Reply to Francesco Paoli's Comments on *Fuzzy Logic and Higher-Order Vagueness*

NICHOLAS J.J. SMITH

I am extremely grateful to Francesco Paoli for his very interesting and valuable comments on my paper. In what follows, I shall try to respond to his objections. As will be evident from the fact that it frequently directs the reader to [2] for longer discussions of issues and more detailed presentations of arguments, my paper is not fully self-contained: at a number of places, it stands in need of the further support provided by the longer work [2]. Accordingly—and because I have very limited space here—these replies will sometimes take the form of indicating where in [2] considerations which address the objection under discussion are presented.

1 Multidimensional predicates and closeness

In my paper, I propose that F is vague if and only if it satisfies the following condition, for any objects a and b:

Closeness If a and b are very close/similar in respects relevant to the application of F, then 'Fa' and 'Fb' are very close/similar in respect of truth.

Paoli objects that while this definition is adequate for predicates such as 'tall' and 'old' which are associated with natural metrics, it does not seem to extend to multidimensional predicates such as 'beautiful' and 'clever' which lack obvious metrics. He asks "What does it mean for a and b to be very close/similar in respects relevant to the application of 'clever'?"

I offer a response to this sort of worry in [2, p. 145, n. 31]. The example considered there is the predicate 'table', but the idea applies just as well to 'clever'. The basic thought is that a and b are very close/similar in respects relevant to the application of a multidimensional predicate such as 'clever' iff they are very close along each dimension considered individually.

Of course it will often be difficult to specify the dimensions. In the case of cleverness, for example, it is clear that height is not a relevant respect (the fact that Bill is tall and Ben short has no bearing on the question whether each is clever) while speed and accuracy at mental arithmetic is relevant (the fact that Bill can quickly and accurately add two digit numbers in his head, while Ben cannot, does have a bearing on the question whether each is clever)—but these observations still leave us far from a complete analysis of the respects relevant to application of the predicate 'clever'. However, competent speakers surely have an implicit grasp of which factors are relevant to the application of multidimensional predicates: otherwise they would not be able to use these predicates successfully.

In any case, the key point here is that we can say what it *means* for a and b to be very close/similar in respects relevant to the application of a multidimensional predicate such as 'clever'—it means that they are very close along each dimension considered individually—even if we cannot (without a great deal of difficulty) make explicit exactly what the relevant dimensions are in any given case. Difficulties in specifying the dimensions—which are certainly real (even though, as competent speakers, we must have some implicit grasp of which factors are relevant)—do not translate into a problem with the very idea of applying Closeness to multidimensional predicates.

2 Vagueness and borderline cases

I argue in my paper, using as an example the made-up predicate 'is schort', that having borderline cases is not sufficient for being vague. Paoli objects to the example as follows:

> a necessary condition for a to be a borderline case for F is that F is actually *defined* for a... Here, on the other hand, the definition reads as though 'schort' makes sense exactly of those people who are either less than four feet or more than six feet tall.

We need to distinguish two senses in which a predicate might not be defined for certain objects. First, there is the sense Paoli is using: a predicate F is not defined for an object a if Fa is nonsense (a 'category mistake'). The relationship between 'schort' and persons between four and six feet in height is meant to be different, however. The predicate is 'not defined' for these cases in a quite different sense: namely, we have *said nothing at all* about whether the predicate applies to these cases—and what we have said (that it applies to objects less than four feet in height, and does not apply to objects more than six feet in height) has no implications regarding these remaining cases. In other words, the definition of the predicate is *silent* regarding these cases. That is why they are borderline cases: they are cases where we do not know what to say (which is quite different from cases for which a predicate is not defined in the *first* sense: here we know *not* to apply the predicate). At the same time, the sharp delineation of these cases prevents the predicate from being vague. Hence the claim that a predicate can have borderline cases without being vague.

Paoli asks for a natural language predicate—not an artificial one—that has borderline cases but is not vague. It is hard to think of such a predicate, precisely for the reason that it would have to possess two features:

1. there are cases regarding which the definition of the predicate is simply silent (i.e. says nothing, one way or the other, regarding the application of the predicate)

2. this range of cases is sharply delineated

There are plenty of naturally occurring predicates with the first feature. For example, when terms are defined in the rules of sports or games (e.g. 'fault' in tennis) certain situations are simply *not envisaged at all*, and so nothing is said about them, one way or the other. (For example, if the tennis ball *passes through* the net without the ball or the net being disrupted in any way—perhaps due to some sort of quantum effect—is this a fault?) However, the set of cases about which the definition remains silent is not generally sharply bounded. (If it were, it would be natural to complete the definition by saying something about these cases, one way or the other.)

Nevertheless, I think that the clear *possibility* of a predicate (such as 'is schort') whose definition says nothing either way about some sharply-bounded set of cases is enough to show that having borderline cases is insufficient for vagueness. It does not seem to me that a naturally occurring example of such a predicate is required to make this point.

3 Worldly vagueness and semantic indeterminacy

Paoli writes:

> In the standard fuzzy perspective, we do not say many things at once: we say just one thing about a property which neither definitely applies nor definitely fails to apply. On [Smith's] hybrid approach, on the other hand, vagueness would seem to depend from the latter phenomenon as much as from semantic indeterminacy. On the basis of what Smith says, I do not have sufficient evidence to assess whether this is the correct interpretation. If so, perhaps this concurrence calls for more substantial justification.

It is correct that my view—fuzzy plurivaluationism—combines semantic indeterminacy (of the sort found in classical plurivaluationism) and (as I call it) worldly vagueness (of the sort found in the standard fuzzy picture). I make a detailed case for this combination of features in [2]. However, I do not think that *vagueness* depends on semantic indeterminacy: in my view, as explained in detail in [2, §6.1.3], I regard semantic indeterminacy as a separate phenomenon, distinct from vagueness.

4 Higher-order vagueness

Paoli raises a version of one problem that has often gone under the heading 'higher-order vagueness'—the problem of "a sharp drop-off between the region of definitely positive (or negative) instances of a predicate and its borderline cases"—asking "what is the first shade which has the property 'red' to degree 1 in every acceptable model?" This is a significant objection to fuzzy plurivaluationism, which requires a detailed reply. I have given such a reply in [2, §6.2.2].

5 Fuzzy metalanguages

In my paper, I describe two different views, both of which could be described as proposing a 'fuzzy metalanguage'. I object to the first on grounds summed up in a quote from Goguen, and to the second on different grounds. Paoli writes:

> I have the impression that the fuzzy metalanguage approach has been dismissed too quickly. Fuzzy class theory, briefly mentioned in [Smith's] paper, has been used [1] in an attempt to frame this position into the precise language of fuzzy mathematics, rather than classical mathematics. This would seem to defuse Goguen's objection.

I quite agree that Běhounek's view [1] is not subject to Goguen's objection. However, as I say briefly in the paper (n. 15), Běhounek's view is not the same as either of the two versions of the 'fuzzy metalanguage' view which I criticise, and it is not—unlike those two views—intended as a solution to the problem of artificial precision (as presented in §1 of my paper). If my aim had been to say that there is no good view that can reasonably be described as involving a 'fuzzy metalanguage', then it would be quite correct to say that I had not made the case: that the fuzzy metalanguage approach had been dismissed too quickly. However, my aim was more modest: to reject two particular versions of the 'fuzzy metalanguage' approach as solutions to the problem of artificial precision (as presented in §1 of my paper). As things stand, I still believe that such rejection is warranted.

BIBLIOGRAPHY

[1] Libor Běhounek. A model of higher-order vagueness in higher-order fuzzy logic. Typescript (available at `http://at.yorku.ca/c/a/s/u/34.dir/casu-34.pdf`); abstract of a paper presented at *Uncertainty: Reasoning about Probability and Vagueness*, Institute of Philosophy, Academy of Sciences of the Czech Republic, Prague, 5–8 September, 2006.

[2] Nicholas J.J. Smith. *Vagueness and Degrees of Truth.* Oxford University Press, Oxford, 2008.