THE JOURNAL OF PHILOSOPHY
VOLUME CII, NO. 8, AUGUST 2005

A PLEA FOR THINGS THAT ARE NOT QUITE ALL THERE: OR, IS THERE A PROBLEM ABOUT VAGUE COMPOSITION AND VAGUE EXISTENCE?

Although they might not express themselves in quite this way, nonphilosophers tend to think that *mereological composition is a vague matter*: sometimes it occurs, sometimes it does not, and sometimes it *sort of* occurs. For example, when I am building a boat, at first the timbers that I have acquired for the job do *not* jointly compose an entity; in the end they do—they compose the boat that I have built; and in between they *sort of* or *more or less* or *to some extent* compose an entity, which in turn *sort of* or *more or less* or *to some extent* exists—this entity being the boat I am building. This idea seems innocuous enough. However, the orthodox view among philosophers is that composition can never be a vague matter, because vague composition entails vague existence (the common-sense view agrees with this step), and vague existence is impossible if not nonsensical. Let us call the following the *orthodox argument*:[1]

O1: Whenever there is a case of vague composition, there is a case of vague existence.
O2: There are no cases of vague existence (because vague existence is at least impossible, if not nonsensical).
OC: Therefore there are no cases of vague composition.

* I am grateful to Ed Mares and Huw Price for their encouragement and support of this project. Versions of the paper were presented at the Annual Conference of the Australasian Association of Logic in November 2002, and at the Australian National University in January 2005; I thank the audiences on those occasions for their comments and questions. Finally, thanks to the anonymous referees for their detailed comments.

[1] We may attribute this argument to "Davore Lewder" (the fusion of David Lewis and Theodore Sider). The actual arguments of Lewis, *On the Plurality of Worlds* (New York: Blackwell, 1986), pp. 212–13, and Sider, *Four-Dimensionalism: An Ontology of Persistence and Time* (New York: Oxford, 2001), pp. 126–27, will be discussed in section VI below.

0022-362X/05/0208/381–421

The validity of the argument being beyond dispute, there are three approaches we can take: deny one (or both) of the premises, or accept the conclusion.

Accepting OC. If composition is never vague, we have three options: sharp boundaries, nihilism, and unrestricted composition. The *sharp boundaries* view has it that in a series of cases, each of which is extremely similar to its successor and/or predecessor in all respects relevant to whether or not composition occurs, there are adjacent cases such that in one, composition occurs, while in the other, it does not. For example, start at case 0 with a chair with *n* parts; move to case 1 by moving part 1 one nanometer from the others;[2] move to case 2 by moving part 2 one nanometer from the others; ... move to case $n + 1$ by moving part 1 another nanometer from the others; and so on.... Continue the series until all the parts are widely separated from one another. According to the sharp boundaries view, at some particular point in the series, the chair suddenly ceases to exist. *Nihilism* is the view that composition *never* occurs: there are no composite objects. *Unrestricted composition* is the view that composition *always* occurs: for any bunch of objects whatsoever, there exists an object that is composed of just the objects in the bunch. The sharp boundaries view has won few converts.[3] Nihilism is doing slightly better: once defended by Peter Unger[4] and rejected by almost everyone else, it has recently been reinvigorated by Gideon Rosen and Cian Dorr.[5] Unrestricted composition is doing best of all, having been famously and influentially defended by David Lewis, and more recently defended by (among others) Michael Rea[6] and Theodore Sider.

Three ways, then, to accept OC: and each horribly implausible! In philosophy, familiarity tends to breed acquiescence rather than contempt, but the reader should not need much reminding that each of these views is intuitively repulsive. During the demolition process, my house existed right up until the point when a particular board was removed, at which time it ceased to exist entirely?! My house does not exist at all (only timbers arranged house-wise)?! Not only my house exists, but also another object, composed of my house together with Phar Lap's heart and Australia II's winged keel?! I do not mean to suggest that these incredulous questions constitute any sort of *argument* against the views in question—I simply wish to remind the

[2] For ease of exposition, I assume the parts are numbered.

[3] See Sider, p. 124, for an argument against the view.

[4] "I Do Not Exist," in Michael C. Rea, ed., *Material Constitution: A Reader* (Lanham, MD: Rowman and Littlefield, 1997), pp. 175–90.

[5] "Composition as a Fiction," in Richard M. Gale, ed., *The Blackwell Guide to Metaphysics* (Malden, MA: Blackwell, 2002), pp. 151–74.

[6] "In Defense of Mereological Universalism," *Philosophy and Phenomenological Research*, LVIII (1998): 347–60.

reader that each of these views is prima facie absurd. So why has acceptance of OC become orthodox? Because so many philosophers have been convinced that they cannot have what pre-philosophical intuition desires: a vague restriction on composition. They have been convinced of this because they accept the orthodox argument. But can we deny one of the argument's premises?

Denying O1. Why think that vague composition entails vague existence? Well, suppose there are three teacups in the cupboard. Do they jointly compose a fourth entity, Cup? Suppose they do not. It is still logically possible that Cup exists, even though the three cups do not compose Cup; but once we abandon the claim that the cups compose Cup, we have no reason at all to believe in Cup any more. Cup seems to exist only to the extent that the cups compose Cup. If the cups *sort of* compose an entity Cup, this is not a matter of (a) there being four entities, with a vague relation holding between one of them and the other three; rather, it is a matter of (b) there being three definite entities, and one shady one which only *sort of* exists. Of course, (b) is not the only consistent way of thinking of matters in this area—(a) is a *consistent* option—but it is by far the most natural way. If we want to defend an intuitively satisfying view of composition, we should not reject O1.[7]

Denying O2. Can we then make sense of the idea of vague existence? The prevailing attitude among philosophers is that we cannot. The aim of this paper, however, is to show that we can: my aim is to *make good sense* of vague composition, vague existence, and the relations between them. I take the state of play to be as follows. What we would have *liked* to be able to say, when we came to philosophy, is that com-

[7] I am not aware of anyone who argues that vague composition does not entail vague existence. Michael Morreau, "What Vague Objects Are Like," this JOURNAL, XCIX, 7 (July 2002): 333–61, argues that *parthood* is vague, but that *neither* composition nor existence is vague. A referee commented that the link between vague composition and existence is not straightforwardly intuitive, for instead of saying that "insofar as the timbers sort of, or more or less, compose the boat, the boat sort of, or more or less, exists...one might as well say that insofar as the timbers sort of, or more or less, compose a boat, they sort of, or more or less, fall into the extension of the predicate 'boat'." Consider the claim that they (that is, the timbers) fall into the extension of the predicate 'boat'. This might mean one of two things. First, it might mean that each of the individual timbers falls into the extension of 'boat'. However, no one would be tempted to say this, so let us set this reading aside. Second, it might mean that the sum of the timbers falls into the extension of 'boat'. However, we cannot claim this—or deny it—unless we assume that the sum of the timbers exists. But ordinary persons will hesitate over precisely this question of whether the sum of the timbers exists at all, when the timbers are only partially assembled. It thus seems that the question of existence, and the question of predication, are not on a par. Ordinary persons will hesitate over both, but their hesitation over the existence question is first order (it is simply unclear, when the boat is half built, whether the timbers compose anything), whereas their hesitation over the predication question is second order (any answer to the question presupposes an affirmative answer to the existence question—a question over which they are already hesitating).

position is sometimes vague, and that when it is, existence is vague, too. However, as philosophers, we can see no way of making clear sense of this idea, and so we abandon it in favor of one or other way of accepting OC. But in fact, the original idea makes perfectly good sense—or so I shall argue here. My aim is to make this view a serious contender among theories of composition; as for why we should actually accept the view to be presented here, once it has been clearly stated and shown to make good sense, I take the reason to be simply that it is what we wanted all along, but thought we could not have![8] Of course, once the neglected view is on the table and receives serious consideration, it may turn out that there is some subtle reason why it should be rejected in favor of a view that has *less initial* appeal. But such considerations are premature here: for now, what I want to show is that—contrary to received wisdom—the neglected view *makes perfectly good sense.*

In what follows, I shall be employing fuzzy logic/set theory as my working framework for handling vagueness. I employ the fuzzy framework here because I think it is the correct framework for the treatment of vagueness.[9] This is a minority view among contemporary philosophers. To those who oppose this view, I respond first that I defend the fuzzy approach to vagueness at length elsewhere, and do not wish to repeat myself here, and second that everything that I do in this paper could also be done within the framework of a linguistic theory of vagueness—in particular, within the *degree-theoretic* form of supervaluationism.[10] I leave the details of such alternative formulations to readers who have sympathy with the positive views of this paper, but not with the background theory of vagueness employed here.

[8] Lewis himself admits as much, noting that "To restrict composition in accordance with our intuitions would require a vague restriction": he argues that we cannot have such a restriction; but he does not deny that a vague restriction is what intuition desires. Note that Lewis does not merely say that *if* we are to have a restriction, then intuitively, it should be a vague one: he also notes that intuitively, we want a restriction: "We are happy enough with mereological sums of things that contrast with their surroundings more than they do with one another; and that are adjacent, stick together, and act jointly. We are more reluctant to affirm the existence of mereological sums of things that are disparate and scattered and go their separate ways" (*op. cit.*, p. 212).

[9] More subtly, I think it is the best of the major current theories. In my "Vagueness and Blurry Sets," *Journal of Philosophical Logic*, XXXIII (2004): 165–235, I present a new theory of vagueness which is (in a way explained clearly in that paper) of the same abstract kind as the fuzzy theory, while being significantly different in its concrete details—and it is *this* theory that I think is the correct framework for the treatment of vagueness. The views of the present paper could be recast within this new theory of vagueness—but I do not do this here because the views of the present paper and the new theory of vagueness are not a package deal: as discussed further in the text below, one can develop the views of the present paper within a variety of background frameworks for handling vagueness.

[10] For this view, see Hans Kamp, "Two Theories about Adjectives," in Edward L. Keenan, ed., *Formal Semantics of Natural Language* (New York: Cambridge, 1975), pp.

Before presenting the details of my proposal, I shall present a sketch of the overall picture.

First thought: intuitively, there is no such thing as the mereological fusion of the Empire State Building (E) and the Sydney Opera House (S): there simply is no object which has E and S as parts, and which is such that any of its parts overlaps one of E or S. However, there *could be* such a thing: just transport S and attach it to E (in an architecturally coherent and structurally sound way) and such an object would exist. So the sum of E and S is a possible object, but not an actual one. More generally, for any bunch of objects at all, their mereological fusion is a *possible* object, but in general does not *actually* exist.

Second thought: when I build a boat, I begin by acquiring the timbers that I shall need, and I stack them in numerous piles in my shed, garage, and garden. At this stage, the timbers do not jointly compose a further object. When the job is complete, the timbers do compose a further object: my boat. In between, as the timbers are assembled and fastened together, they gradually come to compose a further thing, and this further thing gradually comes into existence.

Combining these two thoughts: (1) I start with a picture familiar from the semantics of quantified modal logic, in which we have an overall domain D of all possible objects, and a subset D_w of this domain for each possible world w, consisting of the things which exist *at world* w. I alter the picture as follows: D remains a crisp set, but each D_w becomes a *fuzzy* subset of D—this is where the idea of vague existence comes in (it can be a matter of degree whether a certain object exists at a certain world). (2) I distinguish *two* parthood relations: *notional* and *concrete*. Notional mereology is classical extensional mereology with unrestricted composition. In accordance with the first thought above, this operates on the background set D of all possible objects: for any bunch of possible objects, there is a possible object which is the (notional) fusion of that bunch. Concrete parthood, on the other hand, is vague. It has a lot to do with contact and adhesion. For example, a given plank is a *concrete* part of my boat to the extent that it is fastened to the rest of the boat. At first, the plank is not a concrete part of my boat at all, but as I tack it in position, then glue it, then

123–55; "The Paradox of the Heap," in Uwe Mönnich, ed., *Aspects of Philosophical Logic* (Boston: Reidel, 1981), pp. 225–77, see pp. 234–35; Lewis, "General Semantics," in his *Philosophical Papers*, Volume I (New York: Oxford, 1983), pp. 189–232, see pp. 228–29, and "Survival and Identity," in his *Philosophical Papers*, Volume I, pp. 55–77, see pp. 69–70. Briefly, the idea is this: start with the (ordinary) supervaluationist's admissible interpretations—thus accepting the fundamental supervaluationist idea that vagueness is a linguistic matter, an extreme form of ambiguity—but then instead of saying that a sentence is (super)true if it is true on all admissible interpretations (and so on), say that a sentence's degree of (super)truth is the size (as given by a normalized measure function) of the set of its admissible interpretations on which it is (classically) true.

screw it, and then the glue cures, it gradually becomes a concrete part of my boat to a greater and greater degree, until finally it is a definite concrete part. (3) A notionally composite object (an object in D with more than one notional part) fades into/out of concrete existence (at a given world) as its notional parts become concrete parts to a greater/lesser degree (at that world). Thus, consider ES, the notional fusion of E and S. It does not actually exist, but as S is moved to New York and placed on top of E, and the process of joining them progresses—that is, as S and E, the *notional* parts of ES, become *concrete* parts of ES to a greater degree—ES comes gradually into (actual, concrete) existence; in other words, it becomes a member of the domain of the actual world (which is a fuzzy subset of D) to a greater degree. Or again, consider the notional fusion B of the planks which will eventually make up my boat. Before work begins, B does not actually exist. As B's notional parts are assembled and fastened together—that is, as they become concrete parts of B to a greater degree—B comes gradually into actual, concrete existence, until in the end it is a definite member of the domain of the actual world.

The paper proceeds as follows. Sections I, II, and III present the core of my view: they discuss, respectively, vague existence, notional mereology, and concrete mereology. Section IV discusses how to extend the core view to handle mereologically variable objects (objects which change their parts over time). Sections V and VI respond to key objections to my view, and section VII briefly discusses some possible extensions and augmentations of the view. Finally, the appendix discusses some further technical details of my treatment of vague existence.

I. VAGUE EXISTENCE

One standard approach to the semantics of quantified modal logic involves an overall domain D of possible objects, and for each possible world w, a domain D_w consisting of the objects which exist in w. Put most basically, the way I propose to handle vague existence is to follow this approach, but make D_w a *fuzzy* subset of D, rather than a regular (classical, crisp) subset. This has the effect that we can say that some possible object (such as the boat I am building) exists to an intermediate degree in the actual world—and to an increasing degree as work progresses, until finally the boat exists definitely (utterly, fully, to degree 1) in the actual world.

I begin by recalling the classical modal model theory which I shall take as my starting point. We consider a standard first order language, with identity, and with standard modal vocabulary, including an existence predicate E.[11] A classical modal model $\mathcal{M}$ is a five-tuple $(W,R,$

[11] More precisely, the symbols of the language are: the unary propositional connectives $\neg$ and $\Box$, and the binary propositional connective $\vee$; the quantifier $\forall$; the parentheses (and); infinitely many variables $x_1, x_2, \ldots$; infinitely many predicate letters A^n_k, for

$D,Q,V\rangle$. W is a set of worlds. R is a binary relation on W (the accessibility relation). D is a set of objects (the domain of all possible objects). Q is a function which assigns to each $w \in W$ a subset D_w of D (D_w is the set of things that exist in world w). V is a valuation function which assigns to each n-ary predicate an extension (a set of n-tuples of members of D) in each world w.[12] We stipulate that in the case of the identity predicate, V assigns it the real identity relation on D (at every world), and in the case of the existence predicate, V assigns it D_w at w. A *value assignment* based on a model $\mathcal{M} = \langle W,R,D,Q,V\rangle$ is a function v which assigns a member of D to every variable x. $V_v^M(\alpha,w)$ is the truth value of the wf α at the world w in the model $\mathcal{M}$ when the individual variables are given the values assigned to them by v. This truth value is either 1 (true) or 0 (false). The truth value of any wf at any world is given by the following clauses:[13]

- $V_v^M(Px_1,...,x_n,w) = V^M(P,w)(v(x_1),...,v(x_n))$
  that is, $Px_1...x_n$ is true at w iff $\langle v(x_1),...,v(x_n)\rangle$ is in the extension of P at w.
- $V_v^M(\neg\alpha,w) = 1 - V_v^M(\alpha,w)$
  that is, $\neg\alpha$ is true at w iff α is false at w.
- $V_v^M(\alpha \vee \beta,w) = \bigvee\{V_v^M(\alpha,w), V_v^M(\beta,w)\}$
  that is, $\alpha\vee\beta$ is true at w iff α or β is true at w.
- $V_v^M(\Box\alpha, w) = \bigwedge\{V_v^M(\alpha,w'): \forall w' \text{ with } wRw'\}$
  that is, $\Box\alpha$ is true at w iff α is true at all worlds accessible from w.
- $V_v^M(\forall x\alpha,w) = \bigwedge\{V_\rho^M(\alpha,w): \forall\rho \text{ such that } \forall y \neq x, \rho(y) = v(y)\}$
  that is, $\forall x\alpha$ is true at w iff α is true at w relative to all assignments which differ from v at most in what they assign to x.

This semantics countenances a domain for each world, with each of these domains being a subset of an overall domain of possibilia; yet the quantifiers $\exists$ and $\forall$ range over the entire domain of possibilia, rather than being restricted, at each world, to the domain of that world. Although not a new combination of features—a system of this sort is presented, in favorable terms, by George Hughes and Max Cresswell[14]—

$n,k \geq 1$ (superscripts represent number of arguments; subscripts are index numbers); the binary identity predicate =; and the unary existence predicate E. Well-formed formulas are defined as follows: if $x_1,...,x_n$ are variables and P is an n-ary predicate then $Px_1,..., x_n$ is a wf; if $\mathcal{A}$ and $\mathcal{B}$ are wfs and y is a variable, then $(\neg\mathcal{A}),(\mathcal{A}\vee\mathcal{B}),(\Box\mathcal{A})$ and $(\forall y(\mathcal{A}))$ are wfs; nothing else is a wf. In practice, parentheses will be omitted when this will cause no confusion. The following are defined symbols: $\exists y\mathcal{A} := \neg\forall y\neg\mathcal{A}$; $\mathcal{A}\rightarrow\mathcal{B} := \neg\mathcal{A}\vee\mathcal{B}$; $\mathcal{A}\wedge\mathcal{B} := \neg(\neg\mathcal{A}\vee\neg\mathcal{B})$; $\Diamond\mathcal{A} := \neg\Box\neg\mathcal{A}$; $\Pi y\mathcal{A} := \forall y(Ey\rightarrow\mathcal{A})$ (actualist universal quantification); $\Sigma y\mathcal{A} := \exists y(Ey\wedge\mathcal{A})$ (actualist existential quantification).

[12] Formally, we can think of V as mapping each pair $\langle P,w\rangle$ of an n-ary predicate P and a world w, to the characteristic function of a subset of D^n, that is, to a function from the set of n-tuples of members of D to $\{0,1\}$.

[13] $\bigvee$ and $\bigwedge$ are, respectively, the supremum (least upper bound) and infimum (greatest lower bound) operations on the reals (they map sets of real numbers to real numbers).

[14] *A New Introduction to Modal Logic* (New York: Routledge, 1996), pp. 303–04.

it is not a common one. It is, however, a highly attractive combination. The simplest semantics for quantified modal logic has one overall domain of possibilia, over which the quantifiers $\exists$ and $\forall$ range, and has no separate subdomains for each world. Given some standard further details, this semantics validates the Barcan Formula ($\Diamond\exists x\Phi \rightarrow \exists x\Diamond\Phi$). Those with actualist tendencies find this objectionable, because they think (to borrow an example from Timothy Williamson[15]) that while it is true that Wittgenstein could have had a son ($\Diamond\exists xPx$), it is not true that there exists something which could have been Wittgenstein's son ($\exists x\Diamond Px$). Saul Kripke[16] presented models with a domain for each world, in which the quantifiers $\exists$ and $\forall$ range, at each world, only over the domain of that world. This semantics does not validate the Barcan Formula. However, this semantics does not genuinely satisfy the concerns underlying the actualists' opposition to the Barcan Formula—because the possibilia are still there in the model, and quantification over possibilia is a feature of the *metalanguage* in which the Kripke semantics is presented.[17] The fact that the object language quantifiers do not reach the mere possibilia is like a gag order preventing us talking about some toxic waste, rather than a proper clean-up job which actually gets rid of it—the waste is still there, and it is spoken of in the text of the gag order. All is not lost, however, for closer inspection reveals that the waste is not toxic at all—that is, mere possibilia are not objectionable entities. In particular, they are *not* the Meinongian horrors of which Quine[18] sought to keep our doorways free. As both Linsky and Zalta, and Williamson make clear, Wittgenstein's possible son is not a shadowy entity who *is* a man, a miner, in therapy, and so on, but an entity which *could have been* a man, a miner, in therapy, and so on. Williamson argues that actualists are *wrong* to wish to rid the modal landscape of such entities, and Linsky and Zalta argue that actualists can *accept* such entities; both papers then argue, on this basis, for a return to the simple semantics, with one overall domain of possibilia, and no separate subdomains for each world. I accept the first part of the argument (the taming of the possibilia[19]), but think the second part (jettisoning the subdomains) is overkill. There are indeed technical advantages in having the quanti-

[15] "Bare Possibilia," *Erkenntnis*, XLVIII (1998): 257–73.

[16] "Semantical Considerations on Modal Logic," *Acta Philosophica Fennica*, XVI (1963): 83–94.

[17] Bernard Linksy and Edward N. Zalta, "In Defense of the Simplest Quantified Modal Logic," *Philosophical Perspectives*, VIII (1994): 431–58, and Williamson, also make this point.

[18] "On What There Is," in his *From a Logical Point of View* (New York: Harper, 1953), pp. 1–19.

[19] It is unnecessary here to enter into the vexed, and largely terminological, dispute over whether possibilia are acceptable *to actualists*.

fiers range over the overall domain,[20] but this does not mean we should get rid of the subdomains for each world. It *simply* means that, at each world, ∃ and ∀ should range over the entire domain, not over that world's subdomain. As mentioned, this *is* a feature of the semantics presented above. Now what of the subdomains? Are they not otiose, if they do not figure in the truth definitions for ∃ and ∀? No. On the contrary, they provide the basis for equipping the modal language with certain essential expressive resources, namely, actualist quantification and an existence predicate. Not only is there an *intuitively* clear distinction, among the overall domain of possibilia, between the ones which exist *around here*, which exist *in this world*, which *actually* exist, which are *part of the furniture of the world*—say it how you will—and the rest, but furthermore, Linsky and Zalta and Williamson *themselves* make use of this very distinction in presenting their accounts (Linksy and Zalta talk of "contingently nonconcrete" objects, and Williamson talks of "mere possibilia"). They thus make a distinction which their own semantics does not recognize. Much better, then, to retreat from Kripke not right back to the simplest semantics (without subdomains), but to the sort of semantics presented above. This gives us actualist quantification (Π,Σ) *and* possibilist quantification (∀,∃), and an existence predicate with which to distinguish the things which are part of the furniture of the world from the rest—with the attendant double advantage of technical tractability *and* expressive power.

I now turn to the task of fuzzifying the classical model theory presented above. A fuzzy model is exactly the same as a classical model, except for the following two changes. (i) Q assigns to each w in W a *fuzzy* subset D_w of D (rather than a crisp subset). D_w is still to be interpreted as the set of things that exist in w. This means that a thing may now exist at a world *to an intermediate degree*. (ii) V is a valuation function which assigns to each n-ary predicate a *fuzzy set* of n-tuples of members of D (rather than a crisp subset), in each world w.[21] We still stipulate (a) that in the case of the identity predicate, V assigns it the real identity relation on D (at every world)—*and we assume that real identity is always nonvague*—and (b) that in the case of the existence predicate, V assigns it D_w at w—which, on the other hand, is now a *fuzzy* set.

These two changes are the only ones we make. Thus, in particular, W (the set of all worlds), R (the accessibility relation), and D (the do-

[20] See, for example, Hughes and Cresswell, chapters 13–16.

[21] Formally, we can think of V as mapping each pair (P,w) of an n-ary predicate P and a world w, to a fuzzy subset of D^n, that is, to a function from the set of n-tuples of members of D to $[0,1]$.

main of all possible objects) remain nonfuzzy.[22] The notion of a value assignment is also unchanged. As before, $V_v^M(\alpha,w)$ is the truth value of the wf α at the world w in the model $\mathcal{M}$ when the individual variables are given the values assigned to them by v. *The clauses giving the truth conditions of the different sorts of wf are written exactly as above*; however, given the changes just made, the truth values of wfs are now members of [0,1], rather than {0,1}. Thus, for example, the degree of truth of $\alpha \wedge \beta$ at w is the minimum of the degrees of truth of α and β at w, and—because necessity functions like universal quantification over accessible worlds, and universal quantification functions like generalized conjunction—the degree of truth of $\Box\alpha$ at w is the infimum of the degrees of truth of α at all worlds accessible from w. I discuss some further details of fuzzy modal logic in the appendix.

To sum up the idea of the present section: we have an overall set of all possible objects, over which our existential quantifier $\exists$ ranges. Everything in this set exists *simpliciter*, in the sense of being in the range of $\exists$, and outside of this set, there is simply *nothing at all*. So existence, in the sense of $\exists$, is an all-or-nothing matter. However, for each possible world w, there is a *property* which things in the overall domain of possible objects may or may not possess: the property of existing at world w. (For example, you and I both have the property of existing at the actual world, while Santa Claus does not.[23]) Each of these properties is now a *vague* matter: objects need not possess these properties either outright or not at all—they may possess them to any intermediate degree. This means that when it comes to giving an inventory of the contents of the actual world (or any other world), some of the possible things in the overall domain are definitely in the inventory, and some are definitely out, while others are *sort of* in and *sort of* out. In short, the set of things which exist *in our world* has blurred boundaries.

II. NOTIONAL MEREOLOGY

For any bunch at all of possible things, there is a possible thing which is their fusion. Thus we want to allow ourselves the resources of classical mereology with unrestricted composition—and accordingly, in this section, I present a version of classical mereology. Of course we do not want to assume that the domain of the *actual* world is closed

[22] Fuzzifying R is an interesting idea, but is not relevant for present purposes. This idea is mentioned in Graeme Forbes, "Thisness and Vagueness," *Synthese*, LIV (1983): 235–59, p. 257, n. 12 (where it is attributed to Nathan Salmon), and carried out in Nobu-Yuki Suzuki, "Kripke Frame with Graded Accessibility and Fuzzy Possible World Semantics," *Studia Logica*, LIX (1997): 249–49. Forbes there, and in *The Metaphysics of Modality* (New York: Oxford, 1985), introduces a version of counterpart theory in which the counterpart relation is a matter of degree, but—in contrast to the present paper—the existence-at-a-world relation is *not* a matter of degree.

[23] Sorry, kids.

under mereological fusion, but we do wish to assume that the domain of all possible objects is so closed: fusing some actual objects may take us to a nonactual, merely possible object; but fusing some possible objects should never lead beyond the possible objects.

In presenting our system of mereology, we begin with a primitive (undefined) relation of parthood; we then define various other useful notions in terms of this one; and we then impose some fundamental principles governing this relation.[24]

We have a primitive binary relation of *proper parthood,* symbolized by $\sqsubset$, so $x \sqsubset y$ means that x is a proper part of y. $\sqsubset$ is a *non*fuzzy relation (or a fuzzy relation which always holds to degree 1 or degree 0). We say that x is an *improper* part of y just in case x and y are identical (compare the notion of an improper subset). Thus we define the relation of *proper or improper parthood,* symbolized by $\sqsubseteq$, as follows:

$$x \sqsubseteq y := x \sqsubset y \vee x = y.$$

In future, 'part' (used on its own) will mean proper or improper part. Two objects *overlap* just in case they have a part in common.[25] Thus we define the relation $\boxdot$, where $x \boxdot y$ means that x and y overlap, as follows:

$$x \boxdot y := \exists z(z \sqsubseteq x \wedge z \sqsubseteq y).$$

Two objects are *disjoint* just in case they do *not* overlap. Thus we define the relation $\boxtimes$, where $x \boxtimes y$ means that x and y are disjoint, as follows:

$$x \boxtimes y := \neg x \boxdot y.$$

An object is a mereological *atom* or *simple* just in case it has no proper parts.[26] Thus we define the property A, where Ax means that x is a mereological atom or simple, as follows:

[24] The cognoscenti will recognize that not all of the fundamental principles discussed below need be included as axioms in a formal theory of mereology, for some can be derived from combinations of others. For some different axiomatizations of classical mereology see Henry S. Leonard and Nelson Goodman, "The Calculus of Individuals and Its Uses," *Journal of Symbolic Logic,* v (1940): 45–55; Alfred Tarski, "Foundations of the Geometry of Solids," in his *Logic, Semantics, Metamathematics* (New York: Oxford, 1956), pp. 24–29; Alfred Breitkopf, "Axiomatisierung einiger Begriffe aus Nelson Goodmans *The Structure of Appearance,*" *Erkenntnis,* XII (1978): 229–47; Peter Simons, *Parts: A Study in Ontology* (New York: Oxford, 1987); and Achille Varzi, "Mereology," in Edward N. Zalta, ed., *The Stanford Encyclopedia of Philosophy* (Fall 2004).

[25] Thus any object overlaps itself, as it has an improper part—itself—in common with itself.

[26] Note the restriction to *proper* parts in this definition: every object has at least one *proper or improper* part, namely itself—so if we did not have this restriction, then *by definition* there would be no atomic objects. Given the definition in the text, it may still of course be the case that *as a matter of fact* there are no atomic objects.

$Ax := \neg\exists y(y \sqsubset x)$.

Given a set S of objects, we say that an object x is a *fusion* or *sum* of S just in case:

(1) everything in S is part of x, that is, $\forall y \in S(y \sqsubseteq x)$; and
(2) every part of x overlaps something in S, that is, $\forall y(y \sqsubseteq x \rightarrow \exists z \in S(y \boxdot z))$.

Now for our fundamental principles of notional mereology. These fall into four groups.

Partial ordering. First, a minimal requirement on any relation that is to deserve the title 'parthood' is that it partially order its field. Thus we stipulate that the proper parthood relation is *irreflexive* (nothing is a proper part of itself):[27]

N1: $\neg x \sqsubset x$

and *transitive* (if x is a proper part of y and y is a proper part of z, then x is a proper part of z):

N2: $x \sqsubset y \wedge y \sqsubset z \rightarrow x \sqsubset z$.

From these it follows that $\sqsubset$ is *asymmetric* (if x is a proper part of y then y is not a proper part of x). Given N1 and N2 and the definitions above, it follows that the parthood relation $\sqsubseteq$ is reflexive (everything is part of itself), transitive (if x is part of y and y is part of z, then x is part of z) and antisymmetric (if x is part of y and y is part of x, then x and y are identical). Thus the parthood relation is also a partial order.[28]

Supplementation. If a part of an object does not exhaust the object—that is, if it is a *proper* part—then it would seem that there must be *other* proper parts (or at least one, at any rate) which make up the difference. Thus we impose a principle which says that any object which has one proper part also has another proper part, disjoint from the first one:

N3: $x \sqsubset y \rightarrow \exists z(z \sqsubset y \wedge z \boxtimes x)$.

Extensionality. A further requirement is that the system of mereology is extensional, that is, that things with exactly the same proper parts are identical:[29]

[27] Note that in what follows, I shall often omit initial universal quantifiers for the sake of increased readability.

[28] $\sqsubset$ is a *strict* partial order and $\sqsubseteq$ is a *nonstrict* partial order; compare $<$ and $\leq$ on the real numbers.

[29] Assuming of course that they have proper parts at all: for we do not want to say that there cannot be two distinct simple objects x and y, that is, objects having no proper parts; yet note that in this case x and y have exactly the same set of proper parts, namely the empty set. Note also that the restriction to proper parts is essential:

N4: $\neg Ax \wedge \neg Ay \wedge \forall z(z \sqsubset x \leftrightarrow z \sqsubset y) \rightarrow x = y$.

Existence of Fusions. The final principle guarantees the existence of a unique mereological sum or fusion for every set of objects:[30]

N5: For every set *S*, there exists exactly one *x* such that *x* is a fusion of *S*.

In the present section, I have introduced a version of standard classical extensional mereology with unrestricted composition. This does not mean, however, that I have ended up accepting OC (the orthodox conclusion that composition can never be vague). I am committed to the view that *notional composition* can never be vague, but in the next section I shall introduce a new notion of *concrete composition*, which *can* be vague.[31]

III. CONCRETE MEREOLOGY

In the classical mereology of the previous section, for any set of things, there exists a sum of those things—no matter how scattered or disparate those things may be. Ordinarily, however, we would say that my hair is part of me until I cut it off, after which it is part of me no longer: and it is not that we think that the sum of the rest of me and my detached hair exists, but is not me; we think that there simply is no such thing as the sum of me and my detached hair. Considerations of this sort have led many theorists to abandon classical mereology in favor of weaker theories of mereology—in particular, theories in which composition is restricted. I think that a better way to accommodate these considerations is to posit *another* relation as well as $\sqsubset$. Let us refer to $\sqsubset$ as the relation of *notional* proper parthood. We shall now introduce a new primitive binary relation $\lhd$ of *concrete* proper parthood. When I cut off my hair, my hair is no longer a *concrete* part of me: my hair, together with my other parts, still notionally compose a possible entity, but they no longer concretely compose an actual entity. Notional parthood and composition will remain as in the previous section, and the considerations which have led some mereologists to place restrictions on composition will instead find outlet in the treatment of *concrete* parthood and composition. Two points about concrete parthood deserve special mention. First, concrete parthood

---

for any two distinct objects automatically do not have exactly the same set of proper-or-improper parts (for each is a part of itself, but given antisymmetry, both cannot be parts of the other).

[30] Note that I say "every set"—including the empty set—hence giving rise to the fusion of the null set (or "null fusion"). This is a personal preference *and is inessential to what follows.* Note also that if we wanted a first order theory of mereology, we could replace N5 with an axiom schema using a (schematic) unary predicate in place of '*S*'.

[31] For further discussion of the objection that I am simply a universalist about composition, see the end of section III.

is a *vague* matter. As one of my hairs slowly falls out, it goes *gradually*, by degrees, from being a part of me to not being a part of me. Second, concrete parthood is not absolute: it varies from world to world (some planks might concretely compose a ship in world w, but not in world w', where they remain disassembled); and even within a world, it varies from time to time (and thus the domain of a world varies from time to time, as composite objects come into and go out of existence in that world—but more on this below).

Turning to the second point first: we already have the semantic machinery (section I) to handle relativization of concrete parthood to worlds, but we shall need some new machinery to handle the relativization to times. It is a straightforward matter to augment our models so that sentences are evaluated at world-time pairs rather than simply at worlds. I shall now briefly describe how this may be done; readers uninterested in the details may safely skip the following paragraph.[32]

One thing we do *not* want to assume is that time is exactly the same in all possible worlds: time might begin and end at different points in different worlds; in some worlds time might not end or begin at all; in some worlds it might be discrete, in others dense; and so on. So we shall say that each world w comes equipped with a *time* $\mathcal{T}_w = (T_w, <_w)$, which consists in two things: a set T_w (of time instants), and a binary relation $<_w$ on that set (the earlier-later relation).[33] Now one possibility we would not like to rule out is branching time, that is, the case where one instant has several alternative futures.[34] However, given that we already have possible worlds, we do not need branching time *within* a possible world: each alternative future of a particular time can be confined to its own possible world. Thus we can say that $<_w$ is always *linear* (for any two distinct points of time, one precedes the other), as well as *irreflexive* (no point of time precedes itself) and *transitive* (if point x precedes point y and point y precedes point z, then point x precedes point z). Our semantic definitions of section I undergo the obvious minor changes: (i) The function Q now assigns subsets of D not to each world, but to each world-time pair (w,t) where

[32] The reader may wonder why I did not build in time right from the start, in section I. The reason I adopt a two-stage approach is that the framework of fuzzy quantified modal logic introduced in section I is both easier to comprehend initially, and of more *general* interest than the augmented framework which includes times as well as worlds—and with that framework in place, the addition of times is then a simple matter.

[33] I allow the case of the null time: the null set of times together with the null relation thereon.

[34] One possibility we shall *not* countenance, however, is that there might be more than one time dimension in some worlds. On this idea, see, for example, Jack W. Meiland, "A Two-Dimensional Passage Model of Time for Time Travel," *Philosophical Studies*, XXVI (1974): 153–73. (The difference between branching time and two-dimensional time is like the difference between a branching *path* and a *plane*.)

w is a world in W and t is a time in T_w, with the subset $D_{(w,t)}$ assigned being the set of things that exist at time t in world w. (ii) The valuation function V assigns to each n-ary predicate a fuzzy set of n-tuples of members of D, in each world w *at each time* t (in T_w); so V is now a function not from pairs (P,w), but from triples (P,w,t). The clauses giving the truth values for compound wfs are now as follows:[35]

- $V^M_v(Px_1,\ldots,x_n,w,t) = V^M(P,w,t)(v(x_1),\ldots,v(x_n))$
- $V^M_v(\neg\alpha,w,t) = 1 - V^M_v(\alpha,w,t)$
- $V^M_v(\alpha \vee \beta,w,t) = \bigvee\{V^M_v(\alpha,w,t),\ V^M_v(\beta,w,t)\}$
- $V^M_v(\Box\alpha,w,t) = \bigwedge\{V^M_v(\alpha,w',t')\text{: } \forall w' \text{ with } wRw' \text{ and } \forall t' \in T_{w'}\}$
- $V^M_v(\forall x\alpha,w,t) = \bigwedge\{V^M_\rho(\alpha,w,t)\text{: } \forall\rho \text{ such that } \forall y \neq x,\ \rho(y) = v(y)\}$

With our extended semantic apparatus in place, we now need some principles governing the concrete parthood relation $\lhd$: principles which are similar enough to our principles governing the notional parthood relation $\sqsubset$ to make $\lhd$ recognizable as a relation of *parthood*, but different enough to accommodate the fact that concrete parthood is a matter of degree and varies from world to world and from time to time, and the fact that concrete composition is not unrestricted. I shall consider our five principles of notional mereology in turn, considering in each case whether we want an analog of the principle for concrete mereology, and if so, what the analog should be. First, however, we impose a principle linking notional and concrete parthood: the principle that nothing can be a positive-degree concrete part of an object, at any world or time, unless it is a *notional* part of that object. Before stating this formally, I need to explain my notation. I employ both an infix notation and a prefix notation, as the context requires. These have distinct, though related, meanings. Infix notation: '$x \lhd y$' is a *sentence*, which has a particular truth value at each world-time (relative to a model and value assignment). Prefix notation: $\lhd_{(w,t)}(x,y)$ is the degree to which x is a concrete part of y at time t in world w; so '$\lhd_{(w,t)}(x,y)$' (which will sometimes be abbreviated to '$\lhd(x,y)$') is a *name* of a real number between 0 and 1 inclusive. (Compare: 'Bob is bald' is a sentence which, let us say, has degree of truth 0.5. 'Bob's degree of baldness' is a name of the number 0.5.) Our linking principle can now be expressed as follows:[36]

[35] Note that necessity is interpreted as truth *at all times* in all accessible worlds. A notion of *historical necessity* could easily be introduced, as well as the usual temporal operators *F*, *P*, and so on, but as none of these will play a role in what follows, I shall not discuss them here. For further discussion, see Richmond H. Thomason, "Combinations of Tense and Modality," in D. Gabbay and F. Guenthner, eds., *Handbook of Philosophical Logic*, Volume II (Boston: Reidel, 1984), pp. 135–65.

[36] As in the previous section, initial universal quantifiers will not be written explicitly, but should be assumed.

NC: $\neg(x \sqsubset y) \rightarrow \forall w,t[\lhd_{(w,t)}(x,y) = 0]$.

Thus notional parthood is one of the determinants of concrete parthood (but not the only one): nonparts in the notional sense are not even in the running to be parts in the concrete sense.[37]

Partial ordering. Some version of the partial order requirement should govern concrete parthood, generalized to cover the fact that concrete parthood is a vague and nonabsolute (that is, world- and time-relative) relation:

C1: Irreflexivity: $\lhd_{(w,t)}(x,x) = 0$.

This says that the degree to which any object is a concrete part of itself is 0, at every world and every time.

C2: Transitivity: $\lhd_{(w,t)}(x,z) \geq \bigwedge\{\lhd_{(w,t)}(x,y), \lhd_{(w,t)}(y,z)\}$.

This says that at any world and time, the degree to which x is a concrete part of z is greater than or equal to the minimum of the degrees to which x is a concrete part of y and y is a concrete part of z. We do not make the inequality here an equality, because of the following sort of example. Suppose we are building a boat c, and all we have left to do is step the mast. The mast-step a is fastened in place, but the mast b has yet to be fitted to the mast-step. The degree to which a is a concrete part of the mast-step/mast unit ab is 0. The degree to which the mast-step/mast unit ab is a concrete part of the boat c is low, maybe 0 (because only the mast-step part of the mast-step/mast unit is fastened in place). Yet the degree to which the mast-step a is a concrete part of the boat c is 1.

Together with the fact that $0 \leq \lhd(x,y) \leq 1$, C2 implies:[38]

$\lhd(x,y) = 1 \wedge \lhd(y,z) = 1 \rightarrow \lhd(x,z) = 1$.

It follows from C1 and C2 that $\lhd$ is asymmetric (that is, $\lhd(x,y) > 0 \rightarrow \lhd(y,x) = 0$).[39] If we define the relation $\unlhd$, where $x \unlhd y$ means that x is a proper or improper concrete part of y, as follows:

$x \unlhd y := x \lhd y \vee x = y$

then it follows from C1 and C2 that $\unlhd$ is reflexive (that is, $\unlhd(x,x) =$

[37] Thus, in particular, notional simples never have any proper concrete parts at any world-time.

[38] Here and elsewhere, for the sake of increased readability, I omit not only initial universal quantifiers, but also w and t subscripts.

[39] Proof: Suppose that $\lhd(x,y) = m$ and $\lhd(y,x) = n$, with $0 < m,n < 1$; then by transitivity $\lhd(x,x) \geq \bigwedge\{m,n\} > 0$, contradicting irreflexivity.

1),[40] antisymmetric (that is, $\trianglelefteq(x,y) > 0 \wedge \trianglelefteq(y,x) > 0 \rightarrow x = y$)[41] and transitive (that is, $\trianglelefteq(x,z) \geq \bigwedge\{\trianglelefteq(x,y), \trianglelefteq(y,z)\}$).[42]

Supplementation. Do we want a principle which ensures that no object can have one proper concrete part without also having another proper concrete part which is disjoint from the first? There are actually several possible principles here. If we define the relation $\odot$, where $x \odot y$ means that x and y concretely overlap, as follows:

$$x \odot y := \exists z(z \trianglelefteq x \wedge z \trianglelefteq y)$$

then we can consider as possible principles each of the eight claims derived from the following formula by choosing either the condition above the line, or the condition below the line, at each of the three points at which such a choice can be made:

$$\triangleleft(x,y)\frac{>0}{=1} \rightarrow \exists z\left(\triangleleft(z,y)\frac{>0}{=1} \wedge \odot(z,x)\frac{=0}{<1}\right)$$

Why might one think that one of these claims is true? Well, one might think that something becomes a concrete part of an object when it is *fastened* to other parts of the object, and that *one* object is not enough for fastening to occur (just as one hand cannot clap, and one person cannot tango)—you need at least two objects, so that they can be fastened *together.* I am not convinced by this, however. For example, suppose that we are building a sculpture on a plinth. The plinth is not part of the sculpture; it is just the base on which the sculpture will sit. Suppose the bottom-most part b of the sculpture is attached to the plinth, but is not attached to any other parts of the sculpture, and those other parts are not attached to each other or to the plinth. In this case, it seems to me that b is a degree-1 proper concrete part of the sculpture, and that the sculpture has no positive-degree concrete parts which are disjoint from b. Thus, I think we do not want an analog of supplementation for concrete parthood.[43]

[40] Proof: $x \trianglelefteq y := x \triangleleft y \vee x = y$, so the degree to which $x \trianglelefteq x$ holds is the maximum of the degree to which $x \triangleleft x$ holds (which is 0) and the degree to which $x = x$ holds (which is 1), which is 1.

[41] Proof: Suppose $\trianglelefteq(x,y) > 0$ and $\trianglelefteq(y,x) > 0$ and $x \neq y$; then $\triangleleft(x,y) > 0$ and $\triangleleft(y,x) > 0$, contradicting asymmetry of $\triangleleft$.

[42] Proof: Case (i): $x = y = z$; reduces to $1 \geq \bigwedge\{1,1\}$ which is true. Case (ii): $x \neq y \neq z$; reduces to $\triangleleft(x,z) \geq \bigwedge\{\triangleleft(x,y), \triangleleft(y,z)\}$, that is, transitivity of $\triangleleft$. Case (iii): $x \neq y = z$; reduces to $\triangleleft(x,z) \geq \triangleleft(x,y)$, but $y = z$ so $\triangleleft(x,z) = \triangleleft(x,y)$, so this is true. Case (iv): $x = y \neq z$; reduces to $\triangleleft(x,z) \geq \triangleleft(y,z)$, but $x = y$ so $\triangleleft(x,z) = \triangleleft(y,z)$, so this is true.

[43] A referee expressed the following intuition about the statue example: "It seems to me that a natural thing to say about that example is that after the feet (say) have been attached to the plinth, the sculpture exists *simpliciter* and the feet constitute the whole of the sculpture as it is at that time." This intuition *can* be accommodated within the framework of this paper, via the idea (to be introduced below) that at the time in question, the set of statue parts containing only the feet has degree of importance 1.

Extensionality. Note for a start that we do not want to say that if there is a time in a world such that for all z, the degree to which z is a concrete part of x at that world-time is the same as the degree to which z is a concrete part of y at that world-time, then $x = y$. For example, suppose I am building a dinghy, and I have all the parts of my boat cut and shaped, and numbered in accordance with my plan, so I know where each part is supposed to go. As it happens, I have accidentally cut *two* foredecks, and given them the same number. Let one of these foredecks be a and the other b. I am at the stage of building the boat where all parts have been fastened except for the foredeck. Now, consider the notional fusion A of the set containing all the other boat parts and a, and the notional fusion B of the set containing all the other boat parts and b. For every thing z, the degree to which z is a concrete part of A is the same as the degree to which z is a concrete part of B (note that a is a degree-0 concrete part of both, and so is b), yet we do not want to say that $A = B$.

But do we want to say that if, for *every* time t in every world w and for every z, the degree to which z is a concrete part of x at (w,t) is the same as the degree to which z is a concrete part of y at (w,t), then $x = y$ (provided x and y are not atomic)? That is, do we want to impose the following as a principle of concrete mereology?:

$$\neg Ax \wedge \neg Ay \wedge \forall z,w,t(\lhd_{(w,t)}(z,x) = \lhd_{(w,t)}(z,y)) \rightarrow x = y$$

For this to come out false, we would need two distinct nonatomic objects x and y which had exactly the same concrete parts to exactly the same degrees at every world-time. Now, given the extensionality of notional mereology, if x and y are distinct, one must have a notional part the other lacks; and then given NC and the supposition that x and y have exactly the same concrete parts to exactly the same degrees at every world-time, one of x and y must have a *notional* part that is a *degree-0 concrete* part of it at *every* world-time. Thus, if the principle under consideration is sometimes false, then there is a notional fusion which is never fully realized, at any time in any world: some of its possible parts are never (at any world or time) brought together as concrete parts. Now, this conflicts with the idea of a *possible* thing as something which is *actual at some world*. But this is not a knock-down objection, and we might wish to accept that the claim under consideration can be false—in which case we should not, of course, adopt it as a principle of concrete mereology. On the other hand, we might think this principle can never be false. But even so, we *still* should not adopt it as a principle of concrete mereology. This is because it has nothing to do with concrete mereology as such: it has to do with making sure that our space of possible worlds is big enough and varied enough—and it is no skin off the nose of concrete mereology *per se* if there is in fact just one possible world! So either way, we should not adopt this extensionality claim as a principle of concrete mereology.

Existence of Fusions. Our final principle of notional mereology told us under what conditions there exists a fusion of a given set of objects (the answer was: under *all* conditions—notional composition is unrestricted). What we need now is an analogous principle for concrete mereology. This principle should relate concrete parthood to *actual* or *concrete* existence, that is, *existence at a world-time,* as opposed to existence simpliciter, that is, membership of the overall domain D. For we already know that for any bunch of things, there is a possible object which is their notional fusion: what we now want to know is under what conditions (specified in terms of facts about concrete parthood) this notional fusion *concretely* or *actually* exists at a time in a world.

For an *atomic* object x in D we stipulate that for every world w and every time t, $D_{(w,t)}(x)$ is either 1 or 0 (where $D_{(w,t)}(x)$ is the degree to which x exists at (w,t), that is, the degree to which x is a member of the fuzzy set $D_{(w,t)}$, this set being the domain of the world-time (w,t), that is, the set of objects which actually or concretely exist at (w,t), this sort of existence being potentially a vague matter, as discussed in section I). That is, there cannot be vagueness about whether a simple object concretely exists at a time in a world: it exists fully (definitely, to degree 1), or not at all (to degree 0). As for whether or not a given simple *does in fact* exist at a particular world-time, this is a matter for modal geographers: it does not fall within the scope of the theory of concrete mereology, and so we do not introduce any principles concerning the concrete existence of notional simples. (Compare the fact that in notional mereology, we say nothing concerning what, if any, simples exist: we say only, for example, that for any set of simples, there exists a fusion of that set.)

Nonatomic objects (that is, objects with notional proper parts), on the other hand, fade into/out of concrete existence as their notional parts become concrete parts to a greater/lesser degree. But what exactly is the relationship between concrete parthood and concrete existence? As a first try, we might think that if $y_1,\ldots,y_n$ are the notional proper parts of x, then to say that x concretely exists is to say that y_1 is a concrete part of x, *and* y_2 is a concrete part of x, *and* $\ldots y_n$ is a concrete part of x. This leads to the following principle:

$$(1)\ D_{(w,t)}(x) = \bigwedge_{\{y_i\,:\,y_i \sqsubset x\}} \lhd_{(w,t)}(y_i,x)$$

This says that the degree to which an object exists at a world-time is the infimum of the degrees to which each of its notional parts is a concrete part of it at that world-time. However, it is a consequence of this principle that if one notional part y_i of x is completely destroyed

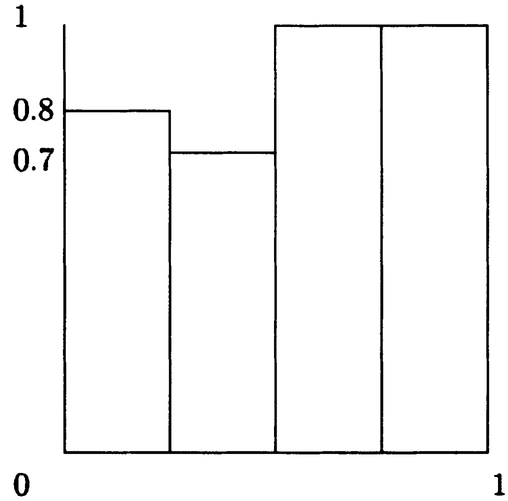

Figure 1

or disconnected at t in w, so that $\lhd_{(w,t)}(y_i,x) = 0$, then x exists to degree 0 at t in w. This is too extreme: a better idea is that where x has n notional simple parts, each simple part can contribute one nth of the concrete existence of x. How much it contributes depends on its degree of concrete parthood of x: if all x's notional parts are degree-1 concrete parts, x concretely exists to degree 1; if one atom is destroyed and becomes a degree-0 concrete part, x loses $^1/_n$ of its concrete existence; if the atom gets somewhat disconnected and becomes a degree-0.5 concrete part of x, x loses $^{0.5}/_n$ of its existence; and so on. This leads us to the following principle:

$$(2)\ D_{(w,t)}(x) = \sum_{i=1}^{n} \lhd_{(w,t)}(y_i, x) \cdot \frac{1}{n}$$

where $y_1,\dots,y_n$ are all the simple notional proper parts of x (that is, all the y_i such that Ay_i and $y_i \sqsubset x$).

We can add a further subtlety. It might be (for example) that although your transistor radio has nine hundred and eighty parts, some of them are more important than others: some are just for decoration, while others are crucial parts. So it might be that removing one part affects the (actual, concrete) existence of the radio more than removing another part. So we assign each simple notional part y_i of the radio x a *degree of importance to x*, in symbols $\langle y_i \rangle_x$. We allow this degree of importance to vary from world to world and from time to time, that is, we read $\langle y_i \rangle_x$ as $\langle y_i \rangle_{x,(w,t)}$ (although the subscript (w,t) will generally not be written explicitly, for the sake of increased readability). For each y_i and each x, $0 \leq \langle y_i \rangle_x \leq 1$ (at every world-time), and where there are n notional parts $y_1 \dots y_n$ of x in total, $\Sigma_{i=1}^{n} \langle y_i \rangle_x = 1$ (at each world-time). Thus, we arrive at the following principle:

$$(3)\ D_{(w,t)}(x) = \sum_{i=1}^{n} \lhd_{(w,t)}(y_i, x) \cdot \langle y_i \rangle_{x,(w,t)}$$

where $y_1 \ldots y_n$ are all the simple notional proper parts of x. Note that (2) is simply the special case of (3) where every part has the same degree of importance, that is, $\frac{1}{n}$.

To get a feel for what is going on here, consider figure 1, which depicts a particular notional fusion x at a particular world-time. Each bar of the graph represents one of the notional parts of x. The width of the bar represents the importance of the part to x (at the given world-time), and the height of the bar represents the degree to which that part is a *concrete* part of x (at the given world-time). x's degree of existence (at the given world-time) is represented by the area under the graph: in particular, when all the bars reach to 1, x concretely exists to degree 1. In the case shown in the diagram, we have an object with four notional atomic parts; each part has the same degree of importance; two parts are degree-1 concrete parts, one is a degree-0.8 concrete part, and one is a degree-0.7 concrete part; the degree of concrete existence of the object is $0.25 \times (0.8 + 0.7 + 1 + 1) = 0.875$.[44]

So far our proposal covers only notional fusions that (i) have simple parts (they are not made of gunk), and (ii) have *finitely* many simple parts.[45] Dealing with the second point first, we may generalize the finite, bar-graph case to the infinite case in a standard way. Intuitively, as the number of parts increases to infinity, the bars on the graph become as thin as points, the stepped line across their tops becomes a curve, and we integrate rather than forming a sum.[46] Formally, the details are as follows (they may be skipped by readers so inclined, without jeopardizing their understanding of the rest of the paper).

Consider a notional fusion x. Let S_x be the set of all notional atomic parts of x. Let F_x be a σ-field of subsets of S_x. Let μ be a measure on F_x, and furthermore let μ be normalized, that is, $\mu(S_x) = 1$. μ is to be interpreted as a measure of importance; as before, we allow μ to vary from world to world and from time to time, that is, we read μ as $\mu_{(w,t)}$. Let $f^x_{(w,t)} : S_x \rightarrow [0,1]$ be the function which assigns to each $y \in S_x$ its degree of concrete parthood of x at t in w, that is, $f^x_{(w,t)}(y) = \lhd_{(w,t)}(y,x)$. We require that $f^x_{(w,t)}$ be a measurable function, that

[44] Any reader who finds it absurd to be so precise about the degree of existence of an object is objecting not to the views presented in this paper as such, but to the background framework for the treatment of vagueness: the fuzzy framework. See the discussion in the introductory section above.

[45] There are at least two ways in which the finiteness assumption might fail: an object with finite (or infinite) extension might have infinitely many parts which do not have positive extension; and an object with infinite extension might have infinitely many parts which do have positive extension.

[46] In the finite case the vertical axis represents (roughly speaking) a mass-like quantity, whereas in the infinite case it represents a density-like quantity.

is, measurable relative to F_x.[47] Now where $\int$ is the Lebesgue integral, we say:[48]

$$(4)\ D_{(w,t)}(x) = \int_{S_x} f^x_{(w,t)} d\mu$$

[47] This constrains our choice of F_x and $f^x_{(w,t)}$, but there is no reason to think that this constraint will conflict with natural choices of F_x and $f^x_{(w,t)}$. Typically, although perhaps not always, F_x will simply be the set of all subsets of S_x, and then $f^x_{(w,t)}$ will automatically be measurable relative to F_x.

[48] A referee thought that the integral should be taken not over the set S_x but over x itself, and that μ should take notional parts of x (rather than sets of such parts) as arguments. The thought could be that there is a *technical* error in my proposal, or that it faces a *conceptual* problem. First, there is no technical error in my proposal. Distinguish the function $f^x_{(w,t)}$—which assigns to each notional part of x its degree of concrete parthood of x at t in w—and the measure function μ. We integrate the function f, relative to the measure μ. f should (and does) take parts of x as arguments, and therefore μ should (and does) take *sets* of parts of x as arguments, so that we can talk about the sizes of the preimages under f of given subsets of [0,1] ([0,1] being the codomain of f, and the preimage under f of a given subset of [0,1] being the *set* of parts which get mapped to a value in that subset). (Compare the way in which in probability theory, we have a sample space, a random variable—a function which assigns real number values to *elements* of the sample space—and a probability measure—a function which assigns sizes to *sets* of elements. In determining the expectation of the random variable, we integrate it with respect to the measure.) Second, conceptual issues. Intuitively, μ is supposed to represent the measure of importance of *parts* of x—and yet it assigns values to sets of parts, not parts themselves. This is no deep problem: we regard the degree of importance of an individual *part* as the value which μ assigns to its *singleton.* Shifts of this sort happen all the time when we model intuitive phenomena in a precise way, and while they may cause confusion if we do not note them carefully, once noted they are no cause for concern. On the other hand, there *is* a deep conceptual problem with the alternative proposal that μ take parts, not sets of parts, as arguments. Measures must be additive. On my proposal (where μ takes sets of parts as arguments), μ is additive over unions: the measure of the union of two sets of parts is the sum of the measures of those two sets of parts. On the alternative proposal (where μ takes parts as arguments), the referee suggests—and this would seem to be the only option—that μ be additive over mereological sums: the measure of the mereological sum of a and b is the sum of the measures of a and b. But this yields results which are unacceptable in the context of the present project. My aim is to capture ordinary intuitions about vague composition and existence. One such intuition is that sums of *scattered* concretely-existing objects do not concretely exist (cf. *datum 2* below). Another is that an object which does not concretely exist (that is, its degree of concrete existence is zero) cannot be a concrete part of any other object (that is, to any positive degree—cf. *datum 3* below). The fundamental thought in this paper is that an object concretely exists to the extent to which its notional parts are *concrete* parts of it. Now these ideas will not mesh with the idea that the mereological sum of any two important parts is important—that is, with the idea that μ be additive over mereological sums. For consider, say, a car. The steering wheel and the engine are important parts of the car: the car will not fully concretely exist until they are concrete parts of it. Their sum is a scattered object which does not concretely exist at all: to make it exist we would have to attach the steering wheel directly to the engine. Now, if—as the proposal under consideration implies—their sum is an important part of the car—because it is the sum of two important parts—then the car will not fully exist until this sum is a concrete part of the car; but this cannot happen until the sum concretely exists—and this cannot happen until the steering wheel is directly attached to the engine. So the car cannot fully exist unless the steering wheel is removed from its usual mounting point and attached directly to the engine. This is absurd. Moreover, we clearly face a situation in which the car can *never* fully concretely exist: for whatever sums of its important parts we make concretely exist, we will invariably take others out of concrete existence in the process.

Let us turn now to the first point: the assumption of atomism. In order to accommodate the case of gunk, we have to abandon the restriction to atomic parts. What we do is proceed exactly as above, except that we let S_x be the set of *all* notional parts of x (rather than the set of all notional *atomic* parts of x). This change gives rise to a subtlety; let us consider a specific example. We are building a boat, and it is made of gunk: the planks are not simple notional parts, and nor are the cellulose molecules of which each plank is composed (and so on)—rather, the notional parts go down forever. Now suppose the boat is complete: all planks are fastened in place. We certainly want to say that the boat concretely exists to degree 1 (call this *datum 1*). Consider the notional part *ab* of the boat which consists of two widely separated planks *a* and *b*, say the port shear strake and the starboard garboard strake.[49] This object *ab* has a low—indeed zero—degree of concrete existence (call this *datum 2*), because the notional parts of *ab* have not been brought into contact and fastened together, that is, they are not *concrete* parts of *ab*: we do not want to abandon this claim, for one of our main motivations was to accommodate the intuition that such scattered objects do not actually exist. Now, a very plausible principle is the following: if the degree of concrete existence of an object x at (w,t) is 0, then for any object y, the degree to which x is a concrete part of y at (w,t) is 0 (call this *datum 3*). Now, we face an apparent problem. We are integrating our concrete-parthood function over the set of *all* parts (not just simple ones): *including ab*. But by datum 2, *ab* does not concretely exist, and so by datum 3, *ab* cannot be a concrete part of the boat. Hence, it seems that the integral will not equal 1: some of the notional parts we are integrating over are not concrete parts, so the boat will not fully concretely exist—contradicting datum 1.

In fact, there is no problem here: the three data are actually compatible. We simply say—very plausibly—that the measure of importance at (w,t) of any notional fusion of parts which are *scattered* at (w,t) is zero. Note here that we must be careful to distinguish $\{a,b\}$ and $\{ab\}$ (the former set contains two elements, *a* and *b*; the latter set contains one element *ab*, which is the notional fusion of *a* and *b*). As both the sets $\{a\}$ and $\{b\}$ have a positive measure of importance, $\{a,b\}$ must also have a positive measure of importance, as measures are additive. But this does *not* mean that $\{ab\}$ must have a positive measure of impor-

[49] The port shear strake is the topmost plank, adjacent to the deck, on the port side; the starboard garboard strake is the bottommost plank, adjacent to the keel, on the starboard side; so if you imagine the port shear strake and the starboard garboard strake remaining in position (perhaps held by wires) while all other parts of the boat are removed, these planks would be separated by a significant expanse of empty space.

tance. We are quite free to say that $\{a\}$, $\{b\}$ and $\{a,b\}$ all have positive measure, while $\{ab\}$ has zero measure—and indeed this is eminently plausible. (Imagine a marine engineer inspecting a ship and giving it a positive survey report, and then waking up in a cold sweat in the night thinking 'I checked that the rudder was secure, and also the keel, but I forgot to check their fusion!' The rudder is a crucially important part, so too the keel—but their fusion, *considered as an object in its own right,* is not important at all.) Now, one of the key properties of the Lebesgue integral is that it "ignores" sets of measure zero: two functions which differ in value only over a set of measure zero have the same integral. This means that although *ab* is a concrete part to degree 0, and although *ab* is in the set of parts over which we integrate, this does not affect the degree of concrete existence of the boat: the degree to which *ab* is a concrete part of the boat is entirely irrelevant to the value of the integral—and the same goes for any other notional part of the boat which is a fusion of scattered parts (that is, parts which are scattered at time *t* in world *w*).[50]

[50] A referee raised the following example. "Suppose that object *p* is a rectangular prism that is the concrete fusion of four nonoverlapping duplicate cubes of homogeneous, atomless, gunk (*a*, *b*, *c*, and *d*) arranged in a line. Suppose that the domain of the importance measure μ is the set of all notional parts of *p*. What values should μ assign to *a*, *b*, *c*, and *d*? It might seem natural to say $\mu(a) = \mu(b) = \mu(c) = \mu(d) = \frac{1}{4}$. But then what value is μ going to assign, say, to the concrete proper part of *p* that consists of the fusion of *a* and *b*? It will have to be zero, if we are to avoid saying that *p* has a degree of existence greater than 1, yet *a* and *b* are in direct contact with one another and their fusion seems like a perfectly good, and important, part of *p*. (This is quite unlike the case of the notional fusion of the port shear and starboard garboard strakes.)" There is no problem here for my view. Rather, the problem is an incoherence in the set of intuitions which says *both* that $\mu(a) = \mu(b) = \mu(c) = \mu(d) = \frac{1}{4}$ *and* that the fusion of *a* and *b* is a perfectly good, and important, part of *p*. The total measure of importance of *all* the notional parts of *p* must be 1. (When I talk of the measure of importance of a part, I mean the value which μ assigns to the set containing only that part—see note 48.) Parts of no importance at all get measure 0. The rest—the not-totally-unimportant parts—get measures which sum to 1. So if you think that there are more than four important parts (you think *a*, *b*, *c*, *d and* the fusion of *a* and *b* are all important, to some degree) then you simply should not divide the total measure of importance of 1 equally among only *four* of these parts. (Compare dividing up a cake at the birthday party of twins Bill and Ben: you give Bill and Ben half each, and then think that all the guests should have some too—but there is none left! This does not show that the cake was too small: it shows that you divided it in the wrong way.) You should say *for example* that each of *a*, *b*, *c*, and *d* has measure $\frac{8}{49}$, that each of *ab* (the notional fusion of *a* and *b*), *bc*, and *cd* has measure $\frac{4}{49}$ (while scattered fusions such as *ac* and *ad* have measure 0), that each of *abc* and *bcd* has measure $\frac{2}{49}$ (while *acd* and *abd* have measure 0), and that *abcd* has measure $\frac{1}{49}$. This renders *ab* half as important as *a* and *b*, *abc* half as important as *ab*, and so on (but of course, if your intuition concerning the relative importance of these parts is different, you can choose the numbers differently); it renders scattered fusions totally unimportant; and the measures of all not-totally-unimportant parts sum to 1. Of course you might not agree at all with this particular choice of numbers, but the general point remains: divide the total measure of importance of 1 among the parts which you think are not-totally-unimportant so that each of them gets some; do not first divide it among only *some* of these parts, and then find there is none left for other parts which you think are not-totally-

Now, in fact the proposal just described for gunk also works for the infinite nongunk case. Thus our final principle of concrete mereology is (4) above, except that S_x is the set of *all* notional parts of x:

$$\text{C3: } D_{(w,t)}(x) = \int_{S_x} f^x_{(w,t)} d\mu$$

Although there are further subtleties to be discussed in later sections, the basics of my proposal are now on the table. Our quantifiers $\exists$ and $\forall$ range over D, and they are classical. Our existence predicate E has $D_{(w,t)}$ as its extension at (w,t), and this is a fuzzy subset of D. We say that an object exists *simpliciter* if it is in the range of the quantifier $\exists$, that is, if it is in D, and that it concretely or actually exists at (w,t) to the extent that it is a member of the domain of (w,t), that is, $D_{(w,t)}$. Concrete existence can be a vague matter, but existence simpliciter is always a nonvague, all-or-nothing matter. For *any* set of things that exist simpliciter, there exists simpliciter a notional fusion of that set; however, in general these fusions *exist to degree 0 at the actual world.* A notional fusion exists at a world-time to the extent to which its notional parts are *concrete* parts of it at that world-time. This view offers a double advantage: it makes perfectly clear sense, *and* it satisfies widespread intuitions about composition and existence. It makes sense because the quantifiers and the background set D are classical: vague existence is not mysterious at all—no more so than the notion of a fuzzy subset. On the other hand, the view satisfies our intuitions, because it allows (for example) that the planks that I have acquired *gradually* come to compose a boat, which comes *gradually* into existence. Of course, the notional fusion of the planks that I have acquired exists all along as a possible object—but it only comes gradually to exist as part of the furniture of the actual world, alongside the shed in which I am building it, myself, and all the other actual, concrete inhabitants of our world. It is precisely this double picture of existence that allows my proposal to *both* make perfectly clear sense, *and* satisfy widespread intuitions about vague composition and existence.

At this point, the reader might be thinking that I am not opposed to the orthodox view at all: I am in fact an advocate of unrestricted composition (and hence my view does *not* accommodate the common intuitions about composition with which—as noted at the outset—unrestricted composition conflicts). It is true that I advocate unrestricted composition (I think that arbitrary notional fusions exist

---

unimportant. (This talk of us choosing the numbers we like is not meant to imply that importance measures—and hence degrees of concrete existence of objects—are subjective matters. We can all say what we think is the most reasonable importance measure in a given case—just as we can all say how many marbles we think the jar contains—but for all that, there may (or may not) just *be* one *correct* measure.)

simpliciter); but it is false that I am not opposed to the orthodox view (and that my view does not accommodate the common intuitions with which unrestricted composition conflicts). Let me explain. What conflicts with the common intuitions about composition is universalism *end of story*: the claim that not only tables and chairs exist, but also arbitrary fusions thereof (and so on), and that is all there is to it. Now in order *not* to be a universalist *end of story* one can either keep one's story the same length and make it nonuniversalist, or one can retain the universalism but not make it the end of the story—that is, one can add to the story. The problem with the first option—with having just one notion of existence and one notion of composition (like the universalist *end of story*), but *not* being a universalist about the existence of fusions—is that in order to satisfy our intuitions, we need a vague restriction on the existence of fusions (existence *simpliciter*, that is—for ex hypothesi, that is the only notion of existence available), and no one has been able to see how to make sense of this idea (myself included). Thus, I take the second option: I start with universalism about the existence simpliciter of notional fusions, but I then add to this a view about concrete composition and concrete existence. Thus, according to my view, arbitrary notional fusions do exist simpliciter, *but they do not also exist in the further sense in which tables and chairs exist*: notional fusions are there in the domain of all possible objects, but (in general) they do not exist in our world—*they are not present in our world alongside the tables and chairs and so on which do inhabit our world.* This seems to me to capture entirely the intuition that (in general) arbitrary fusions do not exist: *the intuition being that, unlike tables and chairs, arbitrary fusions are not part of the furniture of our world—that is, would not appear in a complete inventory of the contents of our world.* Thus I am a universalist, but I am not a universalist *end of story*—and this difference is crucial. Universalism *end of story* flouts intuition; nonuniversalism *end of story* is obscure if not nonsensical; only the present combination of universalism *plus* a further story about vague restrictions on the *concrete* existence of notional fusions offers the double advantage of *both* making perfect sense *and* being intuitively satisfying.

IV. PERSISTENCE

Consider all the objects which are (according to common sense) parts of my boat at t_1; let a be one of these objects (a plank), and let A be the notional fusion of all these objects. Suppose that a is disconnected or destroyed in such a way that it becomes a degree-0 concrete part of A, and suppose that at time t_2, the missing part of my boat is replaced by b. Consider all the objects which are parts of my boat at t_2; let B be the notional fusion of all these objects. At t_2, A no longer fully concretely exists, whereas B now does (and we may suppose that at t_1, B did not fully concretely exist, whereas A did). Thus, if we want to say that my boat—the same one as before—still fully exists, after a has been replaced by b, then we cannot identify my boat with A (or with B).

More generally, if we wish to avoid *mereological essentialism* for ordinary persisting objects such as houses and boats,[51] then we cannot identify such objects with notional fusions. But as things stand, we have nothing else in our picture with which to identify them. We have allowed that a fusion of planks can fade into and out of (actual, concrete) existence as its notional parts become concrete parts to a greater or lesser degree. But there is another way in which objects can vary over time: an object can be composed of different material at different times—it can have different parts at different times. What we now need to do is make room for this second sort of variation with time. There are two different ways of doing this: a way which involves distinguishing between material objects and the matter which constitutes them (persistence and change involve an object being differently *constituted* at different times); and a way which involves distinguishing between four-dimensional objects and their temporal parts (persistence and change involve an object having different *temporal parts* at different times).[52] I shall consider these ways in turn.

IV. 1. Matter versus Material Objects. The first way of proceeding retains the above account of notional mereology as it is, and adds something new. The thought is this: so far, what we have in D are *items of matter*; but we also need to make room for *material objects*. So we add a new domain D' of material objects. D' contains every function from the set of all world-time pairs (w,t), with $w \in W$ and $t \in T_w$, to D. The function associated (or identified) with a material object tells us which item of matter—that is, which member of D—constitutes that material object at each time in each world.[53]

Our model theory undergoes the following changes: (i) ν now as-

[51] That is, the claim that such objects have their parts essentially, and hence cannot survive the gain or loss of parts; see Roderick M. Chisholm, *Person and Object: A Metaphysical Study* (London: George Allen and Unwin, 1976), chapter III; and Simons, p. 177 (Simons uses an alternative terminology of "mereologically variable" and "mereologically constant" objects).

[52] For general discussions of these two sorts of view, see, for example, David Wiggins, "On Being in the Same Place at the Same Time," *Philosophical Review*, LXXVII (1968): 90–95; and Mark Johnston, "Constitution Is Not Identity," *Mind*, CI (1992): 89–105 (on the first sort of view); and Sider (on the second sort).

[53] I take the functions in D to be total functions, each of which assigns the null fusion to world-times at which the material object associated with the function does not exist. We stipulate that the null fusion exists to degree 0 at every world-time. Those who find the null fusion uncongenial could instead consider partial functions, each of which assigns *nothing* to world-times at which the material object associated with the function does not exist. This would require certain alterations in what follows, but none that involves any substantial difficulty.

signs members of $D \cup D'$ (not D) to variables, but V still assigns n-tuples of members of D (not $D \cup D'$) to predicates (at world-times). We replace the clause:

$$V_v^M(Px_1,...,x_n,w,t) = V^M(P,w,t)(v(x_1),...,v(x_n))$$

with the clause:

$$V_v^M(Px_1,...,x_n,w,t) = V^M(P,w,t)(m_{(w,t)}(v(x_1)),...,m_{(w,t)}(v(x_n)))$$

where:

$$m_{(w,t)}(v(x)) = \begin{cases} v(x) \text{ if } v(x) \in D \\ f(w,t) \text{ if } v(x) = f \in D' \end{cases}$$

Thus, the truth (at a given world-time) of something said of a *material object* is determined by, as it were, passing the statement on to the *matter* which constitutes the object at that world-time, and seeing how true the statement is of that matter (at that world-time).[54] (ii) Q now assigns subsets of $D \cup D'$ (not D) to each (w,t). For $f \in D'$, we stipulate that $D_{(w,t)}f = D_{(w,t)}f(w,t)$, that is, for a material object f in D', concrete existence is governed by the principle that the degree to which f concretely exists at (w,t) is the same as the degree to which its constituting matter at (w,t), that is, $f(w,t)$, concretely exists at (w,t). So the account of concrete existence in section III is taken to apply directly only to items of matter (members of D); for material objects (members of D'), we now have an indirect account, parasitic on the direct one.[55]

Returning to the case of my boat, we find the case is problematic no more. My boat is not a mereologically static item of matter—a member of D. Rather, it is a material object—a member of D'. So the degree to which my boat exists at a time (in the actual world) is the degree to which its constituting matter exists at that time, where the latter is determined in accordance with the account presented in section III. At t_1, my boat is constituted by A, which concretely exists to degree 1 at t_1, so my boat concretely exists to degree 1 at t_1. (No matter that B does not concretely exist to degree 1 at t_1, because at t_1, my boat is not constituted by B.) At t_2, my boat is constituted by B, which concretely exists to degree 1 at t_2, so my boat concretely exists to degree 1 at t_2. (No matter that A does not concretely exist to degree 1 at t_2, because at t_2, my boat is no longer constituted by A.) Thus my boat—one and the same boat—exists to degree 1 at t_1 and

[54] With the exception, of course, of any statements involving the constitution function f itself (for example, 'This material object is constituted by that item of matter at time t in world w').

[55] Note that on this approach, corresponding to each item of matter a in D, there is a material object which is composed of that matter at all world-times: it is the constant function f_a in D' which assigns a to all pairs (w,t).

at t_2; the trick is that this one boat is constituted by different matter at t_1 and at t_2.[56]

IV.2. Temporal Parts. The other way to proceed involves making notional parthood a world-time-indexed relation, rather than an absolute one.[57] We abandon the absolute relation $\sqsubset$ in favor of the indexed relation $\sqsubset_{(w,t)}$, where $x \sqsubset_{(w,t)} y$ means that x is a notional proper part of y *at time t in world w*. Note that although this relation is now (like the concrete parthood relation) nonabsolute, it is still (*unlike* the concrete parthood relation) *non*vague. The definitions, and first four principles, of section II undergo the following modifications:

- proper-or-improper part at (w,t): $x \sqsubseteq_{(w,t)} y := x \sqsubset_{(w,t)} y \vee x = y$
- overlap at (w,t): $x \boxdot_{(w,t)} y := \exists z (z \sqsubseteq_{(w,t)} x \wedge z \sqsubseteq_{(w,t)} y)$
- disjointness at (w,t): $x \boxtimes_{(w,t)} y := \neg x \boxdot_{(w,t)} y$
- atom or simple at (w,t): $A_{(w,t)} x := \neg \exists y (y \sqsubset_{(w,t)} x)$
- fusion at (w,t): $xF_{(w,t)}S$ (that is, x is a fusion-at-(w,t) of a set S) just in case:

  1. everything in S is part-at-(w,t) of x, that is, $\forall y \in S(y \sqsubseteq_{(w,t)} x)$; and
  2. every part-at-(w,t) of x overlaps-at-(w,t) something in S, that is, $\forall y(y \sqsubseteq_{(w,t)} x \rightarrow \exists z \in S(y \boxdot_{(w,t)} z))$.[58]

N1′: $\neg x \sqsubset_{(w,t)} x$
N2′: $x \sqsubset_{(w,t)} y \wedge y \sqsubset_{(w,t)} z \rightarrow x \sqsubset_{(w,t)} z$
N3′: $x \sqsubset_{(w,t)} y \rightarrow \exists z (z \sqsubset_{(w,t)} y \wedge z \boxtimes_{(w,t)} x)$
N4′: $\neg A_{(w,t)} x \wedge \neg A_{(w,t)} y \wedge \forall z, w', t' (z \sqsubset_{(w',t')} x \leftrightarrow z \sqsubset_{(w',t')} y) \rightarrow x = y$.

Let F be the set of all (total) functions from the set of all pairs (w,t) (with $w \in W$ and $t \in T_w$) to the power set of D; that is, each function in F assigns a subset of D to each world-time pair. Our principle governing the existence of fusions is then:

N5′: For every f in F, there exists exactly one x such that for every world-time (w,t), x is a fusion at (w,t) of $f(w,t)$.

The idea is this: for each world-time, pick a bunch of objects; there then exists an object which has as its parts at each world-time exactly the things which you just associated with that world-time. Thus, for example, there is an object which consists of my now-parts now, my tomorrow-parts tomorrow, and so on (that object is me), and there are also

[56] Other puzzle cases are handled in a similar way. For example, suppose that after t_2 I dismantle my boat and assemble all the pieces at t_3 into a sculpture that does not float and looks like a random hunk of junk, but is structurally sound. At t_3, B once again concretely exists to degree 1—but it would seem that my boat does not exist at all (supposing I have not rebuilt it from other parts). No problem: B is not the constitutor of my boat at t_3; the sculpture is made of the right stuff to be my boat, but it is not made in the right way.

[57] Sider, p. 57, discusses temporally relativized parthood.

[58] This definition follows Sider, p. 58.

gerrymandered objects such as one which consists of the Eiffel Tower's now-parts now and the Opera House's tomorrow-parts tomorrow.

Turning now to concrete mereology, our linking principle becomes:

$$\text{NC}': \neg(x \sqsubseteq_{(w,t)} y) \rightarrow \lhd_{(w,t)}(x,y) = 0.$$

This says that if x is *not* a notional part of y *at a given world-time*, then x is a concrete part of y *to degree 0* at that world-time. C1 and C2 remain as above. As for C3, which governs the relationship between concrete parthood and concrete existence, we retain our earlier principle with just one change: rather than the set of all notional parts *simpliciter* of x, S_x is now the set of all notional parts *at* (w,t) of x:

$$\text{C3}': D_{(w,t)}(x) = \int_{S_x} f^{x}_{(w,t)} d\mu$$

Thus the degree to which an object x concretely exists at (w,t) is the degree to which its notional parts at (w,t) are concrete parts of it at (w,t).

Note that on this second approach we make no distinction between matter and material objects: we just have D, not D and D'. The idea is that I, for example, am the object in D which is the notional fusion of my notional now-parts now, my notional yesterday-parts yesterday, and no parts at all at times before I was born, and that I concretely exist now to the degree to which my notional now-parts are now concrete parts of me, and I exist to degree 0 before I was born (because the fusion of no parts is the null object, which exists to degree 0 at every world-time). Returning to the case of my boat: there are functions in F which assign to each time in the actual world all the objects which are (according to common sense) parts of my boat at that time, and for each such function f there is an object in D which at every world-time (w,t) is the fusion of $f(w,t)$; my boat is one of these latter objects. At t_1 it consists of all the objects which are (according to common sense) parts of my boat at t_1, and as these parts are all degree-1 concrete parts of my boat at this time, it concretely exists to degree 1. At t_2 it consists of a different bunch of objects (a has been replaced by b) and as these parts are all degree-1 concrete parts of my boat at this later time, it again concretely exists to degree 1. Thus my boat—one and the same boat—exists to degree 1 at t_1 and at t_2; the trick this time is that this one boat has different notional parts at t_1 and at t_2.

The temporal parts approach to persistence just outlined faces a subtlety which is worth discussing before we move on. Consider a particular tree, which—on the temporal parts approach—consists of (among other things) the fusion-at-t of certain tree-parts at t and the fusion-at-t' of certain tree-parts at t' (where t' is after t); consider also a particular lamp, which consists of (among other things) the fusion-at-t of certain lamp-parts at t and the fusion-at-t' of certain lamp-parts at t'; and consider finally a gerrymandered object X which consists of (among other things) the tree-parts at t and the lamp-parts at t'.

Suppose the tree is quite intact at t and the lamp quite intact at t', so that the tree has degree-1 concrete existence at t and the lamp has degree-1 concrete existence at t'. In this case, the gerrymandered object X has degree-1 concrete existence at t and at t' also. Now you might say that this is no problem: while we do not want to admit the degree-1 concrete existence at t of something which consists at t of the tree-parts at t and the lamp-parts at t, the object X is quite different, and entirely unobjectionable: all it takes for this object concretely to exist at t is for the tree concretely to exist at t; the latter is unproblematically the case; hence X's concrete existence at t is unproblematic. Alternatively, one might wish to deny the concrete existence of X at t. If so, all is not lost on the present framework: one can have what one wants by imposing the principle that (for any x and y and any world-time (w,t)) if x is a concrete part of y at (w,t), *then unless* x *is moved between* t *and* t' (or disturbed in some other way relevant to concrete parthood), x is a concrete part of y at (w,t'). With this principle in place, we find that X concretely exists to degree 0 at t (even though the tree concretely exists to degree 1 at t): the tree-parts at t are *not* concrete parts of X at t, because if they were they would have to be concrete parts of X at t' (given our principle, and assuming that the tree does not move between t and t'), but they are *not* concrete parts of X at t', because (and here I appeal to the linking principle NC′) they are not even notional parts of X at t' (at t', X consists of the lamp-parts at t').

Although formally quite different, both approaches to persistence just outlined have an important feature in common. The degree to which a persisting object concretely exists at a time t is the same as the degree to which its temporal part/constitutor at t concretely exists at t—where the *latter* degree is determined as in section III. Thus, when it comes to the concrete existence of an ordinary persisting object O at a time t, there are *two* variables: first there is the question of the degree of concrete existence of different notional fusions at t; second, there is the question of which of these notional fusions is O's temporal part/constitutor at t.[59]

[59] Suppose that I am building a boat, and I have all my timber scattered in the shed. Let X be the notional fusion of the timbers (either simpliciter, if we are working in the first framework, or at the particular time in question in the actual world, if we are working in the second framework). Each timber is a degree-0 *concrete* part of X at the time in question, and so the degree of concrete existence of X at the time in question is 0. The degree of existence of my boat at this time is, of course, also 0. But here we face a subtle question. Does my boat exist to degree 0 at this time because it *has no* temporal part/constitutor at this time (that is, its temporal part/constitutor at this time is the null fusion), or does it exist to degree 0 at this time because its temporal part/constitutor at this time is X, which exists to degree 0 at this time? That is, at a time before it has been built, but after its component planks have been collected in my shed, does my boat have no temporal part/constitutor, or is its temporal part/constitutor the fusion at this time of the component planks? I find the former option more congenial, but nothing hangs on this.

V. COUNTING VAGUE EXISTENTS

One of the major problems with vague composition and existence is supposed to be that they lead to indeterminacy or vagueness about *how many things there are in the world*—and this is supposed to be impossible/ obscure/nonsensical. This sort of worry is central to Lewder's argument for unrestricted composition (as we shall see in section VI), and—referring to Lewis's part of that argument—Rosen and Dorr write that a vague principle of composition "is objectionable if construed as a serious theoretical claim. It seems to entail the deeply obscure doctrine that it is a vague matter how many things there are" (*op. cit.*, p. 31).

There is certainly an issue about what to say in answer to the question 'How many things are there in the world?', when things can have vague existence—but it is in no way a deeply puzzling or problematic issue.[60] Suppose that we are constructing desks to fill a school hall. We have made a certain number, and a few more are half built. How many concretely existing desks are there in the room? It is not clear how to answer this question. But note that the problem is simply an instance of a general problem which arises when we have fuzzy sets. Suppose there are five definitely bald men in the room, and a few more who are bald to various intermediate degrees. How many bald men are there in the room? What we have here is, in essence, exactly the same problem as we have in the case of the school desks. So our counting problem is a problem that arises for anyone who countenances fuzzy sets: it is *not* a special problem to do with vague *existence*.

What are we to say in answer to the problem? One option is to introduce some reasonable method for counting fuzzy sets. One thought which springs to mind is that if there were three definitely bald men and one half-bald man, we might say that the number of bald men is three and a half. Following this thought, we could say that when counting bald men, a degree-n bald man adds n to the count of bald men, so if there are three degree-1 bald men, one degree-0.3 bald man, one degree-0.2 bald man, and the rest are degree-0 bald men, then the number of bald men is 3.5. Now we might say this, but when we spell out the thought, as I just have, I think we see that it is best

[60] To clarify: there is no deeply puzzling or problematic issue here when we approach vague existence as I have above—that is, if we think of '(concretely) exists' as a predicate. This is not to say that there is no deeply puzzling or problematic issue in this area for someone who thinks that the unrestricted existential quantifier $\exists$ is vague. Such a person really would have a big problem here—and as a referee pointed out, "people arguing against the possibility of counting vague existents have in mind a purely Quinean (that is, logical) notion of existence." So I do not mean to suggest that I am solving this big problem. Rather, the point is that my approach to vague existence never runs into this big problem in the first place. This is a very good thing, because the problem is (as far as I can see) insuperable—that is, I *agree* with those who deny that $\exists$ can be vague.

regarded as a *joke.* We might then look for some *other* reasonable method for counting fuzzy sets. Alternatively, we might refuse to introduce any such method, saying simply that we *cannot* count the bald men (for example). I believe the latter option is the correct one. This option faces two apparent problems, which I shall now address.

First there is the problem of expressive power. If I cannot count the membership of fuzzy sets, then there are huge expanses of reality which I am simply powerless to describe (for example, I cannot say how many bald men there are in the room); this is a bad thing; so we should introduce methods for describing these parts of reality. But the problem is not as bad as it may seem. First, we can *precisify.* We can count any crisp set, so we can count the things which are bald to degree 1, the things which are bald to a degree greater than 0.3, the things which are between 0.2 and 0.4 bald, and so on. Second, consider the numerical sentences of first-order logic:

(1) There is exactly one P: $\exists x(Px \wedge \forall w(Pw \rightarrow w = x))$
(2) There are exactly two Ps: $\exists x \exists y(Px \wedge Py \wedge x \neq y \wedge \forall w(Pw \rightarrow w = x \vee w = y))$
(3) There are exactly three Ps: $\exists x \exists y \exists z(Px \wedge Py \wedge Pz \wedge x \neq y \wedge x \neq z \wedge y \neq z \wedge \forall w(Pw \rightarrow w = x \vee w = y \vee w = z))$
⋮

In a classical context, for any interpreted predicate P (that is, for any set P), either none of these sentences will be true, or exactly one will be true—and if the nth sentence is true, there are exactly n Ps. When P is a fuzzy set, this breaks down; but nevertheless, we are not left with nothing to say: we can calculate the truth value of each of these numerical sentences, and we can then say that the degree of truth of the claim that there are exactly n Ps is the degree of truth of sentence n.[61] Thus, in response to the problem of expressive power: we can count all the degree-n bald men, for each n; and we can give the degree of truth of each claim 'there are exactly n men'; and once we have done these things, it seems to me we have given as much information as anyone could reasonably want—we do not need to then go on to give the number of bald men *simpliciter.*

The second problem is a Quinean one. You might think that the problem with not being able to count the concretely existing boats, the bald men, and so on, is that these things will then get mixed up with one another—they will not be able to be individuated properly—and then, following Quine, you might think that this is such a serious

[61] Compare the use of such numerical sentences in the theory of indeterminate identity developed in Terence Parsons, *Indeterminate Identity: Metaphysics and Semantics* (New York: Oxford, 2000).

problem that we cannot seriously speak of such things at all. But in the present case we run into no such problems. There is no problem of individuation, no confusion about identity: the impossible-to-count concretely existing boats do not get mixed up with one another, because when we try to count them, we are not trying to count objects *simpliciter*, we are trying to count a fuzzy subset of a background set of objects (the overall domain *D*, which is a regular, classical set), and identity and individuation are clear and precise matters in this background set. We cannot count the concretely existing boats *not* because each of them is (as it were) a fuzzy dot which blurs into other dots in such a way that we cannot get a handle on how many dots there are—this is indeed deeply obscure, if not outright nonsense; rather each is (continuing the metaphor) a precise dot, quite distinct from all other dots and in no danger of being confused with them—but the borders of the *set* of things we are counting is fuzzy, so that some of these precise dots lie in the penumbral region of the set, and hence we do not know whether or not to include them in our count.

In sum, the response to the counting problem is that there is no answer to the question 'How many *P*s?' when *P* is a fuzzy subset—although the precisified questions 'How many degree-*n* *P*s?', 'How many degree-*n*-or-less *P*s?', and so on, all have answers, and the claims 'There is exactly one *P*', 'There are exactly two *P*s', and so on, all have unique fuzzy truth values (so even though there is no uniquely correct answer to the simple question 'How many *P*s?', some possible answers are better—that is, more true—than others). Of course, the *problem* was supposed to be precisely that there is no answer to the question 'How many *P*s?' when *P* is a fuzzy subset: so to be quite explicit, the response is that this is not a problem at all.

VI. LEWDER'S ARGUMENT

I have set my views in opposition to Davore Lewder's "orthodox argument," which is a rough compilation of two arguments in the literature. I shall now respond to these two arguments.

Lewis has a famous argument for unrestricted composition:[62]

(1) A restriction on composition must be vague or nonvague.
(2) "To restrict composition in accordance with our intuitions would require a vague restriction." So a *nonvague* restriction would fail to satisfy the intuitions motivating restriction in the first place, and hence "would be gratuitous."
(3) "But if composition obeys a *vague* restriction, then it must sometimes be a vague matter whether composition takes place or not. And that is impossible." The reason it is impossible is that:

[62] Quotations are from Lewis, *Plurality*, pp. 212–13.

(a) "The only intelligible account of vagueness locates it in our thought and language...Vagueness is semantic indecision"; and
(b) "The question whether composition takes place in a given case... can be stated in a part of language where nothing is vague. Therefore it cannot have a vague answer."

(4) So we cannot have a vague restriction on composition, and we have no use for a nonvague one. Conclusion: composition is unrestricted.

An obvious line of objection to this argument is this: Why should we accept that the predicate 'is part of' is not vague (this predicate being in the part of language needed to ask the question whether composition takes place in a given case—see step 3b)? For it seems very natural to say, for example, that the hair that Tibbles is in the process of shedding is only *sort of* part of Tibbles. Enter Sider, who argues that it can never be a vague matter whether or not composition takes place, "for if it were vague whether a certain class had a fusion then it would be vague how many concrete objects exist" (*op. cit.*, pp. 126–27). In the terms of my presentation of Lewis's argument above, Sider is defending step 3, but in a new way—one which avoids the need for a direct argument that the predicate 'is part of' is nonvague. Sider argues as follows. Let the *concrete* objects be those objects which are *not* sets, classes, numbers, properties, and so on (add your favorite "abstract" objects to the list).

> Suppose now for *reductio* that...it can be vague whether a given class has a fusion. In such a case, imagine counting all the concrete objects in the world. One would need to include all the objects in the class in question, but it would be indeterminate whether to include another entity: the fusion of the class. Now surely if [the claim that it can never be vague whether a given class has a fusion] can be violated, then it could be violated in...a world with only finitely many concrete objects. That would mean that some *numerical sentence*—a sentence asserting that there are exactly *n* concrete objects, for some finite *n*—would be indeterminate. But numerical sentences need contain only logical terms and the predicate '*C*' for concreteness (a numerical sentence for $n = 2$ is: $\exists x \exists y[Cx \,\&\, Cy \,\&\, x \neq y \,\&\, \forall z(Cz \rightarrow [x = z \vee y = z])])$ (*op. cit.*, p. 127).

Now given the linguistic theory of vagueness, a numerical sentence can only be indeterminate if one or more of its components has multiple admissible interpretations. Sider argues, however, that all components are precise: apart from the logical vocabulary, which is precise,[63]

[63] Although I agree with Sider that the logical connectives are precise, Sider begs the question when he says that it "is overwhelmingly plausible that the boolean operators lack precisifications" (p. 128). Of course the *Boolean* operators lack precisifications: but reading the logical operators in Sider's numerical sentences *as Boolean operators* is *already* choosing one way of precisifying them. In general, the different precisifications of the logical operators correspond to the different possible systems of semantics for the logical connectives: for example, in the case of the negation

there is only the predicate '*C*', and it is precise because "it was defined by a list of predicates for fundamental ontological kinds that do not admit of borderline cases" (*op. cit.*, p. 127). The conclusion is that it *cannot* be vague whether a certain class has a fusion.

Although I have employed a nonlinguistic theory of vagueness in this paper, *this is not the source of my disagreement with Lewis and Sider.*[64] Rather, my disagreement is as follows. First, Lewis: obviously, I think that the question whether composition takes place in a given case can*not* be stated in a part of language where nothing is vague—for 'part' (that is, '*concrete* part') is vague.[65] Second, Sider: Sider says that if composition were vague (in a finite world), then some numerical sentence would be indeterminate. Is this true? Suppose that the numerical sentences involve *Sider's* predicate '*C*' for concreteness. In this case, it does *not* follow from my views about vague composition that some numerical sentence will be indeterminate: the overall domain D will be precisely cleaved into two crisp subsets, containing the concrete (in Sider's sense) and the nonconcrete objects respectively; the fact that (for example) some table only concretely exists—in *my* sense of 'concretely exists'—to degree 0.5 at the present time in the actual world does not make it any less than *definitely* a concrete object in *Sider's* sense of 'concrete'. Suppose instead that the numerical sentences involve *my* predicate '*E*' for concrete existence. In this case it *does* follow from my views about vague composition that some numerical sentence will be indeterminate (at some times in some worlds); but as shown in foregoing sections, this is perfectly possible—and it is even possible on a linguistic theory of vagueness.[66]

VII. FURTHER ISSUES

In this section I discuss four further issues connected with the views presented in this paper. Some of these issues merit a more extended treatment in future work, but here I can only touch on them briefly.

VII.1. Ambiguity. Suppose that I am building a boat: currently it is about half built, and I have not yet bought all the timber required to finish it. In this case when I speak of 'the boat I am building', it

---

operator, we have a three valued Boolean reading, a three valued de Morgan reading, a two valued classical reading, and so on.

[64] It had better not be: for recall that I noted at the outset that everything that I do in this paper could also be done within the framework of a linguistic theory of vagueness (in particular, within the degree-theoretic form of supervaluationism).

[65] How could 'part' be vague on a linguistic theory of vagueness? What would be its precisifications? Well, each precisification would involve a precise answer to the question as to how much contact (or in general, proximity) and adhesion is required between objects in order for them to be concrete parts of their notional fusion.

[66] In the latter case each precisification would involve a precise answer to the question as to how much of an object has to be concretely assembled in order for the object to exist concretely.

seems that there is no clear referent for this phrase. There are many boats in many possible worlds, each of which may, for all I know, turn out to be the boat I end up building. This in itself is not a great problem: we may suppose that there is one particular boat that I will, as a matter of fact, end up building, and the fact that I do not yet *know* which boat it is need have no effect on the *semantics* of the situation. But the problem gets worse: for suppose that as a matter of fact I shall never finish my boat. Nevertheless, it still seems to make good sense to speak of 'the boat I am building' (or later, of 'the boat I was building before I gave up'). Here we may use supervaluations. When I use a term, such as 'the boat I am building', which has multiple admissible referents (in the present case, each possible completion of my boat is an admissible referent)—or when I use a predicate with multiple admissible extensions—we have a notion of an *admissible interpretation* of my utterance. Each admissible interpretation is a model of the sort discussed above; in each admissible interpretation, each initially polysemic term is assigned a unique referent/extension (in an acceptable way: this is what makes the interpretation an *admissible* one).[67] Now, suppose I make an utterance involving one or more polysemic terms. This utterance has a particular degree of truth in each admissible interpretation. Overall, it is true simpliciter, or supertrue, if it is true on *every* admissible interpretation. Now, we are working in a degree-theoretic environment, so if $\{\mathcal{M}_i\}$ is the set of all admissible interpretations of my utterance, and f_i is the truth value of my utterance on $\mathcal{M}_i$, then the supertruth value of my utterance with respect to the supervaluation $\{\mathcal{M}_i\}$ is $\bigwedge\{f_i\}$ (the universal quantifier in 'supertruth is truth in *every* admissible interpretation' thus receiving its standard treatment as generalized conjunction). The case where my utterance contains no polysemic terms becomes the special case where the supervaluation has one member, and in this case the supertruth value of my utterance is the same as its truth value on this interpretation.

VII.2. Quasidentity. Lewis[68] discusses objects which are not identical, but which almost are, in the sense that they almost entirely overlap, and he argues that sometimes when we individuate objects, we are not concerned with strict identity, but with overlap. In the present framework, identity is an entirely classical, nonvague matter—but as well as nonvague identity, we also have a vague relation: (concrete) overlap. We thus have the resources to follow Lewis and talk *loosely* about de-

[67] See Kit Fine, "Vagueness, Truth, and Logic," in Rosanna Keefe and Peter Smith, eds., *Vagueness: A Reader* (Cambridge: MIT, 1997), pp. 119–50.

[68] "Many, but Almost One," in John Bacon, Keith Campbell, and Lloyd Reinhardt, eds., *Ontology, Causality, and Mind: Essays in Honour of D.M. Armstrong* (New York: Cambridge, 1993), pp. 23–38.

grees of 'identity'—we must simply realize that when we talk this way, we are really using 'identical' to mean overlapping. Lewis thinks that total overlap coincides with (strict) identity. Is this the case in the present framework? Certainly two nonidentical objects can entirely concretely overlap at some world-time. But does total overlap at every world-time imply (strict) identity? Maybe as a matter of fact, but not as a matter of principle: for recall the discussion of extensionality in section III, in which I argued that we should *not* adopt as a fundamental principle of concrete mereology the claim that if two objects have the same concrete parts to the same degrees at every world-time, then they are identical.

VII.3. Essential Parts. We could define the *essential* parts of x as the things which are positive-degree concrete parts of x at every world-time at which x exists to degree 1 (or, more stringently, at every world-time at which x exists to any degree greater than 0). Thus, an object cannot fully exist (or, on the stronger definition, exist at all) without its essential parts. Note that this provides an instant tie-in with the notion of a part's degree of importance, introduced in section III: essential parts cannot have zero importance (and on the stronger definition, the set of all essential parts of an object has degree of importance 1).

VII.4. Contact and Adhesion. I have not said much about *what makes it the case* that some notional part of a notional fusion is a *concrete* part of that fusion (at some world-time). I assume that concrete parthood often has a lot to do with contact and adhesion; however, I am not in a position here to offer a *theory* of the relationship between concrete parthood on the one hand, and physical contact and adhesion on the other. Suppose, by way of analogy, that we are theorizing the relationship between the colors of objects and the emotions they provoke in viewers. There are facts in the offing such as that no object can be both green and red. There are two attitudes we can take to these facts. (i) We can regard them simply as facts. If, according to our theory, only red provokes emotion A and only green provokes emotion B, then as a matter of fact, no uniformly colored object will simultaneously produce both emotion A and emotion B. Now this is an interesting fact, but not a theorem of our theory, given that the facts about the relationships between the colors have the status of untheorized, background facts. (ii) We can produce a 'logic of color' and graft it onto our theory. If it is a theorem of the logic of color that no object can be both green and red, and we have as theorems of our theory that only red provokes emotion A and only green provokes emotion B, then it will follow as a theorem that no uniformly colored object can provoke both emotion A and emotion B. Returning to the case of concrete mereology, I am taking the analog of approach (i) to facts about the relationship between contact, adhesion, and concrete parthood: these facts have the status of untheorized, background facts which have implications for, but are not part of the content of, the theory offered here.[69]

VIII. CONCLUSION

In this paper, I have tried to present a view which both makes perfectly clear sense, and satisfies widespread intuitions about composition and existence. On the first point: the views presented herein appeal only to tried and tested logical techniques; there are no obscure attempts to introduce, for example, new forms of vague unrestricted quantification. On the second point: first, it seems to me that the exclusion of objects such as the fusion of the Empire State Building and the Sydney Opera House from the domain of the actual world is exclusion enough to satisfy widespread intuitions about the nonexistence of fusions—not only do we do not *need* to go on to deny that such objects are present in the overall domain of possible objects, this would in fact be a counterintuitive move (given that we think it is *possible* for the fusion of the Empire State Building and the Sydney Opera House to exist: just bring the two together in an architecturally coherent and structurally sound way); and second, the view presented here allows us to say, for example, that as we transform a pile of planks into a boat, the planks *gradually* come to compose a further object, which *gradually* comes into existence.

NICHOLAS J.J. SMITH

University of Sydney

APPENDIX: FUZZY MODAL CONSEQUENCE RELATIONS

Standard discussions of modal logic focus on the notion of *valid formula* or *tautology*. A wf α is said to be *valid in a model* $\mathcal{M}$ iff $V_v^{\mathcal{M}}(\alpha,w) = 1$ for every $w \in W$ and every assignment v based on $\mathcal{M}$. A *frame* is a pair (W,R) with W and R as above; a model $\mathcal{M} = (W,R,D,Q,V)$ is said to be *based on* the frame (W,R); and a wf α is said to be *valid on a frame* (W,R) iff it is valid in every model based on that frame. I focus instead

[69] There is a substantial literature on *mereotopology*, which seeks to integrate mereological notions such as *parthood* with topological notions such as *contact* and *boundary* (see, for example, Barry Smith, "Mereotopology: A Theory of Parts and Boundaries," *Data and Knowledge Engineering*, xx (1996): 287–303; and Roberto Casati and Varzi, *Parts and Places: The Structures of Spatial Representation* (Cambridge: MIT, 1999), and the many references therein). The sort of theory I envisage—which treats of the relationship between concrete parthood, contact, and adhesion—would need to incorporate the insights of mereotopology, but would also need to go further, in discussing the nontopological concept of *adhesion*.

on the notion of *valid inference* or *consequence.* We can distinguish a number of consequence relations, arising from the various combinations of the two basic parameters *local–global* and *fuzzy–classical*:

- $\Gamma \models^{g\text{-}c}_{\mathcal{F}} \alpha$ (classical global consequence)
  there is no classical model $\mathcal{M}$ based on a frame in class $\mathcal{F}$, such that every member of Γ is true at every world in $\mathcal{M}$, and α is false at some world in $\mathcal{M}$.
- $\Gamma \models^{g\text{-}f}_{\mathcal{F}} \alpha$ (fuzzy global consequence)
  there is no fuzzy model $\mathcal{M}$ based on a frame in class $\mathcal{F}$, such that every member of Γ is >0.5 true at every world in $\mathcal{M}$, and α is <0.5 true at some world in $\mathcal{M}$.
- $\Gamma \models^{l\text{-}c}_{\mathcal{F}} \alpha$ (classical local consequence)
  there is no classical model $\mathcal{M}$ based on a frame in class $\mathcal{F}$, such that there is a world in $\mathcal{M}$ at which every member of Γ is true, and α is false.
- $\Gamma \models^{l\text{-}f}_{\mathcal{F}} \alpha$ (fuzzy local consequence)
  there is no fuzzy model $\mathcal{M}$ based on a frame in class $\mathcal{F}$, such that there is a world in $\mathcal{M}$ at which every member of Γ is >0.5 true, and α is <0.5 true.

It is not hard to prove that $\Gamma \models^{g\text{-}c}_{\mathcal{F}} \alpha$ iff $\Gamma \models^{g\text{-}f}_{\mathcal{F}} \alpha$, and $\Gamma \models^{l\text{-}c}_{\mathcal{F}} \alpha$ iff $\Gamma \models^{l\text{-}f}_{\mathcal{F}} \alpha$, that is, the classical and fuzzy consequence relations are identical in each of the local and global cases.[70]

We get notions of *tautology* when we set $\Gamma = \varnothing$. Of the four notions we get from the above notions of consequence by setting $\Gamma = \varnothing$, it is easy to see that the local and global classical notions coincide, and the local and global fuzzy notions coincide. Given that the classical and fuzzy consequence relations are identical in each of the local and global cases, it follows that there is just one class of tautologies for any class of frames. Thus where $\mathcal{F}$ is the set of all frames, the set of classical $\mathcal{F}$-tautologies is the set of formulas provable in K, and *so is the set of fuzzy $\mathcal{F}$-tautologies*; where $\mathcal{F}$ is the set of reflexive transitive frames, the set of classical $\mathcal{F}$-tautologies is the set of formulas provable in S4, and *so is the set of fuzzy $\mathcal{F}$-tautologies*; and so on. Thus, even when we fuzzify our modal semantics, we can retain our usual systems of modal proof theory, and the usual correspondences between frame conditions and modal axioms. This makes the fuzzy modal logic presented here very tractable.

There are stricter notions of local and global fuzzy consequence:

- $\Gamma \models^{g\text{-}f\text{-}s}_{\mathcal{F}} \alpha$ (strict fuzzy global consequence)
  there is no fuzzy model $\mathcal{M}$ based on a frame in class $\mathcal{F}$, such that

[70] Apply the basic idea behind the proof at pp. 193–95 of Smith, "Vagueness and Blurry Sets."

every member of Γ is 1 true at every world in $\mathcal{M}$, and α is not 1 true at some world in $\mathcal{M}$.

- $\Gamma \models^{l\text{-}f\text{-}s}_{\mathcal{F}} \alpha$ (strict fuzzy local consequence)
  there is no fuzzy model $\mathcal{M}$ based on a frame in class $\mathcal{F}$, such that there is a world in $\mathcal{M}$ at which every member of Γ is 1 true, and α is not 1 true.

These do *not* coincide with, respectively, the global and local classical notions. The reason I favor the consequence relations discussed above—the ones which coincide with the classical consequence relations—is as follows. In vague contexts, there is a natural distinction between *inference grade* statements and *assertion grade* statements. A natural response to Sorites reasoning is that we cannot simply keep taking the output of the inductive premise and feeding it back into the inductive premise at the next stage of the slide down the series—or at least we can, but our conclusions become progressively shakier. 'Man 1 is bald' is certainly true. 'For any n, if man n is bald then man $n + 1$ is bald' seems a safe enough assertion. From these it follows that man 2 is bald. Now if we feed this output back in, we get 'man 3 is bald'; if we feed this output back in, we get 'man 4 is bald'; and so on. But intuitively, our conclusions become more and more shaky. A natural thought is that while an output at one stage may be safe enough to assert, it may not be safe enough to serve as the start of the next stage of reasoning: it is like a terminating pass, as opposed to a pass which enables you to enroll in the next level of studies. So there seems to be, as I said, a natural distinction between *inference grade* statements and *assertion grade* statements. Now if we take a statement to be inference grade if it is strictly greater than 0.5 true, and assertion grade if it is at least 0.5 true, then we can say that a valid inference in the sense introduced above always yields at least an assertion grade conclusion, when the premises are all inference grade (in the local case, if all the premises are inference grade at a world, then the conclusion is at least assertion grade at that world; in the global case, if all the premises are inference grade at every world, then the conclusion is at least assertion grade at every world). This fits perfectly with the intuition that sorites reasoning is *valid*, even though it cannot be continued indefinitely and still yield secure results.