Reply to Libor Běhounek's Comments on *Fuzzy Logic and Higher-Order Vagueness*

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I am extremely grateful to Libor Běhounek for his very interesting and valuable comments on my paper. In what follows, I shall try to respond to his objections. As will be evident from the fact that it frequently directs the reader to [1] for longer discussions of issues and more detailed presentations of arguments, my paper is not fully self-contained: at a number of places, it stands in need of the further support provided by the longer work [1]. Accordingly—and because I have very limited space here—these replies will sometimes take the form of indicating where in [1] considerations which address the objection under discussion are presented.

1 What determines the set of acceptable models?

According to fuzzy plurivaluationism, which is the view defended in my paper (and originally proposed, and defended at greater length, in [1]), a vague discourse is associated not with a single correct or intended fuzzy model, but with multiple acceptable fuzzy models. Běhounek is sympathetic with this general idea, but proposes a different mechanism by which the class of acceptable models is determined.

On my view (see §5 of my paper, and [1, §§6.1.1–6.1.2]), the main (but not the only) meaning-determining facts are speakers' usage dispositions, and the acceptable models of a discourse are those that meet all the constraints imposed by the meaning-determining facts. Such constraints include paradigm case constraints (if speakers would all unhesitatingly apply the predicate 'tall' to the object *a* in normal conditions, then any acceptable model must assign 'tall' a function which maps *a* to 1), ordering constraints (if persons *a* and *b* are of the same sex and roughly the same age, and *a*'s height is greater than *b*'s, then any acceptable model must assign to the predicate 'is tall' a function which maps *a* to a value greater than or equal to the value to which it maps *b*), and so on. Běhounek expresses a worry about this way of picking out the acceptable models, and proposes an alternative. The worry is that the set of acceptable models so determined will not be sharp. I offer my reply to this concern in [1, §6.2.2].[1] The alternative proposal is that the acceptable models are those on which certain axioms are fully true. These axioms are thought of as formalisations in fuzzy logic of meaning postulates for vague predicates.

[1] Běhounek refers to this section of my book, writing "Smith justifies the assumption of sharpness of the set of fuzzy models, de facto, by Ockham's razor. . . Such an explanation is, however, just meta-theoretical: it does not offer a deeper explanation within the theory itself as to why the semantics of vague predicates should be a sharp rather than unsharp set of fuzzy models." I disagree with this characterisation of my reply: I do try to offer an 'internal' reason why the set of acceptable models should be sharp; see in particular the discussion at [1, pp. 312–313].

I have a concern of my own about the alternative proposal. Let us say that a set S of models is axiomatisable if there is a set F of formulas such that S includes all and only the models on which every formula in F is fully true. I certainly agree that if the set of acceptable models of a discourse is axiomatisable, then approaching it via its axioms can be beneficial, allowing us to bring to bear useful logical tools. However, I see no reason to think that the set of acceptable models of a discourse will always be axiomatisable. To give the flavour of the kind of concern I have in mind here, consider for a moment classical first order logic. It seems to me that it could be part of the meaning of a certain predicate P—common knowledge to speakers—that there can be only finitely many P's. However, as is well known, there is no set F of wffs (of first order logic) such that on every (classical) model on which all the wffs in F are true, the extension of P is a finite set.

So much for giving the general flavour of my worry about Běhounek's proposal that "Smith's plurivaluations are thus exactly the classes of models of formal fuzzy theories that straightforwardly formalize the meaning postulates of vague predicates"—let's turn now to a second example, of more direct relevance to the discussion of fuzzy approaches to vagueness. As already mentioned, one form of constraint on acceptable interpretations concerns paradigm cases. For example, Běhounek gives the examples:

- Michael J. Fox is not tall.
- Christopher Lee is tall.

He suggests that these can be axiomatized as $\neg Ha$ and Hb, respectively. But there is a problem here: there are models that make Hb true on which the name b does not refer to Christopher Lee. Thus, the following two constraints on models are not equivalent:

1. Ensure that Christopher Lee (i.e. the man himself) is (fully) in the extension of H on the model.
2. Ensure that the formula Hb is (fully) true on the model.

To satisfy the second condition, we need only ensure that the referent of b—whatever it is on the model in question—is in the extension of H on the model. To satisfy the first condition, we have to ensure that Christopher Lee himself is in the extension of H on the model.

This issue cannot be resolved simply by adding more axioms which constrain the interpretation of the name b: for as is well known, if some propositions (in this case the expanded set of axioms) are all true in a model $\mathfrak{M}$, then they are likewise all true in any model which is isomorphic to $\mathfrak{M}$. The best that a set of axioms can do is fix its models up to isomorphism. But it seems that the set of *acceptable* models of a discourse need not be closed under isomorphism. Suppose we have an acceptable model of a vague discourse which includes the word 'tall'. In this model, Christopher Lee is in the extension of the predicate 'tall'. Now consider an isomorphic model, whose domain contains only (say) numbers. It would seem that this is *not* an acceptable model of the original vague discourse.

If we formulate meaning postulates for some of the vague terms in a discourse, then we know that the acceptable models of the discourse will be *among* the models of the theory whose axioms formalise those meaning postulates. This is certainly useful knowledge. What is not clear, however, is that given a vague discourse, we can always find a set of axioms such that the models of those axioms are *precisely* the acceptable models of the discourse. Whether or not we can do this depends, of course, on the discourse, and on the chosen logic. The general worry, however, is as follows. For a given logic, there will be a kind of transformation of models such that the class of models of any set of axioms will be closed under such transformations. (For example, in the case of classical predicate logic, models of a theory are closed under isomorphisms; in the case of certain modal logics, models are closed under bisimulations [2, ch. 3]; and so on.) The worry is that the set of acceptable models of a discourse will *not* be closed under such transformations. If so, then the set of acceptable models cannot be specified as the set of models of certain axioms. It is for this reason that I prefer my way of specifying the acceptable models—as those not ruled out as incorrect by the constraints on models generated by the meaning-fixing facts (which comprise, inter alia, the usage dispositions of speakers)—to the alternative way proposed by Běhounek.

2 Vagueness, closeness and semantic indeterminacy

In my paper, I propose that F is vague if and only if it satisfies the following condition, for any objects a and b:

Closeness If a and b are very close/similar in respects relevant to the application of F, then 'Fa' and 'Fb' are very close/similar in respect of truth.

Běhounek makes a number of objections to this definition of vagueness. First, he says that "it is itself based on the vague terms *very close/similar*, and thus already its application requires an apparatus for handling vagueness". However, as explained at [1, p. 145], the relation of absolute closeness or similarity, as it features in Closeness, is to be regarded as *precise* (even if the *ordinary* notion of absolute similarity in F-relevant respects is vague, for some predicates F; see [1, p. 145, n. 30]). Second, Běhounek objects that the definition is trivially true, on the basis of the suggestion that "similarity in respects relevant to the application of F coincides with similarity in respect of truth". I argue against this suggestion at [1, pp. 179–80]. Third, Běhounek argues that there are counterexamples to the definition: predicates which are vague, but which do not conform to Closeness. The only example discussed at any length is that of "the predicate 'pregnant' (when understood as *stipulated* to be bivalent, for instance for legal purposes)". I agree that this predicate does not count as vague according to the Closeness definition—but I find it hard to understand why Běhounek thinks that it is *intuitively* vague. This may well be related to the point to be discussed next (where, it seems, there is a terminological dispute between us over the meaning of the term 'vague').

Běhounek suggests: "Rather than graduality, it is the semantic indeterminacy ... which is essential for vagueness." I disagree. I argue that standard examples of vague predicates—'tall', 'bald' and so on—exhibit two distinct features: they conform to

Closeness (they exhibit graduality, in Běhounek's terms); and they have multiple acceptable extensions—one on each acceptable model (they exhibit semantic indeterminacy). I attach the term 'vagueness' to the first feature (see [1, §6.1.3]). It seems that Běhounek wants to attach it to the second feature. This may seem like a trivial terminological dispute—but actually I think that there are good reasons for going my way here, not Běhounek's. I take it as a datum that the term 'vague' should be applied to predicates which exhibit three features: they admit of borderline cases; they generate Sorites paradoxes; and they have blurred boundaries. (If someone did not agree with *this*, then that *would* I think be a mere terminological dispute of no deep interest.) I argue that if a predicate satisfies Closeness, then it will exhibit these three features [1, §§3.5.2–3.5.4]. I also argue that if a predicate is semantically indeterminate, then it need *not* exhibit these three features [1, p. 137]. Hence, we should, in the first instance, attach the term 'vague' to predicates which conform to Closeness—not (contra Běhounek) to predicates which merely exhibit semantic indeterminacy.

BIBLIOGRAPHY

[1] Nicholas J.J. Smith. *Vagueness and Degrees of Truth.* Oxford University Press, Oxford, 2008.
[2] Johan van Benthem. *Modal Logic for Open Minds.* CSLI, Stanford, 2010.