

## FREGE'S JUDGEMENT STROKE

Nicholas J.J. Smith

This paper brings to light a new puzzle for Frege interpretation, and offers a solution to that puzzle. The puzzle concerns Frege's judgement-stroke ('|'), and consists in a tension between three of Frege's claims. First, Frege vehemently maintains that psychological considerations should have no place in logic. Second, Frege regards the judgement-stroke—and the associated dissociation of assertoric force from content, of the act of judgement from the subject matter about which judgement is made—as a crucial part of his logic. Third, Frege holds that judging is an inner mental process, and that the distinction marked by the judgement-stroke, between entertaining a thought and judging that it is true, is a psychological distinction. I argue that what initially looks like confusion here on Frege's part appears quite reasonable when we remind ourselves of the differences between Frege's conception of logic and our own.

### I. The Tension

This paper is concerned with an apparent tension in Frege's work. On the one hand, Frege vigorously opposes any incursion on the part of psychology into the realm of logic. His attitude is summed up in the statement: 'it is the business of the logician to conduct an unceasing struggle against psychology and those parts of language and grammar which fail to give untrammelled expression to what is logical' [11, pp. 6–7]. On the other hand, his logic includes a symbol—the judgement-stroke ('|')—that apparently marks the difference between entertaining a thought, and judging that the thought is true—where to make a judgement is '[i]nwardly to recognize something as true' [11, p. 2].<sup>1</sup> Why would Frege, of all people, think that *logic* should find a place for the apparently *psychological* distinction between inward recognition of the truth of something and lack of such inward recognition?

To feel the genuine tension here, consider the following trio of claims:

- [1] In logic we must reject all distinctions that are made from a purely psychological point of view. What is referred to as a deepening of logic by psychology is nothing but a falsification of it by psychology. [19, p. 142]
- [2] Both grasping a thought and making a judgement are acts of a knowing subject, and are to be assigned to psychology. [31, p. 253]

<sup>1</sup> The same formulation occurs on p. 7. Cf. 'When we inwardly recognize that a thought is true, we are making a judgement' [19, p. 139].

[3] We . . . require another special sign to be able to assert something as true. For this purpose I let the sign “|—” precede the name of the truth-value . . . I distinguish the *judgement* from the *thought* in this way: by a *judgement* I understand the acknowledgement of the truth of a *thought*. The presentation in Begriffsschrift of a judgement by use of the sign “|—” I call a . . . *proposition*. I regard this “|—” as composed of the vertical line, which I call the *judgement-stroke*, and the horizontal line . . . the *horizontal*. [33, p. 38]<sup>2</sup>

1 says that logic must shun psychological distinctions; 2 says that the distinction between grasping a thought and making a judgement is a psychological distinction; 3 introduces a means of marking this distinction *in logic*.

Following Wittgenstein's contemptuous dismissal of Frege's judgement-stroke as 'logically quite meaningless: in the works of Frege (and Russell) it simply indicates that these authors hold the propositions marked with this sign to be true' [48, §4.442], the general consensus among commentators has been that the judgement-stroke is superfluous, a mere folly on Frege's part. Dudman, for instance, writes, 'Peano perceived immediately that Frege's judgement-stroke is otiose and thus anticipated Wittgenstein's *Tractatus* criticism' [7, p. 26]. Dudman thinks that a judgement-stroke-like operator might well find a role in a system of formal *dialectic*—just not in *logic*. But if the judgement-stroke is 'otiose' from the logical point of view; if it is 'logically quite meaningless'; if it merely indicates that Frege holds the propositions marked with it to be true; if its significance is restricted solely to the realm of dialectic; then it is a mystery how Frege—the arch anti-psychologist—could have committed the folly of giving the judgement-stroke a place in his logic.

The mystery is no small one. When Russell wrote to Frege pointing out that a contradiction could be derived from the latter's Basic Laws [43], Frege saw astonishingly quickly,<sup>3</sup> and to a far greater depth than Russell, the extent of the modifications to his system required to deal with Russell's problem (in effect, abandoning his life's work): 'It seems, then . . . that my Rule V . . . is false . . . It is all the more serious since, with the loss of my Rule V, not only the foundations of my arithmetic, but also the sole possible foundations of arithmetic, seem to vanish' [21, pp. 127–8]. Frege is justly famous for squarely facing Russell's difficulty. Now compare this response of Frege's with his response to the following criticism of *Grundgesetze* from Peano:

On page 9 he introduces one notation |—*a* to mean “*a* is true”, and another notation —*a* to indicate “the truth of *a*” (*a* being a proposition). I fail to see the purpose of these conventions, which have nothing corresponding to them in *Formulaire*. After all, the particular position a proposition occupies in a given formula shows unequivocally what it is that is being asserted about it in that formula. [42, p. 29]

If the judgement-stroke really does serve no logical purpose, then it can be removed from Frege's system without causing any damage (unlike Basic Law V); Frege is all in favour

<sup>2</sup> Except where explicitly stated otherwise, all italics in quotations are from the originals.

<sup>3</sup> Russell's letter is dated 'Friday's Hill, Haslemare, 16 June 1902', Frege's 'Jena, 22 June 1902'.

of minimising primitive logical symbolism;<sup>4</sup> and Frege is, in Dummett's words, 'vehement in his insistence that psychological considerations are irrelevant to logic' [8, p. xxxiii]—in the Introduction to the very work Peano is criticising, Frege writes 'And this brings me to what stands in the way of the influence of my book among logicians: namely, the corrupting incursion of psychology into logic' [33, p. 12]. Why, then, does Frege respond to Peano with the counter-accusation that Peano should have a sign corresponding to the judgement-stroke, given that Peano does 'acknowledge the distinction between the case in which a thought is merely expressed without being put forward as true, and that in which it is asserted' [18, p. 35]? Dudman simply calls this 'doggedness' on Frege's part [7, p. 26]—but clearly, more needs to be said.

In the later part of his career, in a piece of only ten lines entitled 'What may I regard as the Result of my Work?', Frege devotes two lines to the comment 'strictly I should have begun by mentioning the judgement-stroke, the dissociation of assertoric force from the predicate' [26, p. 184]. Obviously Frege regarded the judgement-stroke as a very important element in his logic. Hence if one accepts the standard view of the judgement-stroke as 'Frege's folly', one still faces the task of explaining how Frege, of all people, could have committed his folly. Conversely, the existence of the latter mystery may make us wary of the standard account: perhaps the judgement-stroke is not superfluous after all?

There are, then, two options: either Frege made an egregious blunder (viz., opposing the importation of psychological considerations into logic while importing some himself)—in which case we need to explain how this could have happened; or the judgement-stroke does *not* mark a psychological distinction—or at least, does not mark a psychological distinction of the sort whose importation into logic Frege opposed—in which case we need to explain just what the judgement-stroke is for, and (perhaps) why it is not superfluous.

Section II of this paper looks at Frege's anti-psychologism. In section III, Frege's various introductions of his judgement-stroke are presented. Section IV tries to determine what purpose Frege wished the judgement-stroke to serve; some existing accounts are criticised. Finally section V consists in an attempt to resolve the apparent tension between Frege's wish to have a sign that serves the purpose outlined in section IV, and his anti-psychologism.

## II. Frege's Anti-Psychologism

My aim in the present section is not to give a complete account of Frege's anti-psychologism, but merely to motivate my claim that it is *not good enough* to say of Frege's judgement-stroke simply that it marks a psychological distinction of no relevance to logic.

<sup>4</sup> 'This seems to me essential if our trains of thought are to be relied on; for only what is finite and determinate can be taken in at once, and the fewer the number of primitive sentences, the more perfect a mastery can we have of them' [12, p. 39]. See also, for example, [12, pp. 35–6], [10, p. 17] (on minimising the number of rules of inference), and [13, p. 48]: 'What strikes one in all this is the superfluity of signs.' Frege does not, however, regard minimisation of primitives as the *summum bonum*—see [18, p. 35].

Anti-psychologism is not a major theme of *Begriffsschrift*, the work in which Frege first presents his judgement-stroke.<sup>5</sup> Frege's anti-psychologism first emerges clearly in an unpublished piece entitled 'Logic' [11], written some time between 1879 (the year of publication of *Begriffsschrift*) and 1891.<sup>6</sup> There a theme emerges which Frege reiterates throughout his career:

Logic, like psychology, has for its subject-matter things that cannot be perceived by the senses. There is a sharp divide, however, marked by 'true'. . . . Psychology is only concerned with truth in the way every science is, in that its goal is to extend the domain of truths; but in the field it investigates it does not study the property 'true' as, in its field, physics focuses on the properties 'heavy', 'warm', etc. This is what logic does. [11, pp. 2–3]<sup>7</sup>

Frege also writes:

Now the grounds which justify the recognition of a truth often reside in other truths which have already been recognized. But if there are any truths recognized by us at all, this cannot be the only form that justification takes. There must be judgements whose justification rests on something else, if they stand in need of justification at all.

And this is where epistemology comes in. Logic is concerned only with those grounds of judgement which are truths. To make a judgement because we are cognisant of other truths as providing a justification for it is known as *inferring*. There are laws governing this kind of justification, and to set up these laws of valid inference is the goal of logic. [11, p. 3]

This leads to a more focussed anti-psychologism:

The task of logic being what it is, it follows that we must turn our backs on anything that is not necessary for setting up the laws of inference. In particular we must reject all distinctions in logic that are made from a purely psychological standpoint and have no bearing on inference. . . . Therefore let us only distinguish where it serves our purpose.

<sup>5</sup> In the Preface he writes: 'we divide all truths that require justification into two kinds, those for which the proof can be carried out purely by means of logic and those for which it must be supported by facts of experience. . . . [I]t is not the psychological genesis but the best method of proof that is at the basis of the classification' [10, p. 5]; and later, in regard to his employment of just one rule of inference (*modus ponens* or the *rule of detachment*): 'With this restriction to a single mode of inference, however, we do not intend in any way to state a psychological proposition; we wish only to decide a question of form in the most expedient way' [10, p. 17]. That, however, is the extent of Frege's distancing of his own project in *Begriffsschrift* from the tasks of psychology.

<sup>6</sup> The editors of *Posthumous Writings* write: 'In this piece . . . we clearly have a fragment of what was intended as a textbook on logic' [35, p. 1].

<sup>7</sup> Compare the opening passage of 'Thoughts', one of Frege's last three published works: 'Just as "beautiful" points the ways for aesthetics and "good" for ethics, so do words like "true" for logic. All sciences have truth as their goal; but logic is also concerned with it in a quite different way: logic has much the same relation to truth as physics has to weight or heat. To discover truths is the task of all sciences; it falls to logic to discern the laws of truth' [30, p. 351].

The so-called deepening of logic by psychology is nothing but a falsification of logic by psychology. [11, p. 5]

The idea, then, is that nothing irrelevant to inference is relevant to logic. This idea had already found expression in *Begriffsschrift* ('I decided to forgo expressing anything that is without significance for the *inferential sequence*' [10, p. 6]; 'Everything necessary for a correct inference is expressed in full, but what is not necessary is generally not indicated' [10, p. 12]), and also appears elsewhere, for example in a paper, written shortly after *Begriffsschrift*, which Frege tried unsuccessfully to publish: 'One must always hold fast to the fact that a difference is only logically significant if it has an effect on possible inferences' [12, p. 33, n. \*]. And thus when Frege writes 'Rejection of psychological distinctions. . . Isolating what is psychological, by consciously marking it off. Warning against confusing points of view and switching from one question to another' [11, p. 2], the enemy is not so much psychology *per se*, as psychological considerations *that have no bearing on inference*.

Already we can see how great a strain is involved in the view that the judgement-stroke marks a psychological distinction of a kind relevant to dialectic, perhaps, but not to logic. Frege was on the lookout for just such distinctions, and was keen to banish them from the realm of logic.

Half way through his Introduction to *Grundgesetze*, Frege has a footnote: 'Mathematicians reluctant to venture into the labyrinths of philosophy are requested to leave off reading the Introduction at this point' [33, p. 12, n. 7]. From this point on, the Introduction consists in a diatribe—in places verging on a rant (see for instance p. 22: 'At this I almost feel like losing my temper entirely and shouting at him . . .')—directed against the 'psychological logicians'. Frege writes:

the prevailing logic . . . seems to be infected through and through with psychology. If people consider, instead of things themselves, only their subjective *simulacra*, their ideas of them, then naturally all the more delicate distinctions within the subject matter are lost, and others appear in their place that are logically completely worthless. [33, p. 12]

He states, 'I take it as a sure sign of a mistake if logic has need of metaphysics and psychology—sciences that require their own logical first principles' [33, p. 18], and goes on, 'All psychological considerations, with which our logic-books of today are swollen, then prove to be irrelevant' [33, p. 22]; 'psychological considerations have no more place in logic than they do in astronomy or geology' [33, p. 23].

Also in his Introduction to *Grundgesetze*, Frege elaborates on the following theme—again one which recurs throughout his writings:<sup>8</sup>

laws of logic . . . have a special title to the name "laws of thought" . . . But the expression "law of thought" seduces us into supposing that these laws govern thinking in the same way as laws of nature govern events in the external world. In that case they

<sup>8</sup> See for instance [30, pp. 351–2] and [19, pp. 145–9].

can be nothing but laws of psychology: for thinking is a mental process. . . . the psychological logicians confuse [something's being taken to be true with its being true]. . . . All I have to say is this: being true is different from being taken to be true, whether by one or many or everybody, and in no case is to be reduced to it. . . . I understand by 'laws of logic' not psychological laws of takings-to-be-true, but laws of truth. . . . These mixings together of wholly different things are to blame for the frightful unclarity that we encounter among the psychological logicians. [33, pp. 12–5]

Again, we see here the potential strain involved in saying that Frege's judgement-stroke merely marks the propositions which Frege takes to be true (Wittgenstein's view): for Frege is adamant that truth and people's takings-to-be-true are very different things, and that logic is concerned with the former.

There are many other places in which Frege voices his anti-psychologism, but I think I have said enough to serve my present purpose.<sup>9</sup> It is simply not good enough to proceed smugly in the supposition that we have seen something that Frege missed—namely, that the judgement-stroke marks a merely psychological distinction that is of no interest to logic. If one wishes to argue that the judgement-stroke does indeed have this status, then one needs in addition to offer some account of how *Frege*—who opened *our* eyes to the need to separate logic and psychology—failed to see this. Alternatively, of course, one might argue that the judgement-stroke is *not* 'Frege's folly'.

### III. Introducing the Judgement-Stroke

This section sets down the data for the discussion to follow. The data consist in Frege's various introductions of his judgement-stroke. Because we are concerned with a puzzle of interpretation, it is essential that we do not *begin* with paraphrases—hence the lengthy quotations.

There are two major occasions on which Frege introduces the judgement-stroke: *Begriffsschrift* and *Grundgesetze*. Associated with each are minor occasions on which Frege more or less repeats what he says about the judgement-stroke on one of the major occasions. (The question as to how many *distinct* accounts of the judgement-stroke Frege puts forward is one to which we shall come.) Frege first introduces the judgement-stroke in *Begriffsschrift*:

A judgement will always be expressed by means of the sign

┌—,

which stands to the left of the sign, or combination of signs, indicating the content of the judgement. If we *omit* the small vertical stroke at the left end of the horizontal one,

<sup>9</sup> For a few more instances of Frege's anti-psychologism see [17, pp. 208 ('how difficult it is for the light of truth to penetrate the fog that rises from the mixture of psychology and logic'), 209 ('the devastation caused by the irruption of psychology into logic')], [19, pp. 143, 145–6, 149 ('purify logic of all that is alien and hence of all that is psychological')] and [30, pp. 368, 401].

the judgement will be transformed into a mere combination of ideas, of which the writer does not state whether he acknowledges it to be true or not. For example, let

|—A

stand for the judgement “Opposite magnetic poles attract each other”; then

—A

will not express this judgement; it is to produce in the reader merely the idea of the mutual attraction of opposite magnetic poles, say in order to derive consequences from it and to test by means of these whether the thought is correct. When the vertical stroke is omitted, we express ourselves *paraphrastically*, using the words “the circumstance that” or “the proposition that”. . . . *The horizontal stroke* that is part of the sign |— *combines the signs that follow it into a totality, and the affirmation expressed by the vertical stroke at the left end of the horizontal one refers to this totality.* Let us call the horizontal stroke the *content stroke* and the vertical stroke the *judgement stroke*. [10, pp. 11–2]

Later Frege writes, ‘If there is no judgement stroke, then here—as in any other place where the ideography is used—no judgement is made. —A merely calls upon us to form the idea that A does not take place, without expressing whether this idea is true’ [10, p. 18].

Shortly after *Begriffsschrift*, in two papers intended for publication, and in a third, published paper, Frege offers similar formulations: ‘The *judgement-stroke* is placed vertically at the left hand end of the content-stroke, it converts the content of possible judgement into a judgement’ [12, p. 11, n. \*\*\*]; ‘in order to put a content forward as true, I make use of a small vertical stroke, the judgement stroke, as in |—3<sup>2</sup>=9 whereby the truth of the equation is asserted, whereas in —3<sup>2</sup>=9 no judgement has been made’ [13, p. 51]; ‘If I wish to assert a content as correct, I put the judgement stroke on the left end of the content stroke: |—2+3=5 . . . Through this mode of notation I meant to have a very clear distinction between the act of judging and the formation of a mere assertible content’ [15, p. 94].

The next place in which Frege introduces the judgement-stroke is ‘Function and Concept’:

If we write down an equation or inequality, e.g. 5>4, we ordinarily wish at the same time to express a judgement; in our example, we want to assert that 5 is greater than 4. According to the view I am here presenting, ‘5>4’ and ‘1+3=5’ just give us expressions for truth-values, without making any assertion. This separation of the act from the subject matter of judgement seems to be indispensable; for otherwise we could not express a mere supposition—the putting of a case without a simultaneous judgement as to its arising or not. We thus need a special sign in order to be able to assert something. To this end I make use of a vertical stroke at the left end of the horizontal, so that, e.g., by writing

|—2+3=5

we assert that 2+3 equals 5. Thus here we are not just writing down a truth-value, as in

2+3=5,

but also at the same time saying that it is the True. [16, p. 149]

This is very similar to the account that appears a few years later in *Grundgesetze*:

We have already said that in a mere equation there is as yet no assertion; “2+3=5” only designates a truth-value, without its being said which of the two it is. . . . We therefore require another special sign to be able to assert something as true. For this purpose I let the sign “|—” precede the name of the truth-value, so that for example in

“|—2<sup>2</sup>=4”,

it is asserted that the square of 2 is 4. . . . I regard this “|—” as composed of the vertical line, which I call the *judgement-stroke*, and the horizontal line, which I will now simply call the *horizontal*. . . . Of the two signs of which “|—” is composed, only the judgement-stroke contains the act of assertion. [33, pp. 37–9]

#### IV. The Purpose of the Judgement-Stroke

Now that we have before us Frege's various introductions of the judgement-stroke, we may enquire as to its intended purpose. I find it useful to proceed via a criticism of Dudman's view on this matter.<sup>10</sup>

According to Dudman, Frege offers—and conflates—two inconsistent accounts of the judgement-stroke. According to the first account, which Dudman calls the ‘Geach version’ (in reference to views put forward by Geach [38]), the judgement-stroke is simply an index of assertion: ‘it signals by its presence or absence whether or not a given conceptual content (of a kind capable in principle of being put forward as true) is in fact being put forward as true’ [6, p. 153]. According to the second account of the judgement-stroke, which Dudman calls the ‘Black version’ (in reference to views put forward by Black [2, p. 227]), the judgement-stroke converts designations into assertions. As we saw in §III above, according to the Frege of ‘Function and Concept’ and *Grundgesetze*, ‘2+3=5’ merely designates a truth value, whereas “|—2+3=5” does not designate anything; it asserts something’ [32, p. 34, n. \*].

On both accounts, we have an assertion if and only if the judgement-stroke is present. The difference between the two accounts is that on the second (Black) account, the judgement-stroke ‘alter[s] semantic status’ [6, p. 153]—it converts a designation into something that is not a designation—whereas on the first (Geach) account, ‘the

<sup>10</sup> As far as I am aware, Dudman is the only other person to have devoted a substantial paper to Frege's judgement-stroke.



expressions to which it is appropriate to prefix an assertion-sign are, both when they include it and when they lack it . . . alike expressions of conceptual contents of the sort which are in principle capable of being held true' [6, p. 153].

Dudman sees Frege presenting the second (Black) account in 'Function and Concept' and *Grundgesetze* (in the passages quoted in the second half of §III above); he sees Frege presenting the first (Geach) account in *Begriffsschrift* and related articles (in the passages quoted in the first half of §III above), and in what Dudman calls Frege's 'mature works' (those between 1891 and Russell's Paradox) [6, p. 158]—and on pp. 157–9 Dudman does indeed produce four quotations from the later works in which Frege says things about the judgement-stroke that sound rather like the things he says in earlier works. The idea is that Frege held the Geach view throughout his career, and in the later part of his career held the Black view *as well*:

In Frege's mature works, quite evidently, there are *two* explanations of the judgement-stroke to be found. *First* there is the one sketched by Professor Black, according to which indicative sentences combine with judgement-strokes to form assertions—and an assertion is not a name of anything. On the other hand, an indicative sentence unadorned by a judgement-stroke serves merely to name an object. Accordingly, the role of the judgement-stroke is to convert mere designations into truth-claims. This doctrine . . . is peculiar to the mature works and appears for the first time in 1891.

The *second* explanation of the judgement-stroke to be found in Frege's mature works is the Geach one, the old "index of assertion" one familiar from *Begriffsschrift* days. [6, p. 159]

Dudman then contends that Frege *conflates* the two accounts; as evidence, Dudman quotes the passage from 'Function and Concept' quoted in §III above [6, pp. 159–60].

While the passage from 'Function and Concept' in question does indeed bear similarities both to passages in which Dudman sees the Geach account, and to passages in which Dudman sees the Black account, we cannot speak of 'conflation' here—for, *contra* Dudman, Frege only ever offered a single account of his judgement-stroke. The account he offered was the Geach account. The passages Dudman cites in support of the Black account in fact express the Geach view: they simply do so in a slightly different way from the passages which Dudman cites in support of the latter—to be precise, they express the Geach view in the context of Frege's new terminology of *sense*, *Meaning* and *truth value*.<sup>11</sup> As for the Black account, according to which the judgement-stroke *converts* a name into something that is not a name, I shall argue that it is nowhere endorsed by Frege.<sup>12</sup>

<sup>11</sup> When writing *about* Frege, I follow Evans [9] in using 'Meaning' for Frege's 'Bedeutung'. (When quoting *from* Frege I simply reproduce the cited translations.)

<sup>12</sup> I use the term 'the Black account' to refer to the account elaborated by Dudman and attributed by him to Black. I do not myself attribute this view to Black, but nor do I deny that Black held the view—Black's comment ('Frege's introduction of the assertion-sign may be viewed as an unsuccessful attempt to restore to the propositional sign, which he had degraded to a mere designation, its truth-claiming aspect' [2, p. 227]) is simply too brief to enable me to form an opinion either way.

Dudman himself criticises the Black view—but he attributes it to Frege nevertheless. He writes:

the Black version, taken literally, implies that asserted sentences are neither true nor false—a result totally at odds with Frege's manifest intentions.<sup>13</sup> To be true (false) in the mature system is to be a *name* of the True (or the False). But according to the Black version asserted sentences are not names at all: the judgement-stroke “does not serve, in conjunction with other signs, to designate an object, ‘|—2+3=5’ does not designate anything; it asserts something”. [6, p. 161]<sup>14</sup>

Dudman continues that Frege ‘is bound to concede that asserted sentences are just as much proper names as are unasserted ones: he cannot withhold the status of names from asserted sentences on pain of depriving them of sense and reference’ [6, p. 161]. But Frege *does* concede that asserted sentences are just as much proper names as are unasserted ones! Dudman is confusing *asserted sentences* with *assertions*. The following:

|—2+3=5

is an *assertion*; the *asserted sentence* here is ‘2+3=5’, and it is just as much a name in its occurrence above as it would be if written down all by itself. One can assert an *asserted sentence* (one simply asserts the sentence the previous assertion of which rendered it an *asserted sentence*—if this were not possible then the sentence could never have become an asserted sentence in the first place), but one cannot assert an *assertion*. One cannot *assert*

|—2+3=5

for this is already an *assertion* of ‘2+3=5’. The judgement-stroke is part of the assertion, but it is not part of the asserted sentence. The assertion (asserted sentence plus judgement-stroke plus content-stroke or horizontal) is not a name; the asserted sentence is (still) a name. The judgement-stroke thus does not—as the Black view would have it—*convert* a name into something else; rather, it combines with a name to form something that is not a name. (One thing that makes this all a bit tricky is that

|—2+3=5

*considered as a line of a work written in Begriffsschrift*,<sup>15</sup> has just the same status as

2+3=5

*considered as one complete line of a serious work written in English.*)

<sup>13</sup> Here, as elsewhere—for example when Dudman says that Frege expresses two inconsistent views in the one passage of ‘Function and Concept’—it is a mystery why Dudman does not give greater weight to the thought that perhaps Frege did *not* hold the Black view.

<sup>14</sup> Dudman is quoting [32, p. 34, n. \*].

<sup>15</sup> That's Begriffsschrift named with emphasis, not *Begriffsschrift*—i.e. Frege's logical system, not the work in which he first expounds that system.

With the distinction between assertions and asserted sentences in mind, we can see that Frege does indeed concede that asserted sentences (but *not* assertions) are just as much proper names as are unasserted sentences. As we saw in §III, Frege writes:

by writing

$$\ulcorner 2+3=5$$

we assert that  $2+3$  equals 5. Thus here we are not just writing down a truth-value, as in

$$2+3=5,$$

but also at the same time saying that it is the True. [16, p. 149]

and again:

“ $2+3=5$ ” only designates a truth-value, without its being said which of the two it is. . . .

We therefore require another special sign to be able to assert something as true. For this purpose I let the sign “ $\ulcorner$ ” precede the name of the truth-value [33, pp. 37–9]

None of this would make any sense if ‘ $2+3=5$ ’ were not a name of a truth value *both* when appearing alone *and* when appearing immediately after ‘ $\ulcorner$ ’. Frege says that by writing

$$\ulcorner 2+3=5$$

we are not *just* writing down a truth-value, we are *also* saying that *it* is the True. If Dudman were right, however—if Frege held the Black view, held that asserted sentences were not names but something else—then Frege would here say instead that in writing

$$\ulcorner 2+3=5$$

we are *not* writing down a truth-value (which is almost the opposite of saying that we are *not just* writing down a truth-value), and the second part of the sentence (containing ‘it’) would be absent altogether (on pain of being nonsense). Again, in the second passage Frege says that ‘ $2+3=5$ ’ only designates a truth value, whereas

$$\ulcorner 2+3=5$$

says *which* truth value *it* designates. This would make no sense if the addition of ‘ $\ulcorner$ ’ stopped ‘ $2+3=5$ ’ designating anything at all. (Of course the whole assertion—including the judgement-stroke—does not designate anything.)<sup>16</sup>

<sup>16</sup> After formulating my criticism of Dudman, I discovered that Stoothoff offers a similar one. Stoothoff writes, “[Dudman] appears to interpret Frege as saying that the sentence ‘ $2+3=5$ ’, as it occurs in ‘ $\ulcorner 2+3=5$ ’, does not designate anything, is not a name of the True. But Frege neither says nor implies this. He says only that ‘ $\ulcorner 2+3=5$ ’ designates nothing, from which it does not follow that ‘ $2+3=5$ ’ is non-designatory as it occurs in ‘ $\ulcorner 2+3=5$ ’ [46, p. 166]. Stoothoff says nothing else concerning this matter; in particular, he does not point out that so far from saying or implying that ‘ $2+3=5$ ’ is non-designatory as it occurs in ‘ $\ulcorner 2+3=5$ ’, Frege says or implies the opposite.

Dudman continues from where we left him a moment ago with the suggestion that instead of running the Black line, what Frege *should* have done is take up the position adopted by Church in *Introduction to Mathematical Logic* [3], according to which all sentences—asserted and unasserted—are names. Dudman concludes that because Frege is committed to the Church view, 'he is not entitled to . . . his Black account of the assertion-sign. . . the Black version (which, as we have seen, Frege expounds in "Function and Concept" and *Grundgesetze* i) is incompatible with other, more central, tenets of the mature period' [6, p. 161]. Would anything be sufficient to make Dudman see his *ponens* as a *tollens* (so to speak)? The passages from 'Function and Concept' and *Grundgesetze* which Dudman mentions here are the ones quoted in §III above and again more recently—passages which not only do not yield the Black account, but make absolutely no sense on that account. Frege's view is precisely the one to which Dudman says Frege is committed—the so-called Church view—and it is a mystery to me how Dudman could have persisted in thinking that Frege did not hold this view.<sup>17</sup>

I suggest, then, that Frege's view of the judgement-stroke is constant throughout his career. As his views on the nature of sentences change, so vary his formulations of the role of the judgement-stroke, but the basic idea is always the same: *Frege sees the judgement-stroke as a mark of assertion*. Note that 'mark' here needs to be understood in the right way. Her heavily lined brow marks Maisy as a thinker of deep thoughts, but she could think those thoughts without showing any signs of doing so. My use of the phrase 'I apologise', on the other hand, not only indicates that I am apologising—it effects my apology. If I showed no signs of apologising, I would not be apologising. The judgement-stroke marks assertion in this second way: it indicates that what follows it is being asserted, but it also effects the assertion. The situation of the judgement-stroke is in this respect precisely that of such ordinary indicators of assertion as tone of voice. My tone indicates that I am asserting that you need to rest, not asking whether you need to; but at the same time, in the absence of this or any other indicator of assertion, no assertion would have been made.

Initially Frege thinks of signs as having 'contents', a notion which is left rather vague. He writes, 'Not every content becomes a judgement when |— is written before its sign; for example, the idea "house" does not. We therefore distinguish contents that *can become a judgement* from those that *cannot*. . . . *Whatever follows the content stroke must have a content that can become a judgement*' [10, pp. 11–2]. The judgement-stroke never occurs except to the immediate left of the content-stroke; the content-stroke never occurs except to the left of a sign for a content that can become a judgement. When only the content-stroke appears, the content in question is merely put forward for consideration; when the judgement-stroke also appears, the content in question is put forward as being true. (See the quotations in the first part of §III above.)

Frege subsequently abandons the notion of a sign's having a *content* in favour of a pair of more precise notions: a sign's having a *sense* and a *Meaning*. Now without *contents*, there is no role for the *content-stroke*—and indeed the latter is reborn in the later works as the *horizontal*. In the Introduction to *Grundgesetze* Frege writes:

<sup>17</sup> Saying that Frege held the Church view is putting things backwards, of course: Church describes himself as 'adopt[ing] a theory due to Frege' [3, p. 23].

The old signs that appear here outwardly unchanged, and whose algorithm has also hardly changed, are nonetheless provided with different explanations. The former ‘content-stroke’ reappears as the ‘horizontal’. These are consequences of a thoroughgoing development of my logical views. Formerly I distinguished two components in that whose external form is a declarative sentence: (1) the acknowledgment of truth, (2) the content that is acknowledged to be true. The content I called a ‘possible content of judgement’. This last has now split for me into what I call ‘thought’ and ‘truth-value’, as a consequence of distinguishing between sense and denotation [Meaning] of a sign. In this case the sense of a sentence is a thought, and its denotation a truth-value. Over and above this is the acknowledgment that the truth-value is the True. That is, I distinguish two truth-values: the True and the False. [33, pp. 6–7]

(Note that Frege does *not* say that the *judgement-stroke* now receives a different explanation; this is a very strong indication that Frege had not altered his basic view of the judgement-stroke. Dudman [6, p. 159] actually quotes the above passage from *Grundgesetze*; however, because of his unshakeable conviction that Frege held the Black view in later writings, he takes the above passage as evidence that in those later writings Frege *also* held the Geach view—the view expressed in earlier writings.) Later in *Grundgesetze* Frege goes on:

I distinguish the *judgement* from the *thought* in this way: by a *judgement* I understand the acknowledgement of the truth of a *thought*. The presentation in *Begriffsschrift* of a judgement by use of the sign “|—” I call a *proposition of Begriffsschrift* or briefly a *proposition*. I regard this “|—” as composed of the vertical line, which I call the *judgement-stroke*, and the horizontal line, which I will now simply call the *horizontal*.<sup>\*</sup> . . . I regard [the horizontal] as a function-name, as follows:

—Δ

is the True if Δ is the True; on the other hand it is the False if Δ is not the True. [33, p. 38]

Frege always demands that functions must be defined for *any* object taken as argument.<sup>18</sup> Hence the horizontal—being a function-symbol—may occur to the left of any name, and in each case the resulting name (i.e. the name made up of the horizontal and the original name) must have a Meaning. (Contrast the content-stroke, which could only occur to the

\* [Frege’s footnote] I used to call it the *content-stroke*, when I still combined under the expression “possible content of judgement” what I have now learned to distinguish as truth-value and thought.

<sup>18</sup> See for instance [16, p. 148]: ‘The requirement of the sharp delimitation of concepts thus carries along with it this requirement for functions in general that they must have a value for every argument’; also [28, p. 244]: ‘The requirement that a concept have sharp boundaries corresponds to the more general requirement that the name of a function of one argument, when supplemented with a meaningful proper name, must in turn yield a meaningful proper name. And the same holds *mutatis mutandis* for functions of two arguments’ (cf. also p. 241).

left of signs for contents that can become judgements—sentences, in effect.) Thus Frege writes:

By our stipulation  $\neg 2^2=5$  is the True; thus:

$$|\neg 2^2=5,$$

in words:  $2^2=5$  is not the True; or: the square of 2 is not 5.

So also:  $|\neg 2$ . [33, p. 40]

Here we have something we did not have in the early works—assertions of the form  $|\neg 2$ , in addition to familiar assertions of the form  $|\neg 2^2=5$ —but the change is due to a change in the conception of ‘—’, not a change in the conception of ‘|’ (the judgement-stroke). The judgement-stroke is doing just what it always did: marking assertion.

But assertion of *what*? Of sentences? Is ‘ $\neg 2$ ’ a sentence?<sup>19</sup> It doesn't really matter whether we call it a sentence or not—it *is* a name of a truth-value (of the False [33, p. 39]). ‘Sentences’ (what gets asserted) may not be quite what they used to be, and assertion itself has a new look: in line with the new conception of ‘sentences’, to assert one is to say that the truth-value it names is the True. But amongst all this change, the judgement-stroke is a pillar: as always, its role is to enable us to make assertions: ‘We thus need a special sign in order to be able to assert something. To this end I make use of a vertical stroke at the left end of the horizontal’ [16, p. 149]; ‘We therefore require another special sign to be able to assert something as true. For this purpose I let the sign “ $|\neg$ ” precede the name of the truth-value . . . Of the two signs of which “ $|\neg$ ” is composed, only the judgement-stroke contains the act of assertion’ [33, pp. 37–9]. *What is asserted* and *what assertion is* may not be quite the same as before, but as always, the role of the judgement-stroke is to make and mark assertions.<sup>20</sup>

The last lines of Dudman's paper on Frege's judgement-stroke read:

The Geach version . . . would appear to be logically independent of the rest of Frege's semantics. It seems possible to maintain that assertoric force is something over and above the “content” of a sentence, so that the same “content” may be put now with and now without assertoric force, without committing oneself at all as to the nature of

<sup>19</sup> Dummett thinks so (although note his ‘in effect’): ‘in *Grundgesetze* [the horizontal (which for some reason Dummett calls the ‘content-stroke’, even when writing about *Grundgesetze*)] in effect turns any singular term into a sentence’ [8, p. 315]. Heck and Lycan's ‘inclination is to deny that there is any determinate answer’ to the question [39, p. 492].

<sup>20</sup> It should be noted that Frege does occasionally mention other roles for the judgement-stroke. First, ‘In the concept-script the judgement-stroke, besides conveying assertoric force, serves to demarcate the scope of the roman letters’ [23, p. 195]. This fits with what Frege says in *Begriffsschrift*: ‘If a Latin letter occurs in an expression that is not preceded by a judgement-stroke, the expression is meaningless’ [10, p. 25]. Second, ‘With this judgement-stroke I close off a sentence, so that each condition necessary for its holding is also effectively to be found within it; and by means of the self-same sign I assert the content of the sentence thus closed off as true’ [20, p. 247]. It is not quite so clear that this fits with what Frege says in *Begriffsschrift*: ‘The horizontal stroke that is part of the sign  $|\neg$  combines the signs that follow it into a totality, and the affirmation expressed by the vertical stroke at the left end of the horizontal one refers to this totality’ [10, p. 12].

such “contents”—in particular without prejudice to the doctrine that sentences name truth-values and express thoughts. For this reason, as well as because of its greater longevity, I think the Geach account ought to be regarded as Frege’s *official* version of the role of the judgement-stroke. [6, p. 161]

Apart from the implication that Frege held more than one view of the judgement-stroke, I agree entirely.<sup>21</sup>

#### IV. The Judgement-Stroke as Predicate?

Before leaving the question of the intended purpose of the judgement-stroke, there is one more issue that needs to be discussed. At one point in *Begriffsschrift* Frege writes:

We can imagine a language in which the proposition “Archimedes perished at the capture of Syracuse” would be expressed thus: “The violent death of Archimedes at the capture of Syracuse is a fact”. To be sure, one can distinguish between subject and predicate here, too, if one wishes to do so, but the subject contains the whole content, and the predicate serves only to turn the content into a judgement. *Such a language would have only a single predicate for all judgements, namely, “is a fact”*. We see that there cannot be any question here of subject and predicate in the ordinary sense. *Our ideography is a language of this sort, and in it the sign |— is the common predicate for all judgements*. [10, pp. 12–3]

A number of commentators have swooped on this passage. Currie writes, ‘Frege’s exposition of [the judgement-stroke] is not very clear, and he says at one point that the judgement stroke can be read as a predicate . . . but ‘A is a fact’ is just a sentence which can be asserted or not. . . . Things become clearer in the *Basic Laws*. . . . In his later work, Frege abandoned the view that the judgement stroke is a predicate’ [4, pp. 113–4]. Geach’s view is similar; he writes that Frege made the ‘mistake’ of regarding ‘his assertion sign [as] a “common predicate” in all assertions . . . But “the circumstance that *p* is one that actually obtains,” like “it is true that *p*,” hardly differs from plain “*p*,” and any such proposition may unequivocally occur now asserted, now unasserted. In later works Frege saw his mistake, and gave up any attempt to explain the assertion sign by classifying it as a predicate’ [38, pp. 457–8]. Medlin, too, criticises Frege for his ‘interpretation of “|—” as a predicate’ [41, pp. 13–4], and Dudman is also there getting the boot in, accusing Frege of a ‘slip’: ‘in the opening sections of *Begriffsschrift* Frege tells us

<sup>21</sup> Stoothoff objects to Dudman here that ‘An adequate, sufficiently comprehensive, explanation of the judgement-stroke must have the form: by prefixing ‘|—’ to ‘ $\Delta$ ’ we indicate acknowledgment that  $\Delta$  is the True. . . . But this requires, or presupposes, the doctrine that sentences are names of truth-values’ [46, p. 167]. I disagree. For a start, an explanation of Stoothoff’s form would be an explanation of the judgement-stroke only as it appears in Frege’s later writings (that is, in a way, Stoothoff’s point)—but there is no evidence that Frege ever changed his view of the judgement-stroke (except perhaps on the minor point noted in the previous footnote). In any case, an adequate, sufficiently comprehensive explanation of the judgement-stroke is this: the judgement-stroke is a device for asserting, and marking the assertion of, ‘sentences’ (whatever exactly ‘sentences’ are, and whatever exactly assertion of them consists in).

that the assertion-sign may be read as a *predicate*; and if it is a predicate it cannot be an index of assertion' [6, p. 153].

The first point to note is that Frege does *not* say that *the judgement-stroke* is a predicate. He says, rather, that '—' is a predicate, and '—' consists of the judgement-stroke *together with* '—'. The second point to note is that Frege says 'there cannot be any question here of subject and predicate in the ordinary sense'. Thus Frege does *not* say that the judgement-stroke is a predicate, nor even, without qualification, that '—' is a predicate. When we look closely at the passage, we see that Frege is simply repeating, in different words, the view of the judgement-stroke recently attributed to him in the present paper. He says that one can distinguish subject and predicate 'if one wishes to do so', but that 'the predicate serves only to turn the content into a judgement'—i.e. serves only to do what Frege has previously told us the judgement-stroke does—and that hence 'We see that there cannot be any question here of subject and predicate in the ordinary sense'. In other words, you can, if you wish, *call* a mark of assertion a 'predicate', but then you are not using the term 'predicate' in the normal way. Frege is *not*, then, saying or implying here that the judgement-stroke is anything other than a mark of assertion (and a mark of assertion is *not* a predicate, in the *ordinary* sense of 'predicate').

#### V. Releasing the Tension

I have been arguing that Frege only ever held one view concerning the function of the judgement-stroke: the view that the judgement-stroke is a mark of assertion.<sup>22</sup> The question now is why Frege *wanted* a sign in his logical system that performs this function.

Nowadays we have two measures of the merit, from a logical point of view, of an argument: validity and soundness. Validity has nothing to do with the truth or falsity of the premises; even soundness has nothing to do with whether or not we *recognise* that the premises are true. Frege, however, had a different view. Consider the following passage, from a letter to Hugo Dingler (Frege is commenting on Dingler's statement that 'If we succeed in inferring logically from a group of premises that a certain statement both holds and does not hold for one of the concepts contained in the premises, then I say: *This group of premises is contradictory, or contains a contradiction*'):

Is this case [Dingler's] at all possible? If we derive a proposition from true propositions according to an unexceptionable inference procedure, then the proposition is true. Now since at most one of two mutually contradictory propositions can be true, it is impossible to infer mutually contradictory propositions from a group of true propositions in a logically unexceptionable way. On the other hand, we can only infer something from true propositions. Thus if a group of propositions contains a proposition whose truth is not yet known, or which is certainly false, then this proposition cannot be used for making inferences. If we want to draw conclusions from the propositions of a group, we must first exclude all propositions whose truth is doubtful. . . . It is necessary to recognize the truth of the premises. When we infer, we recognize a truth on the basis of other previously recognized truths according to a logical law. Suppose we have arbitrarily formed the propositions

<sup>22</sup> But see footnote 20 for a minor proviso.



'2<1'

and

'If something is smaller than 1, then it is greater than 2'

without knowing whether these propositions are true. We could derive

'2>2'

from them in a purely formal way; but this would not be an inference because the truth of the premises is lacking. And the truth of the conclusion is no better grounded by means of this pseudo-inference than without it. And this procedure would be useless for the recognition of any truths. So I do not believe that your case . . . could occur at all. [36, pp. 16–7]

The first thing to note about this passage is how uneasily it seems to sit with a section of Frege's Appendix to Volume Two of *Grundgesetze*, in which he considers Russell's Paradox. There Frege writes that 'it will be useful to track down the origin of this contradiction in our signs', and then adds, concerning 'the derivation that follows': 'in consideration of the doubtful truth of it all I shall omit the judgement-stroke' [33, p. 130]. Then comes a derivation, followed by the claim: 'The propositions ( $\zeta$ ) and ( $\eta$ ) contradict one another. The error can be only in our Law (Vb), which must therefore be false' [33, p. 132].

Isn't Frege doing here exactly what he later tells Dingler cannot be done? I don't think so. For note that in the section of *Grundgesetze* just referred to, not only are there no judgement-strokes, but furthermore the derivation is not set out as a formal inference in Begriffsschrift. Instead of formulas separated by

(I<sub>g</sub>): -----

or

(II<sub>b</sub>): \_\_\_\_\_

etc., Frege has formulas separated by *phrases*—not part of Begriffsschrift—such as 'from which by (I<sub>g</sub>) there follows' and 'whence'. This indicates that Frege does *not* think of this section of *Grundgesetze* as setting out an inference: what we have here is a 'pseudo-inference', and Frege is not pretending otherwise.<sup>23</sup>

<sup>23</sup> It is important to distinguish pseudo-inferences (on which more below) from inferences with conditional premises. Frege writes: 'But it might perhaps be asked, can we not . . . draw consequences from a sentence which may be false, in order to see what we should get if it were true? Yes, in a certain sense this is possible. From the premises

If  $\Gamma$  holds, so does  $\Delta$

*continued*

Dingler replies to Frege, 'It seems to me that the 'truth' of the premises is completely irrelevant to the validity of the inference' [36, p. 18]. Certainly this is the view we hold today. What was Frege's view? Is Frege concerned with the *truth* of the premises, or with our *knowing* or *recognising* that the premises are true? In the passage just quoted he writes 'we can only infer something from true propositions'—but he immediately glosses this with 'Thus if a group of propositions contains a proposition whose truth is not yet known, or which is certainly false, then this proposition cannot be used for making inferences'. For the next few lines Frege is clearly concerned with *recognition* of truth, until he writes 'this would not be an inference because the truth of the premises is lacking'—but again, he follows this with 'this procedure would be useless for the recognition of any truths'. When Frege replies to Dingler's reply, he writes that we cannot infer anything from a proposition 'as long as we do not *know* that it is true' [36, p. 20, my emphasis]. In 'Compound Thoughts', too, Frege says that 'before *acknowledging* its truth, one cannot use a thought as premise of an inference, nor can one infer or conclude anything from it' [30, p. 402, my emphasis]. In a letter to Jourdain Frege writes, 'From false premises nothing at all can be concluded'—but again he follows this immediately with 'A mere thought, which is not *recognized* as true, cannot be a premise. Only after a thought has been *recognised* by me as true, can it be a premise for me. Mere hypotheses cannot be used as premises' [36, p. 182, my emphases]. Elsewhere, however, Frege writes simply 'Only true thoughts can be premises of inference' [25, p. 335]; 'only true thoughts are admissible premises of inferences' [24, p. 180]; and, in 'Negation', 'Of course we cannot infer anything from a false thought' [30, p. 375].

So is it that the premises of an inference must be true, or must they be *recognised* as true? Stoothoff thinks that premises, for Frege, must be 'acknowledged' as true—where 'acknowledge' is clearly a 'try' verb rather than a 'got it' verb, as Ryle would put it [44, p. 152], for Stoothoff speaks of 'false premises which are (mistakenly) acknowledged as true' [45, p. 408]. Stoothoff writes, 'certainly [Frege] admitted the possibility of inference from a thought whose truth is mistakenly acknowledged' [45, p. 407], and explains away Frege's unglossed statements to the effect that only true thoughts can be premises of inferences as 'infelicitous over-compressions' in which 'true thoughts' should be read as 'thoughts acknowledged as true' [45, p. 408]. Dummett, too, thinks that Frege's real point is that we can infer only from premises which we *take to be* true, and that Frege 'misstates his point by saying that we make inferences only from what is *true*' [8, p. 314]. Why the misstatement? Dummett: 'I think the answer is that taking something to be true is a

<sup>23</sup> *continued*

If  $\Delta$  holds, so does  $E$

we can infer

If  $\Gamma$  holds, so does  $E$

. . . without knowing whether  $\Gamma$  is true or false. But we must notice . . . the condition 'If  $\Gamma$  holds' is retained throughout' [28, pp. 244–5]. See also [36, pp. 182–3] and [30, pp. 402–3]. In this connection consider the following passage from *Begriffsschrift*: '— $A$  . . . is to produce in the reader merely the idea of the mutual attraction of opposite magnetic poles, say in order to derive consequences from it and to test by means of these whether the thought is correct' [10, p. 11, my emphasis].

psychological matter, and Frege had set his face against the importation of psychology into logic' [8, p. 313].

I disagree with Stoothoff and Dummett's interpretation. There is no textual evidence to support the claim that Frege admitted the possibility of inference from a thought whose truth is mistakenly acknowledged, while there is—as we have seen—evidence to suggest that Frege admitted inference *only* from true thoughts. And here is some more, overwhelming evidence: 'If a proposition uttered with assertoric force expresses a false thought, then it is logically useless and cannot strictly speaking be understood' [36, p. 79]. I suggest, then, that for Frege what is required is *both* truth of premises *and* acknowledgment of that truth: 'What is to serve as the premise of an inference must be true. Accordingly, in presenting an inference, one must utter the premises with assertoric force, for the truth of the premises is essential to the correctness of the inference' [36, p. 79].

We are close now to seeing why Frege felt it necessary to include the judgement-stroke in his logical system, even though he thought that 'Judging (or recognising as true) is certainly an inner mental process' [36, p. 78]. Recall Frege's view of the task of logic (the following passage was quoted in §II above):

Logic is concerned only with those grounds of judgement which are truths. To make a judgement because we are cognisant of other truths as providing a justification for it is known as *inferring*. There are laws governing this kind of justification, and to set up these laws of valid inference is the goal of logic. [11, p. 3]

It is important to distinguish here between *inferences* and *the laws of inference*. Currie cites the following passage as an indication that Frege 'on occasion draws back from [the] view of inference as involving premises which are known to be true' [4, p. 117]:

The task of logic is to set up laws according to which a judgement is justified by others, irrespective of whether these are themselves true. [27, p. 175]

But in this passage Frege is talking about the *laws* of inference, not about *inferences*. Inferences require true premises, but truth or falsity of premises is irrelevant to the laws of inference. If this were not so, it would make no sense for Frege to characterise a pseudo-inference as a purely formal derivation that lacks true premises—the implication being that there is nothing wrong with the *derivation*, the problem being that the derivation proceeds from false premises. I take it that Frege's idea is that a pseudo-inference conforms perfectly to the *laws* of inference, but is not actually an *inference*, because its premises are false. So far from the idea that inferences must have true premises being incompatible with the view that the laws of inference are not choosy between true and false premises, Frege's distinction between inferences and pseudo-inferences seems to presuppose that the two have something in common (conformity to the laws), while at the same time one lacks something the other possesses (true premises). In the letter to Dingler quoted recently, Frege writes, 'When we infer, we recognize a truth on the basis of other previously recognized truths according to a logical law.' As far as the logical law—the law of inference—is concerned, premises can be true or false; as far as the inference itself is concerned, the premises must be true.

Logic, then, aims to set up laws of valid inference—and truth or falsity of premises is irrelevant here. But the logical language must be capable of expressing actual

inferences—and here, the truth-value of the premises *is* relevant. Before we can infer one statement from another, we need to know not simply *what the first statement says*, but also *that what it says is true*: unless we acknowledge the truth of the premises the inference is not a real inference, it is a pseudo-inference. Now our convention could be that everything written in the logical language is taken as asserted—but that will not do, for when I write ‘If *A* then *B*’ I *write* both ‘*A*’ and ‘*B*’ but I *assert* neither. So we have a choice: assertion as default, and a special sign to indicate supposition; or lack of assertion as default, and a special sign to indicate assertion. Frege, needless to say, takes the latter route. Of course, we might not have an explicit sign at all—we might simply take it as obvious what is asserted and what is not. Thus Peano: ‘the particular position a proposition occupies in a given formula shows unequivocally what it is that is being asserted about it in that formula’ [42, p. 29]; and Vaught, in a recent text on set theory: ‘All expressions in the language of mathematics can be divided (in an extremely important way) into three classes: (1) *asserting* expressions; (2) *naming* expressions; and (3) “neither of these”: The reader will at once be able to classify in this way the following expressions: . . .’ [47, p. 7]. For Frege, however, this is insufficiently rigorous: as he puts it in *Begriffsschrift*, in *Begriffsschrift* ‘*nothing is left to guesswork*’ [10, p. 12]; in a piece entitled ‘On Mr. Peano’s Conceptual Notation and My Own’ Frege writes of his ‘endeavour to have every objective distinction reflected in symbolism’ [20, p. 247]; and in ‘On the Scientific Justification of a Conceptual Notation’ he writes, ‘We need a system of symbols from which every ambiguity is banned’ [14, p. 86]. I take it—and I take it to be very important—that in shunning context as an indicator of assertoric force and introducing the judgement-stroke, Frege was not thinking that the judgement-stroke should do *more* than is done by tone of voice and context in ordinary discourse; rather he was simply thinking that the judgement-stroke should do *exactly* what these ordinary devices do—mark assertion—but do it in an unambiguous way, a way that leaves no room for guesswork. In particular the judgement-stroke is *not* meant to do something impossible—for example get assertoric force *inside* the content of what is asserted.<sup>24</sup> Frege explicitly says that ‘the word “true” seems to make the impossible possible: it allows what corresponds to the assertoric force to assume the form of a contribution to the thought’, but that in fact ‘the attempt miscarries’ [29, p. 252]. Nor is the judgement-stroke meant to provide a magical antidote to false assertion: *of course* an actor could precede a formula with a judgement-stroke on a blackboard on the stage without herself asserting the formula, just as she could utter a sentence in a sincere tone of voice without thereby asserting the sentence herself<sup>25</sup>

<sup>24</sup> I am thinking here of Wittgenstein’s comment: ‘Thus “[—]” is no more a component part of a proposition than is, for instance, the proposition’s number. It is quite impossible for a proposition to state that it itself is true’ [48, §4.442]. Note that in any case Wittgenstein is on the wrong track here: Frege writes that ‘the assertion is not to be found in the word “true”, but in the assertoric force with which the sentence is uttered’ [29, p. 251].

<sup>25</sup> I am thinking here of Anscombe’s comment: ‘Frege has two arguments for [the] necessity [of the assertion sign], one weak and the other strong. [¶] The weak argument is from the necessity of a distinction between entertaining an hypothesis . . . and asserting a proposition. He says that an actor on the stage, for example, is not asserting. At that rate, it would be an inexcusable *faux pas* to make an actor write the assertion sign before a proposition on a blackboard in a play! This argument need not delay us’ [1, p. 113]. See also Davidson [5, p. 113], Dummett [8, pp. 310–1], and Medlin [41, p. 17]: ‘Unless the context is unusual, our words will have “assertive force”. And if the context deprives our words of this force, then no assertion sign will restore it.’

(although admittedly Frege would rather ignore this possibility: ‘assertoric force is closely bound up with the indicative mood of the sentence that forms the main clause. Of course in fiction even such sentences are uttered without assertoric force; but logic has nothing to do with fiction’ [22, p. 198]). This is to say that *wide* contextual factors can still override the judgement-stroke—how could that possibly fail to be the case?—while still it being the case that *within* a passage of Begriffsschrift, the judgement-stroke alone is the unambiguous marker of assertoric force.

It is important to note that use of an assertion sign for the reasons just outlined does not represent an incursion of psychology into logic—even though, for Frege, judgement is a psychological matter. In order for an inference to be possible, the truth of the premises has to be acknowledged (according to Frege). When one uses the judgement-stroke, one *expresses* one’s belief that a certain proposition is true—but one does not *say that* one believes that the proposition is true. Rather, one says simply *that the proposition is true*—or more correctly, one asserts the proposition—and that is precisely what needs to be done if anything is to be inferred from the proposition.

Jourdain once wrote to Frege, asking him, ‘will you tell me . . . whether you now regard assertion (|—) as merely psychological’ [36, p. 78]. Here is Frege’s response:

Judging (or recognizing as true) is certainly an inner mental process; but that something is true is independent of the recognizing subject; it is objective. If I assert something as true I do not want to talk about myself, about a process in my mind. And in order to understand it one does not need to know who asserted it. Whoever understands a proposition uttered with assertoric force adds to it his recognition of the truth. If a proposition uttered with assertoric force expresses a false thought, then it is logically useless and cannot strictly speaking be understood. A proposition uttered without assertoric force can be logically useful even though it expresses a false thought, e.g., as part (antecedent) of another proposition. What is to serve as the premise of an inference must be true. Accordingly, in presenting an inference, one must utter the premises with assertoric force, for the truth of the premises is essential to the correctness of the inference. If in representing an inference in my conceptual notation one were to leave out the judgement strokes before the premised propositions, something essential would be missing. And it is good if this essential thing is visibly embodied in a sign and not just added to it in the act of understanding according to a tacit convention; for a convention according to which something has to be added in that act of understanding under certain circumstances is easily forgotten even if it was once stated explicitly. And so it happens that something essential is completely overlooked because it has not found an embodiment. But what is essential to an inference must be counted as part of logic. [36, pp. 78–9]

Here we have it all: when one sets out an *inference* (as opposed to a pseudo-inference) one must not only say *what* the premises say, but also *that* what they say is true—or rather one must utter (or write) the premises with assertoric force (this is where our view of inference diverges from Frege’s: on our view, all one needs in order to begin inferring is the *content* of the premises); the judgement-stroke allows one to do this (other devices would also suffice, but it is better to make the device explicit than to rely on a tacit convention); finally, doing this is *not* reporting a fact about one’s own psychology (hence a fact of no interest to logic).

The source of the apparent tension between Frege's anti-psychologism, and his insistence that the judgement-stroke is an essential part of Begriffsschrift, is, then, the fact that Frege's conception of logic was rather different from our own. Perhaps we do not readily notice the difference because without Frege we would not understand logic in the way we now do. When it comes to the laws of inference we agree with Frege; but when it comes to actual inferences we disagree. For us, inference is about making moves of a certain sort; if the moves are all in accordance with the laws, it does not matter where one starts or where one ends up. Frege had a different picture: inference is a matter of going from truths, acknowledged as such, to other truths. For Frege inference is about *advancing* from known truths to further truths, building up an edifice that is perfectly secure—and as Frege says above, 'what is essential to an inference must be counted as part of logic'. For us, this is not what the logical enterprise is about. This is hardly surprising: following the failure of Frege's logicist programme, the axioms of set theory—the foundation of mathematics—are chosen upon the basis of what can be derived from them. There is no universal agreement as to what the axioms should be, and certainly, there is no longer any question of starting from elementary certainties and building mathematics step by logical step.

Given *our* conception of logic, whether or not Frege puts forward certain propositions as true is—as Wittgenstein says—of no interest to logic. But given Frege's conception of logic it is of great interest. As to the relative merits of the two conceptions of logic, the first point to note is that Frege's view is in no way psychologistic. The second point to note is that if Frege's project in the foundations of mathematics had not (yet) failed, logic might still be seen in Frege's way.<sup>26</sup>

Princeton University

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#### REFERENCES

1. Anscombe, G.E.M., *An Introduction to Wittgenstein's Tractatus* (London: Hutchinson University Library, 1959).
2. Black, Max, *A Companion to Wittgenstein's 'Tractatus'* (Ithaca, NY: Cornell University Press, 1964).
3. Church, Alonzo, *Introduction to Mathematical Logic*, Revised Edition (Princeton, NJ: Princeton University Press, 1956).
4. Currie, Gregory, *Frege: An Introduction to his Philosophy* (Sussex: Harvester Press, 1982).
5. Davidson, Donald, 'Moods and Performances' (1979), in his *Inquiries into Truth and Interpretation* (Oxford: Clarendon Press, 1984), 109–21.
6. Dudman, V.H., 'Frege's Judgement-Stroke', *Philosophical Quarterly* 20 (1970), 150–61.
7. ———, 'Peano's Review of Frege's Grundgesetze', *Southern Journal of Philosophy* 9 (1971), 25–37.
8. Dummett, Michael, *Frege: Philosophy of Language*, Second Edition (Cambridge, MA: Harvard University Press, 1981).
9. Evans, Gareth, *The Varieties of Reference*, ed. John McDowell (Oxford: Clarendon Press, 1982).
10. Frege, Gottlob, *Begriffsschrift, a formula language, modeled upon that of arithmetic, for pure thought* (1879), trans. Stefan Bauer-Mengelberg, in Jean van Heijenoort, ed., *Frege and Gödel:*

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- Two Fundamental Texts in Mathematical Logic* (Cambridge, MA: Harvard University Press, 1970), 1–82.
11. \_\_\_\_\_, 'Logic' (1879–91), in [35, pp. 1–8].
  12. \_\_\_\_\_, 'Boole's logical Calculus and the Concept-Script' (1880–1), in [35, pp. 9–46].
  13. \_\_\_\_\_, 'Boole's logical Formula-language and my Concept-script' (1882), in [35, pp. 47–52].
  14. \_\_\_\_\_, 'On the Scientific Justification of a Conceptual Notation' (1882), in [34, pp. 83–9].
  15. \_\_\_\_\_, 'On the Aim of the "Conceptual Notation"' (1882–3), in [34, pp. 90–100].
  16. \_\_\_\_\_, 'Function and Concept' (1891), in [37, pp. 137–56].
  17. \_\_\_\_\_, 'Review of E.G. Husserl, *Philosophie der Arithmetik I*' (1894), in [37, pp. 195–209].
  18. \_\_\_\_\_, 'Letter from Sig. G. Frege to the Editor [Peano]' (1896), in [7, pp. 31–6].
  19. \_\_\_\_\_, 'Logic' (1897), in [35, pp. 126–51].
  20. \_\_\_\_\_, 'On Mr. Peano's Conceptual Notation and My Own' (1897), in [37, pp. 234–48].
  21. \_\_\_\_\_, 'Letter to Russell' (1902), in [40, pp. 126–8].
  22. \_\_\_\_\_, 'A brief Survey of my logical Doctrines' (1906), in [35, pp. 197–202].
  23. \_\_\_\_\_, 'Introduction to Logic' (1906), in [35, pp. 185–96].
  24. \_\_\_\_\_, 'On Schoenflies: *Die logischen Paradoxien der Mengenlehre*' (1906), in [35, pp. 177–83].
  25. \_\_\_\_\_, 'On the Foundations of Geometry: Second Series' (1906), in [37, pp. 293–340].
  26. \_\_\_\_\_, 'What may I regard as the Result of my Work?' (1906), in [35, p. 184].
  27. \_\_\_\_\_, '17 Key Sentences on Logic' (1906), in [35, pp. 174–5].
  28. \_\_\_\_\_, 'Logic in Mathematics' (1914), in [35, pp. 203–50].
  29. \_\_\_\_\_, 'My basic logical Insights' (1915), in [35, pp. 251–2].
  30. \_\_\_\_\_, 'Logical Investigations: I Thoughts; II Negation; III Compound Thoughts' (1918–26), in [37, pp. 351–406].
  31. \_\_\_\_\_, 'Notes for Ludwig Darmstaedter' (1919), in [35, pp. 253–7].
  32. \_\_\_\_\_, *Translations from the Philosophical Writings of Gottlob Frege*, ed. Peter Geach and Max Black (Oxford: Basil Blackwell, 1952).
  33. \_\_\_\_\_, *The Basic Laws of Arithmetic: Exposition of the System*, ed. and trans. Montgomery Furth (Berkeley: University of California Press, 1964).
  34. \_\_\_\_\_, *Conceptual Notation and Related Articles*, ed. and trans. Terrell Ward Bynum (Oxford: Clarendon Press, 1972).
  35. \_\_\_\_\_, *Posthumous Writings*, ed. Hans Hermes, Friedrich Kambartel and Friedrich Kaulbach, trans. Peter Long and Roger White (Chicago: University of Chicago Press, 1979).
  36. \_\_\_\_\_, *Philosophical and Mathematical Correspondence*, ed. Gottfried Gabriel, Hans Hermes, Friedrich Kambartel, Christian Thiel and Albert Veraart, trans. Hans Kaal (Chicago: University of Chicago Press, 1980).
  37. \_\_\_\_\_, *Collected Papers on Mathematics, Logic, and Philosophy*, ed. Brian McGuinness, trans. Max Black, V.H. Dudman, Peter Geach, Hans Kaal, E.-H. Kluge, Brian McGuinness and R.H. Stoothoff (Oxford: Basil Blackwell, 1984).
  38. Geach, P.T., 'Assertion', *Philosophical Review* 74 (1965), 449–65.
  39. Heck, William C. and William G. Lycan, 'Frege's Horizontal', *Canadian Journal of Philosophy* 9 (1979), 479–92.
  40. Heijenoort, Jean van, ed., *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931* (Cambridge, MA: Harvard University Press, 1967).
  41. Medlin, Brian, 'The Theory of Truth-Functions', *Australasian Journal of Philosophy* 42 (1964), 1–21 and 183–98.
  42. Peano, G., Review of Frege's *Grundgesetze* (1895), in [7, pp. 27–31].
  43. Russell, Bertrand, 'Letter to Frege' (1902), in [40, pp. 124–5].
  44. Ryle, Gilbert, *The Concept of Mind* (New York: Barnes and Noble, 1949).
  45. Stoothoff, R.H., 'Note on a Doctrine of Frege', *Mind* 72 (1963), 406–8.
  46. \_\_\_\_\_, 'Dudman on Frege's Judgement-Stroke', *Philosophical Quarterly* 21 (1971), 166–7.
  47. Vaught, Robert L., *Set Theory: An Introduction* (Boston: Birkhäuser, 1985).
  48. Wittgenstein, L., *Tractatus Logico-Philosophicus*, trans. D.F. Pears and B.F. McGuinness (London: Routledge, 1974).