as necessary preservation of correspondence truth, then it seems that not only will Pedersen's argument above be valid, but so will a variant in which we replace premise (1) by 'If there are trees, drunk driving is illegal'—whereas surely this argument should be deemed *invalid*. This brings us to a further key point about the classical notion of validity. Consider these inferences: (A) 'The glass contains water. The glass does not contain H_2O. ∴. The glass contains water.' (B) 'The glass contains water. The glass does not contain H_2O. ∴. The glass contains H_2O.' In both cases it is impossible for the premisses both to be true (assuming water is necessarily H_2O). This does not however render both arguments classically valid: (A) is valid and (B) is not. This is so even though in (B) it is impossible for the first premise to be true and the conclusion false. But how can this be?—*if*, as many contributors to this debate claim, validity is necessary truth preservation:

> the Tarskian idea that validity is necessary truth-preservation (Beall, 2000, 381)
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> the classical account of validity, according to which an argument is valid on condition that the truth of the premises necessitates the truth of the conclusion (Tappolet, 2000, 383)
>
> the standard characterization of validity as necessary truth preservation (Cotnoir, 2013, 565).

Well, validity—on the classical conception—is not simply a matter of necessary truth preservation. For an argument to be valid it must be necessarily truth preserving and furthermore this fact must hold in virtue of the *form* of

the argument—i.e. it is not something about the subject matter of the argument that ensures that it is necessarily truth preserving (e.g. the premises talk about water and the conclusion talks about H_2O): it is simply the way the argument is put together that guarantees that the premises cannot be true and the conclusion false. Despite a recent tendency—in introductory logic textbooks, and in papers such as those quoted above—to introduce validity in terms of necessary truth preservation (alone), historically it was generally clear that the notion of validity requires more than this: it requires that the argument be necessarily truth preserving thanks to its form or structure. This view can be found in Tarski's seminal discussion of logical consequence, where it is presented as the traditional, intuitive conception:

> I emphasize...that the proposed treatment of the concept of consequence makes no very high claim to complete originality. The ideas involved in this treatment will certainly seem to be something well known...Certain considerations of an intuitive nature will form our starting-point. Consider any class K of sentences and a sentence X which follows from the sentences of this class. From an intuitive standpoint it can never happen that both the class K consists only of true sentences and the sentence X is false. Moreover, since we are concerned here with the concept of logical, i.e. *formal*, consequence, and thus with a relation which is to be uniquely determined by the form of the sentences between which it holds, this relation cannot be influenced in any way by empirical knowledge, and in particular by knowledge of the objects to which the sentence X or the sentences of the class K refer...The two circumstances just indicated...seem to be very characteristic and essential for the proper concept of consequence...(Tarski, 1956, pp.414–5)

and indeed the idea goes back to Aristotle.[10] Once we are clear that validity is a matter of necessary truth preservation *in virtue of form*, the apparent problem posed by arguments (A) and (B) disappears. Both arguments are necessarily truth preserving—it is impossible for the premises to be true and the conclusion false—but only argument (A) is so in virtue of its form; hence only (A) is valid.

The lesson carries over to the case of alethic pluralism and mixed inferences. Suppose we take a single kind of truth T_i—from the many counte-

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[10]For further discussion and references see Smith, N. J. J. (2012a, Ch.1, §1.4).

nanced by the pluralist—and define validity as follows: an argument is valid iff

> (P) in virtue of the form of the argument, it is impossible for the premisses all to be T_i while the conclusion is not T_i.

The fact that there can be arguments—mixed inferences—whose component propositions cannot (all) be T_i does not pose a problem for this definition. Some such arguments will be valid—those where the *form* of the argument guarantees that it is impossible to make all the premisses T_i while not making the conclusion T_i—and some of them will not be valid—those where this *is* possible *and* those where it is not possible *but* this impossibility holds in virtue of the particular content of the premisses and conclusion rather than in virtue of the form of the argument. For example, if we define validity as preservation of correspondence truth in virtue of form, then Pedersen's example argument is valid (the impossibility of the premisses but not the conclusion being correspondence true holds in virtue of the form of the argument, given that the form is 'If A then B, $A/ \therefore B$' and assuming that a correspondence true conditional cannot have a correspondence true antecedent without having a correspondence true consequent) while my variant of his example is not (the impossibility of the premisses but not the conclusion being correspondence true holds not in virtue of the form of the argument but in virtue of the content of the second premise—in particular its making a claim that falls in the domain of coherence truth).

Consider Tappolet's original example of the wet cats. The reason we think this argument is valid is because of its form: 'All A's that are B are C. This A is B. $\therefore$ This A is C.' In virtue of its form, it is impossible to make the premisses true without making the conclusion true. This holds whatever we mean by 'true'—assuming only that predication and quantification interact with truth in standard ways. It also holds whatever we put in for A, B and C. Thus, in particular, it holds even if we (a) employ a particular sense of 'true' (one out of the many countenanced by the pluralist) and (b) substitute for A, B and C in such a way that the premisses and the conclusion cannot all be true in this sense. The argument was already valid, in virtue of its form: putting in particular premisses with particular contents will not change this fact.[11]

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[11]One contribution to this literature which does mention the idea that validity has something to do with logical form is Cotnoir (2013)—but there are several problems with his discussion. First, Cotnoir's position is unclear. As quoted above, he first invokes "the standard characterization of validity as necessary truth preservation" [565]. He later notes that "logical consequence

So, pluralists can define a notion of validity for any notion of truth that they countenance: validity$_i$ is a matter of necessary preservation of truth$_i$ in virtue of form. Whether an argument is valid$_i$ depends on how the logical operators interact with truth$_i$. If the logical operators behave in the same ways with respect to two notions of truth (e.g. $\alpha \wedge \beta$ is T_1 iff α and β are both T_1, and likewise $\alpha \wedge \beta$ is T_2 iff α and β are both T_2, etc) then the two corresponding notions of validity will coincide (extensionally). If the logical operators do not behave in the same ways with respect to two notions of truth (e.g. $\alpha \wedge \beta$ is T_1 iff α and β are both T_1, but $\alpha \wedge \beta$ is T_2 iff either α and β are both T_2 or one of them is T_1 and the other is T_2) then the two corresponding notions of validity might not coincide. (To flesh out the example a little further: if we have two kinds of truth, T_1 and T_2, and two kinds of falsity, F_1 and F_2, and a conjunction operation $\wedge$ that interacts with them as follows:

$\wedge$	T_1	F_1	T_2	F_2
T_1	T_1	F_1	T_2	F_2
F_1	F_1	F_1	F_2	F_2
T_2	T_2	F_2	T_2	F_2
F_2	F_2	F_2	F_2	F_2

then the argument $\alpha \wedge \beta$ / $\therefore$ α will be valid$_1$, i.e. necessarily T_1 preserving in virtue of form—because the only way that $\alpha \wedge \beta$ can have the value T_1 is if α and β both have the value T_1—but not valid$_2$, because it is possible for $\alpha \wedge \beta$ to have the value T_2 while α does not have the value T_2: if α is T_1 and β is T_2 then $\alpha \wedge \beta$ is T_2.) This is not the place to explore such options further: the details will depend on how many notions of truth a pluralist

is a *formal* notion. Validity in formal logic is *independent* of content" [573]. He then continues: "Pluralists (even strong pluralists) are not barred from thinking that valid inference depends only on the logical *form* of an argument, and not on the content of the particular premises of an instance of an argument form." In fact, not only are they not barred from this—they (like everyone else) are required to think it, if they want to conform to the classical/Tarskian conception of validity. Second, Cotnoir's comments about form are brief and occur entirely within the context of responding to a particular objection to his algebraic definition of validity (discussed in n.4). Although he says that "arguments with the right sort of formal structure are valid *regardless* of whether the premises are interpreted as being from the same domain or entirely different domains. Whether an inference is 'mixed' or not has no effect on the question of its validity" [573], Cotnoir does not see that this sort of point leads to a stand-alone response to the problem of defining validity for alethic pluralism: i.e. the kind of response given in §4 of this paper—which is distinct from and independent of the response given in §3. Third, Cotnoir does not appreciate that the mixed nature of an inference *can* in certain circumstances show up at the level of form, and hence (contra the claim just quoted) have an effect on the question of validity: see §4.1 below.

countenances and on why these are countenanced (i.e. what the various truth values represent and what work they are supposed to do).[12]

4.1 Mixed inferences in virtue of form

So far we have considered mixed inferences of the kind mentioned in the literature—e.g. Tappolet's and Pedersen's examples. These inferences are mixed (with respect to some notion of truth T_i) in virtue of their *content*. Such inferences cannot pose a problem for the strategy for defining validity presented in §4, because whether an argument is valid (in the sense corresponding to T_i, i.e. necessarily T_i preserving in virtue of form) is a matter of the *form* of the argument. In order to determine whether an argument is valid, we need to look at its form—but once we have abstracted to the level of form, we have left behind the fact that the inference is mixed.

We can however get the fact that an argument is mixed to show up at the level of *form* by introducing certain kinds of logical operators.[13] For example, suppose that we have two kinds of truth, T_1 and T_2, and one kind of falsity, F. Suppose that the operators † and ‡ are defined so that †α only ever takes the values T_1 or F and ‡α only ever takes the values T_2 or F.[14] In that case the argument †A, ‡B / ∴ C will be both T_1 valid and T_2 valid: it is impossible *in virtue of the form of the argument* for both premises to be T_1 (without the conclusion being T_1) and similarly for T_2.

I take this to be an observation, not an objection to the strategy for defining logical consequence presented in §4. Consider some comparison cases. In fuzzy logic, it is quite standard to define consequence as necessary preservation of truth degree 1 (in virtue of form) and no problem is posed for this definition by the fact that we can define operators $\textcircled{n}$ such that $\textcircled{n}\alpha$ has degree of truth n if α does and otherwise has degree of truth 0—so that (e.g.) the argument $\textcircled{1}A$, $\textcircled{5}B$ / ∴ C is then valid. Likewise in classical logic, no problem is posed for the classical definition of consequence as necessary preservation of truth (in virtue of form) by the fact that we can define an op-

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[12]Another question for the pluralist is how to define soundness. Classically, an argument is sound if it is valid *and* all its premises are in fact true. There are many possibilities for a pluralist definition of soundness: 'valid' in the classical definition can be replaced by any of the notions of validity discussed above and 'true' can be replaced by any of the kinds of truth countenanced by the pluralist (in which case an inference whose premises are mixed with respect to the notion of truth employed will not be sound) or (e.g.) by a disjunction of some or all of them.

[13]Thanks to Dave Ripley for helpful discussion here.

[14]Beyond these facts, the particular details of the definitions of † and ‡ do not matter here.

erator $\perp$ such that $\perp\alpha$ is false whatever the value of α—so that the argument $\perp A/ \therefore B$ is then valid.[15]

5   Conclusion

I promised strategies for defining validity for alethic pluralists and generally useful reminders about the nature of consequence. Let me gather the reminders here. First, consequence certainly has something to do with 'truth preservation', but this has two equally acceptable interpretations, which coincide in the classical context: preserving a single truth value; or preserving height in an ordering. Second, the conditional often used to spell out the idea of truth preservation—necessarily, if the premises are true then the conclusion is true—is treated in practice as a material conditional: the key point is the impossibility of making the premises but not the conclusion true; there is no implication that the premises can all be true. Third, necessary truth preservation is not enough for validity: logical consequence is a matter of necessary truth preservation *in virtue of form*. Once we have these points clearly in view—and once we appreciate that MVL offers other options for defining logical consequence apart from the idea of preserving designated values—it becomes apparent that there are several workable strategies for defining validity in the context of alethic pluralism.

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[15]Of course some think that this very example—and/or its close relatives $\perp/ \therefore B$ (where $\perp$ here is a nullary operator, rather than a unary operator as above), $A \wedge \neg A/ \therefore B$ and $A, \neg A/ \therefore B$—*is* a problem for the classical definition of validity. Such considerations are however orthogonal to my point in §4, which is that the classical idea of defining validity as necessary preservation of a particular truth value X in virtue of form is not automatically vitiated by the positing of multiple kinds of truth (together with the idea that certain sentences can possess only certain kinds of truth). Given that the aim is to maintain the viability of a classical idea, certain kinds of dialetheists and others who *already* have problems with the classical approach will not find that those problems miraculously disappear when that approach is translated to the context of MVL or alethic pluralism—but they are of course free to try to translate their favoured fixes to these contexts.

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