The Principle of Uniform Solution (of the Paradoxes of Self-Reference)

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Graham Priest (1994) has argued that the following paradoxes all have the same structure: Russell's Paradox, Burali-Forti's Paradox, Mirimanoff's Paradox, König's Paradox, Berry's Paradox, Richard's Paradox, the Liar and Liar Chain Paradoxes, the Knower and Knower Chain Paradoxes, and the Heterological Paradox. Their common structure is given by Russell's Schema: there is a property ϕ and function δ such that

(1) $w = \{x|\phi(x)\}$ exists, and

(2) if y is a subset of w, then

(i) $\delta(y) \notin y$, and

(ii) $\delta(y) \in w$.

Thus, for example, in the case of Burali-Forti's Paradox, $\phi(x)$ is the property of being an ordinal, w is the set of all ordinals, and $\delta(y)$ is the least ordinal greater than every member of y.

Priest spends the main part of his paper showing that the other paradoxes mentioned above also fit Russell's Schema. I have nothing to say against this part of Priest's paper; the part with which I wish to take issue comes near the end, where Priest introduces the *Principle of Uniform Solution* (PUS): if two paradoxes are of the same kind, then they should have the same kind of solution. What is it, according to Priest, for two paradoxes to be of the same kind? It is for there to be a certain structure that produces contradiction, and for the two paradoxes to share that structure. Russell's Schema is a contradiction-producing structure, and all the paradoxes mentioned above share this structure; hence these paradoxes are all of the same kind. Hence, by the PUS, these paradoxes should all have the same kind of solution. "Any solution that can handle only some members of the family is bound to appear somewhat one-eyed, and as not having got to grips with the fundamental issue" (Priest 1994, p. 32). Generally speaking, however, logicians have adopted two different kinds of solution: one for the set-theoretic paradoxes (Russell's, Burali-Forti's and Mirimanoff's), which is of the Zermelo-Fraenkel sort, and involves denying the existence of the totality w of clause (1) of Russell's Schema; and one for the semantic paradoxes, which involves denying the T-schema (or relevant cognate principle), used in establishing clause (2) of Russell's Schema. Whether or not each of these two sorts of solution works in its intended realm, it does nothing towards solving the paradoxes in the other realm. "Hence, the PUS, in conjunction with the [demonstration that the paradoxes of self-reference have a common structure], is sufficient to sink virtually all orthodox solutions to the paradoxes" (Priest 1994, p. 33).

Consider a solution *S* of the Liar Paradox which consists in denying that the T-schema holds in full generality. Priest's criticism of *S* is that it says nothing about Russell's Paradox (which makes no mention of any semantic principle such as the T-schema), even though Russell's Paradox and the Liar Paradox are the same kind of paradox. There is a problem with Priest's line of argument here, stemming from the fact that two objects can be of the same kind at some level of abstraction and of different kinds at another level of abstraction. For example, Bill's loving Ben is a different thing both from Bob's loving Maisy and from Nancy's standing next to Susan. At a certain level of abstraction, however, Bill's loving Ben and Bob's loving Maisy are the same: both consist in one person's loving another. At a further level of abstraction, Bill's loving Ben, Bob's loving Maisy, and Nancy's standing next to Susan are all the same: all three consist in a pair of persons instantiating a relation (not any particular relation this time). At a further level of abstraction all three are the same as a rock's being opposite a hard place: all four consist in a pair of objects (not of any particular kind this time) instantiating a relation. Returning to the case at hand, one cannot argue from the fact that two paradoxes have the same structure at a certain level of abstraction, that their solutions should be the same at a *lower* level of abstraction. Priest describes the orthodox solutions (and I described *S* above) at a far more concrete level than the level at which—in order to establish their similarity—he describes the paradoxes. In the latter case Priest abstracts away from the fact that the Liar Paradox mentions truth but not sets, that Russell's Paradox mentions sets but not truth, and so on, talking instead of a non-specific property ϕ and a non-specific function δ. In the case of the orthodox solutions, however, Priest does not abstract: he considers specifics (whether or not a particular solution involves semantic notions, etc.). But considered at the level of abstraction at which the paradoxes are shown to be the same, each solution *does* apply to *all* the paradoxes of self-reference. (For example, at this level of abstraction *S* is "Deny clause (2) of Russell's Schema"—and this is a possible solution to *any* paradox which instantiates Russell's Schema.) This could not fail to be the case: all the paradoxes instantiate Russell's Schema, so any solution to any of them must, considered at an abstract level, circumvent that Schema; but circumventing the Schema is all that is needed to solve any of the remaining paradoxes. Conversely, considered at the more concrete level (the level at which Priest describes the orthodox solutions), the paradoxes of self-reference are *not* all the same, precisely because some mention truth while others do not, and so on.

An analogy may be useful. Consider a botanist attempting to graft two different types of tree in order to create a new type. She finds that the new trees are perfectly healthy, but none of their limbs will grow more than one node, the consequence being that the trees are stunted. Consider also a zoologist, attempting to breed an endangered species of animal. She finds that the offspring are perfectly healthy, but that no male line will stretch for more than two generations, the consequence being that the population will not grow in the way she desires. Considered at an abstract level, both the botanist and the zoologist face the same problem: a problem involving tree structures of a certain sort which are subject to a particular kind of branching limitation. Now suppose that the botanist discovers that her problem is caused by the presence in the atmosphere of her glasshouse of a certain chemical, generated by her air-conditioning plant. Her solution is to remove this chemical from the air. Given that her problem and the zoologist's are of the same abstract kind, does it follow that the botanist's solution is a poor one unless the zoologist's problem is caused by the presence of the same chemical in the atmosphere of her enclosures, and is solved by the removal of this chemical? Of course not. At a more concrete level, the solution to the zoologist's problem might be quite different from the botanist's solution, even though at a certain level of abstraction, they face the same problem.

Similarly, there is no reason at all why on a more concrete level, the solution to the Liar Paradox should not make specific use of the notion of truth, even though at a certain level of abstraction the Liar Paradox is of the same kind as Russell's Paradox, which does not (at the more concrete level) involve truth. This is not to say that there is something wrong with the PUS. Paradoxes which share a characterization at a certain level of abstraction should indeed have solutions which likewise share a characterization at that level of abstraction. However, unless two paradoxes are also the same at a more concrete level, their solutions need not be the same at that concrete level either. In the present case the former requirement is met: the paradoxes have a common structure—Russell's Schema—and so do the orthodox solutions—they are all Russell's Schema-circumventers. Of course, some solutions circumvent Russell's Schema by denying clause (1), while others deny clause (2)—but if *this* fact were in conflict with the PUS, then the PUS would be utterly implausible. Compare: a combination of too much food and too little exercise causes one to put on weight. Is your solution to your weight problem—exercising more—a poor one unless *everyone* who solves his weight problem solves it your way, rather than by eating less? Of course not. What *is* compelling is the idea that if various persons are all overweight because they eat too much and exercise too little, then the solution to *all* their weight problems must be the same in that it involves altering the food/exercise ratio—but this ratio need not be altered in any particular way. What the various persons have in common is a certain food/exercise ratio—not a certain food intake or a certain exercise level. Hence what their solutions must have in common is a certain adjustment of the ratio, rather than a certain level (or adjustment thereof) of eating or of exercise. Similarly, what the paradoxes of self-reference have in common is that they instantiate Russell's Schema. Hence what their solutions must have in common is simply that they all circumvent this schema.

At least, this is what all *consistent* solutions to the paradoxes must have in common. What about Priest's *inconsistent* solution? The paradoxes of self-reference are generated by theories which, at first sight, appear to be true: naive set theory generates Russell's Paradox; naive semantic theory (according to which declarative sentences are true or false but not both, a declarative sentence is true if and only if what it says is the case, and a declarative sentence may say simply that it itself is false) generates the Liar Paradox; and so on. Priest has shown that all these paradoxes have the same structure: Russell's Schema. Priest's solution to the paradoxes does not involve circumventing Russell's Schema: rather, in each case it involves accepting the contradiction which the schema generates (and thus may be likened—in terms of my earlier example—to the solution to one's weight problem which consists in learning to love one's body as it is). The contradiction which we are to accept is different in each case, but once we know what the contradictions involved in the various paradoxes are, Priest can say one thing of all the paradoxes: "accept the contradictions". Now this statement is not in itself a complete response to the paradoxes: it needs to be (and is) backed up by a theory which explains how we can accept some contradictions as true without having to accept as true any statement whatsoever. The point to note is that this background theory (Priest's paraconsistent logic) is a *general* one: it is not specific to any particular paradox.

The proponent of a consistent solution to the paradoxes is in a somewhat different position. She too can say one thing of all the paradoxes: "the contradictions are avoided". Again, this statement is not in itself a complete response to the paradoxes: if one wishes to provide a consistent solution to the paradoxes, then as well as saying that Russell's Schema is circumvented, one must also say in each case precisely *how* it is circumvented—that is, one must offer new theories to replace the naive theories which generate the paradoxes. This involves work at the concrete level, and the work involved in solving Russell's Paradox (i.e. in finding a consistent replacement for naive set theory) is quite distinct from the work involved in solving the Liar Paradox (i.e. in finding a consistent replacement for naive semantic theory).[1] Thus, the difference between the orthodox consistent approach to the paradoxes and Priest's inconsistent approach is that what is required to back up the basic inconsistent response ("accept the contradictions") is *one* general theory, whereas what is required to fill out the basic consistent response ("the contradictions are avoided") is a *host* of specific theories.[2] But this difference does not in any way count against the orthodox consistent solutions to the paradoxes: for on the one hand, the reason why the work involved in providing a consistent solution to Russell's Paradox is distinct from the work involved in providing a consistent solution to the Liar Paradox is *precisely* that the *paradoxes themselves* are different at the concrete level: some involve sets while others involve truth (and so on), so a new theory of sets is required in some cases and a new theory of truth (or, more generally, of semantics) in other cases; and on the other hand, whatever the differences at the concrete level between some new set theory and some new semantic theory, if each theory really does fail to generate the paradox it is intended to avoid (Russell's Paradox and the Liar Paradox respectively), then *at the level of abstraction* at which the *paradoxes* share a characterization ("satisfying Russell's Schema"), the *solutions* will *also* share a characterization ("circumventing Russell's Schema"). Thus the orthodox solutions to the paradoxes are as alike as they need to be: *they are as alike as the paradoxes themselves*—alike in *structure*. In fact Priest's solutions to the various paradoxes are themselves alike only in structure. Priest's solution to Russell's Paradox is to accept the existence of a set which both is and is not a member of itself, but accepting the existence of such a set does not help with the Liar Paradox, which does not mention sets. Priest's solution to the Liar Paradox—accepting the existence of a sentence which is both true and false—is the same as his solution to Russell's Paradox *only* at a certain level of abstraction, at which the two solutions are "accept the truth of a contradiction generated in accordance with Russell's Schema".

[1] The aim is not to find just *any* consistent replacement for the naive theory in question: the aim is to find the *correct* theory of sets (of semantics, etc.), the thought (with which Priest disagrees) being that no theory which entails a contradiction is correct.

[2] Basically, the starting point for the paradoxes is a bunch of naive theories which generate contradictions and a logical theory (classical logic) which cannot accommodate true contradictions. The orthodox consistent approach to the paradoxes involves replacing the naive theories with theories that are compatible with classical logic, while Priest's inconsistent approach involves replacing classical logic with a paraconsistent logic which is compatible with the naive theories.

(The fact that Priest's solution to Russell's Paradox is not the same at the concrete level as his solution to the Liar Paradox is quite compatible with the fact, noted above, that these two solutions are backed up by the *same* abstract logical theory.)

In sum, I do not seek to question Priest's main result—the demonstration that the paradoxes of self-reference share a common structure; nor do I wish to question the Principle of Uniform Solution. I wish only to deny that "the PUS, in conjunction with the main result … is sufficient to sink virtually all orthodox solutions to the paradoxes" (Priest 1994, p. 33). On the contrary, the demonstration and the PUS, as correct and as interesting as they may be in themselves, count not at all against any orthodox solution to any of the paradoxes of self-reference.[3]

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REFERENCE

Priest, Graham 1994: "The Structure of the Paradoxes of Self-Reference". *Mind*, 103, pp. 25–34.

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