Groups, Sets, and Wholes

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As he recalls in his *Naive Physics*, Paolo Bozzi's experiments on naïve or phenomenological physics were partly inspired by Aristotle's spokesman Simplicio in Galileo's *Dialogue*. Aristotle's 'naïve' views of physical reality reflect the ways in which we are disposed perceptually to organize the physical reality we see. In what follows I want to apply this idea to the notion of a group, a term which I shall apply as an umbrella expression embracing ordinary visible collections (of pieces of fruit in the fruit bowl), but also families, populations, kinds, categories, species and genera. I will try to determine to what extent we can understand what groups, in this broad sense, have in common and how they are distinguished from two sorts of entities with which they are standardly confused, namely sets and wholes.

A set in the mathematical sense was initially conceived by Cantor as "a collection into a whole of definite and separate objects of our intuition or thought." While set theory itself has since departed in several ways from this conception, it still comes close to capturing what is involved when we talk informally of groups in the sense intended here. 'Group' is thus for all its broadness still narrower than 'set' as the latter term is nowadays standardly understood in mathematical contexts. For there are no empty groups, no groups of groups, no infinite groups, no complement groups, and no arbitrary unions or intersections of groups.

Groups, like wholes and unlike sets, are concrete denizens of reality: their elements or members are in every case real things or objects. There is nothing in the realm of groups or wholes analogous to the empty set ϕ, and there is no counterpart, either, of monsters such as:

$$\{\phi, \{\phi, \{\phi, \{\phi, \{\phi\}\}\}\}\}, \{\phi\}\}$$

and other so-called 'pure' sets. Groups thus belong to the realm of 'naïve' ontology; sets, rather, to the world of overly sophisticated ontologies (to which belong also the so-called non-existent objects of Alexius Meinong).

Groups are distinguished from both sets and wholes (as conceived, respectively, by Cantor and his successors, and by Lesniewski, and his successors) also in this: that where sets and wholes can be compounded out of members or parts arbitrarily, groups have a certain natural rounded-offness or

completeness. If *G* is a group, then the result of adding or subtracting some single object to *G* is very rarely itself a group (or if it is, then this is because *G* itself has changed with time). For groups are distinguished from sets as standardly conceived also in this: that, like all entities in physical reality, they are subject to change: above all, they may gain and lose members while preserving their identity.

Groups share this with sets: that they are made up of members or elements – and in this they are contrasted with wholes as understood by Lesniewski and other mereologists. This means that groups, like sets, are marked by the factor of granularity. The *parts* of the members of a group are not themselves members of the group in the way in which the parts of the parts of a whole *are* parts of the whole. Each group or set, we might say, divides up its respective domain into whole units or members in such a way that the proper parts of the latter are as it were traced over. Each group or set is laid across reality like a grid consisting (1) of a number of slots or pigeonholes each (2) occupied by some member. To say that a set is *determined by its members* is to say that it is (i) associated with a specific number (perhaps zero) of slots, each of which (ii) must be occupied by some specific member. A set is thus specified in a double sense. A group, in contrast, can survive a turnover in its instances, and so it is specified in neither of these senses, since both (i) the number of slots (always greater than one) *may vary with time,* and so also may (ii) the stock of individuals (always concrete) which occupies these slots. The family group which is the Cabots of Massachussets can survive the change in the stock of its instances which occurs when Henry and Mabel die, just as Henry and Mabel themselves can similarly survive changes in the stock of cells or molecules by which they are constituted.

We can distinguish further between two kinds of groups. On the one hand are what we might call bona fide (or Aristotelian) groups – groups which exist in reality independently of human cognition. Examples are: the planets of the solar system, the group of rabbits on the Island of Gozo, the species cat, the genus mammal. On the other hand are what we might call fiat (or Borgesian or Cambridge) groups – groups which exist only as the product of human fiat, and thus of some cognitive process: the European Commission, rabbits in Trentino, the Republican voters in Dade County, the Finnish diaspora – all of these are groups whose boundaries exist as a result of human decision, convention or practice, not in reflection of the underlying characteristics of the entities involved, though all are often more or less arbitrarily delineated sub-groups of bona fide populations (for example the population of all human beings).

Bona fide groups would exist, and would be set into relief in relation to their surroundings, even independently of all human intervention, whether physical or intellectual. Fiat groups, in contrast, are spatial shadows of human activity: they begin to exist and are sustained in existence only as a result of

certain cognitive acts, practices or institutions on the parts of human beings. There are no fiat groups in the extra-human world. It is thus a sad reflection on the exact philosophy of our day that its two principal formal tools – set theory and mereology – apply, at best, to counterparts of fiat groups. A formal theory adequate to the realm of bona fide groups does not yet exist.

Bozzi, P. (1990). *Fisica ingenua. Oscillazioni, piani inclinati e altre storie: studi di psicologia della percezione*. Milano: Garzanti