## The Arbitrariness of Belief

## Martin Smith

In Knowledge and Lotteries, John Hawthorne offers a diagnosis of our unwillingness to believe, of a given lottery ticket, that it will lose a fair lottery - no matter how many tickets are involved. According to Hawthorne, it is natural to employ parity reasoning when thinking about lottery outcomes: Put roughly, to believe that a given ticket will lose, no matter how likely that is, is to make an arbitrary choice between alternatives that are perfectly balanced given one's evidence. It's natural to think that parity reasoning is only applicable to situations involving lotteries dice, spinners etc. - in short, situations in which we are reasoning about the outcomes of a putatively random process. As I shall argue in this paper, however, there are reasons for thinking that parity reasoning can be applied to any proposition that is less than certain given one's evidence. To see this, we need only remind ourselves of a kind of argument employed by John Pollock and Keith Lehrer in the 1980s. If this argument works, then believing any uncertain proposition, no matter how likely it is, involves a (covert) arbitrary or capricious choice - an idea that contains an obvious sceptical threat.

## I. PARITY REASONING

Suppose I'm confronted by an exclusive and exhaustive set of hypotheses. Suppose, further, that my evidence does not discriminate in any way between these hypotheses, but provides equal support to each and every one. I will take it for granted, as my starting point, that I could never be justified in believing, of any one of these hypotheses, that it doesn't obtain. If the hypotheses are exclusive, exhaustive and I have no evidence favouring any one over any other, then I should refrain from making any such judgments and remain open minded about each of them. If I were to simply
reject one of these hypotheses, there would be something arbitrary about my doing so - this would almost be a paradigm case of an arbitrary decision.

In saying such things, I don't mean to commit myself to the so-called 'principle of indifference'. This principle is conventionally formulated in something like the following way: If I'm confronted by a set of exclusive, exhaustive hypotheses that are equally supported by my evidence then I should divide my confidence evenly amongst them - assign the very same degree of confidence to each. If there can be more than one way of partitioning up the epistemic space into exclusive, exhaustive and equally supported hypotheses then, as is well known, this principle can potentially place inconsistent requirements upon us. Whether this is indeed possible will depend, I think, upon just how we cash out the notion of 'equal support' - but this topic is not my concern here. My claim concerns belief simpliciter and not degrees of confidence: If I'm confronted by a set of exclusive, exhaustive hypotheses that are equally supported by my evidence then I should not believe simpliciter, of any given hypothesis, that it does not obtain. I can clearly hold back from doing so and, if all the hypotheses are evidentially balanced, I ought to.

My claim could, perhaps, be backed by an argument of the following kind: If I had justification for believing, of any one hypothesis, that it did not obtain then I would have to have justification for believing the same thing about each hypothesis, given that they enjoy equal evidential support. But it would then follow, given a multiple premise closure principle for justification, that I have justification for believing that none of the hypotheses obtain - but I know this to be impossible since the hypotheses are exhaustive. By reductio, I do not have justification for believing,
of any one hypothesis, that it does not obtain. I'm inclined to think that this reasoning is perfectly sound - but I also think that the conclusion itself has a direct intuitive plausibility (and is perhaps just as intuitively plausible as the premises used in this argument).

Suppose a fair circular spinner, divided into six equal sectors numbered 1 to 6 , is spun and lands just out of view. Suppose I'm interested in whether the spinner landed on a number between 1 and 5. It's very natural for me to reason in this way: There are six alternative outcomes, I have absolutely no evidence favouring any one of these over any other, and the proposition in question amounts to the denial of one such outcome. If I were to believe or accept that the spinner landed on a number between 1 and 5 , then this would seem tantamount to taking a biased stance toward the set of six alternatives. Why should I single out one case for special or differential treatment? To do so seems arbitrary.

This kind of thinking is akin to what John Hawthorne calls 'parity reasoning' (Hawthorne, 2004, pp16). To engage in parity reasoning is to conceive of a proposition as subsuming a number of exclusive, exhaustive and evidentially balanced cases - or, as Hawthorne refers to them, parity cases. To believe or accept a proposition, so conceived, is tantamount - or at least appears tantamount - to taking an arbitrarily biased stance toward the parity cases, and this is something that we are naturally disinclined to do ${ }^{1}$.

[^0]There is something undeniably reassuring about the possibility of applying parity reasoning to an uncertain proposition. Parity reasoning allows us to package or quantify our uncertainty in a way that seems to make it more tractable - easier to deal with. Parity reasoning can encourage us to exploit degrees of confidence in managing our uncertainty. If asked whether the spinner landed on a number between 1 and 5 , it would be natural for me to reply that I'm confident that this is so or that this is likely to be so or, indeed, that this has a 5 in 6 chance of being so. Once again, traditional concerns about the principle of indifference should prompt some caution about the idea that I must always distribute my degrees of confidence evenly over a parity partition. Nevertheless, this does seem mandated in the case described.

When it comes to belief simpliciter, though, parity reasoning can engender a rather sceptical frame of mind - a frame of mind in which belief seems like a peculiarly ill-fitting kind of commitment. If asked whether the spinner landed on a number between 1 and 5, I would not reply with an unqualified 'yes'. To answer 'yes' would be to reject one of the parity cases.

The applicability of parity reasoning seems unaffected by the sheer number of parity cases that one confronts. Suppose I'm facing one thousand cases rather than six - suppose I hold a single ticket in a fair one thousand ticket lottery. If I'm asked whether my ticket will lose, and I am thinking about the situation in parity terms, then I wouldn't simply answer 'yes'. After all, some ticket has to win and there is nothing at all that sets my ticket apart - it could just as easily win as any other. When
thinking in this way, I might answer that I'm very confident that my ticket will lose or that it's very likely that my ticket will lose or even that it's $99.9 \%$ likely that my ticket will lose - but a simple 'yes' would seem quite unwarranted.

There is, however, a rather different way of thinking, according to which what makes a belief reasonable or appropriate is a suitably low risk of error. If I'm thinking in this way then I would have no hesitation in answering 'yes' when asked if my ticket will lose the lottery. After all, the chance that this belief will turn out to be false is exceedingly low. That these two ways of thinking - parity reasoning and risk minimisation reasoning - are in tension with one another should come as no particular surprise to those familiar with the lottery paradox.

The negation of a single parity case can be exceedingly likely, if there are enough cases in the partition. Since I'm assuming here that one can never be justified in believing the negation of a parity case, I am effectively assuming that high likelihood, at least when purchased in this way, does not suffice for justification. I will have a little more to say about risk minimisation reasoning at the end of the paper.

It's natural, in any case, to think that relatively few propositions can be conceived as an inexhaustive disjunction of parity cases. That is, it's natural to think that the scope of parity reasoning is relatively circumscribed - limited to cases involving spinners, lotteries, dice or, more generally, cases in which we are reasoning about the outcomes of a putatively random process. Some epistemologists have speculated that the scope of parity reasoning is restricted in essentially this way
(Goldman, 2008, pp467). And certainly our inclination to employ parity reasoning, and to enter into the kind of mind-set that goes along with it, does not extend far beyond cases of this kind.

As Hawthorne suggests, however, it may be that the scope of legitimate parity reasoning dramatically outstrips our initial inclinations to apply it - an idea that contains an obvious sceptical threat (see Hawthorne, 2003, pp14-16, Hawthorne and Lasonen-Aarnio, 2009). If, for instance, I drop a marble, it is natural to think that I can justifiably believe that it will soon land on the floor. But, as Hawthorne and Lasonen-Aarnio point out:

Consider extremely unlikely and bizarre 'quantum' events such as the event that a marble I drop tunnels through the whole house and lands on the ground underneath, leaving the matter it penetrates intact. The situation can be redescribed as a cosmic lottery with immensely many tickets. In this lottery, holding a winning ticket means having one's marble tunnel through the house. Re-describing the situation as a kind of lottery invites thinking of the actual world as surrounded by a sphere of equally close worlds, among them worlds in which the marble does tunnel.
(Hawthorne and Lasonen-Aarnio, 2009)

There's something quite compelling about this - if I do accept that there is a non-zero objective chance of my marble tunnelling through the house and accept that this makes my situation relevantly similar to a lottery, then I will lose any tendency to simply believe or accept that the marble will land on the floor. I would retreat instead to something probabilistically qualified ('It's overwhelmingly likely that the marble will land on the floor' or some such). If legitimate, this generalisation of parity reasoning would appear to extend, at the very least, to all contingent propositions about the future.

In this paper I shall present a somewhat different argument to similar effect namely, that parity reasoning is far more widely applicable than we might initially expect. This argument, though, proceeds on essentially a priori grounds and purports to show that parity reasoning can in fact be applied to any uncertain proposition whatsoever. If this is correct then believing anything that is uncertain involves an arbitrary selection between parity cases. This argument has no particular novelty - it is, in effect, a re-heated version of some reasoning deployed by Keith Lehrer (1980) and John Pollock (1983). The trick devised by Lehrer and Pollock, and borrowed for the purposes of my argument, is to divide up a given proposition into a desired number of subcases by exploiting a range of independent states of affairs.

This argument, like Lehrer and Pollock's original, assumes that any body of evidence will be associated with a single evidential probability distribution over propositions. The assumption is, doubtless, quite a strong one - though it is quite commonly made in one form or another. The argument could, I think, survive various weakenings of this assumption, but I won't explore this further here.

## II. PARITY REASONING GENERALISED

In probing the limits of parity reasoning, we might wonder initially whether there is any way of modifying the simple spinner case described above in such a way as to disable its use. Suppose I do have a reason for preferring one of my six original cases over the others. Suppose the six sectors into which the spinner is divided are not in fact equal in area - the sector numbered 1 has fifteen times the area of the remaining
five sectors (all of which are equal). Suppose I know this. Would this prevent the use of parity reasoning? It seems that it would not. One very natural way for me to accommodate this evidence is simply to imagine another fourteen lines on the spinner - that is, think of the spinner as divided into twenty virtual segments, each of which are equal in area and fifteen of which are associated with the number 1.

These twenty new cases are parity cases for me - they are exclusive, exhaustive and equally supported by my evidence. As long as I think about my situation in these terms, I would still be reluctant, as in the original spinner example, to think that I could justifiably believe that the spinner landed on a number between 1 and 5. If you were to ask me whether the spinner landed on a number between 1 and 5, I may reply that there is a $19 / 20$ or $95 \%$ chance.

In order to break the cycle of parity reasoning, I would need to encounter a reason for preferring a given outcome that is not simply a reason for probabilistically weighting it more heavily than the others. Clearly I am inclined, at least, to treat some reasons in this way - I won't go on denying forever that I know how the spinner has landed. I won't continue to deny this after seeing the spinner for instance, or after hearing someone report the outcome. Perception and testimony are fallible of course. After seeing or hearing how the spinner landed, the other outcomes do, in some sense, remain possible for me - it is just that they come to be conceptualised in a rather different way.

It's natural to think that the evidence provided by, for instance, perception or testimony can render propositions probable without accruing them some greater share
of an underlying set of parity cases. The mechanism of probabilification for these kinds of evidence does seem different - more impressionistic. Perhaps, then, the probabilistic support afforded by perception, testimony and the like can suffice for epistemic justification.

Consider a new example: Suppose that a generally trustworthy friend tells me that Vientiane is the capital of Laos making it likely, though less than certain, that it is. The most salient partition of the open possibilities distinguishes just two cases one in which Vientiane is the capital of Laos and one in which it isn't. The first case, given my testimonial evidence, is far more likely than the second. As such, there does not appear to be anything arbitrary about my embracing the first and rejecting the second. Such an attitude could, it seems, be perfectly justified in virtue of the distribution of probabilities.

Suppose, for ease, that the proposition $(\varphi)\langle$ Vientiane is the capital of Laos〉, has an evidential probability of 0.9 , given my friend's testimony. Now consider the following nine propositions: $\left(\psi_{1}\right)$ <The last single digit number to be rolled on a fair 10-sided die, somewhere on Earth, was one〉 ... $\left(\psi_{9}\right)\langle$ The last single digit number to be rolled on a fair 10 -sided die, somewhere on Earth, was nine $\rangle$. The cases represented by these propositions are genuine parity cases for me - given my evidence they are exclusive and exhaustive and equally supported. Each has an evidential probability of $1 / 9$ for me. Presumably, these propositions are probabilistically independent of $\varphi$, in which case, for any $1 \leq \mathrm{i} \leq 9$, the conjunction $\varphi \wedge \psi_{\mathrm{i}}$ will have an evidential probability of 0.1 - which, of course, is also the evidential probability of $\sim \varphi$.

Consider now the ten cases represented by the propositions $\varphi \wedge \psi_{i}$, for each $i$, $1 \leq \mathrm{i} \leq 9$, and $\sim \varphi$. Evidently, these propositions are exclusive and exhaustive given my evidence and, given the original probability assignment to $\varphi$, equally likely. Proposition $\varphi$ is equivalent, given my evidence, to the disjunction of the first nine cases. I would be inclined to take my friend at his word - to accept that Vientiane is the capital of Laos and thus, in effect, to reject one of these cases. If these are truly parity cases then I could not be justified in rejecting one case and, thus, could not be justified in believing that Vientiane is the capital of Laos on my friend's say-so.

These numbers were, of course, selected for convenience - but there is a general recipe for representing any proposition with a rational evidential probability value less than one as a corresponding proportion of exclusive, exhaustive and equiprobable cases ${ }^{2}$. If $\varphi$ has an evidential probability of $r / k$ (for $r$ and $k$ positive integers with $\mathrm{r}<\mathrm{k}$ ), we identify a set of $\mathrm{r}(\mathrm{k}-\mathrm{r})$ exclusive and exhaustive propositions, $\psi_{1} \ldots \psi_{\mathrm{r}(k-\mathrm{r})}$ that are probabilistically independent of $\varphi$ and equiprobable, given one's evidence. Identifying such a set may sound like a tall order. In general, though, sets meeting these specifications (for any r and k ) will be readily available, provided we are willing to make their members suitably gerrymandered. At the end of this section,

[^1]I shall have a little more to say about just what this＇availability＇assumption amounts to．

This set of propositions $\left\{\psi_{1}, \ldots, \psi_{\mathrm{r}(\mathrm{k}-\mathrm{r})}\right\}$ can be divided without remainder into $r$ disjoint subsets $S_{1} \ldots S_{r}$ ，each of which having k－r members．For any such set $S_{i}$ ，the probability that some member of $S_{i}$ is true will be $1 / r$ ．Given that $S_{i}$ is a set of exclusive propositions，each probabilistically independent of $\varphi$ ，it follows that the proposition $\left\langle\right.$ Some member of $S_{i}$ is true〉 will be probabilistically independent of $\varphi^{3}$ in which case the proposition $\varphi \wedge\left\langle\right.$ Some member of $S_{i}$ is true $\rangle$ will have a probability of $\mathrm{r} / \mathrm{kx} 1 / \mathrm{r}=1 / \mathrm{k}$ ．

The set of propositions $\left\{\psi_{1} \ldots \psi_{\mathrm{r}(\mathrm{k}-\mathrm{r})}\right\}$ can also be divided without remainder into k －r disjoint subsets $\mathrm{T}_{1} \ldots \mathrm{~T}_{\mathrm{k}-\mathrm{r}}$ ，each having r members．For any such set $\mathrm{T}_{\mathrm{i}}$ ，the probability that some member of $\mathrm{T}_{\mathrm{i}}$ is true will be $1 /(\mathrm{k}-\mathrm{r})$ in which case the proposition $\sim \varphi \wedge\left\langle\right.$ Some member of $\mathrm{T}_{\mathrm{i}}$ is true〉 will have a probability of $(\mathrm{k}-\mathrm{r}) / \mathrm{k} \mathrm{x}$ $1 /(k-r)=1 / k$ ．Now consider the set of propositions $\left\{\varphi \wedge\left\langle\right.\right.$ Some member of $S_{1}$ is true $\rangle$, $\ldots, \varphi \wedge\left\langle\right.$ Some member of $\mathrm{S}_{\mathrm{r}}$ is true $\rangle, \sim \varphi \wedge\left\langle\right.$ Some member of $\mathrm{T}_{1}$ is true $\rangle, \ldots, \sim \varphi \wedge$〈Some member of $\mathrm{T}_{\mathrm{k}-\mathrm{r}}$ is true〉\}. This set will contain k propositions，they will be exclusive，exhaustive and equiprobable and $\varphi$ will be true according to $r$ of them－ that is，it will be equivalent，given my evidence，to a disjunction of the first r cases．

[^2]Here is a more graphic way of presenting this result: Imagine the scope of epistemic possibility left open by some evidence as represented by a space, with a proposition subsuming a region proportional to its likelihood given the evidence. What the preceding argument shows is that, provided a proposition $\varphi$ excludes some region (no matter how small), there is always a way of constructing a grid upon the total epistemic space, with cells smaller than or equal to the excluded region, such that the proposition and its negation effectively tessellate with it:


Any attitude that preferences the large subsumed region over the small excluded one is exposed, once the grid is superimposed, as an attitude that simply preferences certain regions over other regions of the exact same size.

Before moving on, I shall say a little more about the 'availability' assumption noted earlier - that is, the assumption that auxiliary propositions with the right characteristics to partition the target proposition and its negation into equiprobable cases will always be available in principle. Despite the examples used above, it is wrong to think that this assumption rests upon empirical facts - even mundane ones such as the fact that lotteries have been held, dice have been rolled etc. The assumption that a proposition can always be partitioned up in the right way can, in
effect, be guaranteed by certain structural constraints upon the set of propositions over which evidential probability functions are defined.

Provided that this set of propositions is closed under countable disjunction and evidential probability functions are countably additive, the only constraint needed is this: For any proposition $\varphi$ with a non-zero evidential probability, there is always a logically stronger proposition that is less likely than $\varphi$, but more likely than zero. What is being assumed, in effect, is that the domain of an evidential probability function be atomless in the sense that any proposition with a non-zero probability can always be divided up into less likely propositions that have a non-zero probability. On its face, the availability assumption would appear to be a good deal stronger than this assumption - and, indeed, would appear to be an assumption of a very different sort. That the latter suffices for the former is proved in the appendix - this is one novel contribution to the argumentative technique of Lehrer and Pollock.

## III. THE INFALLIBILIST OPTION

I claimed at the outset that one could never justifiably believe a proposition that was equivalent, given one's evidence, to an inexhaustive disjunction of parity cases. The argument of the previous section, when combined with this claim, can be thought of as constituting a distinctive kind of sceptical paradox. Naturally, though, there are various options open to us when it comes to resisting the sceptical conclusion.

Let's say that a set of hypotheses are equiprobable cases, relative to a body of evidence, iff (i) given that evidence, the propositions are exclusive and exhaustive and
(ii) the evidence makes each proposition equally probable. What the argument of the preceding section demonstrates is that any uncertain proposition (with a rational evidential probability value) will be equivalent, given one's evidence, to an inexhaustive disjunction of equiprobable cases. But equiprobable cases are not yet parity cases. Driving a wedge between these two notions, however, involves conceding that there is a kind of evidential support that is not reducible to probabilification. More precisely, any equiprobable partition will count as a parity partition given the following assumption: If a body of evidence makes two propositions $\varphi$ and $\psi$ equally probable, then that evidence provides $\varphi$ and $\psi$ with equal support. If we reject this assumption then equiprobable partitions need not count as parity partitions.

If there is no gap between parity and equiprobable partitions, then any proposition with a rational evidential probability less than one is equivalent, given one's evidence, to an inexhaustive disjunction of parity cases. If we accept that we can never be justified in believing such a proposition, we wind up with a kind of infallibilism: One is justified in believing a proposition $\varphi$, only if the evidential probability of $\varphi$ is one. On this view, one can only be justified in believing a proposition if one's evidence leaves no room for its falsity.

Given the supposition that one can never be justified in believing an inexhaustive disjunction of parity cases, the argument of the previous section presents us with a dilemma: Either evidential certainty is necessary for justification or justification turns, in part, upon factors orthogonal to evidential probability - factors
that cannot be analysed in broadly logical or probabilistic terms ${ }^{4}$. I won't discuss this second horn at length here - most of my attention, in this final section, will be directed toward the prospects for the first horn.

Suffice it to say, if we do grasp the second horn then we will need to concede that justification-conferring kinds of evidence - perceptual and testimonial evidence amongst them - work in a different way to the kind of 'evidence' that we have for lottery propositions. If I have a perceptual experience as of a red wall before me then, naturally, the evidential probability that there is a red wall before me dramatically increases. But this increase in evidential probability cannot, on this way of thinking, exhaust the impact or the bearing of the evidence - otherwise the argument of the previous section would secure a foothold. If my perceptual evidence confers justification for believing that there is a red wall before me, then there must be something more to it - something that couldn't be emulated by purely probabilistic evidence for the same proposition.

I think this idea is, indeed, quite intuitive. It's intuitive that perceptual evidence, for instance, does more than simply 'stack the odds' in favour of certain propositions and against others. The challenge, of course, is to say just what this 'something more' is. What is it that perceptual evidence is supposed to be doing exactly, beyond giving the evidential probabilities a boost? Without some substantial

[^3]answer to these questions, this option is simply not viable. In any case, I won't pursue this matter further here - but see Smith (2010).

If we wish to maintain that perceptual evidence, testimonial evidence and the like do work in the same way as probabilistic evidence - by simply impacting evidential probabilities - then we must grasp the other horn of the dilemma and maintain that justification requires an evidential probability of one. I am not entirely convinced that this is the wrong conclusion to draw - but I do suspect that it is. In the remainder of the paper, I shall highlight what I see as the primary difficulty associated with it.

One initial and obvious worry is that this conclusion leads to immediate sceptical consequences. If we are only justified in believing propositions that are evidentially certain then, presumably, we are not justified in believing much at all. This sceptical result may yet be resisted though. The idea that the vast majority of our beliefs are underdetermined by our evidence is widespread amongst philosophers and, arguably, a part of common sense - but it has been denied (see for instance Klein, 1981, 1995, Williamson, 2000, chap. 9). If we are prepared to grant that many of our beliefs are evidentially certain then we could embrace infallibilism without thereby embracing scepticism. But even if scepticism could be resisted in this way, the infallibilist still faces a serious and, to my mind, deeper, problem: Infallibilism collapses - or comes close to collapsing - the notions of certainty, knowledge and justification - notions that appear to play very different roles in our epistemic lives ${ }^{5}$.

[^4]Consider a standard kind of case: Suppose John tells me that he was born in 1972 and, having absolutely no reason to think him dishonest or mistaken, I take him at his word. As it happens John's situation is very unusual - in actual fact he was born in 1971 but his parents concealed this fact, destroying all original records of the birth, and concocted a new story. John is completely oblivious to all of this. In this situation, it is not certain for me that John was born in 1972 in which case, given infallibilism, my belief is unjustified. But this strikes us as unduly harsh - surely I drew the appropriate conclusion, given the evidence that was available to me.

Compare my situation with that of Bob who believes that one's year of birth determines one's personality and who has identified in John personality traits that he believes to correspond to the year 1972. Suppose he manages to convince himself, on this basis alone, that John must have been born in 1972. Surely I am doing better epistemically than Bob is - surely my belief is in better epistemic shape. But infallibilism does not appear to leave us with any space for such a verdict.

One might respond to this concern by introducing some new kind of epistemic status in order to take up the theoretical slack. Even if my belief is not, properly speaking, justified, on this way of thinking there may still be something epistemically to be said in its favour - the belief might be described as epistemically blameless or epistemically reasonable or some such (see Williamson, 2000, section 11.4, Sutton,

[^5]2005, section 3.4). This status, then, would be what serves to separate my belief from a misguided or superstitious one.

The question that we then face, of course, is whether one can blamelessly or reasonably believe a proposition that is equivalent to an inexhaustive disjunction of parity cases. If the answer is 'no' then the argument can, of course, simply be re-run to show that blamelessness and reasonableness must also collapse into certainty. If the answer is 'yes' then the epistemic credentials of blamelessness and reasonableness are cast into doubt. It is exceedingly troubling if the only thing to be said on behalf of my belief in the case described is that it possesses a property shared by beliefs that we would generally regard as arbitrary. If blamelessness and reasonableness are to do the work required of them, then they should never be satisfied by arbitrary. But the rejection of one or more parity cases could I think be convicted of much the same kind of arbitrariness as Bob's belief.

It is worth pointing out that some philosophers have defended the view that justified belief requires knowledge - a position that is not far from the sort of infallibilism being countenanced here. This position is unflinchingly endorsed by Jonathan Sutton (2005, 2006) - see also Williamson (2000, pp255-256). Furthermore, the popular knowledge account of assertion, as defended by Williamson (2000, chap. 11), Hawthorne (2003, section 1.3) and DeRose (2002) amongst others, is in the near vicinity. On one way of understanding this view, one has justification for asserting $\varphi$ only if one knows that $\varphi$.

Few philosophers, however, deny that there are epistemic norms of belief or assertion that are weaker than knowledge - that it is possible to discriminate epistemically between believers or asserters who fall short of knowledge. Williamson famously makes recourse to a norm of reasonable assertion, which does not require knowledge (Williamson, 2000, section 11.4). And Sutton (2005, section 3.4) appeals to a norm of epistemic blamelessness - also conceived as a norm less demanding than knowledge ${ }^{6}$. If equiprobable partitions are always parity partitions, then, as I've shown, there is at least a sense in which there cannot be norms like this.

It is not quite right to say that infallibilism leaves us with no resources for epistemically distinguishing believers who fall short of certainty - but it does prevent us from distinguishing between such believers in one very natural way. Consider again the contrast between my belief and Bob's belief as described above. Presumably the evidential probability that John was born in 1972 is much higher for me than it is for Bob - after all, I do have genuine evidence in favour of this conclusion. As such, I come closer to satisfying evidential certainty than Bob does ${ }^{7}$. There is, of course, nothing preventing us from defining various norms of belief in terms of evidential probability thresholds - I would be inclined to concede that beliefs that meet such thresholds are, in some sense, epistemically better off than beliefs that don't ${ }^{8}$. The important point, though, is that any such norms will be satisfied,

[^6]sometimes, by the arbitrary rejection of parity cases. Surely there is an epistemic norm that is quite unlike these - a norm sandwiched, as it were, between arbitrariness and certainty - and much of our ordinary practice of epistemic evaluation betrays a commitment to its existence.

This brings us back to the topic of 'risk minimisation' reasoning, as discussed in the first section. The idea behind such reasoning is that one has justification for believing $\varphi$ provided that the evidential probability of $\varphi$ is sufficiently high (surpasses some threshold $\mathbf{t}<1$ ). As discussed, this is straightforwardly inconsistent with my initial supposition - namely that one can never have justification for believing a proposition that is equivalent to an inexhaustive disjunction of parity cases. In exploring this inconsistency further, it is important to keep in mind that the notion of epistemic justification is, at least, a semi-technical one and no one, as such, has complete propriety over it.

I've granted that evidential probability thresholds do characterise epistemic norms in a broad sense. Beliefs that meet such thresholds are, in some respect, in better epistemic shape than beliefs that don't. Even if one were to insist that the term ‘justification’ be used to pick out some such a norm I would have no deep-seated objection, unless the implication is that this is the only legitimate way in which to epistemically appraise beliefs.

One who claims that we have justification for believing anything that is sufficiently likely, given our evidence, may simply be expressing a preference as to
how the term 'justification' be used - as to the sort of norm with which it is connected. This is quite consistent with there being a significant epistemic norm that is incompatible with arbitrariness, but compatible with uncertainty - and with there being a genuine philosophical project concerned with understanding the nature and possibility of such a norm. To deny that there is any norm like this is, I think, to adopt an attitude that is every bit as stifling as infallibilism - an attitude on which we are robbed of any resources to discriminate between arbitrary and non-arbitrary beliefs that fall short of certainty. Stipulating that the term 'justification' be used in such a way as to apply to some arbitrary beliefs is nothing more than a cosmetic improvement. It remains true, on this picture, that much of our ordinary practice of epistemic evaluation is jettisoned.

It's natural to think that one could never be justified in taking an arbitrarily skewed stance towards a set of hypotheses that are equally supported by one's evidence. It's also natural to think that one can be justified in accepting propositions that are less than certain, given one's evidence. My primary aim here has been to convey just how difficult it is to reconcile these two natural thoughts - just how difficult it is to clear a space in between non-arbitrariness and certainty.

## ACKNOWLEDGEMENTS

An earlier version of this paper was presented at a Basic Knowledge seminar at the University of St Andrews in October 2007. Thanks to all of those who participated on that occasion - in particular, Dylan Dodd, Duncan Pritchard, Jonathan Schaffer, Crispin Wright and Elia Zardini. Thanks also to audiences at the University of

Stirling, University of Glasgow, University of Western Australia and University of Oslo.

## REFERENCES

DeRose, K. (2002) 'Assertion, knowledge and context' Philosophical Review v111(2), pp167-203

Douven, I. and Williamson, T. (2006) 'Generalizing the lottery paradox' British Journal for the Philosophy of Science v57(4), pp755-779

Goldman, A. (2008) 'Knowledge, explanation and lotteries' Noûs v42(3), pp466-481
Hawthorne, J. (2004) Knowledge and Lotteries (Oxford: Oxford University Press)
Hawthorne, J. and Lasonen-Aarnio, M. (2009) 'Knowledge and objective chance' in Pritchard, D. and Greenough, P. eds. Williamson on Knowledge (Oxford: Oxford University Press)

Klein, P. (1981) Certainty: A Refutation of Scepticism (Minnesota University Press)
Klein, P. (1995) 'Scepticism and closure: Why the evil genius argument fails' Philosophical Topics v23, pp213-236

Lehrer, K. (1974) Knowledge (Oxford: Clarendon Press)
Lehrer, K. (1980) 'Coherence and the racehorse paradox' in French, P., Uehling, T. and Wettstein, H. eds. Midwest Studies in Philosophy v5, (Minneapolis: University of Minnesota Press)

Pollock, J. (1983) 'Epistemology and probability’ Synthese v55, pp231-252
Pollock, J. (1986) Contemporary Theories of Knowledge (Rowman and Littlefield)
Pollock, J. (1990) Nomic Probability and the Foundations of Induction (Oxford: Oxford University Press)

Savage, L. (1972) Foundations of Statistics (New York: Dover Publications)
Smith, M. (2010) 'What else justification could be' Noûs v44(1), pp10-31
Sutton, J. (2005) 'Stick to what you know’ Noûs v39(3), pp359-396
Sutton, J. (2007) Without Justification (Cambridge, MA: MIT Press)
Villegas, C. (1964) 'On qualitative probability $\sigma$-algebras’ Annals of Mathematical Statistics v35(4), pp1787-1796

Williamson, T. (2000) Knowledge and Its Limits (Oxford: Oxford University Press)
Wright, C. (2004) 'Warrant for nothing (and foundations for free)?' Aristotelian Society Supplement v78, pp167-212

## APPENDIX: THE AVAILABILITY ASSUMPTION

The argument presented in section II purported to show that any proposition $\varphi$ with a rational probability value was equivalent to a disjunction of exclusive, exhaustive and equiprobable propositions. The argument, as presented, appeared to help itself to a range of further propositions bearing quite specific logical and probabilistic relations to the target proposition $\varphi$. I termed this the 'availability assumption'.

In this appendix I shall give a more formal reconstruction of the argument. I think the exercise is interesting in its own right - but one concrete pay-off is to clarify the precise status of the availability assumption. As it turns out, the assumption can be guaranteed by certain relatively weak constraints on how the underlying set of propositions is structured.

Let Pr be a probability function defined over a set of propositions $\Omega$. We assume that $\Omega$ constitutes a $\sigma$-field - that is, $\Omega$ is closed under negation and countable disjunction and it contains a universal proposition entailed by all others in the set. We assume that Pr is countably additive - that is, if $\psi_{1}, \psi_{2} \ldots$ is a countably infinite sequence of propositions in $\Omega$ such that $\psi_{1} \rightarrow \psi_{2} \rightarrow \ldots$ then $\operatorname{Pr}\left(V_{\mathrm{n} \in \mathbf{Z}} \psi_{\mathrm{n}}\right)=\lim _{\mathrm{n} \rightarrow \infty}$ $\operatorname{Pr}\left(\psi_{\mathrm{n}}\right) . \Omega$ is said to be atomless with respect to $\operatorname{Pr}$ just in case, for any proposition $\varphi$ $\in \Omega$ such that $\operatorname{Pr}(\varphi)>0$, there is a further proposition $\psi \in \Omega$ such that $\psi \rightarrow \varphi$ and $\operatorname{Pr}(\varphi)>\operatorname{Pr}(\psi)>0$. Alternately, any proposition $\varphi \in \Omega$ such that $\operatorname{Pr}(\varphi)>0$ is equivalent to a disjunction $\psi \vee \chi$ such that $\psi, \chi \in \Omega, \operatorname{Pr}(\psi)>0, \operatorname{Pr}(\chi)>0$ and $\operatorname{Pr}(\psi \wedge \chi)=0$. Given that $\Omega$ is atomless with respect to $\operatorname{Pr}$, it follows that, for any proposition $\varphi$ in $\Omega$ such that $\operatorname{Pr}(\varphi)$ is rational, there is a set of exclusive, exhaustive and equiprobable propositions in $\Omega$ such that $\varphi$ is equivalent to a disjunction of its members.

The proof exploits a corollary of a result established by Villegas (see Villegas 1964, Theorem 4, see also Savage, 1972, pp37, 38) - a corollary to the effect that any proposition within an atomless probability space can always be partitioned into n equiprobable sub-propositions, for any positive integer n . The proof relies upon Zorn's Lemma. Let $(\mathrm{S}, \leq)$ be a partially ordered set. A subset C of S is described as a chain iff for all $\mathrm{x}, \mathrm{y} \in \mathrm{C}, \mathrm{x} \leq \mathrm{y}$ or $\mathrm{y} \leq \mathrm{x}$. The lemma states that, if S is a nonempty, partially ordered set, such that every chain in $S$ has an upper bound, then $S$ has a maximal element. Zorn's Lemma is set-theoretically equivalent to the Axiom of Choice. I won't comment further upon its use here. With this background I shall prove the following:

Theorem 1 If $\Omega$ is a $\sigma$-field and $\operatorname{Pr}$ a countably additive probability function such that $\Omega$ is atomless with respect to $\operatorname{Pr}$ then, for any proposition $\varphi \in \Omega$ such that $\operatorname{Pr}(\varphi)=r / k$, for $\mathrm{r}, \mathrm{k}$ integers, there exists within $\Omega$ a set of k propositions that are exclusive, exhaustive and equiprobable (each with a probability of $1 / \mathrm{k}$ ) such that $\varphi$ is equivalent to a disjunction of $r$ of its members.

## Proof

Let $\varphi$ be a proposition in $\Omega$ such that $\operatorname{Pr}(\varphi)=\mathrm{r} / \mathrm{k}$, for $\mathrm{r}, \mathrm{k}$ positive integers. If $\mathrm{r}=1$ then we have it, right away, that $\operatorname{Pr}(\varphi)=1 / \mathrm{k}$. Call a proposition $\psi$ a sub-proposition of $\varphi$ just in case $\psi \rightarrow \varphi$ and an $r$-minor sub-proposition of $\varphi$ just in case, in addition, $\operatorname{Pr}(\psi) \leq \operatorname{Pr}(\varphi) /$ r. By atomlessness, any proposition with positive probability can be divided into two sub-propositions with positive probability. As such, there is a decreasing sequence of sub-propositions of $\varphi, \psi_{1}, \psi_{2} \ldots$ such that $\lim _{n \rightarrow \infty} \operatorname{Pr}\left(\psi_{n}\right)=0$, in which case $\varphi$ is guaranteed to have an r-minor sub-proposition, for any r . Consider the set of all r-minor sub-propositions of $\varphi$. This set can be partially ordered by entailment. If $\chi_{1}, \chi_{2} \ldots$ is a chain of elements within this set (such that $\chi_{1} \rightarrow \chi_{2} \rightarrow \ldots$ ) then $\lim _{\mathrm{n} \rightarrow \infty} \operatorname{Pr}\left(\chi_{\mathrm{n}}\right) \leq \operatorname{Pr}(\varphi) / \mathrm{r}$ in which case, by countable additivity, $\operatorname{Pr}\left(\mathrm{V}_{\mathrm{n} \in \mathrm{Z}} \chi_{\mathrm{n}}\right) \leq$ $\operatorname{Pr}(\varphi) / \mathrm{r}$. In this case, the disjunction of members of any chain of r-minor subpropositions will itself be an r-minor sub-proposition and an upper bound to the chain. By Zorn's lemma, then, the set of r -minor subpropositions of $\varphi$ must have a maximal member. Let $\chi$ be one such member.

Consider the proposition $\varphi \wedge \sim \chi$. By atomlessness, there is a decreasing sequence of sub-propositions of $\varphi \wedge \sim \chi, \delta_{1}, \delta_{2} \ldots$ such that $\lim _{n \rightarrow \infty} \operatorname{Pr}\left(\delta_{\mathrm{n}}\right)=0$. Since $\chi$ is a maximal r-minor sub-proposition of $\varphi$, it follows that $\chi \vee \delta_{\mathrm{n}}$ is not an r -minor subproposition of $\varphi$ for any integer $n$ in which case $\operatorname{Pr}\left(\chi \vee \delta_{\mathrm{n}}\right)>\operatorname{Pr}(\varphi) / \mathrm{r}$. We have $\lim _{\mathrm{n} \rightarrow \infty}$ $\operatorname{Pr}\left(\chi \vee \delta_{\mathrm{n}}\right)=\operatorname{Pr}(\chi)$ in which case, by countable additivity, $\operatorname{Pr}(\chi) \geq \operatorname{Pr}(\varphi) /$ r. Since $\chi$ is an r-minor sub-proposition of $\varphi$ we have $\operatorname{Pr}(\chi) \leq \operatorname{Pr}(\varphi) /$ r. In this case we have $\operatorname{Pr}(\chi)=$ $\operatorname{Pr}(\varphi) / \mathrm{r}=1 / \mathrm{k}$.

If $r=2$ then $\operatorname{Pr}(\chi)=\operatorname{Pr}(\varphi \wedge \sim \chi)=1 / k$. If $r>2$, we seek out a maximal $r-1-$ minor sub-proposition of $\varphi \wedge \sim \chi-$ call it $\lambda$ - which, by the above reasoning, will also have a probability of $1 / \mathrm{k}$. If $\mathrm{r}=3$ then $\operatorname{Pr}(\lambda)=\operatorname{Pr}(\chi)=\operatorname{Pr}(\varphi \wedge \sim \chi \wedge \sim \lambda)=1 / \mathrm{k}$. If $\mathrm{r}>$ 3, we seek out a maximal r-2-minor sub-proposition of $\varphi \wedge \sim \chi \wedge \sim \lambda$ and so on. After $\mathrm{r}-1$ repetitions of this process, $\varphi$ will be divided into r exclusive and exhaustive subpropositions, each with a probability of $1 / \mathrm{k}$. $\varphi$ will be equivalent to the disjunction of these r propositions (and $\varphi$ already will be a proposition with probability $1 / \mathrm{k}$ in case r $=1$ ). We then repeat the same process with respect to $\sim \varphi$, which, after k-r-1 repetitions, will be divided into k-r exclusive and exhaustive sub-propositions, each with a probability of $1 / \mathrm{k} . \sim \varphi$ will be equivalent to the disjunction of these $\mathrm{k}-\mathrm{r}$ propositions. QED


[^0]:    ${ }^{1}$ According to one possible view, even if one's evidence provides equal support to a range of exclusive and exhaustive hypotheses, one may have an epistemic entitlement to reject some but not others, where entitlement is understood as a kind of non-evidential, positive epistemic status (see, for instance, Wright, 2004). If one wished to accommodate this kind of view, one could define parity cases in a slightly more restrictive way: Parity cases are exclusive, exhaustive and equally supported by one's overall epistemic position, where one's epistemic position is determined by the bearing of one's

[^1]:    ${ }^{2}$ This argument cannot be generalised to propositions with irrational evidential probability values. I'm inclined to think that this is not an important shortcoming - given, at any rate, the way that I intend to use the argument here. If we end up drawing a sceptical lesson from this argument - that we cannot have justification for believing propositions with rational evidential probability values less than one then the justificatory status of propositions with irrational evidential probability values (if such there be) will also be placed in jeopardy via the following principle: If one lacks justification for believing a proposition $\varphi$ and the evidential probability of $\varphi$ exceeds the evidential probability of $\psi$, then one lacks justification for believing $\psi$. For any irrational number in the unit interval, there will be a rational number that lies between it and one (a consequence of the Archimedean property of the reals). The principle used here will seem attractive to one who thinks that the probabilistic bearing of a body of evidence effectively exhausts its significance. And, as I will discuss in the next section, an assumption of broadly this sort is required to elicit even the initial sceptical conclusion.

[^2]:    ${ }^{3}$ Suppose that $\chi$ and $\psi$ are exclusive and each independent of $\varphi$ ．The proof that $\varphi$ is independent of $(\psi \vee \chi)$ is straightforward：By the independence assumptions we have $\operatorname{Pr}(\varphi \wedge \psi)=\operatorname{Pr}(\varphi) \cdot \operatorname{Pr}(\psi)$ and $\operatorname{Pr}(\varphi \wedge \chi)=\operatorname{Pr}(\varphi) \cdot \operatorname{Pr}(\chi)$ ．By the exclusiveness assumption we have $\operatorname{Pr}(\psi \vee \chi)=\operatorname{Pr}(\psi)+\operatorname{Pr}(\chi)$ and $\operatorname{Pr}(\varphi \wedge \psi \wedge \chi)=0 . \operatorname{Pr}(\varphi \wedge(\psi \vee \chi))=\operatorname{Pr}((\varphi \wedge \psi) \vee(\varphi \wedge \chi))=\operatorname{Pr}(\varphi \wedge \psi)+\operatorname{Pr}(\varphi \wedge \chi)-\operatorname{Pr}(\varphi \wedge \psi \wedge \chi)=$ $\operatorname{Pr}(\varphi \wedge \psi)+\operatorname{Pr}(\varphi \wedge \chi)=\operatorname{Pr}(\varphi) \cdot \operatorname{Pr}(\psi)+\operatorname{Pr}(\varphi) \cdot \operatorname{Pr}(\chi)=\operatorname{Pr}(\varphi) \operatorname{Pr}(\psi)+\operatorname{Pr}(\chi))=\operatorname{Pr}(\varphi) \cdot \operatorname{Pr}(\psi \vee \chi) \cdot$ QED

[^3]:    ${ }^{4}$ The idea that justification does not supervene upon evidential probability can be developed in two rather different ways. Either the bearing of a body evidence does not supervene upon the evidential probabilities it assigns or epistemic justification does not supervene upon the bearing of one's evidence - that is, justification turns in part upon factors that are non-evidential. As I suggested in footnote 1, if non-evidential justification views are to be accommodated, then parity partitions should be defined in terms of equal support by one's overall epistemic position rather than one's evidence per se. If one held to such a view, one would have reason to deny that equiprobable partitions are parity partitions so defined. Thanks to Duncan Pritchard for discussion of this point.

[^4]:    ${ }^{5}$ Strictly speaking, the kind of infallibilism under consideration here serves to collapse justification and evidential certainty - where this is defined in terms of maximal evidential probability. Evidential certainty defined in terms of entailment by one's evidence may yet remain distinct, since it is possible for propositions not entailed by a body of evidence to, nevertheless, enjoy maximal evidential

[^5]:    probability, given that evidence. It is also possible, of course, for propositions that enjoy maximal evidential probability to be false, at least where infinite probability spaces are concerned. As such, knowledge, too, will remain a distinct status - at least in principle. These points are certainly worth noting - but they do little to ameliorate the concern that infalliblism leaves us with very crude tools of epistemic evaluation.

[^6]:    ${ }^{6}$ In Without Justification (2007), Sutton takes a harder line, insisting that no beliefs that fall short of knowledge can have anything going for them, epistemically speaking. Sutton still supplies a certain means of discriminating between beliefs in this category - very roughly put, some of these beliefs would count as knowledge were their contents probabilistically qualified. This strategy is rather similar to a proposal floated below - namely, that we look to high evidential probability to supply a new epistemic norm - and it shares what I take to be its primary disadvantage - namely, that it will classify beliefs in a way that cross-cuts the intuitive distinction between those that are arbitrary and those that are not.
    ${ }^{7}$ Thanks to Jonathan Schaffer for pressing me on this point.
    ${ }^{8}$ I am inclined to think that evidential probability thresholds are only norms in the same sense that 'driving less than 10 kmph over the speed limit' is a norm - conforming to it provides no guarantee

