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Mathematics and the Theory of Multiplicities: Badiou and Deleuze Revisited

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1. Introduction

Deleuze once wrote that "encounters between independent thinkers always occur in a blind zone," and this is certainly true of the encounter between Alain Badiou and Gilles Deleuze.[1] In 1988, Badiou published *Being and Event*, which attempted to develop an "ontology of the multiple" derived from the mathematical model of axiomatic set theory.[2] Soon afterward, he tells us, he realized—no doubt correctly—that his primary philosophical rival in this regard was Deleuze, who similarly held that "philosophy is a theory of multiplicities,"[3] but whose own concept of multiplicities was derived from different mathematical sources and entailed a different conception of ontology itself. In 1997, Badiou published a study of Deleuze entitled *Deleuze: The Clamor of Being*, in which he confronted his rival directly and attempted to set forth their fundamental differences. The study, Badiou tells us in the introduction, was occasioned by an exchange of letters he had with Deleuze between 1992 and 1994, which focused directly on the concept of "multiplicity" and the specific problem of "an *immanent* conceptualization of the multiple."[4] On the opening page of the book, Badiou notes that "Deleuze's preferences were for differential calculus and Riemannian manifolds ... [whereas] I preferred algebra and sets"[5]—leading the reader to expect, in what follows, a comparison of Deleuze's and Badiou's notions of multiplicity based in part, at least, on these differing mathematical sources.

Yet as one reads the remainder of *Deleuze: The Clamor of Being*, one quickly discovers that Badiou in fact adopted a quite different strategy in approaching Deleuze. Despite the

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announced intention, the book does not contain a single discussion of Deleuze's theory of multiplicities; it avoids the topic entirely. Instead, Badiou immediately displaces his focus to the claim that Deleuze is not a philosopher of multiplicity at all, but rather a philosopher of the "One." Nor does Badiou ever discuss the mathematical sources of Deleuze's theory of multiplicity. Instead, he puts forth a secondary claim that, insofar as Deleuze *does* have a theory of multiplicity, it is not derived from a mathematical model, as is Badiou's own, but rather from a model that Badiou terms variously as "organic," "natural," "animal," or "vitalistic."[6]

Critics have rightly ascertained the obvious aim of this double strategy of avoidance and displacement: since Badiou presents himself as an ontologist of the multiple, and claims that his ontology is purely mathematical, he wants to distance Deleuze as far as possible from both these concerns.[7] To get at what is interesting in the Badiou-Deleuze encounter, however, these all-too-obvious strategies need to be set aside, since the real terms of the confrontation clearly lie elsewhere. Badiou's general philosophical (or meta-ontological) position turns on the equation that "ontology = mathematics," since "mathematics alone thinks being."[8] The more precise equation, however, would be that "ontology = axiomatic set theory," since for Badiou it is only in axiomatic set theory that mathematics adequately "thinks" itself and constitutes a condition of philosophy.[9] Badiou's ontology thus follows a not uncommon reductionist strategy: physics is ultimately reducible to mathematics, and mathematics to axiomatic set theory. From a Deleuzian viewpoint, the fundamental limitation of Badiou's philosophy—but also its fundamental interest—lies in this identification of ontology with axiomatic set theory. Badiou's confrontation with Deleuze must consequently be staged directly on each of these fronts—axiomatics, set theory, and their corresponding ontology—since it is only here that their differences can be exposed in a direct and intrinsic manner.

From this viewpoint, the two essential differences between Badiou and Deleuze immediately come to light. First, for Deleuze, the ontology of mathematics is *not* reducible to axiomatics, but must be understood much more broadly in terms of the complex tension between axiomatics and what he calls "problematics."[10] Deleuze assimilates axiomatics to "major" or "royal" science, which is linked to the social axiomatic of capitalism (and the State), and which constantly attempts to effect a reduction or repression of the problematic pole of mathematics, itself wedded to a "minor" or "nomadic" conception of science. For this reason, second, the concept of multiplicity, even within mathematics itself, cannot simply be identified with the concept of a set; rather, mathematics is marked by a tension between extensive multiplicities or sets (the axiomatic pole) and virtual or differential multiplicities (the problematic pole), and the incessant translation of the latter into the former. Reformulated in this manner, the Badiou-Deleuze confrontation can be posed and explored in a way that is *internal* to both mathematics (axiomatics versus problematics) and the theory of multiplicities (differential versus extensive multiplicities).

These two criteria allows us to assess the differences between Badiou and Deleuze in a way that avoids the red herrings of the "One" and "vitalism." Although Badiou claims that "the Deleuzian didactic of multiplicities is, from start to finish, a polemic against sets,"[11] in fact Deleuze nowhere militates against sets, and indeed argues that that the translation (or reduction) of differential multiplicities to extensive sets is not only inevitable ontologically, but *necessary* scientifically.[12] What separates Badiou and Deleuze is rather the ontological status of events (in Badiou's sense). For Deleuze, mathematics is replete with events, to which he grants a full ontological status, even if their status is ungrounded and problematic; multiplicities in the Deleuzian sense are themselves constituted by events. In turn, axiomatics, by its very nature, necessarily selects against and eliminates events in its effort to introduce "rigor" into mathematics and to establish its foundations. It would be erroneous to characterize the problematic pole of mathematics as "merely" intuitive and operative, while "royal" axiomatics is conceptual and formalizable. "The fact is," writes Deleuze, "that the two kinds of science have *different modes of formalization*.... What we have are two formally different conceptions of science, and ontologically, *a single field of interaction* in which royal science [e.g., axiomatics] continually appropriates the contents of vague or nomad science [problematics], while nomad science continually cuts the contents of royal science loose."[13] The task Deleuze takes upon himself, then, is to formalize the distinction between problematic and axiomatic multiplicities in a purely intrinsic manner, and to mark the ontological and scientific transformations or conversions between the two.

Badiou, by contrast, in taking axiomatics as his ontological model, limits his ontology to the pole of mathematics that is constituted on the elimination of the events, and he therefore necessarily denies events any ontological status: "the event is forbidden, ontology rejects it."[14] As a consequence, he places himself in the paradoxical position of formulating a theory of the event on the basis of an axiomatic viewpoint that explicitly eliminates the event. The event thus appears in Badiou's work under a double characterization. Negatively, so to speak, an event is undecidable or indiscernible from the ontological viewpoint of axiomatics: it is not presentable in the situation, but exists (if it can even be said to exist) on the "edge of the void" as a mark of the infinite excess of the inconsistent multiplicity over the consistent sets of the situation. Positively, then, it is only through a purely subjective "decision" that the hitherto indiscernible event can be affirmed, and made to intervene in the situation. Lacking any ontological status, the event in Badiou is instead linked to rigorous conception of *subjectivity*, the subject being the sole instance capable of "naming" the event and maintaining a fidelity to it through the declaration of an axiom (such as "all men are equal," in politics; or "I love you," in love). In this sense, Badiou's philosophy of the event is, at its core, a philosophy of the "activist subject."

Deleuze and Badiou thus follow opposing trajectories in their interpretations of mathematics. For Deleuze, problematics and axiomatics (minor and major science) together constitute a single ontological field of interaction, with the latter perpetually effecting a repression—or more accurately, an arithmetic conversion—of the former. Badiou, by contrast, grants an ontological status to axiomatics alone, and in doing so, he explicitly adopts the ontological viewpoint of "major" science, along with its repudiation and condemnation of "minor" science. As a result, not only does Badiou insist that Deleuze's concept of a virtual multiplicity "remains inferior to the concept of the Multiple that can be found in the contemporary history of sets,"[15] but he goes so far as to claim "the virtual does not exist,"[16] in effect denying the "problematic" pole of mathematics in its entirety. Interestingly, this contrast between Badiou and Deleuze finds a precise expression in a famous poetic formula. Badiou at times places his entire project under the sign of Lautréamont's poetic paean to "severe mathematics," which Deleuze, for his part, cites critically: "In contrast to Lautréamont's song that rises up around the paranoiac-Oedipal-narcissistic pole [of mathematics]—"*O severe mathematics.... Arithmetic! Algebra! Geometry! Imposing Trinity! Luminous triangle!*"—there is another song: O schizophrenic mathematics, uncontrollable and mad..."[17]

It is this *other* mathematics—problematics, as opposed to axiomatics as a "specifically scientific Oedipus"—that Deleuze attempts to uncover and formalize in his work. The obstacles to such a project, however, are evident. The theory of extensional multiplicities (Cantor's set theory) and its rigorous axiomatization (Zermelo-Frankel, et. al.) is one of the great achievements of modern mathematics, and in *Being and Event* Badiou was able to appropriate this work for his philosophical purposes. For Deleuze, the task was quite different, since he himself had to construct a hitherto non-existent (philosophical) formalization of differential or virtual multiplicities which are, by his own account, selected against by "royal" mathematics itself. In this regard, Deleuze's relation to the history of mathematics is similar to his relation to the history of philosophy: even in canonical figures there is something that "escapes" the official histories of mathematics. At one point, he even provides a list of "problematic" figures from the history of science and mathematics: "Democritus, Menaechmus, Archimedes, Vauban, Desargues, Bernouilli, Monge, Carnot, Poncelet, Perronet, etc.: in each case a monograph would be necessary to take into account the special situation of these scientists whom State science used only after restraining or disciplining them, after repressing their social or political conceptions."[18] Since Badiou has largely neglected Deleuze's writings on mathematics, in what follows I would first like to outline the nature of the general contrast Deleuze establishes between problematics and axiomatics, and then briefly identify the mathematical origins of Deleuze's notion of "multiplicities" With these resources in hand, we will then return to Badiou's specific critiques of Deleuze, partly to show their inherent limitations, but also to identify what I take to be the more relevant points of contrast between their respective philosophical positions.

2. Problematics and Axiomatics

Let me turn first to the problematic-axiomatic distinction. Although Deleuze formulates this distinction in his own manner, it in fact reflects a fairly familiar tension within the history of mathematics, which we must be content to illustrate hastily by means of three historical examples.

1. The first example comes from the Greeks. Proclus, in his *Commentary of the First Book of Euclid's Elements*, had already formulated a distinction, within Greek geometry, between problems and theorems.[19] Whereas theorems concern the demonstration, from axioms or postulates, of the inherent properties belonging to a figure, problems concern the construction of figures using a straightedge and compass.[20] In turn, theorematics and problematics each involve two different conceptions of "deduction": in theorematics, a deduction moves from axioms to the theorems that are derived from it, whereas in problematics a deduction moves from the problem to the ideal accidents and *events* that condition the problem and form the cases that resolve it. "The event by itself," writes Deleuze, "is problematic and problematizing."[21] For example, in the theory of conic sections (Apollonius), the ellipse, hyperbola, parabola, straight lines, and the point are all "cases" of the projection of a circle onto secant planes in relation to the apex of a cone. Whereas in theorematics a figure is defined statically, in Platonic fashion, in terms of its essence and its derived properties, in problematics a figure is defined dynamically by its capacity to be affected—that is, by the ideal events that befall it: sectioning, cutting, projecting, folding, bending, stretching, reflecting, rotating. As a theorematic figure, a circle is an organic and fixed essence, but the morphological variations of the circle (figures that are "lens-shaped," "umbelliform," "indented," etc.) form problematic figures that

are, in Husserl's words, "vague yet rigorous," "essentially and not accidentally inexact."[22] In Greece, problematics found its classical expression in Archimedean geometry (especially the Archimedes of "On the Method"), an "operative" geometry in which the line was defined less as an essence than as a continuous process of "alignment," the circle as a continuous process of "rounding," the square as the process of "quadrature," and so on.

Proclus, however, had already pointed to (and defended) the relative triumph, in Greek geometry, of the theorematic over the problematic. The reason: to the Greeks, "problems concern only events and affects which show evidence of a *deterioration* or a projection of essences in the imagination," and theorematics thus could present itself as a necessary "rectification" of thought.[23] This "rectification" must be understood, in a literal sense, as a triumph of the rectilinear over the curvilinear. The definition of the straight line as "the shortest distance between two points," for example, is understood dynamically in Archimedean geometry as a way of defining the length of a curve in predifferential calculus, such that the straight line is seen as a "case" of the curve; in Euclidean geometry, by contrast, the essence of the line is understood statically, in terms that eliminate any reference to the curvilinear ("a line which lies evenly with the points on itself").[24] In the "minor" geometry of problematics, figures are inseparable from their inherent variations, affections, and events; the aim of "major" theorematics, by contrast, is "to uproot variables from their state of continuous variation in order to extract from them fixed points and constant relations,"[25] and thereby to set geometry on the "royal" road of theorematic deduction and proof. Badiou, for his part, explicitly aligns his ontology with the position of theorematics: "the pure multiple, the generic form of being, *never* welcomes the event in itself as its component."[26]

2. By the seventeenth-century, the tension between problems and theorems, which was internal to geometry, had shifted to a more general tension between geometry itself, on the one hand, and algebra and arithmetic on the other. Desargues' *projective geometry*, for instance, which was a qualitative and "minor" geometry centered on problems-events (as developed, most famously, in the *Draft Project of an Attempt to Treat the Events of the Encounters of a Cone and a Plane*, which Boyer aptly describes as "one of the most unsuccessful great books ever produced"), was quickly opposed (and temporarily forgotten) in favor of the *analytic geometry* of Fermat and Descartes—a quantitative and "major" geometry that translated geometric relations into arithmetic relations that could be expressed in algebraic equations (Cartesian coordinates).[27] "Royal" science, in other words, now entailed an arithmetization of geometry itself. "There is a correlation," Deleuze writes, "between geometry and

arithmetic, geometry and algebra which is constitutive of major science."[28] Descartes was dismayed when he heard that Desargues' *Draft Project* treated conic sections without the use of algebra, since to him "it did not seem possible to say anything about conics that could not more easily be expressed with algebra than without."[29] As a result, Desargues' methods were repudiated as dangerous and unsound, and his practices of perspective banned.

It would be two centuries before projective geometry was revived in the work of Monge, the inventor of descriptive geometry, and Poncelet, who formulated the "principle of continuity," which led to developments in *analysis situs* and topology. Topology (so-called rubber-sheet geometry) concerns the property of geometric figures that remain invariant under transformations such as bending or stretching: under such transformations, figures that are theorematically distinct in Euclidean geometry, such as a triangle, a square, and a circle, are seen as one and the same "homeomorphic" figure, since they can be continuously transformed into one another. This entailed an extension of geometric "intuitions" far beyond the limits of empirical or sensible perception (à la Kant). "With Monge, and especially Poncelet," writes Deleuze, commenting on Léon Brunschvicg's work, "the limits of sensible, or even spatial, representation (striated space) are indeed surpassed, but less in the direction of a symbolic power of abstraction [i.e., theorematics] than toward a trans-spatial imagination, or a trans-intuition (continuity)."[30] In the twentieth-century, computers have extended the reach of this "trans-intuition" even further, provoking renewed interest in qualitative geometry, and allowing mathematicians to "see" hitherto unimagined objects such as the Mandelbrot set and the Lorenz attractor, which have become the poster children of the new sciences of chaos and complexity. "Seeing, seeing what happens," continues Deleuze, "has always had an essential importance, greater than demonstrations, even in pure mathematics, which can be called visual, figural, independently of its applications: many mathematicians nowadays think that a computer is more precious than an axiomatic."[31] But already in the early nineteenth-century, there was a renewed attempt to turn projective geometry into a mere practical dependency on analysis, or so-called higher geometry (the debate between Poncelet and Cauchy).[32] The development of the theory of functions would eventually eliminate the appeal to the principle of continuity, substituting for the geometrical idea of smooth-ness of variation the arithmetic idea of "mapping" or a one-to-one correspondence of points (point-set topology).

3. This double movement of major science toward theorematization and arithmetization would reach its full flowering, finally, in the late nineteenth-century, primarily in response to

problems posed by the invention of the *calculus*. In its origins, the calculus was tied to problematics in a double sense. The first refers to the ontological problems that the calculus confronted: the differential calculus addressed the problematic of *tangents* (how to determine the tangent lines to a given curve), while the integral calculus addressed the problematic of *quadrature* (how to determine the area within a given curve). The greatness of Leibniz and Newton was to have recognized the intimate connection between these two problematics (the problem of finding areas is the inverse of determining tangents to curves), and to have developed a symbolism to link them together and resolve them. The calculus quickly became the primary mathematical engine of what we call the "scientific revolution." Yet for two centuries, the calculus, not unlike Archimedean geometry, itself maintained a problematic status in a second sense: it was allotted a para-scientific status, labeled a "barbaric" or "Gothic" hypothesis, or at best a convenient convention or well-grounded fiction. In its early formulations, the calculus was shot through with dynamic notions such as infinitesimals, fluxions and fluents, thresholds, passages to the limit, continuous variation—all of which presumed a *geometrical* conception of the continuum, in other words, the idea of a process. For most mathematicians, these were considered to be "metaphysical" ideas that lay beyond the realm of mathematical definition. Berkeley famously ridiculed infinitesimals as "the ghosts of departed quantities"; D'Alembert famously responded by telling his students, *Allez en avant, et la foi vous viendra* ("Go forward, and faith will come to you").[33] The calculus would not have been invented without these notions, yet they remained problematic, lacking an adequate mathematical ground.

For a long period of time, the enormous success of the calculus in solving physical problems delayed research into its logical foundations. It was not until the end of the nineteenth-century that the calculus would receive a "rigorous" foundation through the development of the "limit-concept." "Rigor" meant that the calculus had to be separated from its problematic origins in geometrical conceptions or "intuitions," and reconceptualized in purely arithmetic terms (the loaded term "intuition" here having little to do with "empirical" perception, but rather the ideal geometrical notion of continuous movement and space).[34] This "arithmetization of analysis," as Félix Klein called it,[35] was achieved by Karl Weierstrass, one of Husserl's teachers, in the wake of work done by Cauchy (leading Guilio Giorello to dub Weierstrass and his followers the "ghostbusters").[36] Analysis (the study of infinite processes) was concerned with *continuous* magnitudes, whereas arithmetic had as its domain the *discrete* set of numbers. The aim of Weierstrass' "discretization" program was to separate the calculus from the geometry of continuity and base it on the concept of number alone. Geometrical notions were thus reconceptualized in terms of sets of discrete points, which in turn were conceptualized in terms of number: points on a line as individual numbers, points on a plane as ordered pairs of numbers, points in n-dimensional space as n-tuples of numbers. As a result, the concept of the variable was given a static interpretation. Early interpreters had tended to appeal to the geometrical intuition of continuous motion when they said that a variable x "approaches" a limit (e.g., the circle defined as the limit of a polygon). Weierstrass' innovation was to reinterpret this variable x arithmetically as simply designating any one of a collection of numerical values (the theory of functions), thereby eliminating any dynamism or "continuous variation" from the notion of continuity, and any interpretation of the operation of differentiation as a process. Weierstrass, writes Deleuze, "provided what he himself called a 'static' interpretation of the differential and infinitesimal calculus, in which there is no longer any fluction toward a limit, no longer any idea of a threshold, but rather the idea of a system of choice, from the viewpoint of an ordinal interpretation."[37] In Weierstrass limit-concept, in short, the geometric idea of "approaching a limit" was arithmetized, and replaced by static constraints on discrete numbers alone (the episilon-delta method). Dedekind took this arithmatization a step further by rigorously defining the continuity of the real numbers in terms of a "cut": "it is the cut which constitutes ...the idea cause of continuity or the pure element of quantitativity."[38] Cantor's set theory, finally, gave a discrete interpretation of the notion of infinity itself, treating infinite sets like finite sets (the power set axiom)—or rather, treating all sets, whether finite or infinite, as mathematical objects (the axiom of infinity).[39]

Weierstrass, Dedekind, and Cantor thus form the great triumvirate of the program of discretization and the development of the "arithmetic" continuum (the redefinition of continuity as a function of sets over discrete numbers). In their wake, the basic concepts of the calculus—function, continuity, limit, convergence, infinity, and so on—were progressively "clarified" and "refined," and ultimately given a set theoretical foundation.[40] The assumptions of Weierstrass discretization problem—that only arithmetic is rigorous, and that geometric notions are unsuitable for secure foundations—are now largely identified with the "orthodox" or "major" view of the history of mathematics as a progression toward ever more "well-founded" positions.[41] The program would pass through two further developments. The contradictions generated by set theory brought on a sense of a "crisis" in the foundations, which Hilbert's formalist (or formalization) program attempted to repair through *axiomatization*, that is, by attempting to show that set theory could be derived from a finite set of axioms, which were later codified by Zermelo-Frankl (given his theological leanings, even Cantor needed a dose of axiomatic rigor). Gödel and Cohen, finally, in their famous theorems, would eventually expose the internal limits of axiomatization (incompleteness, undecidability), demonstrating, in Badiou's language, that there is a variety of mathematical forms in "infinite excess" over our ability to formalize them consistently.

This historical sketch, though necessarily brief, nonetheless provides a basis from which we can pinpoint the differences between the respective projects of Badiou and Deleuze. In identifying ontology with axiomatic set theory, Badiou is adopting the position of "major" mathematics with its dual programs of "discretization" and "axiomatization." This contemporary orthodoxy has often been characterized as an "ontological reductionism." In this viewpoint, as Penelope Maddy describes it, "mathematical objects and structures are identified with or instantiated by set theorematic surrogates, and the classical theorems about them proved from the axioms of set theory."[42] Reuben Hersh gives it a more idiomatic and constructivist characterization: "Starting from the empty set, perform a few operations, like forming the set of all subsets. Before long you have a magnificent structure in which you can embed the real numbers, complex numbers, quaterions, Hilbert spaces, infinite-dimensional differentiable manifolds, and anything else you like."[43] Badiou tells us that he made a similar appeal to Deleuze, insisting that "every figure of the type 'fold,' 'interval,' enlacement,' 'serration,' 'fractal,' or even 'chaos' has a corresponding schema in a certain family of sets...."[44] Deleuze, for his part, fully recognizes this orthodox position: "Modern mathematics is regarded as based upon the theory of groups or set theory rather than on the differential calculus."[45] Nonetheless, he insists that the fundamental difference in kind between problematics and axiomatics remains, even in contemporary mathematics: "Modern mathematics also leaves us in a state of antinomy, since the strict finite interpretation that it gives of the calculus nevertheless presupposes an axiom of infinity in the set theoretical foundation, even though this axiom finds no illustration in the calculus. What is still missing is the extra-propositional and sub-representative element expressed in the Idea by the differential, *precisely in the form of a problem*."[46]

There are several reasons why Deleuze would refuse Badiou's identification of ontology with axiomatized set theory and maintain the ontological irreducibility of problematics. Most obviously, Badiou's ontology presumes the eventual reduction of physics (and the other sciences) to mathematics, which at present is itself no less a matter of faith than the eighteenth-century belief in the ghosts of infinitesimals.

Freeman Dyson, to give one example among many, has strongly questioned this reductionistic presumption, predicting that "the notion of a final statement of the laws of physics [in a finite set of mathematical equations] will prove as illusory as the notion of a final decision process for all of mathematics."[47] More importantly, within mathematics itself, there are notions that remain outside the grasp of the discretization program—most notably the geometric continuum itself, the non-discrete "continuous continuum," which still maintains its problematic status. "According to this intuitive concept," mused Gödel, "summing up all the points, we still do not get the line; rather the points form some kind of scaffold on the line."[48] Or as Hermann Weyl put it, "in spite of Dedekind, Cantor, and Weierstrass, the great task which has been facing us since the Pythagorean discovery of the irrationals remains today as unfinished as ever; that is, the *continuity* given to us immediately by intuition (in the flow of time and in motion) has yet to be grasped mathematically."[49] (The term "continuum" is still used to denote both types of continuity—the continuous geometric continuum and the discrete arithmetic continuum—even though the two notions differ in kind.) In a seminar, Deleuze noted that "the idea that there is a quantitative becoming, the idea of the limit of this becoming, the idea that an infinity of small quantities tends toward the limit—all these were considered as absolutely impure notions, as non-axiomatic or non-axiomatizable."[50] One of the aims of his own theory of multiplicities is to assess the status of such notions *as* problematic.

A more recent example can help serve to illustrate the ongoing tension between problematics and axiomatics within contemporary mathematics. Even after Weierstrass' work, mathematicians using the calculus continued to obtain accurate results and make new discoveries by using infinitesimals in their reasoning, their mathematical conscience assuaged by the (often unchecked) supposition that infinitesimals could be replaced by Weierstrassian methods. Despite its supposed "elimination" as an impure and muddled metaphysical concept, the ghostly concept of infinitesimals continued to play a positive role in mathematics *as* a problematic concept, reliably producing correct solutions. "Even now," wrote Abraham Robinson in 1966, "there are many classical results in differential geometry which have never been established in any other way [than through the use of infinitesimals], the assumption being that somehow the rigorous but less intuitive ε, δ method would lead to the same result."[51] In response to this situation, Robinson developed his *non-standard analysis*, which proposed an axiomatization of infinitesimals themselves, at last granting mathematicians the "right" to use them in proofs. Using the theory of formal languages, he added to the ordinary theory of numbers a new symbol (which we can call i for infinitesimal), and posited axioms saying that i was smaller than any finite number $1/n$ and yet not zero; he then showed that this enriched theory of numbers is consistent, assuming the consistency of the ordinary theory of numbers. The resulting mathematical model is described as "non-standard" in that it contains, in addition to the "standard" finite and transfinite numbers, nonstandard numbers such a hyperreals and infinitesimals.[52] In the nonstandard model, there is a cluster of infinitesimals around every real number r, which Robinson, in a nod to Leibniz, termed a "monad" (the monad is the 'infinitesimal neighborhood" of r). Transfinites and infinitesimals are two types of infinite number, which characterize degrees of infinity in different fashions. In effect, this means that contemporary mathematics has "two distinct rigorous formulations of the calculus": that of Weierstrass and Cantor, who eliminated infinitesimals, and that of Robinson, who rehabilitated and legitimized them.[53] Both these endeavors, however, had their genesis in the imposition of the notion of infinitesimals *as* a problematic concept, which in turn gave rise to differing but related axiomatizations. Deleuze's claim is that the ontology of mathematics is poorly understood if it does not take into account the specificity and irreducibility of problematics.

With these examples in hand, we can make several summary points concerning the relation between the problematic and axiomatic poles of mathematics, or more broadly, the relation between minor and major science. First, according to Deleuze, mathematics is constantly producing notions that have an objectively problematic status; the role of axiomatics (or its precursors) is to codify and solidify these problematic notions, providing them with a theorematic ground or rigorous foundation. Axiomaticians, one might say, are the "law and order" types in mathematics: "Hilbert and de Broglie were as much politicians as scientists: they reestablished order."[54] In this sense, as Jean Dieudonné suggests, axiomatics is a foundational but *secondary* enterprise in mathematics, dependent for its very existence on problematics: "In periods of expansion, when new notions are introduced, it is often very difficult to exactly delimit the conditions of their deployment, and one must admit that one can only reasonably do so once one has acquired a rather long practice in these notions, which necessitates a more or less extended period of cultivation [*défrichement*], during which incertitude and controversy dominates. Once the heroic age of pioneers passes, the following generation can then codify their work, getting rid of the superfluous, solidifying the bases—in short, putting the house in order. At this moment, the axiomatic method reigns anew, until the next overturning [*bouleversement*] that brings a new idea."[55] Nicholas Bourbaki puts the point even more strongly, noting that "the axiomatic method is nothing but the 'Taylor System'—the 'scientific management'—of mathematics."[56] Deleuze adopts a similar historical thesis, noting that the push toward axiomatics at the end of the nineteenth-century arose at the same time that Taylorism arose in capitalism: axiomatics does for mathematics what Taylorism does for "work."[57]

Second, problematic concepts often (though not always) have their source in what Deleuze terms the "ambulatory" sciences, which includes sciences such as metallurgy, surveying, stone-cutting, and perspective. (One need only think of the mathematical problems encountered by Archimedes in his work on military installations, Desargues on the techniques of perspective, Monge on the transportation of earth, and so on.) The nature of such domains, however, is that they do not allow science to assume an autonomous power. The reason, according to Deleuze, is that the ambulatory sciences "subordinate all their operations to the sensible conditions of intuition and construction—*following* the flow of matter, *drawing and linking up* smooth space. Everything is situated in the objective zone of fluctuation that is coextensive with reality itself. However refined or rigorous, 'approximate knowledge' is still dependent upon sensitive and sensible evaluations that pose more problems than they solve: problematics is still its only mode."[58] Such sciences are linked to notions—such as heterogeneity, dynamism, continuous variation, flows—that are "barred" or banned from the requirements of axiomatics, and consequently they tend to appear in history as that which was superceded or left behind. By contrast, what is proper to royal science, to its theorematic or axiomatic power, is "to isolate all operations from the conditions of intuition, making them true intrinsic concepts, or 'categories.'... Without this categorical, apodictic apparatus, the differential operations would be constrained to follow the evolution of a phenomenon."[59] In the ontological field of interaction between minor and major science, in other words, "the ambulant sciences confine themselves to *inventing problems* whose solution is tied to a whole set of collective, nonscientific activities but whose *scientific solution* depends, on the contrary, on royal science and the way it has transformed the problem by introducing it into its theorematic apparatus and its organization of work. This is somewhat like intuition and intelligence in Bergson, where only intelligence has the scientific means to solve formally the problems posed by intuition."[60]

Third, what is crucial in the interaction between the two poles are thus the processes of translation that take place between them—for instance, in Descartes and Fermat, an algebraic translation of the geometrical; in Weierstrass, a static translation of the dynamic; in Dedekind, a discrete translation of the continuous. The "richness and necessity of translations," writes Deleuze, "include as many opportunities for openings as risks of closure or stoppage."[61] In general, Deleuze's work in mathematical "epistemology" tends to focus on the reduction of the problematic to the axiomatic, the intensive to the extensive, the continuous to the discrete, the nonmetric to the metric, the nondenumerable to the denumerable, the rhizomatic to the arborescent, the smooth to the striated. Not all these reductions, to be sure, are equivalent, and Deleuze analyses each on its own account. Deleuze himself highlights two of them. The first is "the complexity of the means by which one translates intensities into extensive quantities, or more generally, multiplicities of distance into systems of magnitudes that measure and striate them (the role of logarithms in this connection)"; the second, "the delicacy and complexity of the means by which Riemannian patches of smooth space receive a Euclidean conjunction (the role of the parallelism of vectors in striating the infinitesimal)."[62] At times, Deleuze suggests, axiomatics can possess a deliberate will to halt problematics. "State science retains of nomad science only what it can appropriate; it turns the rest into a set of strictly limited formulas without any real scientific status, or else simply represses and bans it."[63] But despite its best efforts, axiomatics can never have done with problematics, which maintains its own ontological status and rigor. "Minor science is continually enriching major science, communicating its intuitions to it, its way of proceeding, its itinerancy, its sense of and taste for matter, singularity, variation, intuitionist geometry and the numbering number.... Major science has a perpetual need for the inspiration of the minor; but the minor would be nothing if it did not confront and conform to the highest scientific requirements."[64] In Deleuzian terms, one might say that while "progress" can be made at the level of theorematics and axiomatics, all "becoming" occurs at the level of problematics.

Fourth, this means that axiomatics, no less than problematics, is itself an inventive and creative activity. One might be tempted to follow Poincaré in identifying problematics as a "method of discovery" (Riemann) and axiomatics as a "method of demonstration" (Weierstrass).[65] But just as problematics has its own modes of formalization and deduction, so axiomatics has its own modes of intuition and discovery (axioms are not chosen arbitrarily, for instance, but in accordance with specific problems and intuitions).[66] "In science an axiomatic is not at all a transcendent, autonomous, and decision-making power opposed to experimentation and intuition. On the one hand, it has its own gropings in the dark, experimentations, modes of intuition. Axioms being independent of each other, can they be added, and up to what point (a saturated system)? Can they be withdrawn (a 'weakened' system)? On the other hand, it is of the nature of axiomatics to come up against so-called *undecid-*

able propositions, to confront *necessarily higher powers* that it cannot master. Finally, axiomatics does not constitute the cutting edge of science; it is much more a stopping point, a reordering that prevents decoded flows in physics and mathematics [= problematics] from escaping in all directions. The great axiomaticians are the men of State within science, who seal off the lines of flight that are so frequent in mathematics, who would impose a new *nexum*, if only a temporary one, and who lay down the official policies of science. They are the heirs of the theorematic conception of geometry."[67]

For all these reasons, problematics is, by its very nature, "a kind of science, or treatment of science, that seems very difficult to classify, whose history is even difficult to follow."[68] Nonetheless, according to Deleuze, the recognition of the irreducibility of problems and their genetic role has become "one of the most original characteristics of modern epistemology," as exemplified in the otherwise diverse work of thinkers such as Canguilhem, Bouligand, Vuillemin, and Lautman.[69] Beyond its significance in the interpretation of mathematics, problematics plays a significant role in Deleuze's theory of Ideas as well as his ontology ("Being" necessarily presents itself under a problematic form, and problems themselves are ontological). In all these domains, Deleuze's theory of problematics is extended in a theory of multiplicities, and it is to the nature of such multiplicities that we now turn.

3. Deleuze's Theory of Multiplicities

One of Badiou's most insistent claims is that Deleuze's theory of multiplicities is drawn from a "vitalist" paradigm, and not a mathematical one. The primary point I would like to establish in what follows is that, contra Badiou, Deleuze's theory is in fact drawn *exclusively* from mathematics—but from its problematic pole. Badiou at least admits that Deleuze's conception of multiplicities is derived in part from the differential calculus, but he concedes this point only to complain that Deleuze's "experimental construction of multiplicities is anachronistic because it is *pre-Cantorian*."[70] Cantor's set theory, however, represents the crowning moment of the tendency toward "discretization" in mathematics (the conception of sets as purely extensional), whereas Deleuze's project, as we have seen, is to formalize the conception of multiplicities that corresponds to the problematic pole of mathematics. In other words, problematics, no less than axiomatics, is the object of pure mathematics. Abel, Galois, Riemann, and Poincaré are among the great names in the history of problematics, just as Weierstrass, Dedekind, and Cantor are the great names in the discretization program, and Hilbert, Zermelo, Frankel, Gödel, and Cohen the great names in the movement toward formalization and axiomatization. Deleuze is fully aware of the apparent "anachronism" involved in delving into the pre-Weierstrassian theories of the calculus (Maimon, Bordas-Demoulin, Wronski, Lagrange, Carnot...). "A great deal of truly philosophical naiveté is needed to take the symbol *dx* seriously," he admits, while nonetheless maintaining that "there is a treasure buried in the old so-called barbaric or prescientific interpretations of the differential calculus, which must be separated from its infinitesimal matrix."[71] The reason Deleuze focuses on role of the differential (*dx*), however, is twofold. On the one hand, in the calculus, the differential is by nature problematic, it constitutes "the internal character of the problem as such," which is precisely why it must *disappear* in the result or solution.[72] On the other hand, whereas Plato used geometry as a model for his conception of transcendent "Ideas" because he saw the latter as *unchanging* theorematic forms, Deleuze uses the calculus as a model for his conception of immanent Ideas because the differential provides him with a mathematical symbolism of the problematic form of *pure change* (Bergson had already spoken of the differential or "fluxion" as a mean of capturing, via mathematics, a vision of the *élan vital*).[73] Deleuze will thus make a strong distinction between "differ-ential relations" and "axiomatic relations."[74] Even in *Difference and Repetition*, however, the calculus is only one of several mathematical domains that Deleuze utilizes in formulating his theory of multiplicities: "We cannot suppose that differential calculus is the only mathematical expression of problems as such.... More recently, other procedures have fulfilled this role better."[75] What is at issue, in other words, is neither the empirical or intuitive origin of mathematical problems (e.g., in the ambulatory sciences) nor the historical moment of their mathematical formalization (pre- or post-Cantorian). "While it is true that the [continuous] continuum must be related to Ideas and to their problematic use," Deleuze writes, "this is on condition that it no longer be defined by characteristics borrowed from sensible or even geometrical intuition."[76] What Deleuze finds in pure mathematics is a rigorous conception of the constitution of problems *as such*, divorced not only from the conditions of intuition, but also from the conditions of their solvability. It is on the basis of this formalization that Deleuze, in turn, will be able to assign a precise status to mathematical notions such as continuous variation and becoming—which can only be comprehended under the mode of problematics. Space precludes a more detailed analysis of Deleuze's theory of multiplicities here; for our purposes, I would simply like to highlight three mathematical domains that have formalized the theory of the problem, and which Deleuze utilizes in formulating his own conception of multiplicities as problematic.[77]

1. The first domain is the theory of *groups*, which initially arose from questions concerning the solvability of certain

algebraic (rather than differential) equations. There are two kinds of solutions to algebraic equations, particular and general. Whereas a *particular* solution is given by numerical values ($x^2 + 3x - 4 = 0$ has as its solution $x = 1$), a *general* solution provides the global pattern of all particular solutions to an algebraic equation (the above equation, generalized as $x^2 + ax - b = 0$, has the solution $x = \div a^2/2 + b - a/2$). But such solutions, writes Deleuze, "whether general or particular, find their sense only in the subjacent problem which inspires them."[78] By the sixteenth century, it had been proved (Tataglia-Cardan) that *general* solvability was possible with squared, cubic, and quartic equations. But equations raised to the fifth power and higher refused to yield to the previous method (via radicals), and the puzzle of the "quintic" remained unresolved for more than two centuries, until the work of Lagrange, Abel, and Galois in the nineteenth-century. In 1824, Abel proved the startling result that the quintic was in fact *unsolvable*, but the method he used was as important as the result: Abel recognized that there was a pattern to the solutions of the first four cases, and that it was this pattern that held the key to understanding the recalcitrance of the fifth. Abel showed that the question of "solvability" had to be determined internally by the *intrinsic* conditions of the problem itself, which then progressively specifies its own "fields" of solvability.

Building on Abel's work, Evariste Galois developed a way to approach the study of this pattern, using the technique now known as *group theory*. Put simply, Galois "showed that equations that can be solved by a formula must have groups of a particular type, and that the quintic had the wrong sort of group."[79] The "group" of an equation captures the conditions of the problem; on the basis of certain substitutions within the group, solutions can be shown to be indistinguishable insofar as the validity of the equation is concerned.[80] In particular, Deleuze emphasizes the fundamental procedure of *adjuction* in Galois: "Starting from a basic 'field' R, successive adjunctions to this field (R', R'', R'''...) allow a progressively more precise distinction of the roots of an equation, by the progressive limitation of possible substitutions. There is thus a succession of 'partial resolvants' or an embedding of 'groups' which make the solution follow from the very conditions of the problem."[81] In other words, the group of an equation does not tell us what we know about its roots, but rather, as George Verriest remarks, "the objectivity of what we do *not* know about them."[82] As Galois himself wrote, "in these two memoirs, and especially in the second, one often finds the formula, *I don't know....*"[83] This non-knowledge is not a negative or an insufficiency, but rather a rule or something to be learned that corresponds to an *objective* dimension of the problem. What Deleuze finds in Abel and Galois, following the exemplary analyses of Jules Vuillemin in his *Philosophy of Algebra*, is "a radical reversal of the problem-solution relation, a more considerable revolution than the Copernican."[84] In a sense, one could say that "unsolvability" plays a role in problematics similar to that played by "undecidability" in axiomatics.

2. The second domain Deleuze utilizes is the calculus itself, and on this score Deleuze's analyses are based to a large extent on the interpretation proposed by Albert Lautman in his *Essay on the Notions of Structure and Existence in Mathematics*.[85] Lautman's work is based on the idea of a fundamental difference in kind between a problem and its solution, a distinction that is attested to by the existence of problems *without* solution. Leibniz, Deleuze notes, "had already shown that the calculus...expressed problems that could not hitherto be solved, or indeed, even posed."[86] In turn Lautman establishes a link between the theory of differential equations and the theory of singularities, since it was the latter that provided the key to understanding the nature of *nonlinear* differential equations, which could not be solved because their series diverged. As determined by the equation, singular points are distinguished from the ordinary points of a curve: the singularities mark the points where the curve changes direction (inflections, cusps, etc.), and thus can be used to distinguish between different *types* of curves. In the late 1800s, Henri Poincaré, using a simple nonlinear equation, was able to identify four types of singular points that corresponded to the equation (foci, saddle points, knots, and centers) and to demonstrate the topological behavior of the solutions in the neighborhood of such points (the integral curves).[87] On the basis of Poincaré's work, Lautman was able to specify the nature of the difference in kind between problems and solutions. The conditions of the *problem* posed by the equation is determined by the existence and distribution of singular points in a differen*t*iated topological field (a field of vectors), where each singularity is inseparable from a zone of objective indetermination (the ordinary points that surround it). In turn, the *solution* to the equation will only appear with the integral curves that are constituted the neighborhood of these singularities, which mark the beginnings of the differen*c*iation (or actualization) of the problematic field. In this way, the ontological status of the problem as such is detached from its solutions: in itself, the problem is a multiplicity of singularities, a nested field of directional vectors which define the "virtual" trajectories of the curves in the solution, not all of which can be actualized. Non-linear equations can thus be used to model objectively problematic (or indeterminate) physical systems, such as the weather (Lorenz): the equations can define the virtual "attractors" of the system (the intrinsic singularities toward which the trajectories will tend in the long-term), but they cannot say in advance which trajectory will be actualized

(the equation cannot be solved), making accurate prediction impossible. A problem, in other words, has an objectively determined structure (virtuality), apart from its solutions (actuality).[88]

3. But "there is no revolution" in the problem-solution reversal, continues Deleuze, "as long as we remain tied to Euclidean geometry: we must move to a geometry of sufficient reason, a Riemannian-type differential geometry which tends to give rise to discontinuity on the basis of continuity, or to ground solutions in the conditions of the problems."[89] This leads to Deleuze's third mathematical resource, the *differential geometry* of Gauss and Riemann. Gauss had realized that the utilization of the differential calculus allowed for the study of curves and surfaces in a purely intrinsic and "local" manner, that is, without any reference to a "global" embedding space (such as the Cartesian coordinates of analytic geometry).[90] Riemann's achievement, in turn, was to have used Gauss's differential geometry to launch a reconsideration of the entire approach to the study of space by analyzing the general problem of *n-dimensional* curved surfaces. He developed a non-Euclidean geometry (showing that Euclid's axioms were not self-evident truths) of a multi-dimensional, non-metric, and non-intuitable "any-space-whatever," which he termed a pure "multiplicity" or "manifold" [*Mannigfaltigkeit*]. He began by defining the distance between two points whose corresponding coordinates differ only by infinitesimal amounts, and defined the curvature of the multiplicity in terms of the *accumulation* of neighborhoods, which alone determine its connections.[91] For our purposes, the two important features of a Riemannian manifold are its variable number of dimensions (its n-dimensionality), and the absence of any supplementary dimension which would impose on it extrinsically defined coordinates or unity.[92] As Deleuze writes, a Riemannian multiplicity is "an n-dimensional, continuous, defined multiplicity.... By *dimensions*, we mean the variables or coordinates upon which a phenomenon depends; by *continuity*, we mean the set of [differential] relations between changes in these variables—for example, a quadratic form of the differentials of the co-ordinates; by *definition*, we mean the elements reciprocally determined by these relations, elements which cannot change unless the multiplicity changes its order and its metric."[93] In his critique of Deleuze, Badiou suggests not only that a Riemannian manifold entails "a neutralization of difference" (whereas Riemannian space is defined differentially) and a "preliminary figure of the One" (whereas Riemannian space has no preliminary unity), but that it finds the "subjacent ontology of its invention" in set theory (whereas its *invention* is tied to problematics and the use of infinitesimals). What Badiou's comments reflect, rather, is the inevitable effort of "major" science to translate an intrinsic manifold into the

discrete terms of an extensive set (though as Abraham Robinson noted, it is by no means clear that results obtained in differential geometry using infinitesmals are automatically obtainable using Weierstrassian methods).[94]

In *Difference and Repetition*, Deleuze draws upon all these resources to develop his general theory of problematic or differential multiplicities, whose formalizable conditions can be briefly summarized as follows. (1) The elements of the multiplicity are merely "determinable," their nature is not determined in advance by either a defining property or an axiom (e.g., extensionality). Rather, they are pure virtualities that have neither identity, nor sensible form, nor conceptual signification, nor assignable function (principle of determinability). (2) They are nonetheless determined reciprocally as singularities in the differential relation, a "non-localizable ideal connection" that provides a purely intrinsic definition of the multiplicity as "problematic"; the differential relation is not only *external* to its terms, but *constitutive* of its terms (principle of reciprocal determination). (3) The values of these relations define the complete determination of the problem, that is, "the existence, the number, and the distribution of the determinant points that precisely provide its conditions" *as* a problem (principle of complete determination).[95] These three aspects of sufficient reason, finally, find their unity in the temporal principle of progressive determination, through which, as we have seen in the work of Abel and Galois, the problem is resolved (adjunction, etc.).[96] The strength of Deleuze's project, with regard to problematics, is that, in a certain sense, it parallels the movement toward "rigor" that was made in axiomatics: it presents a formalization of the theory of problems, freed from the conditions of geometric intuition and solvability, and existing only in pure thought (even though Deleuze presents his theory in a purely philosophical manner, and explicitly refuses to assign a scientific status to his conclusions).[97] In undertaking this project, he had few philosophical precursors (Lautman, Vuillemin), and the degree to which he succeeded in the effort no doubt remains an open question. Manuel DeLanda, in a recent work, has proposed several refinements in Deleuze's formalization, drawn from contemporary science: certain types of singularities are now recognizable as "strange attractors"; the resolution of a problematic field (the movement from the virtual to the actual) can now be described in terms of a series of spatio-temporal "symmetry-breaking cascades," and so on.[98] But as Delanda insists, despite his own modifications to Deleuze's theory, Deleuze himself "should get the credit for having adequately *posed the problem*" of problematics.[99]

4. Deleuze and Badiou

Equipped now with a more adequate understanding of Deleuze's conception of problematics, we can now return to Badiou's critique and see why neither of his two main theses concerning Deleuze articulate the real nature of their fundamental differences. Badiou's thesis that Deleuze is a philosopher of the One is the least persuasive, for several reasons. First, Badiou derives this thesis from Deleuze's concept of univocity, proposing the equation "univocity = the One." But already in Scotus, the doctrine of the "univocity of Being" was strictly incompatible with (and in part directed against) a neo-Platonic "philosophy of the One." Moreover, Deleuze's explicit (and repeated) thesis in *Difference and Repetition* is that the only condition under which the term "Being" can be said in a single and univocal sense is if Being is said univocally of *difference as such* (i.e., "Being is univocal" = "Being is difference").[100] To argue, as Badiou does, that Deleuze's work operates "on the basis of an ontological precomprehension of Being as One" is in effect to argue that Deleuze *rejects* the doctrine of univocity.[101] In other words, "Being is univocal" and "Being is One" are strictly incompatible theses, and Badiou's conflation of the two, as has been noted by several commentators, betrays a fundamental misunderstanding of the theory of univocity.[102] Second, while it is nonetheless true that Deleuze proposed a concept of the One compatible with univocity (e.g., the "One-All" of the plane of immanence as a secant plane cut out of chaos),[103] Badiou seems unable to articulate it in part because of the inconsistency of his own conception of the One, which is variously assimilated to the Neo-Platonic One, the Christian God, Spinoza's Substance, Leibniz's Continuity, Kant's unconditioned Whole, Nietzsche's Eternal Return, Bergson's *élan vital*, a generalized conception of Unity, and Deleuze's Virtual, to name a few.[104] The reason for this conceptual fluidity seems clear: since the task of modern philosophy, for Badiou, is "the renunciation of the One," and since for him only a set theoretical ontology is capable of fulfilling this task, the concept of the "One" effectively becomes little more than a marker in Badiou's writings for *any* non-set-theoretical ontology. But the fact that Augustine—to use a famous example—became a Christian (believer in God) by renouncing his Neo-Platonism (adherence to the One) is enough to show that these terms are not easily interchangeable, and that renouncing the One does not even entail a renunciation of God. Moreover, Kant had already showed that the idea of the "World" is a transcendent illusion: one can only speak of the "whole" of Being ("the totality of what is") from the viewpoint of transcendence; it is precisely the "immanence" of the concept of Being (univocity) that *prevents* any conception of Being as a totality. Third, and most important, the notion of the One does not articulate the difference between Badiou and Deleuze even on the question of "an immanent conception of the multiple." Extensive multiplicities (sets) and differential multiplicities (e.g., Riemannian manifolds) are *both* defined in a purely intrinsic or immanent manners, without *any* recourse to the One or the Whole or a Unity. The real *differend* must be located in the difference between axiomatics and problematics, major and minor science.

Badiou's thesis concerning Deleuze's "vitalism," by contrast, comes closer to articulating a real difference. (Badiou recognizes, to be sure, that Deleuze uses this biological term in a somewhat provocative manner, divorced from its traditional reference to a semi-mystical life-force.) Although Deleuze's formal theory of multiplicities is drawn from mathematical models, it is true that he appeals to numerous non-mathematical domains in describing the intensive processes of *individuation* through which multiplicities are actualized (biology, but also physics, geology, etc.). "Vitalism" enters the picture, in other words, at the level of individuation—hence the distinction, in *Difference and Repetition*, between the fourth chapter on "The Ideal Synthesis of Difference" (the theory of multiplicities, which appeals to mathematics) and the fifth chapter on "The Asymmetrical Synthesis of the Sensible" (the theory of individuation, which appeals to biology). But this distinction is neither exclusive nor disciplinary. Even in mathematics, the movement from a problem to its solutions constitutes a process of actualization: though formally distinct, there is no ontological separation between these two instances (the complex Deleuzian notion of "differen *t/c* ation). As Deleuze explains, "we tried to constitute a philosophical concept from the *mathematical* function of differen*t*iation and the *biological* function of differen*c*iation, in asking whether there was not a statable relation between these two concepts which could not appear at the level of their respective objects.... Mathematics and biology appear here only in the guise of technical models which allow the exposition of the virtual [problematic multiplicities] and the process of actualization [biological individuation]."[105] Deleuze thus rejects Badiou's reduction of ontology to mathematics, and would no doubt have been sympathetic to Ernst Mayr's suggestion that biology might itself be seen as the highest science, capable of encompassing and synthesizing diverse developments in mathematics, physics, and chemistry.[106]

Badiou's resistance to this "vitalism" can be accounted for by his restricted conception of ontology. For Badiou, the term ontology refers uniquely to the discourse of "Being-as-being" (axiomatic set theory), which is indifferent to the question of existence. For Deleuze, by contrast, ontology encompasses Being, beings, *and* their ontological difference (using Heideggerrian language), and the determinations of "Being-as-such" must therefore be immediately related to beings in their existence. This is why the calculus functions as an powerful test case in comparing Deleuze and Badiou. The calculus has been rightly described as the most powerful instrument ever invented for the mathematical exploration of the physical universe. In its initial formulations, however, as we have seen, the calculus mobilized notions that were unjustified from the viewpoint of classical algebra or arithmetic; it was a fiction, as Leibniz said, irreducible to mathematical reality. From these origins, however, one can trace the history of the calculus along two vectors, so to speak: toward the establishment of its foundations, or toward its use in an ever-deepening exploration of existence. The movement toward rigor in mathematics, by "royal" science, was motivated by the attempt to establish a foundation for the concepts of the calculus internal to mathematics itself. Badiou situates his work exclusively on this path, characterizing axiomatic set theory as "rational ontology itself."[107] Deleuze, by contrast, while stressing the foundational necessity of axiomatics, equally emphasizes the role of the calculus in the comprehension of existence. "Differential calculus," he writes, "is a kind of union of mathematics and the existent—specifically, it is the symbolic of the existent. It is because it is a well-founded fiction in relation to mathematical truth that it is consequently *a basic and real means of exploration of the reality of existence*."[108] A law of nature, as Hermann Weyl says, is necessarily expressed as a differential equation, and it is the calculus that establishes this link between mathematics and existence (Einstein's general relativity, for instance, made use of the tensor calculus). While axiomatics established the foundations of the calculus *within* mathematics, it is in the calculus itself that one must seek out the relation of mathematics with *existence* (problematics). This is no doubt the fundamental difference between Badiou and Deleuze: Badiou eliminates existence entirely from his ontology (there is no "being" of matter, life, sensibility...), whereas in Deleuze existence is fully a dimension of ontology as such: "force" is a determination of the being of matter (Leibniz); "vitalism" is a determination of the being of living things (Bergson); "intensity" is a determination of the being of the sensible (Kant); and so on. It is this genetic and problematic aspect of mathematics that remains inaccessible to set theoretical axiomatics.[109]

Badiou's neglect of the "problematic" dimension of Deleuze's thought results in numerous infelicities in his reading of Deleuze. In *Deleuze: The Clamor of Being*, Badiou's approach is guided by the presumption that "the starting point required by Deleuze's method is always a concrete *case*."[110] But this is a false presumption: for Deleuze, the starting point is always the *problem*, and "cases" are themselves derived from problems. The fundamental question is to determine which problems are interesting and remarkable, or to determine what is interesting or remarkable within the problem as such (group theory). If one starts with the case, it is in order to determine the problem to which it corresponds ("the creation of a concept always occurs as the function of a problem").[111] Paul Erdós famously assigned monetary values to mathematical problems, ranging from $10 to $3,000, depending not only on their degree of difficulty but on their importance *as problems*, and he would pay out (often to graduate students) when the problem was solved.[112] Similarly, Poincaré used to say that proving a uninteresting problem was worse than discovering a flaw in one's proof for a remarkable problem: the latter can be corrected, but the former will remain eternally trivial.[113] The truth of a solution, in other words, is less important than the truth or "interest" of the problem being dealt with (a problem always has the solution it "deserves").

Nor can one say—as Badiou frequently does—that Deleuze simply falls back on the "concrete" with the aim of producing phenomenological descriptions of the "figural." Badiou goes so far as to claim that Deleuze's work "does not support the real rights of the abstract" and instead gives itself over to the "seductive scintillations of concrete analysis." At best, Badiou thinks Deleuze draws "powerful metaphors (and yes, I do mean metaphors)" from mathematics and produces little more than "a metaphorizing phenomenology of pure change."[114] Not only does this imply a simplified view of the "concrete" (as Deleuze notes, "the true opposite of the concrete is not the abstract, it's the *discrete*.... Lived experience is an absolutely abstract thing"),[115] it entirely ignores Deleuze's development of a formal theory of problematics, and its complex mathematical sources. As Deleuze writes, "we must not see mathematical metaphors in all these expressions such as 'singular and distinctive points' or 'adjunct fields'.... These are categories of the dialectical Idea, extensions of the differential calculus (*mathesis universalis*) ... corresponding to the Idea in all its domains."[116] This avoidance, in turn, leads Badiou to make several misguided claims. In his book *Bergsonism*, for instance, Deleuze explicitly defines Bergsonian "intuition" as an elaborated method that consists in "the stating and creating of *problems*." [117] Badiou, to support his own theses, ignores this definition, and instead reinterprets intuition as a method that thinks beings as "merely local intensities of the One."[118] Similarly, Deleuze has suggested that "the intuitionist school (Brouwer, Heyting, Griss, Bouligand, etc.) is of great important in mathematics, not because it asserted the irreducible rights of intuition, or even because it elaborated a very novel constructivism, but because it developed a conception of *problems*, and of a *calculus of problems* that intrinsically rivals axiomatics and proceeds by other rules (notably with regard to the excluded middle)."[119] But when Badiou links Deleuze to "the constructivist, and indeed intuitionist vision" of contemporary mathematics, he again ignores the link with problematics, and instead strangely construes the constructivist school as having pursued a purely "descriptive" task that starts from the sensible intuition of "already complex concretions."[120]

Badiou's emphasis on axiomatics also affects his readings of Deleuze's work in the history of philosophy. Badiou, for instance, complains that "Deleuze neglects the function of mathematics in Spinoza," for whom "mathematics alone thinks being."[121] But this is not quite correct either: Deleuze explicitly criticizes Spinoza for allowing his mathematics to assume a purely axiomatic form. "In Spinoza," Deleuze writes, *"the use of the geometric method involves no 'problems' at all."*[122] This is why, in his readings of Spinoza, Deleuze emphasizes the role of the scholia (which are the only elements of the *Ethics* that fall outside the axiomatic deductions, and develop the theme of "affections") and the fifth book (which introduces problematic hiatuses and contractions into the deductive exposition itself).[123] No doubt it is this emphasis on the problematic aspects of the *Ethics* that rendered Deleuze's Spinoza "unrecognizable" to Badiou, who focuses on the theorematic and axiomatic apparatus.[124] Indeed, with regard to problematics, Deleuze suggests that Descartes actually went further than Spinoza, and that Descartes the geometer went further than Descartes the philosopher: the "Cartesian method" (the search for the clear and distinct) is a method for solving problems, whereas the analytic procedure presented is Descartes's *Geometry* is focused on the constitution of problems as such ("Cartesian coordinates" appear nowhere in the *Geometry*).[125] In all these characterizations, one at times senses in Badiou the semi-patronizing attitude of the "royal" scientist, who sees Deleuze's thought mired in problematics and its inferior concepts, and lacking the robustness required for work in "severe mathematics" and its "delicate axiomatics."

But perhaps the most striking omission in Badiou's work, especially given his political interests, is his neglect of Deleuze's political philosophy, since the latter is derived directly from these mathematical models. The central thesis of *Capitalism and Schizophrenia* (whose very title reflects the axiomatics-problematics distinction) is that capitalism itself functions on the basis of an axiomatic—not metaphorically, but literally.[126] This is because capital as such is a problematic multiplicity: it can be converted into *discrete* quantities in our paychecks and loose change, but in itself the monetary mass is *continuous* or intensive quantity that increases and decreases without any agency controlling it. Like the continuum, capital is not masterable by an axiom; or rather, it constantly requires the creation of new axioms (it is "like a power of the continuum, tied to the axiomatic but exceeding it").[127] In turn, capital produces other flows that follow these circuits of capital: flows of commodities, flows of population, flows of labor, flows of traffic, flows of knowledge, and so on—all of which have a necessarily "problematic" status from the viewpoint of the capitalist regime. The fundamental operation of the capitalist State, in Deleuze's reading, is to attempt to control these "deterritorialized" flows by axiomatizing them—but this axiomatization can never be complete, not only because of the inherent limits of any axiomatic, but because new "problematics" are constantly in the process of being created . "The true axiomatic," Deleuze says, "is social and not scientific."[128] To take one well-known example: for Deleuze "minorities" are, in themselves, nondenumerable multiplicities; they can be brought into the capitalist axiomatic by being denumerated, counted, given their identity cards, made a part of the majority (which is a denumerable multiplicity, i.e., a multiplicity of discrete numerical elements); but there is also a power to minorities that comes from *not* entering into the axiomatic, a power that does not reduce minorities to a mere "tear" or "rupture" in the axiomatic, but assigns to them an objective and determinable ontological positivity of their own *as* problematic.[129] "The issue is not at all anarchy versus organization," writes Deleuze, "nor even centralization versus decentralization, but a calculus or conception of *problems* of non-denumerable sets, against the *axiomatic* of denumerable sets. Such a calculus may have its own compositions, organizations, and even centralization; nevertheless, it proceeds not via the States or the axiomatic process but via a pure becoming of minorities."[130]

This brings us back again, finally, to the question of the *event*, which is where the Badiou-Deleuze differend appears in perhaps its starkest contrast. In effect, the respective ontologies of Deleuze and Badiou move in opposing directions: Deleuze's is a "bottom up" ontology (from problematics to discretization-axiomatization), whereas Badiou's is a "top-down" ontology (elaborated exclusively from the viewpoint of axiomatics, denying the existence of problematics). From Deleuze's viewpoint, this denial of problematics constitutes the intractable limitation of Badiou's ontology, which consequently appears in two forms. On the one hand, for Badiou, Being is presented in a purely *discrete* terms: what is "subtracted" from the "count-as-one" rule that constitutes consistent sets (knowledge) is an inconsistent or "generic" multiplicity, the pure discrete multiple of Being, which in itself remains indiscernible, unpresented, and unnameable as such (the void); an event—that which is not "Being-as-Being"—if one occurs, intervenes "on the edge" of this void, and constitutes the condition of a truth-procedure. But this entire characterization revolves in the domain of the discrete: what is truly "unnamed" within it is the entire domain of problematics and its "repressed" notions, such as continuous variation. Such is the substance of the critique Deleuze addresses to Badiou in *What is Philosophy?*. "The theory of multiplicities," he writes, "does not support the hypothesis of an 'any multiplicity whatever,'" that is, a purely "generic" discrete multiplicity.[131] The discretization program found its point of "genesis" in problematics, and in any adequate mathematical ontology there must therefore be "at least two multiplicities, two types, from the outset"—namely the continuous *and* the discrete, the non-metric *and* the metric, and so on. "This is not because dualism is better than unity," continues Deleuze, "but because the multiplicity is precisely what happens between the two," that is, in the movement of conversion that translates the continuous into the discrete, the non-metric into the metric, etc. It is precisely this movement of translation, and Deleuze's own formalization of problematic multiplicities, that we have attempted to sketch out above. On the other hand, for Badiou, the "truth" of Being is presented in a purely *axiomatic* form. As a result, the articulation or "thinking" of a inconsistent multiplicity—the operation of a "truth-procedure"—can only be *subjective*, since it is only by means of a purely subjective "decision" that an event can be affirmed, and the hitherto indistinguishable elements of the multiplicity can be named, thereby altering the "situation" through the declaration of an axiom. Badiou necessarily dissociates this process of subjectivation from ontology itself, since it is only the subject's "fidelity" to the event that allows the elements of the altered situation to achieve consistency. Hence the fundamental *duality* that Badiou posits between "Being" and "Event," and the separation of the articulation of Being from the path of the subject or truth. For Deleuze, by contrast, the genesis of truth (and the genesis of axiomatics itself) must always be found in *problematics*: Being necessarily presents itself under a problematic form, and problems and their ideal events always are ontological, not subjective. The generation of truth, in other words, is derived from the constitution of problems, and a problem always has the truth is "deserves" insofar as it is completely constituted *as* a problem. The greatness of the calculus in mathematics is that it provided a precise symbolism with which it could express problems that, before its invention, could not even have been posed. If Badiou is forced to define truth in purely subjective terms, it is because he wrongly limits his ontology to axiomatics, and denies himself the real ontological ground of truth in problematics.

The path followed by Badiou in *Being and Event*, then, is almost the exact inverse of that followed by Deleuze in *Difference and Repetition*, and the two paths exemplify

Deleuze's own distinction between an immanent and a transcendent ontology. For Deleuze, a purely "immanent" ontology is one in which there is nothing "outside" Being or "other" than Being, and he therefore grants full ontological status to *both* problematics and axiomatics. Since Badiou limits his ontology to axiomatics, he is forced to reintroduce an element of transcendence in the form of the event, which is "supplemental" to ontology, "supernumerary": there can be no ontology of the event, since the event itself introduces a "rupture" into being, a "tear" in its fabric. In *What is Philosophy?*, this is exactly how Deleuze defines the "modern" way of saving transcendence: "it is now from *within* immanence that a breach is expected ... something transcendent is reestablished on the horizon, in the regions of non-belonging," or as Badiou would say, from the "edge of the void."[132] Whereas an immanent ontology "never has a supplementary dimension to that which transpires upon it," an ontology of transcendence "always has an additional dimension; it always implies a dimension supplementary to the dimensions of the given."[133] In this sense, Badiou's is indeed an analogical and reflexive ontology that requires a mechanism of transcendence to "save" the event.[134] Though Badiou is determined to expel God and the One from his philosophy, he winds up reassigning to the event, as if through the back door, many of the transcendent charac-teristics formerly assigned to the divine. In Plotinus, it is the One which is "beyond" Being; in Badiou, it is the event which is "not being-as-being," that "interrupts" Being. In religious life, what is transformative is fidelity to the God; in Badiou, it is fidelity to the event. In Christian theology, it God who creates ex nihilo; in Badiou, it is the subject who proclaims the event and in a sense assumes those once divine powers (as Badiou declares triumphantly, "I conceptualize absolute beginnings!").[135] The primary aim of this paper has been to clarify, in a more adequate manner than Badiou did, the fundamental points of disagreement between the two philosophers. Deleuze, however, often insisted on the irreducibility of "taste" in philosophy, and if these analyses are correct, it would seem that Badiou's taste for discretization and axiomatization in mathematics concealed a deeper taste for the transcendent, and its conceptualizations of total ruptures and absolute beginnings.

Notes

1 Gilles Deleuze, *Foucault*, trans. Sean Hand (Minneapolis: University of Minnesota Press, 1886), 2. Deleuze was speaking of Virilio's relation to Foucault.

2 Alain Badiou, *L'être et l'événement* (Paris: Editions du Seuil, 1988). An English translation of Badiou's magnum opus by Oliver Feltham is forthcoming from Continuum.

3 Gilles Deleuze, "A Philosophical Concept," in *Who Comes After the Subject*, ed. Eduardo Cadava, Peter Connor, Jean-Luc Nancy (New

York: Routledge, 1991), 95.

[4] Alain Badiou, *Deleuze: The Clamor of Being*, trans. Louise Burchill (Minneapolis: University of Minnesota Press, 2000), 4.

[5] Badiou, *Deleuze: The Clamor of Being*, 1.

[6] See Badiou, *L'être et l'événement*, 522: "the latent paradigm in Deleuze is 'natural'.... Mine is mathematical." Similarly, in his review article of Deleuze's book on Leibniz, Badiou writes: "There have never been but two schemas or paradigms of the Multiple: the mathematical and the organicist.... This is the cross of metaphysics, and the greatness of Deleuze ... is to choose without hesitation for the animal" (in Alain Badiou, "Gilles Deleuze, 'The Fold: Leibniz and the Baroque'," in Constantin V. Boundas and Dorothea Olkowski, ed., *Gilles Deleuze and the Theater of Philosophy* [New York and London: Routledge, 1994], 55). This same theme is continued in Badiou's article "De la Vie comme nom de l'Être," in *Rue Descartes* 20 (May 1998), 27–34, which is reprinted in revised form as "L'Ontologie vitaliste de Deleuze," in Alain Badiou, *Court Traité d'ontologie transitoire* (Paris: Seuil, 1998), 61–72.

[7] See, for instance, the articles on Badiou's book by Éric Alliez, Arnaud Villani, and José Gil, collected in *Future Antérieur* 43.

[8] Badiou, *Court Traité*, 72.

[9] See Badiou, *Court Traité*, 51: "A 'crisis' in mathematics is a moment when mathematics is constrained to think its own thought *as the immanent multiplicity of its own unity*. It is at this point, I believe, and at this point alone, that mathematics, that is to say, ontology, functions as a condition of philosophy." For Badiou, philosophy itself is "meta-ontological," since it is the task of philosophy to establish the thesis that mathematics is the discourse of Being-as-such (see Badiou, *L'être et l'événement*, 20).

[10] See Gilles Deleuze, *Difference and Repetition*, trans. Paul Patton (New York: Columbia University Press, 1994), 323, note 22: Given the irreducibility of 'problems' in his thought, Deleuze writes that "the use of the word 'problematic' as a substantive seems to us an indispensable neologism."

[11] Alain Badiou, "Une, multiple, multiplicité(s)," 4. This unpublished text is, to my knowledge, Badiou's only direct discussion of Deleuze's theory of multiplicities. I thank Peter Hallward for making the manuscript available to me. This article has been published under the same title in *Multitudes* 1 (Mar 2000), 195–211; my page references are to the typescript.

[12] See Gilles Deleuze and Félix Guattari, *A Thousand Plateaus*, trans. Brian Massumi (Minneapolis: University of Minnesota Press, 1987), 374: "Only royal science has at its disposal a metric power that can define a conceptual apparatus or an autonomy of science (including the autonomy of experimental science)." On page 486: "Major science has a perpetual need for the inspiration of the minor; but the minor would be nothing if it did not confront and conform to the highest scientific requirements."

[13] Deleuze and Guattari, *A Thousand Plateaus*, 362, 367 (emphases added).

[14] Badiou, *L'être et l'événement*, 205.

[15] Badiou, "Une, multiple, Multiplicité(s)," 4.

[16] Badiou, *Deleuze*, 46: "I uphold that the forms of the multiple are, just like Ideas, always actual and that the virtual does not exist." Deleuze agrees with this characterization of sets: "Everything is actual

in a numerical multiplicity; everything is not 'realized,' but everything there is actual. There are no relationships other than those between actuals." Gilles Deleuze, *Bergsonism*, trans. Hugh Tomlinson and Barbara Habberjam (New York: Zone Books, 1988), 43.

[17] Gilles Deleuze and Félix Guattari, *Anti-Oedipus*, trans. Robert Hurley, Mark Seem, and Helen R. Lane (New York: Viking, 1977), 371–2. For Badiou's appeal to Lautréamont, see *Court Traité*, 72, and "De la Vie comme nom de l'Être," *Rue Descartes*, 34.

[18] Deleuze and Guattari, *A Thousand Plateaus*, 363. See Deleuze's well-known comments on his relation to the history of philosophy in "Letter to a Harsh Critic," in *Negotiations*, trans. Martin Joughin (New York: Columbia University Press, 1995), 5–6. The best general works on the history of mathematics are Carl B. Boyer, *History of Mathematics* (Princeton: Princeton University Press, 1968) and Morris Kline, *Mathematical Thought From Ancient to Modern Times*, 3 vols. (Oxford: Oxford University Press, 1972).

[19] Proclus, *Commentary of the First Book of Euclid's Elements*, trans. Glenn R. Murrow (Princeton, N.J.: Princeton University Press, 1970), 63–7, as cited in *Difference and Repetition*, 163; *A Thousand Plateaus*, 554, note 21; and *Logic of Sense*, trans. Mark Lester, with Charles Stivale; ed. Constantin V. Boundas (New York: Columbia University Press, 1990), 54. See also Deleuze's comments in *The Time-Image*, trans. Hugh Tomlinson and Barbara Habberjam (Minneapolis: University of Minnesota Press, 1989), 174: theorems and problems are are "two mathematical instances which constantly refer to each other, the one enveloping the second, the second sliding into the first, but both very different in spite of their union." On the two types of deduction, see 185.

[20] For instance, determining a triangle the sum of whose angles is 180 degrees is theorematic, since the angles of every triangle will total 180 degrees. Constructing an equilateral triangle on a given finite straight line, by contrast, is problematic, since we could also construct a non-equilateral triangle or a non-triangular figure on the line (moreover, the construction of an equilateral triangle must first pass through the construction of two circles). Classical geometers struggled for centuries with the three great "problems" of antiquity:—trisecting an angle, constructing a cube having double the volume of a given cube, and constructing a square equal to a circle—though it would turn out that none of these problems is solvable using only a straightedge and compass. See E. T. Bell's comments in *Men of Mathematics* (New York: Simon & Schuster, 1937), 31–2.

[21] Deleuze, *Logic of Sense*, 54.

[22] Edmund Husserl, *Ideas: General Introduction to a Pure Phenomenology*, trans. W. R. Boyce Gibson (New York: Macmillan, 1931), §74, 208. See also *Edmund Husserl's Origin of Geometry: An Introduction*, ed. John P. Leavey, Jr. and David B. Allison (Stony Brook, N. Y.: H. Hayes, 1978), which includes Jacques Derrida's important commentary. Whereas Husserl saw problematics as "proto-geometry," Deleuze sees it as a fully autonomous dimension of geometry, but one he identifies as a "minor" science; it is a "proto"-geometry only from the viewpoint of the "major" or "royal" conception of geometry, which attempts to eliminate these dynamic events or variations by subjecting them to a theorematic treatment.

[23] Deleuze, *Difference and Repetition*, 160 (emphasis added).

Deleuze continues: "As a result [of using *reductio ad absurdum* proofs], however, the *genetic* point of view is forcibly relegated to an inferior rank: proof is given that something cannot not be, rather than *that* it is and *why* it is (hence the frequency in Euclid of negative, indirect and *reductio* arguments, which serve to keep geometry under the domination of the principle of identity and prevent it from becoming a geometry of sufficient reason)."

[24] See Deleuze, *Difference and Repetition*, 174: "The mathematician Houël remarked that the shortest distance was not a Euclidean notion at all, but an Archimedean one, more physical than mathematical; that it was inseparable from a method of exhaustion; and that it served less to determine the straight line than to determine the length of a curve by means of a straight line—'integral calculus performed unknowingly'" (citing Jules Houël, *Essai critique sur les principes fondamentaux de la géométrie élémentaire* [Paris: Gauthier-Villars, 1867], 3, 75). Boyer makes a similar point in his *History of Mathematics*, 141: "Greek mathematics sometimes has been described as essentially static, with little regard for the notion of variability; but Archimedes, in his study of the spiral, seems to have found the tangent to the curve through kinematic considerations akin to the differential calculus."

[25] Deleuze and Guattari, *A Thousand Plateaus*, 408–9.

[26] Badiou, *Court Traité*, 71–2.

[27] Boyer, *History of Mathematics*, 393.

[28] Deleuze and Guattari, *A Thousand Plateaus*, 484. On the relation between Greek theorematics and seventeenth-century algebra and arithmetic as instances of "major" mathematics, see Deleuze, *Difference and Repetition*, 160–1.

[29] Boyer, *History of Mathematics*, 394. Deleuze writes that "Cartesian coordinates appear to me to be an attempt of reterritorialization" (Deleuze, seminar of 22 February 1972; transcripts of Deleuze's seminars, by Richard Pinhas, are available on-line at <http://www.webdeleuze.com/sommaire.html>).

[30] Deleuze and Guattari, *A Thousand Plateaus*, 554, note 23, commenting on Léon Brunschvicg, *Les étapes de la philosophie mathématique* (Paris: PUF, 1947; new ed.: Paris: A. Blanchard, 1972). Deleuze also appeals to a text by Michel Chasles, *Aperçu historique sur l'origine et le développement de méthodes en géométrie* (Brussels: M. Hayez, 1837), which establishes a continuity between Desargues, Monge, and Poncelet as the "founders of a modern geometry" (*A Thousand Plateaus*, 554, note 28).

[31] Gilles Deleuze and Félix Guattari, *What is Philosophy?* trans. Hugh Tomlinson and Graham Burchell (New York: Columbia University Press, 1994), 128, translation modified.

[32] See Brunschvicg, *Les étapes de la philosophie mathématique*, 327–31.

[33] See Carl B. Boyer, *The History of the Calculus and Its Conceptual Development* (New York: Dover, 1959), 267. Deleuze praises Boyer's book as "the best study of the history of the differential calculus and its modern structural interpretation" (*Logic of Sense*, 339).

[34] For a discussion of the various uses of the term "intuition" in mathematics, see the chapters on "Intuition" and "Four-Dimensional Intuition" in Philip J. Davis and Reuben Hersch, *The Mathematical Experience* (Boston, Basel, and Stuttgart: Birkhäuser, 1981), 391–405,

as well as Hans Hahn's classic article "The Crisis in Intuition," in J. R. Newman, ed., *The World of Mathematics* (New York: Simon and Schuster, 1956), 1956–76.

[35] Boyer, *The History of Mathematics*, chapter 25, "The Arithmetization of Analysis," 598–619, at 598.

[36] Guilio Giorello, "The 'Fine Structure' of Mathematical Revolutions: Metaphysics, Legitimacy, and Rigour," in *Revolutions in Mathematics*, ed. Donald Gilles (Oxford: Clarendon Press, 1992), 135. I thank Andrew Murphie for this reference. See his article "A Requium for Postmodernism," which is available online at <http://mdcm.arts.unsw.edu.au/homepage/StaffPages/Murphie>.

[37] Seminar of 22 February 1972. See also Deleuze, *Difference and Repetition*, 172: "The limit no longer presupposes the ideas of a continuous variable and infinite approximation. On the contrary, the notion of limit grounds a new, static and purely ideal definition of continuity, while its own definition implies no more than number."

[38] Deleuze, *Difference and Repetition*, 172.

[39] See Penelope Maddy, *Naturalism in Mathematics* (Oxford: Oxford University Press, 1997), 51–2, for a discussion of Cantorian "finitism."

[40] Deleuze provides a summary of these developments in Deleuze, *Difference and Repetition*, 176: "The real frontier defining modern mathematics lies not in the calculus itself but in other discoveries such as set theory which, even though it requires, for its own part, an axiom of infinity, give a no less strictly finite interpretation of the calculus. We know in effect that the notion of limit has lost its phoronomic character and involves only static considerations; that variability has ceased to represent a progression through all the values of an interval and come to mean only the disjunctive assumption of one value within that interval; that the derivative and the integral have become ordinal rather than quantitative concepts; and finally that the differential designates only a magnitude left undetermined so that if can be made smaller than a given number as required. The birth of structuralism at this point coincides with the death of any genetic or dynamic ambitions of the calculus."

[41] For a useful discussion of Weierstrass's "discretization program" (albeit written from the viewpoint of cognitive science), see George Lakoff and Rafael E. Núñez, *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics Into Being* (New York: Basic Books, 2000), 257–324.

[42] Maddy, *Naturalism in Mathematics*, 28.

[43] Reuben Hersh, *What is Mathematics, Really?* (Oxford: Oxford University Press, 1997), 13.

[44] Badiou, *Deleuze: The Clamor of Being*, 47.

[45] Deleuze, *Difference and Repetition*, 180.

[46] Deleuze, *Difference and Repetition*, 178.

[47] Freeman Dyson, *Infinite in All Directions* (New York: Harper & Row, 1988), 52–3. John Wheeler, in *Frontiers of Time* (Austin: Center for Theoretical Physics, University of Texas, 1978), has put forward the stronger thesis that the laws of physics are themselves "mutable" (13).

[48] Kurt Gödel, cited in Hao Wang, *From Mathematics to Philosophy* (New York: Humanities Press, 1974), 86.

[49] Hermann Weyl, *The Continuum: A Critical Examination of the Foundations of Analysis* [1918], trans. Stephen Pollard and Thomas Bole (New York: Dover, 1994), 23–4 (although Weyl still argues for a

discrete interpretation of the continuous continuum). Bertrand Russell makes the same point in his *Principles of Mathematics* (New York: Norton, 1938), 347, citing Poincaré: "The continuum thus conceived [i.e., arithmetically or discretely] is nothing but a collection of individuals arranged in a certain order, infinite in number, it is true, but external to each other. This is not the ordinary [geometric or "natural"] conception, in which there is supposed to be, between the elements of the continuum, a sort of intimate bond which makes a whole of them, in which the point is not prior to the line, but the line to the point. Of the famous formula, the continuum is a unity in multiplicity, the multiplicity alone subsists, the unity has disappeared" (347).

[50] Deleuze, seminar of 29 April 1980.

[51] Abraham Robinson, *Non-Standard Analysis* (Princeton: Princeton University Press, 1966), 83. See also 277: "With the spread of Weierstrass' ideas, arguments involving infinitesimal increments, which survived particularly in differential geometry and in several branches of applied mathematics, began to be taken automatically as a kind of shorthand for corresponding developments by means of the e, d approach."

[52] See Deleuze, *Le Pli*, 177: "Robinson suggested considering the Leibnizian monad as a infinite number very different from transfinites, as a unit surrounded by a zone of infinitely small [numbers] that reflect the converging series of the world."

[53] Hersch, *What is Mathematics, Really?* 289. For discussions of Robinson's achievement, see Jim Holt's useful review, "Infinitesimally Yours," in *The New York Review of Books*, 20 May 1999, as well as the chapter on "Nonstandard Analysis" in Davis and Hersch, *The Mathematical Experience*, 237–54. The latter note that "Robinson has in a sense vindicated the reckless abandon of eighteenth-century mathematics against the straight-laced rigor of the nineteenth-century, adding a new chapter in the never ending war between the finite and the infinite, the continuous and the discrete" (238).

[54] Deleuze and Guattari, *A Thousand Plateaus*, 144.

[55] Jean Dieudonné, *L'Axiomatique dans les mathématiques modernes*, 47–8, as cited in Robert Blanché, *L'Axiomatique*, 91.

[56] Nicholas Bourbaki, "The Architecture of Mathematics," in *Great Currents of Mathematical Thought*, ed. François Le Lionnais; trans. R. A. Hall and Howard G. Bergmann (New York: Dover, 1971), 31.

[57] See Deleuze, seminar of 22 February 1972: "The idea of a scientific task that no longer passes through codes but rather through an axiomatic first took place in mathematics toward the end of the nineteenth-century.... One finds this well-formed only in the capitalism of the nineteenth-century." Deleuze's political philosophy is itself based in part on the axiomatic-problematic distinction: "Our use of the word 'axiomatic' is far from a metaphor; we find *literally* the same theoretical problems that are posed by the models in an axiomatic repeated in relation to the State" (*A Thousand Plateaus*, 455).

[58] Deleuze and Guattari, *A Thousand Plateaus*, 373.

[59] Ibid., 373–4.

[60] Ibid., 374.

[61] Ibid., 486.

[62] Ibid., 486.

[63] Ibid., 362; cf. 144.

[64] Ibid., 485, 486.

[65] Henri Poincaré, "L'oeuvre mathématique de Weierstrass," *Acta mathematica* 22 (1898–1899), 1–18, as cited in Boyer, *History of Mathematics*, 601. Boyer notes that one finds in Riemann "a strongly intuitive and geometrical background in analysis that contrasts sharply with the arithmetizing tendencies of the Weierstrassian school" (*History of Mathematics*, 601).

[66] See Deleuze, *Le Pli: Leibniz et le baroque* (Paris: Minuit, 1988), 64: "axioms concern problems, and escape demonstration."

[67] Deleuze and Guattari, *A Thousand Plateaus*, 461.

[68] Ibid., 361. This section of the "Treatise on Nomadology" (361–74) develops in detail the distinction between "major" and "minor" science.

[69] Deleuze, *Difference and Repetition*, 323, note 22. Deleuze is referring to the distinction between "problem" and "theory" in Georges Canguilhem, *On the Normal and the Pathological*, trans. Carolyn R. Fawcett (New York: Zone Books, 1978); the distinction between the "problem-element" and the "global synthesis element" in Georges Bouligand, *Le déclin des absolus methmatico-logiques* (Paris: Éditions d'Enseignement supérieur, 1949); and the distinction between "problem" and "solution" in Albert Lautman, discussed below. All these thinkers insist on the double irreducibility of problems: problems should not be evaluated extrinsically in terms of their 'solvability' (the philosophical illusion), nor should problems be envisioned merely as the conflict between two opposing or contradictory propositions (the natural illusion) (see *Difference and Repetition*, 161). On this score, Deleuze largely follows Lautman's thesis that mathematics participates in a *dialectic* that points beyond itself to a meta-mathematical power—that is, to a general theory of problems and their ideal synthesis—which accounts for the genesis of mathematics itself. See Albert Lautman, *Nouvelles recherches sur la structure dialectique des mathematiques* (Paris: Hermann, 1939), particularly the section entitled "The Genesis of Mathematics from the Dialectic": "The order implied by the notion of genesis is no longer of the order of logical reconstruction in mathematics, in the sense that from the initial axioms of a theory flow all the propositions of the theory, for the dialectic is not a part of mathematics, and its notions have no relation to the primitive notions of a theory" (13–14). Despite his occasional appeal to Lautman, Badiou is opposed to this Lautmanian appeal to a meta-mathematical dialectic.

[70] Badiou, "Un, Multiple, Multiplicité(s)," 4.

[71] Deleuze, *Difference and Repetition*, 170.

[72] Deleuze, *Difference and Repetition*, 161; see 177–8: "If the differential disappears in the result, this is to the extent that the problem-instance differs in kind from the solution-instance."

[73] Henri Bergson, *The Creative Mind*, trans. Mabelle L. Andison (Totowa, N.J.: Littlefield, Adams & Co., 1975), 33. See also 191: "Metaphysics should adopt the generative idea of our mathematics [i.e., change, or becoming] in order to extend it to all qualities, that is, to reality in general."

[74] See Deleuze, seminar of 29 April 1980.

[75] Deleuze, *Difference and Repetition*, 179.

[76] Ibid., 171.

[77] For analyses of Deleuze's theory of multiplicities, see Robin

Durie, "Immanence and Difference: Toward a Relational Ontology," in *Southern Journal of Philosophy*, 60 (2002), 1–29; Keith Ansell-Pearson, *Philosophy and the Adventure of the Virtual: Bergson and the Time of Life* (London and New York: Routledge, 2002); and Manuel Delanda, *Intensive Science and Virtual Philosophy* (London: Continuum, 2002).

[78] Deleuze, *Difference and Repetition*, 162.

[79] Ian Stewart and Martin Golubitwky, *Fearful Symmetry* (Oxford: Blackwell, 1992), 42.

[80] See Kline, *Mathematical Thought*, 759: "The group of an equation is a key to its solvability because the group expresses the degree of indistinguishability of the roots. It tells us what we do *not* know about the roots."

[81] Deleuze, *Difference and Repetition*, 180.

[82] Ibid., citing C. Georges Verriest, "Evariste Galois et la théorie des équations algébraiques," in *Oeuvres mathématiques de Galois* (Paris: Gauthier-Villars, 1961), 41.

[83] Gilles Deleuze, *Essays Critical and Clinical*, trans. Daniel W. Smith and Michael A Greco (Minneapolis: University of Minnesota Press, 1997), 149, citing a text by Galois in André Dalmas, *Evariste Galois* (Paris: Fasquelle, 1956), 132.

[84] Deleuze, *Difference and Repetition*, 170, referring to Jules Vuillemin, *La philosophie de l'algèbre* (Paris: PUF, 1962). "Jules Vuillemin's book proposes a determination of structures [or multiplicities, in Deleuze's sense] in mathematics. In this regard, he insists on the importance of a theory of problems (following the mathematical Abel) and the principles of determination (reciprocal, complete, and progressive determination according to Galois). He shows how structures, in this sense, provide the only means for realizing the ambitions of a true genetic method." See Gilles Deleuze, "A quoi reconnait-on le structuralisme," in *Histoire de la philosophie 8*, ed. François Châtelet (Paris: Hachette, 1972–73), 315.

[85] Albert Lautman, *Essai sur les notions de structure et d'existence in mathématiques*, vol. 1: *Les schémas de structure*; vol. 2: *Les schémas de genèse* (Paris: Hermann & Co., 1938). Although Badiou occasionally appeals to Lautman (see *Deleuze*, 98), his own ontology seems *opposed* to Lautman's; moreover, Badiou never considers Deleuze's own appropriation of Lautman's theory of differential equations, even though Deleuze cites it in almost every one of his books after 1968.

[86] Deleuze, *Difference and Repetition*, 177.

[87] For discussions of Poincaré, see Kline, *Mathematical Thought*, 732–8; Lautman, *Le Problème du temps* (Paris: Hermann, 1946), 41–3; and Deleuze's seminar of 29 April 1980. Such singularities are now termed "attractors": using the language of physics, attractors govern "basins of attraction" that define the trajectories of the curves that fall within their "sphere of influence."

[88] For this reason, Deleuze's work has been seen to anticipate certain developments in complexity theory and chaos theory. Delanda in particular has emphasized this link in *Intensive Science and Virtual Philosophy* (see note 78). For a presentation of the mathematics of chaos theory, see Ian Stewart, *Does God Place Dice?: The Mathematics of Chaos* (London: Blackwell, 1989), 95–144.

[89] Deleuze, *Difference and Repetition*, 162.

[90] See Lautman, *Essai*, 43: "The constitution, by Gauss and Riemann, of a differential geometry that studies the intrinsic

properties of a variety, independent of any space into which this variety would be plunged, eliminates any reference to a universal container or to a center of privileged coordinates."

[91] See Lautman, *Essai*, 23–4: "Riemannian spaces are devoid of any kind of homogeneity. Each is characterized by the form of the expression that defines the square of the distance between two infinitely proximate points.... It follows that 'two neighboring observers in a Riemannian space can locate the points in their immediate vicinity, but cannot locate their spaces in relation to each other without a new convention.' Each vicinity is like a shred of Euclidean space, *but the linkage between one vicinity and the next is not defined and can be effected in an infinite number of ways. Riemannian space at its most general thus presents itself as an amorphous collection of pieces that are juxtaposed but not attached to each other.*"

[92] See Deleuze, *Difference and Repetition*, 183, 181: A Riemannian multiplicity "is intrinsically defined, without external reference or recourse to a uniform space in which it would be submerged.... It has no need whatsoever of unity to form a system."

[93] Deleuze, *Difference and Repetition*, 182.

[94] Badiou, "Un, Multiple, Multiplicité(s)," 10.

[95] See, in particular, *Difference and Repetition*, 183, although the entirety of the fifth chapter is an elaboration of Deleuze's theory of multiplicities.

[96] Deleuze, *Difference and Repetition*, 210.

[97] See Deleuze, *Difference and Repetition*, xxi: "We are well aware ... that we have spoken of science in a manner which was not scientific."

[98] See Delanda, *Intensive Science and Virtual Philosophy*, 15 (on attractors), and chapters 2 and 3 (on symmetry-breaking cascades).

[99] Delanda, *Intensive Science and Virtual Philosophy*, 102.

[100] See Deleuze, *Difference and Repetition*, 117: "In accordance with Heidegger's ontological intuition, difference must be articulation and connection in itself; *it must relate different to different without any mediation whatsoever.*"

[101] Badiou, *Deleuze: The Clamor of Being*, 20. For Badiou's Neo-Platonic characterization of Deleuze, see 26: "It is as though the paradoxical or supereminent One immanently engenders a procession of beings whose univocal sense it distributes."

[102] This conflation is stated most clearly in Badiou, *Deleuze*, 46: "the univocal sovereignty of the One." For discussions of Badiou's reading of the doctrine of univocity, see Nathan Widder, "The Rights of Simulacra: Deleuze and the Univocity of Being," in *Continental Philosophy Review* 34 (2001): 437–53, and Keith Ansell-Pearson, "The Simple Virtual: A Renewed Thinking of the One," in *Philosophy and the Adventure of the Virtual: Bergson and the Time of Life* (London and New York: Routledge, 2002), 97–114.

[103] Deleuze and Guattari, *What is Philosophy?*, 35, 202–03.

[104] See, for instance, Badiou, "Un, Multiple, Multiplicité(s)," 3: The One "can be called the Whole, Substance, Life, the Body without Organs, or Chaos."

[105] Deleuze, *Difference and Repetition*, xvi, 220–21.

[106] Ernst Mayr, "Is Biology an Autonomous Science?" in *Toward a New Philosophy of Biology: Observations of an Evolutionist*

(Cambridge, Mass.: Harvard University Press, 1988), 8–23.

[107] Badiou, "Un, Multiple, Multiplicité(s)," 6.

[108] Deleuze, seminar of 22 April 1980. See also the seminar of 29 April 1980: "Everyone agrees on the irreducibility of differential signs to any mathematical reality, that is to say, to geometrical, arithmetical, and algebraic reality. The difference arises when some people think, as a consequence, that differential calculus is only a convention—a rather suspect one—and others, on the contrary, think that its artificial character in relation to mathematical reality allows it to be adequate to certain aspects of physical reality."

[109] See Deleuze, *Difference and Repetition*, 178: "Modern mathematics leaves us in a state of antinomy, since the strict finite interpretation that it gives of the calculus nevertheless presupposes an axiom of infinity in the set theoretical foundation, even though this axiom finds no illustration in the calculus. What is still missing is the extra-propositional or sub-representative element expressed in the Idea by the differential, precisely in the form of a problem."

[110] Badiou, *Deleuze: The Clamor of Being*, 14. See Deleuze, *Difference and Repetition*, 192: "Representation and knowledge are modeled entirely upon propositions of consciousness which designate cases of solution, but those propositions by themselves give a completely inaccurate notion of the instance which engenders them as cases."

[111] Deleuze, *Abécédaire*, "H as in 'History of Philosophy" (overview by Charles J. Stivale available on-line at <http://www.langlab.wayne.edu/Romance/FreDeleuze.html>.)

[112] See Paul Hoffman, *The Man Who Loved Only Numbers: The Story of Paul Erdős and the Search for Mathematical Truth* (New York: Hyperion, 1998), 17.

[113] See Deleuze, *Negotiations*, 130: "Poincaré used to say that many mathematical theories are completely irrelevant, pointless. He didn't say they were wrong—that wouldn't have been so bad."

[114] Badiou, *Deleuze*, 1 and 98–9. See also 70, where Badiou links Deleuze with Plato's "metaphorical mathematics." Badiou is referring to Deleuze's notorious distaste for metaphors, but there is no reason to think that distaste disappears here. The concept of the "fold," for instance, is not a metaphor, but a literal topological transformation. Even the concept of the "rhizome," whatever its metaphorical resonance, is directed primarily against the literal uses of "arborescent" schemas in mathematics and elsewhere (tree structures, branches and branchings, etc.).

[115] Deleuze, seminars of 14 March 1978 and 21 March 1978. "The abstract is lived experience. I would almost say that once you have reached lived experience, you reach the most fully living core of the abstract.... You can live nothing but the abstract and nobody has lived anything else but the abstract."

[116] Deleuze, *Difference and Repetition*, 190.

[117] Deleuze, *Bergsonism*, 14.

[118] Ibid.; Badiou, *Deleuze: The Clamor of Being*, 36.

[119] Deleuze and Guattari, *A Thousand Plateaus*, 570, note 61. See also page 461: "When intuitionism opposed axiomatics, it was not only in the name of intuition, of construction and creation, but also in the name of a calculus of problems, a problematic conception of science that was not less abstract but implied an entirely different abstract machine, one working in the undecidable and the fugitive."

[120] Badiou, *Court Traité*, 45. Badiou's claim that Deleuze's methodology relies on intuition is discussed in *Deleuze: The Clamor of Being*, chapter 3, 31–40.

[121] Badiou, *Court Traité*, 72.

[122] Deleuze, *Difference and Repetition*, 323, note 21.

[123] For the role of the scholia, see Gilles Deleuze, *Expressionism in Philosophy: Spinoza*, trans. Martin Joughin (New York: Zone Books, 1992), 342–50 (the appendix on the scholia); for the uniqueness of the fifth book, see "Spinoza and the Three Ethics," in *Essays Critical and Clinical*, esp. 149–50.

[124] Badiou, *Deleuze: The Clamor of Being*, 1. See Badiou's essay on Spinoza, "L'ontologie fermée de Spinoza," in *Court Traité*, 73–93.

[125] See Deleuze, *Difference and Repetition*, 161 and 323, note 21. See also Hersh's comments on Descartes in *What is Mathematics, Really?* 112–13: "Euclidean certainty boldly advertised in the *Method* and shamelessly ditched in the *Geometry*."

[126] See Deleuze and Guattari, *A Thousand Plateaus*, 455: "Our use of the word 'axiomatic' is far from a metaphor; we find *literally* the same theoretical problems that are posed by the models in an axiomatic repeated in relation to the State." In part, this is a historical thesis: it is not by chance that Weierstrass's program of arithmetizing mathematics and Taylor's program of organizing work developed at the same time. See Deleuze, seminar of 22 February 1972: "The idea of a scientific task that no longer passes through codes but rather through an axiomatic first took place in mathematics toward the end of the nineteenth-century, that is, with Weierstrass, who launches a static interpretation of the differential calculus, in which the operation of differentiation is no longer considered as a process, and who makes an axiomatic of differential relations. One finds this well-formed only in the capitalism of the nineteenth-century."

[127] Deleuze and Guattari, *A Thousand Plateaus*, 466.

[128] Deleuze, seminar of 22 February 1972: "The true axiomatic is social and not scientific.... The scientific axiomatic is only one of the means by which the fluxes of science, the fluxes of knowledge, are guarded and taken up by the capitalist machine.... All axiomatics are means of leading science to the capitalist market. All axiomatics are abstract Oedipal formations."

[129] In one text, Badiou seems to recognize the problematic-axiomatic distinction in his own manner: "Today, one starts rather from already complex concretions, and it is a questions of folding or unfolding them according to their singularity, to find the principle of their deconstruction-reconstruction, without being concerned with the plane of the set or a decided foundation. Axiomatics is left behind in favor of a mobile apprehension of surprising complexities and correlations. Deleuze's rhizome wins out over Descartes' tree. The heterogeneous no longer allows us to think the homogeneous" (*Court Traité*, 45). But Badiou nonetheless seems to be moving in a Deleuzian direction when, in his more recent essay on "Being and Appearing," he introduces a minimal theory of relation (through logic and topology), and even assigns the 'event' a minimal ontological status: the event "is being itself, in its fearful and creative inconsistency, or its emptiness, which is the without-place of all place (see *Court Traité*, 200).

[130] Deleuze and Guattari, *A Thousand Plateaus*, 471. And *Anti-Oedipus*, 255: "The theoretical opposition lies elsewhere: it is between,

on the one hand the decoded flows that enter into a class axiomatic on the full body of capital, and on the other hand, the decoded flows that free themselves from this axiomatic."

[131] Deleuze and Guattari, *What is Philosophy?* 152.

[132] Ibid., 46–7.

[133] Deleuze and Guattari, *A Thousand Plateaus*, 266; Deleuze, *Spinoza, Practical Philosophy*, 128.

[134] See Badiou, *Deleuze*, 91: "Deleuze always maintained that, in doing this, I fall back into transcendence and into the equivocity of analogy."

[135] Badiou, *Deleuze*, 91. See also page 64: "Truth must be thought as 'interruption.'"